Introduction to Fluid Machines, and Compressible Flow
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Lecture - 31
Choking in a Converging Nozzle

Good morning I welcome you all to this session. Today we will be discussing choking in a converging nozzle. Last class we have recognized the effects of variation of area in a an isentropic flow, and it is the consequence of such a fix of area variation in an isentropic flow is the very interesting phenomenon in a compressible flow is the choking in a converging nozzle. So, if we see the mass flow rate.

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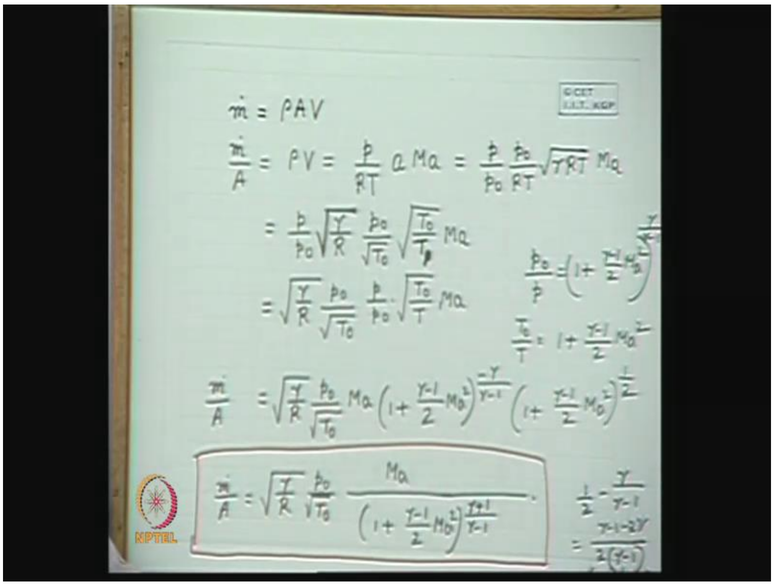

Now, the mass flow rate at any section in a compressible flow could be written as rho into a into v where rho is the density a is the area, and v is the flow velocity. Now, with little algebraic manipulation this can be written like this the mass flow rate per unit area is equal to rho times v which can be written as p by r t considering throughout this course we will consider the flowing medium or the fluid flowing is a perfect gas. So, which obeys this law of equation of state or the law of state p is equal to rho r t. So, I replace rho as p by r t, and v as you know can be replaced as a into mach number where a is the speed of sound.

So, this can be written as little manipulation p by p zero taking p zero there by r t, and we know a this speed of sound can be written as gamma r t where gamma is the ratio of specific heats, and mach. So, therefore, m dot by a can be written as as a continuation of this is p by p zero now this r r goes like this. So, we can write gamma by r under root this root r root, then p zero by we can write a root over t zero. So, this root t, and t makes root t zero. So, that we can write root over t zero.

That means we bring one root over t zero forcefully denominator, and numerator. So, sorry this is root over t root over t zero by t into m a this can be written as this gamma r p zero t zero are constant. So, that we can write this as root over gamma by r well p zero by root over t zero gamma by r p zero root over t zero, then p by p zero into root over t zero by t into m a now we have already recognized that this stagnation properties p zero.

And t zero bears the relationship with the local properties like this p zero by p is one plus gamma minus one by two m a square to the power gamma by gamma minus one this comes basically from t zero by t which was first deduced one plus gamma minus two m a square now if I substitute this p by p zero, and t zero by t quantities in terms of the mach number what we get we get m dot by a is root over gamma by r into p zero by root over t zero into what we get root over p zero m a, and this p by p zero we get that this is I am sorry this is p zero by p, and this is t zero by t all right.

So, p by p zero; that means, one plus gamma minus one by two m a square. So, it will be minus gamma by gamma minus one, and this will be one plus gamma minus one by two all right m a square to the power half now these two if you combine you get half minus gamma by gamma minus one is equal to two gamma minus one well. So, gamma minus one minus two gamma. So, this becomes minus gamma plus one divided by gamma minus one. So, I just work it out . So, therefore, we can write m dot by a becomes root over gamma by r p zero by root over t zero all right into m a this we can take if it is a minus sign. So, that we can write one plus gamma minus one by two m a square raised to the power gamma plus one by gamma minus one.

So, ultimately this is a very important expression for the mass flow rate per unit area in a compressible flow through any variable area duct that mass flow rate per unit area, you see gamma by r p zero by root over t zero m a one plus gamma minus one by two m a square raised to the power gamma plus one by gamma minus one. Now this expression this expression if you plot graphically or if you inspect mathematically possesses a maximum value now what is that maximum value.

First of all recognize which is the variable. So, if we consider m dot m by a as the dependent variable you see p zero t zero are the constants these are the stagnation properties gamma is the fluid property r is also a fluid property that is the characteristic gas constants. So, here mach number is the only independent variable. So, that we can tell that m dot by a is a dependent variable is a function of independent variable mach number, and the functional relationship is such that mach number appears in this way the numerator, and denominator, because of which this function; that means, m dot by a possesses a maximum for a value of for a particular value of m a.

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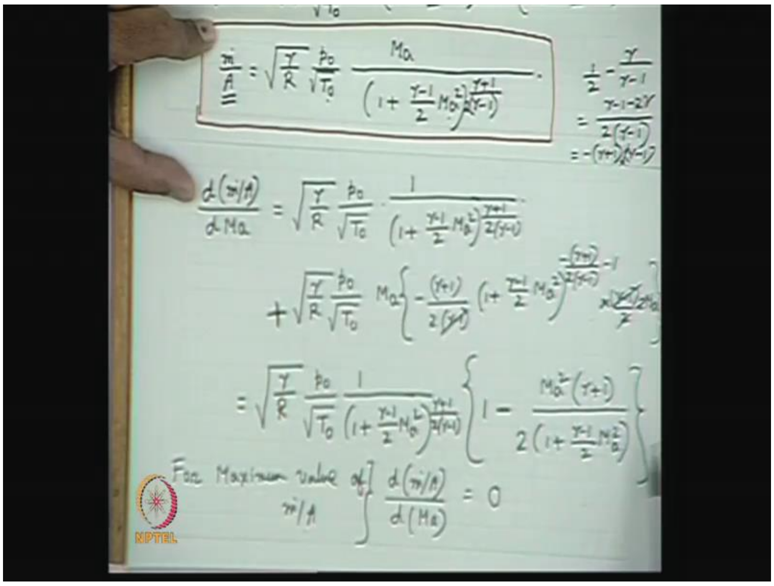

So, our next intension is to find out that maximum value or m dot by a or the value of the argument m a at which this m dot m dot b a possesses the maximum. So, a very simple task is that make this zero the differentiation of this dependent variable with the independent variable d m dot a d m a to make it zero first of all we should find out what is d m dot by d m a d m dot a by d m a; that means, we will have to differentiate this with respect to m a.

Let us do this we will see that gamma by r p zero root over t zero. So, first we take this as the first function whose. So, therefore…

Sir.

Please.

Don’t you think there’ll be one two.

Exponent of.

Exponent of.

Exponent of their there’ll be two gamma minus one sure there will be two gamma minus one. So, there will be two. So, it is all right is it? So, one by one plus gamma minus one by two m a square all right gamma plus one by two gamma minus one the next term will be please tell me minus. So, this may be common does not matter again I can write root over gamma by r p zero by please slowly we can do t zero, then m a remains as it is m a.

So, now what we can do the differentiation of this we make a bracket here. So, differentiation of this is this will be minus; that means, minus gamma plus one divided by two gamma you also check whether any mistake is done by me or not minus. So, this; that means, this quantity one plus gamma minus one by two m a square x two the power n n x rto the power n minus one same formula; that means, minus gamma plus one by two gamma minus one minus one, and this is multiplied with.

Two m a.

Two yes very good. So, gamma minus one by two into two m a all right now first of all I write the expression of this gamma by r p zero by root over t zero, and if I take this as the common. So, one by one plus gamma minus one by two m a square all right gamma plus one divided by two gamma minus one, then what will be there this will be one please one minus this will be what gamma plus one by two gamma minus one. So, I what will be this gamma minus one, and gamma minus one will be this two, and this two will cancel.

So, therefore, one minus m a square what will be the result m a square this will be gamma plus one all right please if any problem you tell me gamma plus.

Sir.

Huh.

After one that will be plus.

Which one will be plus please tell me.

One plus m a square minus one.

Whichever I do not know sir.

Sir lower.

This expression this will be plus why this will be minus. So, you check it this will be plus. So, one minus that will be m a square gamma plus one, now when I divide this this will be cancelled; that means, simply this will come down one plus gamma minus one by two m a square. So, is it all right two will be there, sorry two will be there one plus gamma minus one by two all right this will be the expression now maximum for maximum value of d m dot a by d m a for maximum value of m dot by a for maximum value of m dot by a this has to be zero.

So, definitely this has got a maximum, because the second derivative is the minimum well. So, therefore, the maximum value that we can check we are not going to check it here

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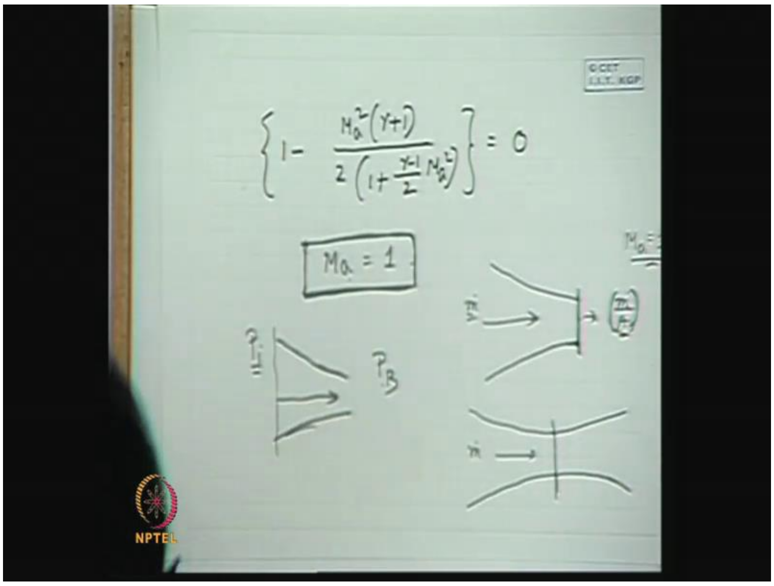

So, if you make it zero, then we will get one minus m a square gamma plus one all right divided by two into one plus gamma this quantity zero, which gives m a is equal to one which gives m a is equal to one ultimately. If you take this two plus gamma minus one m a square minus m a square gamma plus one which will give simply this is a very, very interesting conclusion.

That means the mass flow rate per unit area at any cross section reaches its maximum value, when m a the mach number at this section reached one what is the physical significance of it. Now we will come to the physical significance of it. Now let us consider flow through a nozzle or any divergent it may be a convergent divergent duct, now we know that at a given shape for a given steady condition the mass flow rate is constant through the duct mass flow rate is constant through the duct.

So, the mass flow rate per unit area is is really maximum at the minimum section minimum section is here, and this is the discharge section for a converging nozzle, and this is the throat for a converging diverging duct. So, the mach number one means that if we go on increasing the flow velocity or the subsequent mach number we will see the value of this m dot by a the maximum value of m dot by a at any section; that means, for example, in a converging duct this is at the discharge plane this reaches maximum if we go on varying flow velocity by changing the pressure at the downstream for example.

Then we will see this m dot by a will reach its maximum when this section will attain a velocity corresponding to mach number is equal to one this is the physical significance that mass flow rate per unit area becomes maximum when the mach number at tat section becomes one. Now let us see that till will be explained more clearly, if we observe this now let us consider this thing first of all you must know that certain terminologies that for example, there is a convergent duct first of all we will discuss the convergent duct.

Now, the pressure of the ambience where the convergent duct discharges fluid discharges is discharged from the convergent duct is known as back pressure of the nozzle; that means, the downstream pressure of the nozzle, and the inlet plane pressure is the inlet pressure p I which value we refer as stagnation pressure, if it is coming from a big reservoir with this nomenclature.

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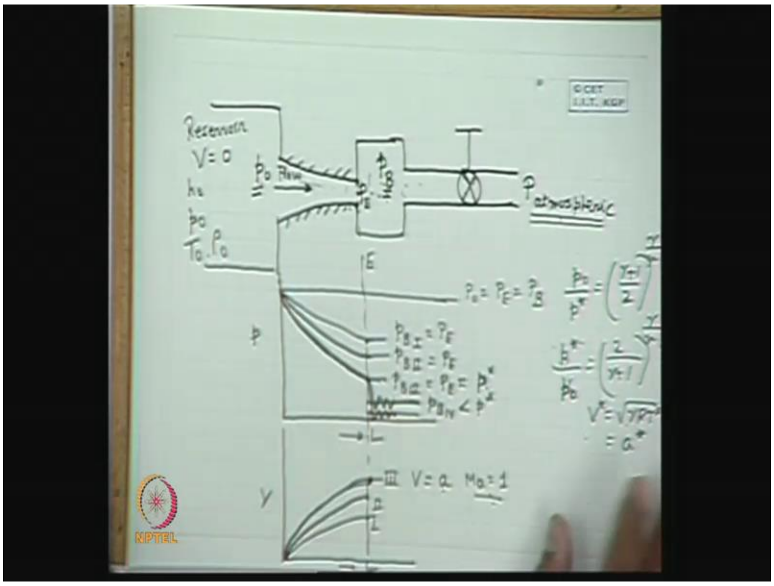

Let us now consider a situation like this let us now consider a situation like this let us now consider a situation like this let us consider a converging nozzle is discharging fluid from a reservoir this is the direction of flow. So, this is the reservoir this is a reservoir where the fluid is substantially addressed, and the properties are corresponding to rho zero t zero p zero t zero rho zero at this stagnation property. Now, therefore, p zero is the pressure that is the inlet pressure to the nozzle this stagnation pressure now you see these the nozzle is discharging into this chamber which is connected like this with a valve to atmospheric pressure.

What is the purpose of it physically what we want to mean that this the if this be the back pressure that is the pressure of the chamber where the fluid is discharged from the convergent nozzle convergent nozzle is isentropic that mean is adiabatic properly insulated this our purpose is to vary this back pressure of the nozzle for a fixed value of this stagnation pressure, that is the inlet pressure to the nozzle. So, this can be done practically like this that if we open the valve if the valve is closed this pressure is equal to the pressure p zero.

So, when we gradually open the valve this pressure is going to be reduced, and the flow in the nozzle will be increased. So, if we widely open the valve this pressure will be ultimately p atmospheric that is atmospheric pressure atmospheric pressure is the minimum pressure which we can achieve, but this pressure can be made higher than the atmospheric pressure at different values by closing the valve gradually or otherwise starting from the fully closed position opening the valve gradually we can reduce the pressure from a very high value which may be equal to p zero under the fully closed position to a value of the p atmospheric pressure.

That means this is the way in practice we can reduce or we can change the back pressure of the nozzle. Now what we will see let us draw two figures one is with the length of the nozzle. Let us consider this is the exit plane exit plane of the nozzle e, and let the pressure at the exit plane be denoted as p e. Now we draw spaces is not much. So, we draw two figures one let we consider the pressure variation along the length, and velocity along the length of the nozzle.

So, now we will see physically what happens when the valve is completely closed there is no flow throughout the fluid will fill, and it is a static fluid filled. So, p zero p b p e all same; that means, we can draw a line like this where this is p zero is equal to p e is equal to p b; that means, the stagnation pressure pressure at the exit plane, and the back pressure all are same, now what happens in practice that if we gradually open the value p b is getting reduced; that means, we gradually open the valve, and set a value of p b which is lower than p zero.

That means we set a value of p b here this is the first one p b one, then what will happen the flow will take place through the nozzle a small flow will take place, and pressure will be gradually decreasing to this, and the exit pressure in the nozzle will match to the back pressure; that means, the exit pressure in this case p b one is equal to p b or rather we should write p is equal to p b one here the p e will equal to p b one, and what will happen the velocity also will increase will from a zero value to a maximum value at the exit plane.

If we further open the valve. So, that we reduce the pressure to a another lower value p b two for example, p b two; that means, second case, then also the nozzle flow will increase, because the pressure difference is more. So, there will be a continuous expansion, and the exit pressure will match to the back pressure of the nozzle; that means, the nozzle plane the exit plane pressure the pressure at the exit pane of the nozzle will be equal to the back pressures.

So, velocity also curve will show like this there will be more velocity at any point in the nozzle the velocity will be more. So, this is the discharge plane. So, now, if we still open the valve again more. So, that p b is further reduced, then it will. So, happen that at some condition let us consider that p b three when the flow rate will continuously increase, and pressure will continuously decrease in such a way; that means, this is one two. So, this three condition represents a flow velocity here v at the discharge plane which is equal to the velocity of this sound or rather we can write mach number is equal to one; that means, if I draw the graph of mach number it will be from zero to some value zero to some value below one. So, this will correspond to a value zero to one there is no space otherwise I could have shown this curve. So, it will be a similar curve like that; that means, if we go on reducing the pressure we will see that a pressure will be reached here which will correspond to a flow velocity which is equal to the mach number one.

It is obvious, because we have already recognized the sonic properties that p by p star if you remember that becomes equal to what is the value gamma plus one by two to the power gamma by gamma minus one or we can write p zero by p star that is p star by p zero p zero is two by gamma plus one to the power gamma by gamma minus one; that means, corresponding to a stagnation pressure p zero there is a value of p star where; that means, there is a value which is p star where the flow velocity will be equal to the velocity of the sound that is v r t star.

That means this is equal to this is known as v star, and v star is equal to this is equal to v star; that means, the mach one is reached, and in that case the back pressure p b three in this case will be equal to the exit plane pressure, and that will correspond to our that will correspond to our p star that is the condition at the corresponding to m a is equal to one now what will happen physically we can still lower the pressure this pressure may be still higher than atmospheric pressure. So, physically we can still go on lowering the pressure or even if it is atmospheric pressure we can attach it to a vacuum pump.

So, physically we can still lower the pressure from this p star which corresponds to a sonic condition here what will happen if we lower this pressure here that is a case four p b four which is lower than the p star, then what will happen further expansion is not possible; that means, the nozzle cannot increase the velocity more than mach one; that means, if we express till if we by reduction of this pressure flow velocity will still increase; that means, the velocity will reach more than the sound velocity.

That means this part the related part of the convergent duct will act as a supersonic flow that will not be done, because the area reduction is not accompanied with acceleration in supersonic flow. So, therefore, the further acceleration is not possible; that means, the velocity here will still remain as a; that means, the mach one will be still remaining, and the corresponding pressure p star will remain like that; that means, by a further reduction in the back pressure the exit plane pressure conditions.

For example the pressure pressure in the exit plane will not reduce further it is not only the pressure, but the velocity corresponding flow properties will remain same as that corresponding to mach one case; that means, in that case the curve will remain same; that means, even if we make another lower back pressure the curve will remain same the pressure curve the velocity curve will remain same. So, what will happen in practice the fluid will suffer a sudden pressure jump from the exit plane to this region in the surrounding ambience.

This will be caused by a serve discontinuity in the pressure field which is a three dimensional phenomena, and through a series of shock waves. So, there is obstination, and finally, that is ends a steady back pressure, and similarly with this back pressure. So, therefore, when the back pressure is reduced below the where critical pressure this pressure is known as the critical pressure that I will come afterwards below the pressure corresponding to the sonic condition at the exit plane the pressure will remain same.

That means which corresponds to this condition, and the velocity will remain same that is equal to root over gamma r t star, and a sudden pressure jump through a series of shock waves will take place. So, the velocity curve, and the pressure curve will remain same let us examine what is the graph of or figure of mass flow rate in this condition in this condition if you you see here this condition, if you draw the mass flow rate you will get a picture like that.

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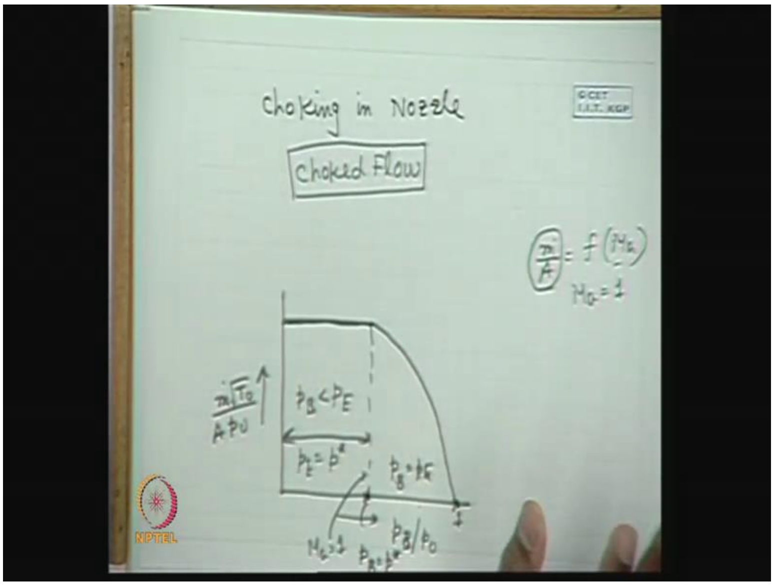

If you draw the mass flow rate as you have already seen that mass flow rate equation per unit area, the mass flow rate equations per unit area here you see the mass flow rate equations per unit area p zero by t zero root over gamma by r.

This quantity is there is value t zero p zero are taking like with mass flow rate per unit area this is the usual convention that t zero root over t zero by p zero is taken, and if we take the if we draw the graph or figure for this mass flow rate variation per unit area with the back pressure p b with the back pressure p b well, then what happens is that let p b by p zero when this is one; that means, here you see that when the ball was completely closed; that means, this is the line.

So, there was no flow no velocity; that means, this starts from this point as we go on reducing the p b the mass flow rate per unit area increases, and it increases up to the point when p b will p b big b are given p b reaches p's asterisk p b reaches p asterisk, then the mass flow rate will not change further, because we have already seen that mess flow rate per per unit area this becomes a function of mach number where it attains the maximum value when mach number becomes equal to one.

That means this is the condition when mach number one is reached; that means, during this condition this this region p b is equal to p b, but during this condition p b is less than p e, and p e becomes equal to p's asterisk; that means, here if you reduce the p the exit pressure at the nozzle discharge plane that is the pressure at the nozzle discharge plane is also reduced, and matched to it p b, and the mass flow rate increases, because these are the situations where you can cause an increase in flow through the nozzle by reducing the pressure.

So, you go on reducing the pressure as it happens for all incompressible flow the flow rate through the nozzle will gradually go on increasing. So, continuous expansion will take place, and a continuous acceleration will take place. So, in those cases the exit plane pressure will always match the corresponding back pressure, but when this pressure reaches the pressure corresponding to the situation when the mach one is reached here, then a further deduction in back pressure that you can do the will not change the exit plane conditions; that means, the exit plane pressure will remain as that corresponding to the mach one situation.

So, this is the situation during when the mass flow rate will not change is remain constant. So, this situation is known as choking choking in nozzle choking in nozzle, and the flow is known as choked flow, then the flow is known as choked flow flow is known as choked flow. So, under this condition the flow is choked choked means now further change in the flow can take place or rather further increase in flow cannot take place by an reduction in back pressure.

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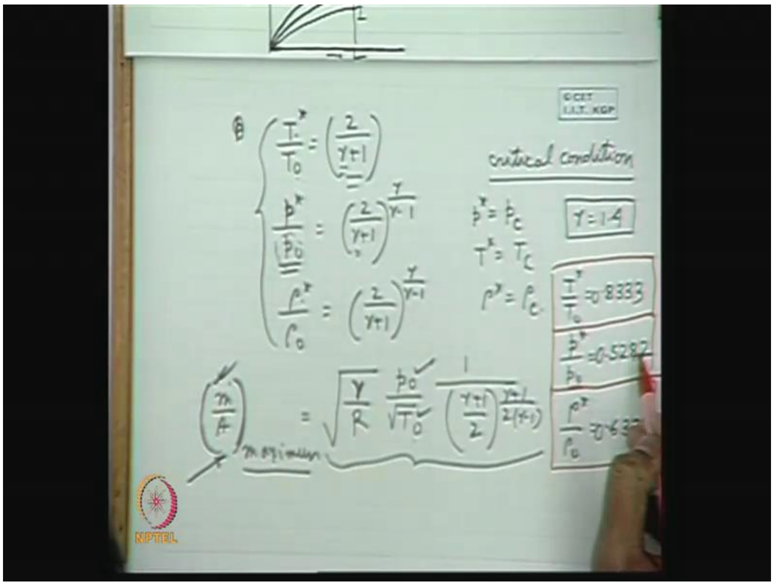

Now, let us examine few more things now we know that the relationship between the stagnation properties sorry sonic sonic, and stagnation properties if we recall we know that if we start from t star t star to t zero is equal to it will be just the reverse two by gamma plus one is it true similarly p star by p zero will be if we see that earlier we have recognized the sonic properties in relation to stagnation properties gamma by gamma minus one. So, rho star by rho zero is equal to two by gamma plus one by gamma minus one.

So, you see now that when it is not the nozzle which is of concern when these properties are fixed; that means, the stagnation state is fixed or sonic state is also fixed for example, for a given stagnation pressure the sonic the pressure for the sonic condition is fixed this condition is known as critical condition critical condition sometimes in respect to nozzle flow this asterisk we can reduce as c that is we write as p c; that means, asterisk can be this star can be written as c. So, that you can replace rho star is rho c; that means, for a given stagnation pressure inlet pressure to the nozzle the critical pressure that is the pressure where the mach one is reached is already fixed provided the gamma is fixed which is a fluid property?

So, it depends entirely similarly for this stagnation temperature is fixed the critical temperature is fixed if the stagnation density is fixed the critical density is fixed; that means, this critical state is fixed corresponding to a stagnation state similarly if we just have a look the mass flow rate equation m dot by a you see it is a function of mach number. So, therefore, if I put m a is equal to one you put m a is equal to one

And find out from here what is the m dot a maximum, please find out the value of m dot a maximum maximum is equal to root over gamma by r p zero by root over t zero. So, simply it will become it is one; that means, two plus gamma minus one that a gamma plus one; that means, it will be two plus one by gamma gamma plus one by two to the power gamma plus pone by two gamma minus one therefore, we see that the maximum mass flow rate that m dot by a per unit area is also fixed by the stagnation properties p zero t zero gamma r are the fluid properties which I want to mean that this reaching or attainment of the critical condition depends on the stagnation conditions along with the corresponding mass flow rate.

So, therefore, the mass flow rate per unit area m dot, and the pressure which will correspond to the critical condition here all will depend pressure or temperature or the density which will correspond to critical condition here, and the corresponding mass flow rate per unit area which will be the maximum above which we cannot go even if we reach the pressure depends all on stagnation condition. Now the question can come that if the critical property critical pressure has reached that p b has reached to this value p c corresponding to p c where p is equal to p c.

In that case if we use the same p c, and if the nozzle is reduced; that means, if I reduce the nozzle area nothing will happen the p c will remain same; that means, the same curve will remain, and the value of m dot by a will remain same as I already write here where I have written in value of this will remain same. So, only thing is that total mass flow rate will change; that means, mass flow rate per unit area is uniquely fixed is uniquely fixed where when while this stagnation properties are fixed. So, while you decrease the area the total mass flow rate will change.

That means it will be reduced if we increase the area it will be increased. So, therefore, the conclusion is that mass flow rate per unit area coming out from a converging nozzle of course, at any given stage this is the maximum here at the exit plane, and this value will reach a further maximum with the change in the sets; that means, set means for a given stagnation pressure if you go on changing the back pressure when the mach number one is reached, and at that condition the pressure is p star which is known the critical conditions or the p c is given by these expressions.

If we consider gamma is one point four which is value as a thumb rule we use that for a diatomic gas gamma is one point four, and if you substitute here this formula always gives t zero becomes point eight three three three p star usually used point five two eight two, and corresponding rho star from this formula becomes point six three three nine point zero point zero point zero point. So, these are the values; that means, if I know a particular stagnation state I can tell the critical. So, therefore for any nozzle.

So, now we can find out that if I have a stagnation pressure I can tell. So, this must be the back pressure point five two eight two times this stagnation pressure for which I can get the maximum flow; that means, at when the pressure will reach to a value of point five two eight two times this stagnation pressure the sonic condition at the discharge plane of the nozzle will arrive. So, a further decrease in back pressure will not increase the flow in the nozzle under this situation the flow though the nozzle is said to be choked; that means, flow is choked.

That means flow is choked; that means flow is choked. So, a further increase in the flow cannot take place a very interesting very interesting physical explanation for this is like that. Now as we have already recognized earlier that a in a compressible flow a disturbance propagates as a pressure wave upstream with a velocity equal to the velocity of sound relative to the fluid medium now you see when the nozzle is choked now we have recognized the choking in the nozzle; that means, if the back pressure is reduced to a value when the sonic condition is reached at the exit plane no further change in the back pressure.

That means if you still reduce the back pressure will not reduce the will not increase the flow rate though the nozzle. Now if you think in this manner that how the floe rate is changed physically when the back pressure in the nozzle is above the critical pressure a reduction in the back pressure, how it has changed the flow when you reduce the back pressure. For example, you operate a valve at the downstream where the people does in practice what happens a disturbance is created at the downstream which is transmitted as a signal in the form of a pressure wave.

Go to the upstream, and changes the flow. So, that it sends a new signal with an increase flow through the nozzle. So, this is precisely the physical picture, but when you reach the sonic condition at the throat, then a further change in the back pressure which can create a disturbance cannot go upstream as a messenger to change the signal to give another increase flow rate, because in that case the absolute velocity of the disturbance becomes zero, because its velocity towards the upstream direction relative to the fluid flow if the sound velocity if you recall that.

So, when the fluid is flowing it is sound velocity it has reached at this end. So, this wave becomes a standstill wave it cannot penetrate it cannot go, because the velocity becomes zero there if we calculate the absolute velocity at the discharge end which is created which will propagate upstream the velocity is zero, because this relative velocity is acoustic velocity it is zero it becomes exactly zero. So, it cannot move upstream to change the condition. So, therefore, the condition in the nozzle remains as it is; that means, the pressure distribution in the nozzle.

The velocity distribution in the nozzle all the conditions, and all the sections remains unchanged along with the mass flow rate through the nozzle, and the mass flow rate under this condition is maximum, and the condition at the discharge plane of the nozzle that is the nozzle orifice or the discharge plane whatever you call is the sonic condition this flow is known as choked flow, and this situation is known as choking in a converging nozzle the flow is choked all right. So, this depends only on the stagnation pressure corresponding to a stagnation conditions stagnation pressure temperature, and density the critical pressure density, and temperature depends. And also the mass flow rate per unit area which is the maximum for that situation any question please.

Thank you.