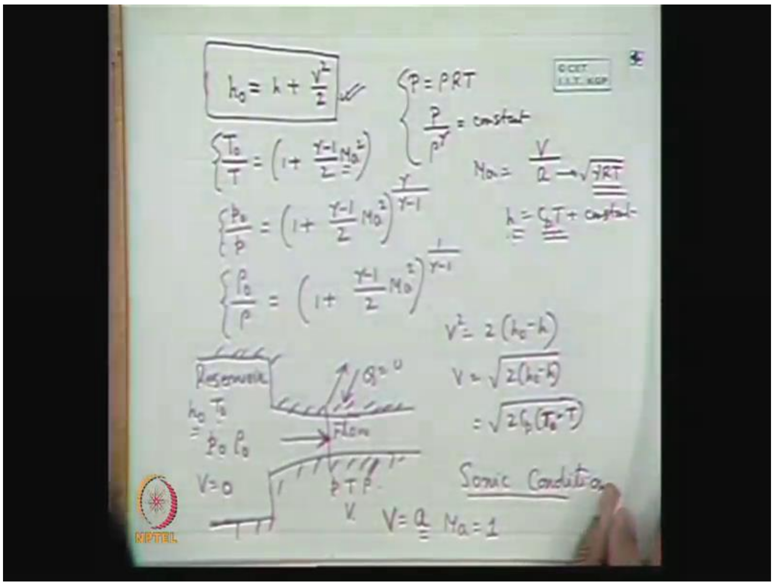
Introduction to Fluid Machines, and Compressible Flow

Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 30
Effects of Area Variation on Properties in an Isentropic Flow

Good afternoon welcome into the session. Last class we discussed the stagnation properties stagnation spaces stagnation temperature stagnation densities that definition, and the expressions for the... So, if we again we capitulate those stagnation properties

(Refer Slide Time: 00:45)



You see that we have already derived that stagnation for example, the stagnation enthalpy is defined like this zero with the suffix for the stagnation quantities is h plus the velocity v square by two the dynamic, and with the head of a perfect gas or an ideal gas as the system which obeys the ideal a gas equation of stayed that p is equal to rho r t, and for isotropic process as you can know p by rho to the power gamma gamma is the ratio of specific heat at constant. We derived the ratios between the stagnation temperatures t zero by t is one plus gamma this is capitulation of this earlier thing mach numbers square of the mach number where this mach number m a is defined as the flow velocity divided by the acoustic velocity or speed of sound which is nothing, but root over gamma r t this is also for an ideal gas as system or the flowing the medium.

Similarly, for a perfect gas p zero by p the ratio of the stagnation pressure to the local pressure is given by gamma minus one by two m a square to the power gamma by gamma minus one, and rho zero by rho the ratio of density stagnation density to the local density is one plus gamma minus one by two m a square one by gamma minus one. So, this have been derived with the head this two question. So, the constants for this definitions are like this the flow should be isentropic; that means, the stagnation temperature stagnation pressure not stagnation in stagnation temperature isentropic is not important, but for stagnation pressure, and the stagnation density the flow should be isentropically brought to rest ok.

Now, let us realize this situation physically for example, if we have large reservoir for example, the flow commences from a large reservoir if we have variable area duct the flow takes place a variable area duck let this is the direction of flow, and if this is large reservoir a large reservoir, and if this duct, and the reservoir is insulated both are insulated there means q is equal to zero there is no q head interactions neither comes out nor goes in, and if we considered the flow to be in visit flow; that means, fluid flowing is in visit flow; that means, the free from fictional effect, then this flow is an isentropic flow.

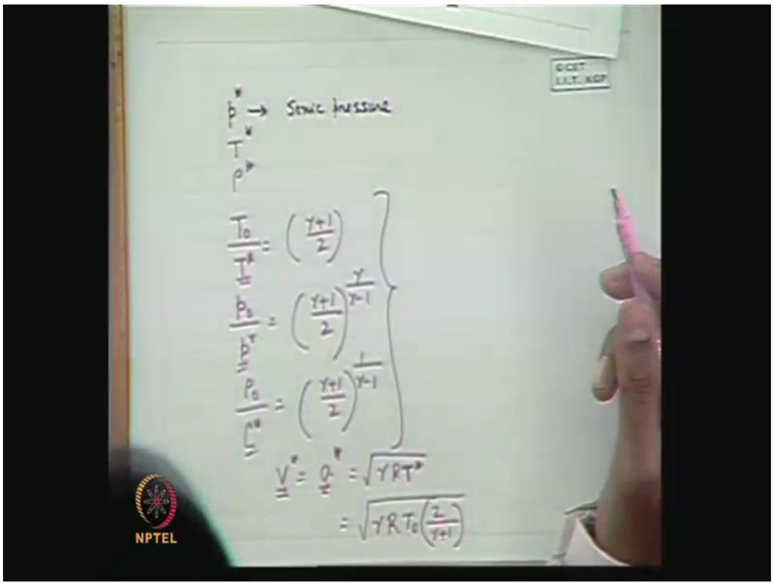
Then for this kind of isentropic flow the conditions at the reservoir where the fluid is addressed are typically the stagnation condition; that means, temperature corresponds to t zero the enthalpy in the reservoir when the fluid was addressed corresponds to eight zero the pressure is p zero, and the density is rho zero, and the most important thing is that at any section where the pressure is p temperature is t corresponding density is rho, and velocity is v the ratios are giving by this, this is the energy equation which is valid if there is no heat flow out or into the system the enthalpy plus the velocity here, and the reservoir v is zero has to be constant. So, there is no restrictions whether the flow will be reversible or not fictional effect will be there or not.

So, therefore, these equation for these equation only constant is the adiabatic flow; that means, the flow should be adiabatic, and with the eight of the perfect gas equation we can derive this equation this is, because for a perfect gas we know h can be expressed as c p t plus some constant arbitrary constant c p times the temperature. So, c p times the temperature enthalpy specific enthalpy. So, with the help of this we can derived that. So, stagnation temperature does not require the condition for isentropic, but stagnation pressure, and stagnation density must require the condition for isentropic. So, stagnation property is as a whole are defined those properties which could arise if the fluid were brought today's isentropically; that means, in the flow situation in an isentropic flow the fluid where the fluid is addressed the situations are the parameters referred to stagnation parameters these are the stagnation parameters we can find out the velocity for example, v square is two h zero minus h or v is equal to root over two h zero minus this simple condition; that means, by virtue of enthalpy of a difference for a perfect gas we can write c p t zero minus t.

So, therefore, at any section the velocity v is achieved by virtue of its change in this stagnation temperature stagnation temperature the simply the index of the energy or the enthalpy in case of an ideal gas another very important condition arises in the isentropic flow is the sonic condition that is another important condition what is meant by sonic condition; that means, that any section; that means, in the duct the flow situation be such that the velocity of flow becomes equal to the acoustic speed or velocity of sound at that section corresponding to the particular properties prevailing at that section; that means, the local properties of p t zero the velocity of sound velocity of sound is given by root over gamma r two.

So, the local section or the locality where v attains a is known as the sonic condition they are the mach number of flow becomes equal to one. So, the sonic condition is a very important condition in sections where the fluid flow achieves the speed of sound with mach number is one, and the properties at that location is are referred as sonic properties, and they are usually conventionally denoted with an asterisk as the super script.

(Refer Slide Time: 06:59)



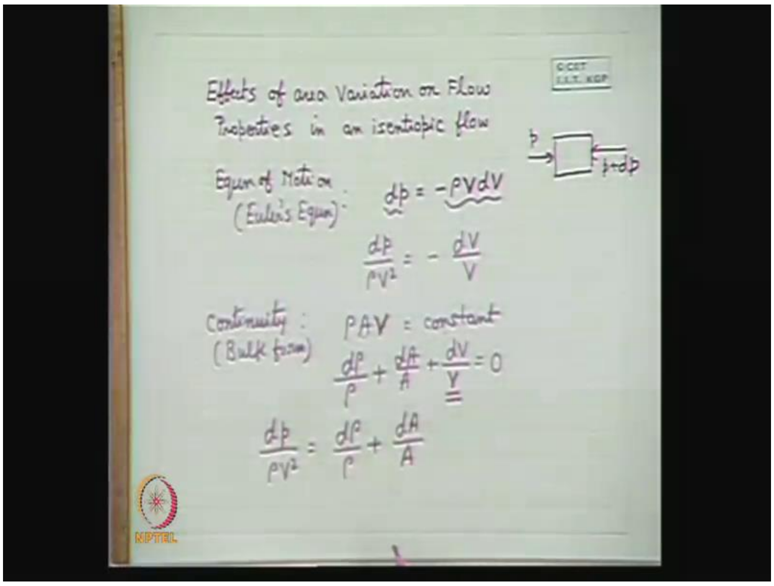
For example p star is the known as sonic pressures. So, pressure p star that is the sonic conditions sonic pressure what is meant by sonic pressure that is the pressure at the section where the sonic velocity is reached similarly t star. Similarly, rho star just like p zero t zero rho zero with zero as suffix conventionally represents the stagnation properties asterisk with super script represent the sonic properties; that means, these are the properties that the section where the sonic velocity that is velocity of sound is reached by the velocity of the fluid. So, to derive an expression of those quantities it is very simple that if we put a m a mach number is equal to one here; that means, if we put mach number is equal to one, then the t correspond to t star, then what is the value of t zero by t star we can find out the ratio, then we can find out t zero by t star becomes equal to if you put mach number is equal to two plus gamma minus one; that means, gamma plus one by two.

So, simply it becomes gamma plus one by two, because mach number is one it is free from mach number is put one. So, similarly for a perfect gas we can write p zero by p star is gamma plus one by two raise gamma by gamma minus one similarly rho zero by rho star is equal to gamma plus one by two to the power one by gamma minus one. So, this defines the sonic properties the ratio of stagnation to sonic properties where the flow velocity as reach the acoustic velocity now what is the flow velocity v v star is equal to a star, and is equal to gamma r t star all star or asterisk whatever you call we call it as star.

So, with a star at the super script means the condition, where the mach number one has reached.

So, at that condition the velocity of the fluid flow, and at the sound speed at that condition is also given as a asterisk mark star a star is root over gamma r t star all right. So, we can find out this t star is t zero by gamma plus one. So, we can also write in terms of the stagnation property t zero t star is t zero into two by gamma plus one. So, one can express also these are liquid algebra manipulation the v star or a star becomes equal to gamma r t zero two by gamma plus one. So, when the mach number one is reached the sonic properties are defined like this as a ratio of the stagnation properties like this. So, these are known as sonic properties, now after this any question?

(Refer Slide Time: 10:14)



So, after this we will see an very important thing we will come to an important directions or important problem that effects of area variation effects of you write this is the topic of the day effects of area variation which is very important, and interesting area variation on flow properties on flow properties on flow properties in an isentropic flow in an isentropic flow effects of area variation on flow properties in an isentropic flow. Let us consider a general situation of an isentropic flow like this; that means, the duct is adiabatic, and we consider in visit flow; that means, free from a reversibility that is is a reversibility adiabatic flow isentropic flow where the area is varying this is a varying area duct.

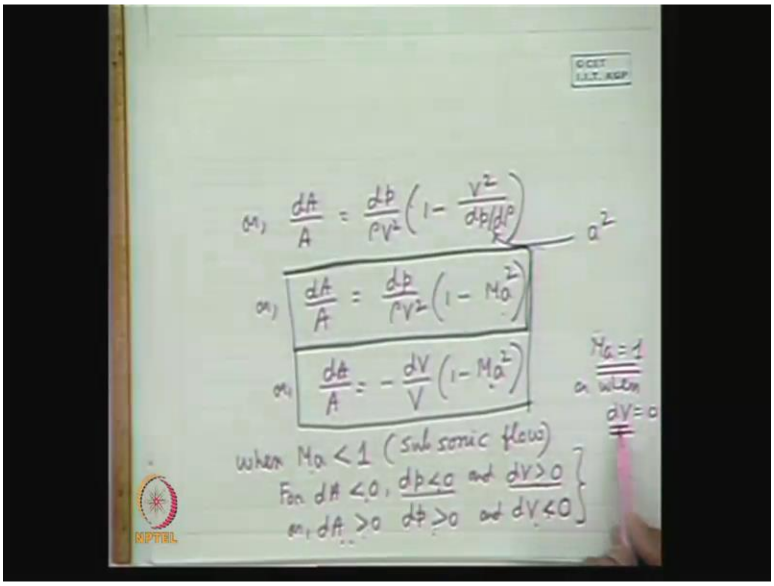
So, we consider analysis for one dimensional flow where velocity is pressures on or functions are the direction of the flow, but is uniform across uniform across a section. So, now, we see that n this case if we write the equation of motion equation of motion equation of motion for an one dimensional in visit flow same to the equation of motion that is your euler's equation euler's equation euler's equation which is the equation of motion for in visit flow equation of motion for in visit flow in one dimensional can be written in a differential form d t is equal to minus rho v d v in a differential form we can write rho v d v.

So, this is the inertial term, and this is the pressure is the pressure this can be found out by making a force balance p p plus d p, and taking this is the inertia of force, and this is the pressure force it is balanced by inertia force is equal to pressure force, because there is no viscous force. So, d p is minus rho v d v if you recall it, it is the differential form of your Euler's equation for one dimensional flow; that means, the equation of motion for an in visit fluid minus rho v d v now if we divide both the sides by rho v square we get d p by rho v square is very simple reduction is minus d v by v now continuity if we write the continuity equation continuity means the integral form of the continuity not differential form the bulk continuity equation that is the bulk bulk form bulk continuity equation we can write the rho density into area into velocity.

That means We are considering the one dimensional approach; that means, any section the velocity is uniform across the section area is the cross sectional area at that section, and rho its density this is equal to constant this is the bulk continuity for a one dimensional flow at that any section rho a v v is the uniform velocity at that section rho is the uniform density, and area of cross section the product of these three is constant; that means, the mass flow rate this implies a mass flow rate now in a differential form this can be written d rho by rho plus d a by a very simple deduction d v by v is zero from continuity equation in this differential form we can substitute d v by v from here.

And we get d p by rho v square is equal to minus d v by v is d rho by rho plus d a by a; that means, d rho by rho plus d a sorry by a all right.

(Refer Slide Time: 14:20)



And next we can wire this d p by rho this we can write d p, if I take this d rho by rho d a by a d a by a in one side we can write d p by rho v square minus d rho by rho or we can write d a by a or or we can write is equal to d p by rho v square we take as common d p by rho v square into one minus v square divided by d p by d rho. So, again we can write d a by a or is equal to d p by rho v square, now what is this value d p by d rho in an isentropic flow.

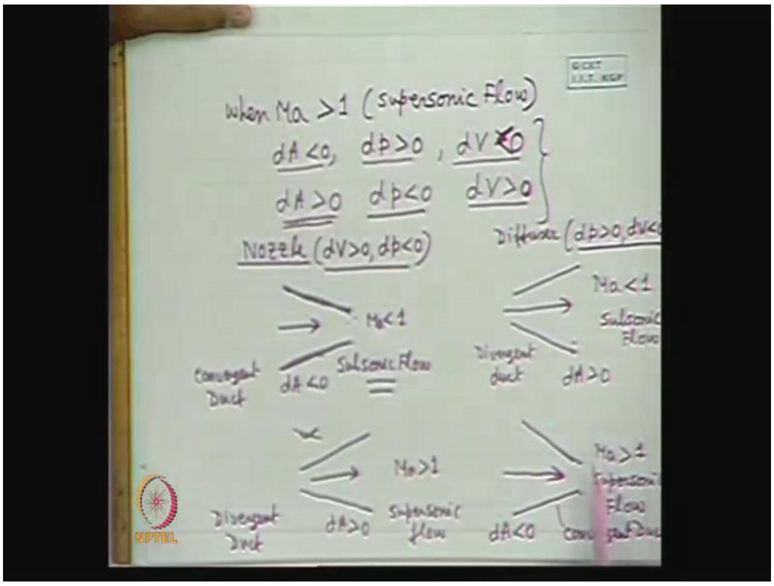
A square.

A square very good that is this is equal to a square, and b square a square is mach number square all right again, we can write another equation or in terms of v d a by a this d p by rho v square d p by rho v square is minus d v by v is see from the equation of motion d p by rho v square is minus d v by v. So, we can write sp minus d v by v. So, this two equations are very very important equation these two these two these two equations are very important equations in compressible fluid flow. So, what do these equation indicate we see now this two equations indicate that when mach number is less than one; that means, subsonic flow subsonic flow when mach number is less than one; that means, in case of subsonic flow, we see that d a, and d p have the same sign d a, and d p has the same sign.

And d a, and d v has the opposite sign; that means, for d a greater than zero d a less than zero for example, d p less than zero, and d v greater than zero or d a greater than zero d p d a greater than zero d p greater than zero, and d v less than d a greater than zero means d v less than zero what does it mean; that means, when m a less than one is subsonic flow when area decreases; that means, d a less than zero d a d p is positive; that means, d a, and d p are at the same sign when area decreases; that means, area less than zero d p also less than zero; that means, the pressure also decreases, but velocity increases similarly when area increases that d a greater than zero, then pressure also increases, and accordingly these two are in opposite signs.

So, pray accordingly the velocity decreases, then it is the usual happening that we already know which take place case in case of incompressible. So, this qualitative trend with which your which we are accounted with the incase of incompressible flow remains the same in case of subsonic flow when mach number less than one, but what happens in case when mach number is greater than one from this we can write when mach number is greater than one.

(Refer Slide Time: 18:17)



When just the reverse you see when mach number is greater than one that is supersonic flow supersonic flow you just see from the equation, when mach number is greater than one we see that when d a is less than zero d p is, because this is negative mach number greater than one.

So, when d a is less than zero d p is greater than zero; that means, when d a is less than zero d p is greater than zero d p is greater than zero, because this is negative similarly when this is negative when d a is less than zero d v is greater than zero; that means, d v is greater than.

Sir less than zero.

Less than zero.

d v is less than zero I am sorry d v is less than zero similarly when d a s greater than zero does the opposite from here we can write, but again we see when d a is greater than zero. So, it has to be positive this is negative d p is less than zero. So, d p is less than zero, and d v is greater than zero; that means, it is just the reverse when area decreases the pressure increases, and the velocity decreases well when the area increases, then what happens the pressure decreases, and the velocity increases just the reverse from that on the subsonic flow let us, then see that in therefore, we see that the change in area in case of subsonic flow, and supersonic flow has the reverse effect has the reverse effect.

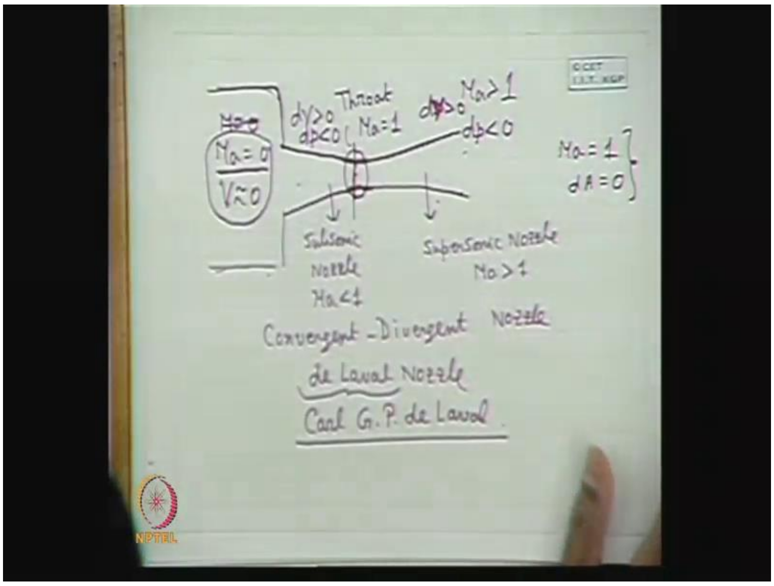
Now, we know the device nozzle what is a nozzle by a definition in the flow of a fluid in the fluid flow nozzle is a device where d v velocity increases d v s greater than zero, and d p is less than zero where the pressure is decrease when velocity is increase now in case of a subsonic flow we see the nozzle action in fluid flow takes place; that means, the velocity increases, and pressure decreases when area decreases; that means, a convergent duct; that means, a convergent duct a convergent duct this is in case of mach number less than one; that means, subsonic flow; that means, in a subsonic flow a nozzle is a convergent duct where d a less than zero this is a convergent duct convergent duct.

But if you make a convergent duct for a supersonic fluid it will not act as a nozzle in case of a supersonic fluid you see the nozzle action; that means, the pressure decrease of pressure an increase of velocity will take place when d a is greater than zero; that means, n case of a sorry in case of a supersonic flow; that means, when m a m a greater than one this is the direction of flow a divergent duct divergent duct divergent duct act as a; that means, in case of supersonic flow act as a nozzle therefore, we see that why the divergent duct act as a nozzle in case of subsonic flow a divergent duct when the area increases act as a super nozzle in case of a supersonic flow the reverse happen in case of diffusers diffusers are those duct where the velocity of the fluid decreases.

And the pressure of the fluid increases; that means, d p greater than zero, and d v less than zero this is the process of diffusion where the pressure of the fluid increases while the velocity decreases, and the duct where it happens. So, is known as diffusion now in case of a subsonic flow you see that the increase in pressure, and decrease in velocity is associated with an increasing area; that means, a diffuser is a divergent duct in case of a subsonic flow; that means, this the direction of flow; that means, in case of a sun sonic flow subsonic flow subsonic flow sp a diffuser is a divergent duct diver gent duct where d a greater than zero here it is also d a greater than zero.

So, d a, but in case of a supersonic flow you see that process of diffusion where d p is greater than zero, and d v is less than zero is associated with d a less than zero; that means, in case of a supersonic flow this is the direction of the flow that m a greater than one; that means, in case of a supersonic flow; that means, in case of a supersonic flow d a less than zero is; that means, a divergent duct act as a divergent duct acts as a diffuser. So, therefore, we see as our convention for the incompressible flow these holds good equally for a subsonic flow that divergent duct is a nozzle, and divergent duct is a diffuser, but for a supersonic flow a divergent duct act as a nozzle, and a convergent duct is acting as a or acts as a diffuser convergent duct act as a diffuser in case of a supersonic flow. So, therefore, we see for a supersonic flow if we have to have a nozzle that nozzle action, then we have to have an divergent duct.

(Refer Slide Time: 24:16)



Now, a situation where the fluid has to for example, increase its velocity continuously from a reservoir a stagnation conditions continuously upto the supersonic region; that means, the initially the mach is equal to zero. If I want a fluid at stagnate from a stagnate condition to reach supersonic velocity; that means, continuous increase of fluid velocity takes place with a continuous decreasing pressure, then what happens in the subsonic region to make the nozzle action we will have to make a convergent duct, but when the fluid velocity will reach sonic after that if you want a still expansion; that means, in pressure, and increase in velocity we will have to provide a divergent duct.

Because we know that in case of supersonic flow in case of supersonic flow a divergent duct for here acts as a where it is here act as a nozzle in case of supersonic flow nozzle is a divergent duct in case of subsonic flow it is a convergent duct. So, therefore, from a very low velocity or from exactly zero mach zero v almost zero, if we want to continuously increase the velocity up to a supersonic level; that means, m a greater than one we will have to provide both convergent, and divergent duct in the convergent portion here the subsonic nozzle it is subsonic nozzles subsonic nozzle where subsonic nozzles where mach number is less than one, and this is known as this is subsonic nozzle this is known as supersonic nozzle where mach number is greater than one.

And the area in between which is the minimum area where the area remains the constant, and becomes minimum this is known as the throat of this nozzle throat portions where mach number is equal to one is reached mach number is equal to one is reached half stream this side the convergent duct where the nozzle action takes place; that means, d v greater than zero, and d p less than zero which the subsonic nozzle, and the downstream of the throat sections the mach from mach number one the mach number increases this is the mach number less than one region; that means, supersonic flow here also d v greater than zero, and d p less than zero.

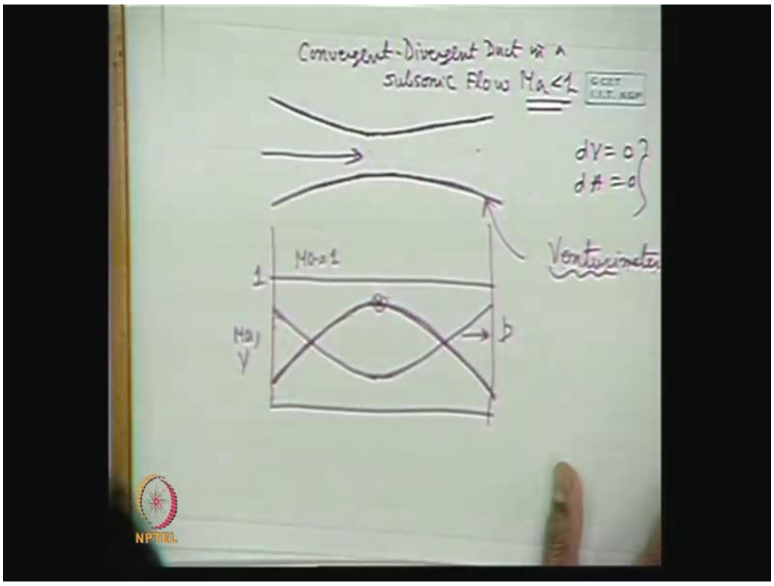
So, these type of ducts where fluid at a velocity very low; that means, mach number much low corresponding to subsonic region it may be if it in zero from a stagnation condition or situation increases continuously upto supersonic velocity is a convergent divergent duct, and it is known as convergent any question you can ask convergent divergent nozzle d p greater than less than zero d v greater than zero I am sorry d v greater than zero very good d v greater than zero here yes all right, this is known as convergent divergent nozzle this is sometimes known as de laval nozzle this laval this is the name of a scientist who first introduce his name is carl g p de laval.

So, carl g p de laval is the man or scientist who first introduced this type of nozzle in relation to a steam turbine it was first used according to the late nineteen century introduce this. So, according to his name this is known as de laval nozzle or convergent divergent nozzle now this section throat section where the area remains constant the sonic condition is achieved how we can prove this it is very simple we can please ask any question if you want to ask please what what happened any question.

No.

Please you ask me now you see that from this equation it is clear that when mach number equal to one d a by a is zero. So, from now this equation this clear that d a becomes zero when mach number equal to one or when d v equal to zero; that means, there is no change in the flow velocity. So, mach number essentially becomes one when d a is equal to zero; that means, from this we can tell that in case of a convergent divergent nozzle this is the section where mach number one is achieved associated with this d a is equal to zero this is the section, but at the same time we see that this d a zero may be achieved without the mach number becoming one; that means, mach number may be less than one greater than one when d v is equal to zero achieved what is the physical significance of it; that means, a throat area may be there even without reaching the mach number one, but satisfying the condition d v is equal to zero, these are very simple things.

(Refer Slide Time: 29:35)

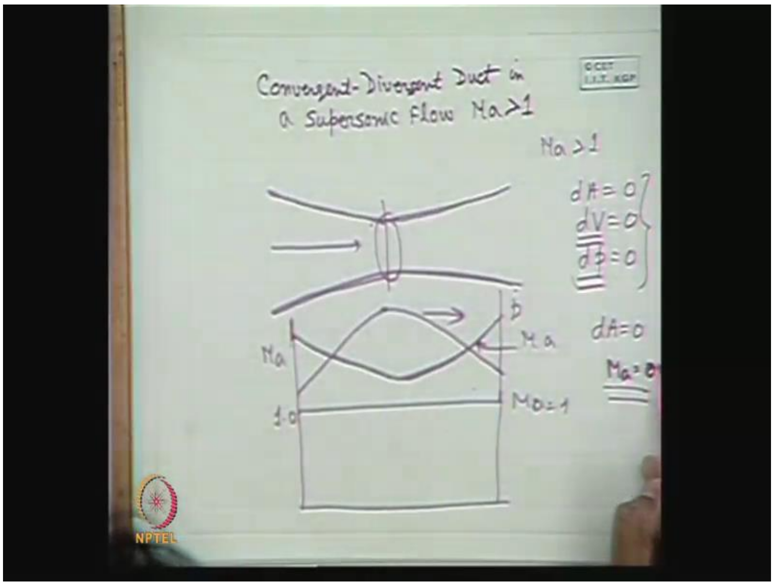


Let us consider completely a supersonic a subsonic flow with a convergent divergent duct a convergent let us consider a convergent divergent duct convergent divergent duct in a subsonic flow in a subsonic in a subsonic flow; that means, m a less than one. So, this is the flow direction. So, if we see the graph for m a you will see like this, and let this is the m a is equal to one line m a is equal to one. So, initially the velocity increases in this duct; that means, the mach number initially increase the mach number this is one the velocity increase, then what will happen the velocity will reach its maximum here mach number, and then it will go on decreasing; that means, this is either the graph of mach number or velocity to a scale.

So, this is the qualitative trained of the mach number. So, mach number will be always below one. So, this convergent divergent duct will act as a nozzle come diffuser; that means, the velocity will increase, and decrease what will be the pressure just the reverse pressure will first decrease if we draw the pressure graph. So, pressure will decrease, and reach the minimum value, and increase like this. So, this is the pressure graph. So, flow velocity, and mach number will flow like that; that means, it will defined by the first convergent part the first off stream convergent part will act as a nozzle.

So, this part is the nozzle that the velocity or the mach number increases, and the pressure decreases mach number is not necessarily reaching one entire it is a subsonic flow, then the maximum velocity is attained at the throat; that means, d v is zero is associated with d a is zero well, and then the rest the last rest downstream part, which is the divergent duct the velocity decreases, and the pressure increases these typical section is known as venturi meter this is also the name of the scientist to first device this type of duct in measuring the flow of fluid in a fluid circuit you know the venturi meter is one of the very accurate flow measuring instruments.

(Refer Slide Time: 31:58)



So, venturi meter convergent, and divergent duct the similar things happens in case almost in case of a convergent, and divergent duct for example, a convergent, and divergent duct convergent, and divergent duct in a supersonic.

Excuse me sir.

Yes well in a supersonic flow where m a is greater than one please.

Sir in the previous case.

Yes here here this figure.

Yes sire previous case.

Previous case where de laval nozzle.

Yes sir.

Yes.

When it reaches that throat it as m a equal to one. So, sir how can you ensure know that is a critical point how can you ensure that flow in supersonic region or it can.

Yes yes.

It can flow in subsonic also.

No it cannot flow in the subsonic region that I will come afterwards. So, whenever the mach one is reached, then if you apply a divergent duct, then from mach one a divergent duct to will always expand; that means, the velocity will always increased; that means, from mach one it cannot reach other way in a subsonic region; that means, you want to tell why not from mach one it will go to the a subsonic velocities; that means, it will act as a diffuser that it cannot go like that it will be, because mach one is reached here when the mach one is reached your mach one is reached d a by a is zero.

So, when d a by a is increased from mach one it goes into the I will discuss that afterwards in isentropic flow calculations you will see cannot come back again to the subsonic region it will automatically go to the supersonic region it will automatically go it cannot come back again to the subsonic region when the critical condition is reached it depends up on the pressure here if you put the this pressure will be such that it will go on expanding; that means, this pressure will be less than this pressure always in this case, because this will be designed in such a way that this pressure will be less than this pressure.

And it will go on expanding; that means, the pressure will go on decreasing on the velocity will increasing this will be make clear afterwards when I will discuss the isentropic flow situations is a good question understand, but whenever the critical condition is reached, then if you apply if you provide a divergent duct the flow will be always reaching the supersonic supersonic situation there is no other way out it cannot go back to again to the subsonic region this will be clear when I will discuss the isentropic flow situation, then what we were discussing well, then convergent divergent duct in a supersonic flow when m a is greater than one this is also very simple; that means, if totally the flow is totally the flow is m a is greater than one in the, then the flow direction is like this, then it will be like this.

If m a one is this line m a is equal to one the flow is initially what will be there the this will be this will act as a diffuser; that means, the velocity will be decreasing, and this part will act as a mosial; that means, this will be accelerating. So, initially there will be decelerating flow this is the mach number graph let this is the mach number, and this is the mach number graph. So, velocity will decrease initially, and then a divergent duct velocity will increase in this subsonic region, because this is the mach one supersonic region I am sorry supersonic region, and similarly the pressure will follow like that when these velocity will increase initially the pressure decrease the pressure will increase, because initially it is diffuser, and then the pressure will decrease.

So, this is the for example, this is the pressure. So, this is pressure one. So, that it is no necessarily that in a convergent divergent duct the critic cal condition has to be reached at the throat here d a zero is associated with d v zero d v zero or d p zero; that means, the maximum or minimum of the velocities are achieved depending upon whether the flow is subsonic of supersonic similarly the minimum or maximum according the just with the reverse sign pressure is associated with, but when the flow is changes from sonic subsonic, then d a is equal to zero is associated with m a is equal to zero; that means, the mach number reaches its maximum there; that means, the mach number is equal to one not reaches its maximum I am sorry the mach number is equal to one sorry d a is zero.

The mach number reaches to one; that means, its well the sonic condition yes the interesting question is that it always when you go give a divergent duct depending upon the design pressure maintained here it will go on that you will be clear when you will be dealing with isentropic flow that entirely depends at this pressure that is known as the design pressure when the back pressure this critical condition corresponds to certain pressure here you understand. So, to make the flow throat it this pressure is very important. So, this pressure cannot be more than this. So, this pressure is less than that, and this pressure has to be set in such a way that there should be undisturbed expansion or undisturbed acceleration in the supersonic region through this duct there is no other way out it cannot go to a lower velocity or in the subsonic region this will be clear I will be discuss this in discussing this isentropic flow well any question please.

Sir at the minimum area.

Yes.

Mac number will be greater than one.

No. So, definitely not minimum area again I am telling this problem mathematics. So, minimum area where d a is zero the throat corresponding to again I am telling this is a very important concept that at the minimum area throat section which is the associated mathematically with d a zero this is achieved when either mach is one or d v or d p is zero well either mach is one or d v or d p is zero; that means, that the throat section either mach number will be one either mach number will be one; that means, fluid is completely accelerating; that means, from a very low velocity it is totally accelerating; that means, in a initially it is acceleration in the subsonic region, then supersonic region in between at the throat mach number one.

Or it is total diffusion from a supersonic flow first in convergent duct that diffusion takes place; that means, a deceleration takes place, then mach number one reaches, then deceleration takes place in the in the divergent duct in the subsonic region with mach number one at the throat this is one situation that corresponds to the mathematical condition that d a is zero associated with it m is equal to one otherwise d a will be zero without achieving or attaining m a is equal to one either d v is zero d v d p is zero either not both d v, and d p will be zero simultaneously.

So, this is the case when mach number is not one; that means, either it is less than one or greater than one; that means, in case of less than one subsonic flow the throat area is associated with the maximum velocity or minimum pressure that is a typically venturi meter first acceleration, then deceleration nozzle, and diffuser or in case of a supersonic fluid it is just the reverse it is first diffuser, and then nozzle; that means, first deceleration, then acceleration with the minimum velocity or maximum pressure at the throat that is the situation mathematically that is the situation that refers mathematically.

Or to the case situation that d a is equal to zero achieved d v or d p is zero, but the most interesting question that I will explain to you while will be deriving the isentropic flow equations that with a convergent divergent duct, and the concept of choking that is the very important thing, then it will be create that mach number one if you reach from a subsonic flow through a convergent duct mach number one at the throat; that means, at the minimum area, then if you go on increasing the area it cannot go to the subsonic flow it will always go to the supersonic all right any question.

Thank you.