Introduction to Fluid Machines and Compressible Flow

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Lecture - 03
Energy Transfer in Fluid Machines

Good morning, welcome you to this session. We will discuss today the energy transfer fluid machines part two in continuation of our earlier discussion. Now in last discussion, we have recognized that the energy transfer to the rotor of the machine by the fluid in terms of the energy per unit weight, which is known as a head can be expressed as…

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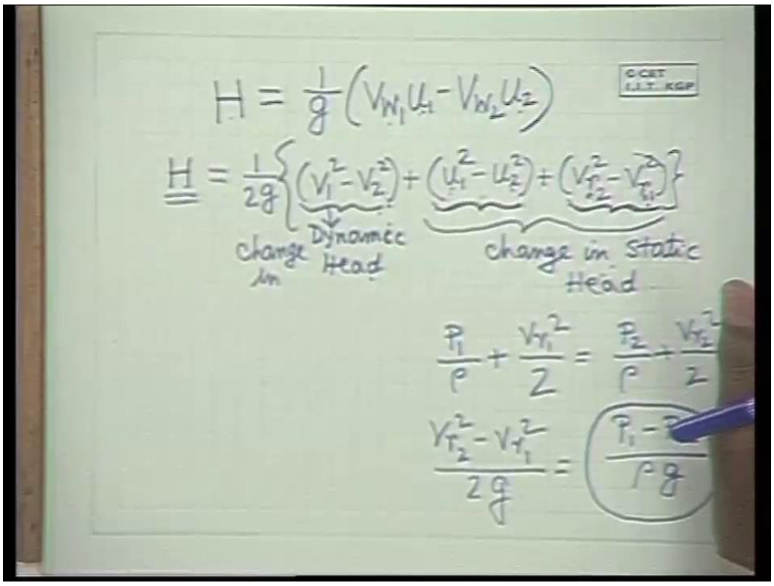

Just let me write 1 by 2 g v 1 square minus v 2 square plus u 1 square minus u 2 square plus v r 2 square minus v r 1 square. So, in last last session, we had recognize that the energy per unit weight that is the head transfer to the fluid rotor can be split out into three distinct components, where the nomenclatures are like this. We can have a recapitulation that where v 1, v 2, u 1, u 2, v r 2, v r 1 are like this.

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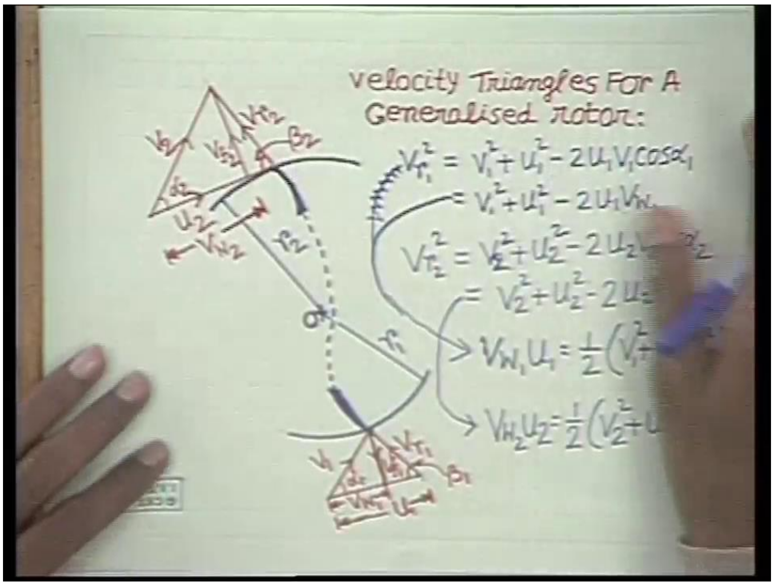

That v 1 and v 2 are the absolute velocities of the fluid at inlet and outlet of the rotor. u 1 and u 2 are the tangential velocities of the rotor at inlet and outlet. These are the rotor velocities, whirling velocities of the rotor at inlet and outlet. And v r 1 and v r 2 are respectably the relative velocity of the fluid with respect to the rotor at inlet and outlet. So, if v 1 v 2, u 1 u 2, v r 2 v r 1 are defined like this. We can express the head that is transferred to the machine by the fluid as it flows through the rotor vanes can be written like this.

Now you see that these three terms have got their different physical implications. Now let us see first what is the terms v 1 minus v 2 square, this implies the change in the velocity head of the fluid or a in change in the kinetic energy per unit weight of the fluid. Or simply it can be told as dynamic head, you can write a change in dynamic head, this one change in dynamic head. So, therefore, due to the change in the dynamic head of the fluid that means, that it is the change in absolute velocity as if flows pass the vanes, the work is being transferred or energy is being transfer to the machine.

Similarly this term represents a change in the head due to the change in its position radial position with respect to the axis of rotation. When a fluid has got a rotational velocity and it changes its radial position with respect to the axis of rotation, there occurs the change in the head or energy in the fluid. Now this term can be better understood, if we see this one.

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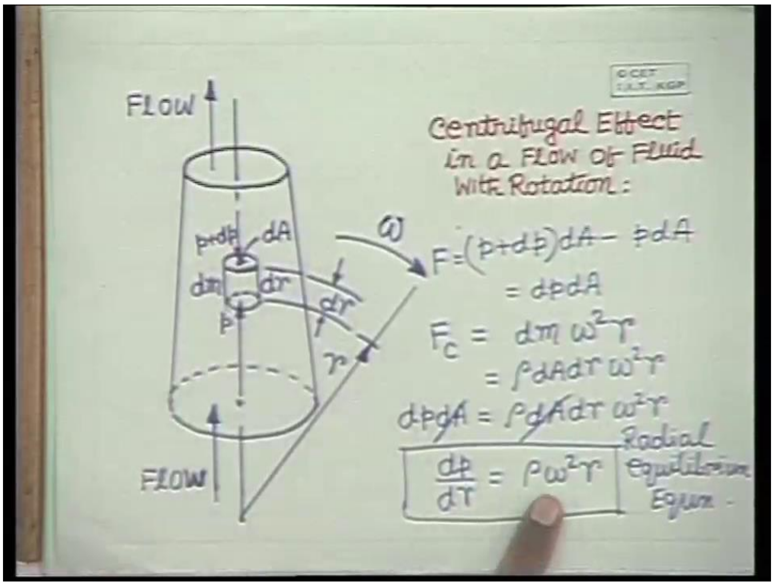

Let us consider a container, where the fluid is flowing in this direction, and container is given an angular rotation omega like this. So, basic objective is to show that when a fluid element under a rotational velocity, changes its position in radial coordinates with respect to the axis of the rotation. Let this is the axis of rotation at this point perpendicular to this plane of the figure, about which the container is rotating. Then we can show that the work is either being done on the fluid element or work is being extracted from the fluid element, how we can show.

Now, let us consider a fluid element at a radius r of thickness - d r, and area - d a. Now you know that whenever there is a rotational flow field it induces a pressure gradient a pressure variation in the flow in the direction of the flow exists. For which the pressure in the positive direction this direction of the r is higher than that at this upstream plane. So, therefore, if we take the force balance of the fluid element, we see that net force acting on the fluid element in the radially inward direction is can be written as p plus d p into d a, where d a is the cross sectional area of the fluid element minus p d a. Well which can be written as d p d a, so what is d p d a is the net force in the radial inward direction. Let we denote it by f that is equal to d p into d a.

Now this radial inward force balances this centrifugal force due to the rotational motion of the fluid element. So, this radial inward force balances the centrifugal force of the fluid element under rotational velocity. So, what is the centrifugal force, let f c for the fluid element it is the elemental mass d m times the linear velocity due to this rotation that is the tangential velocity v square by the radius or the radial location from the axis of rotation. This can be written in terms of the angular velocity as d m omega square r, this is the usual expression of the centrifugal force, which is acted on this fluid element.

Now, if we substitute the mass in terms of the area and the other geometrical dimensions, and the density of the fluid element we can write it rho d a d r. So, rho d a d r is the mass of the fluid element. So, this is the angular velocity square into r. Now at equilibrium these two are equal that means, the fluid motion is possible to in this direction provided there is a balance between the centrifugal force and the inward radial pressure force. So, if we write this, we get the expression d p into d a is equal to rho d a d r omega square into r. So, d a cancels out, well we can write then d p d r is equal to rho omega square r. This equation is a very well known equation in the fluid flow with rotational velocity and is known as radial equilibrium equation. I can write it that radial equilibrium equation.

This equation simply implies that when there is a rotational velocity in a flow field, and fluid flows in the radial direction then a inward radiation pressure gradient is imposed on the flow field which provides the necessary pressure forces to the balance to the centrifugal force. You know that in any rotational motion of in your solid body, there are two forces are in balance with each other. One is the centrifugal force which tends to make it flying away from the path, and another that centripetal force which is a force which makes the possible to have the rotational motion which is inwards towards the center of rotation. So, this centripetal force is provided by the pressure gradient through this pressure force. This is the well known radial equilibrium equation.

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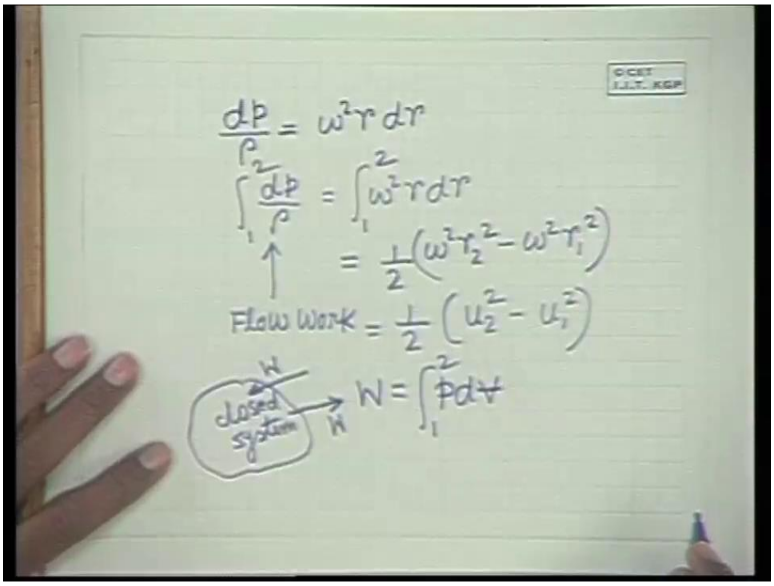

Now, if I write this in a little different form, the same equation can be written as d p by rho is equal to omega square r d r. Now if I integrate this equation, d p by rho integrate this equation omega square r d r between two points one and two, which physically indicates the two points let one is at the inlet and two it is at the outlet. It may be any two points in the flow field, one is an upstream point and two is an downstream point, which may be at the inlet and outlet in a flow passage. Then we can write this one by two d p by d rho is equal to half omega square r 2 square minus omega square r 1 square, which is nothing but half the linear velocity or the tangential velocity due to the rotation at the point two, the section two minus u 1 square.

What is the meaning of this? Now what is this d p by rho from one to two integral, this is the flow work. Now I come to the concept of flow work, now if you recollect thermodynamic general energy equation, you know what is flow work. Let us recapitulate the little bit of thermodynamic concept you know when you have a closed system, and it interacts with the surrounding in terms of work either work is being develop by the system to the surrounding or is absorbed from the surrounding to the system mechanical work if you consider. Most usual form is by the displacement of the system boundary, by the displacement of the system boundary for a close system because the mass within that system is fixed. And under reversible condition this work transfer is written as p d v, where d v is the v cut d v is the change in the volume. So, the integral is made between two state points one and two.

But what happens when the system is an open system that means, in thermodynamics we know there two types of systems one is the closed system at the mass is fixed with the same identity, that is known as control mass system, usually we tell as system. Another system is there where the mass is not fixed with the identity, there is a continuous flow of mass in an flow of mass out, but the volume is controlled, volume is fixed known as control volume system and usually we tell as open system or a control volume.

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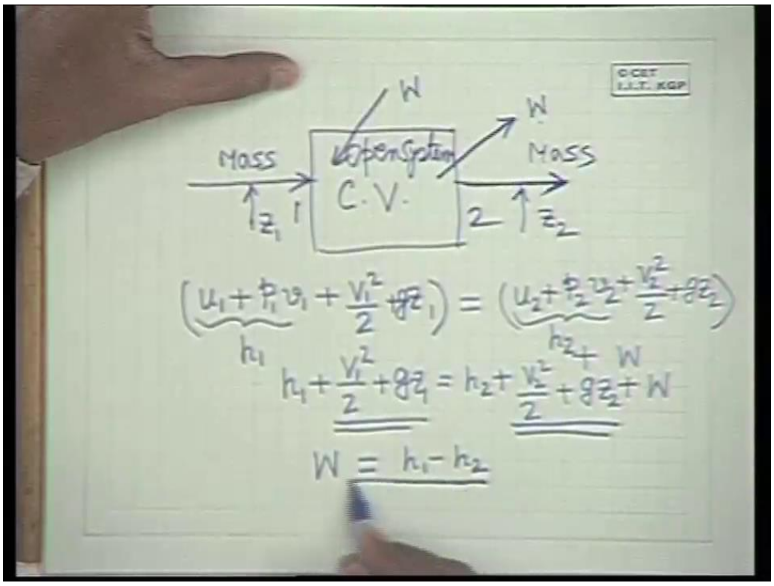

So, in case of an open system or a control volume that means, an open system or a control volume, there is a continuous influx of mass and energy, continuous mass coming and mass going out. Similarly, in this control volume, if it interacts with surrounding in the form of work that means, if it will develops work or it absorb work which comes in our case of fluid machine. A fluid machine is an open system, continuously the fluid goes into the machine at one part, and it goes out of the machine by virtue of with the machine develops work to the surrounding in the form of shaft work. In some machines, it is being develop to the surrounding the shaft work is being obtained by us, and in some cases the machine absorbs the work from the surrounding that means, the work is being put to the shaft in the form of the shaft work. These are case of compressors and pump while their work is obtain in case of turbine.

So, how to find out the this work in this case, in these cases we write the steady flow energy equation, this is the little recapitulation of your thermodynamic concepts. So, that you can recognize or appreciate the term d p by d rho as the flow work. So, what is that if we write the thermodynamic equation general energy equation of thermodynamics at section one and two, then we can write that the internal energy at u 1 associated with the mass flux plus the pressure energy which is written in thermodynamics in terms of the specific volume rather than the density plus we write the kinetic energy v 1 square by two per unit mass basis if we write.

So, if we consider the potential energies at this and this sections are given like this, if we denote z 1 and z 2 are the elevations from reference datum. So, this quantity represents the amount of energy in flux per unit mass with the mass flow coming into the control volume. Similarly the amount of energy going out from the control volume associated with the mass flux out of the control volume per unit mass will be the same energy quantities with their values at the outlet section denoted by the suffix two, per unit mass means this will be g z 1, so this will be g z 2.

If we consider the work is coming out of the open system or control volume, plus the work done per unit mass here w is the work done per unit mass. At the time being we neglect the heat flow; if you consider the heat flow you can take in this heat flow either on the right hand side or on the left hand side depending upon whether you consider the heat is coming in to the system or going out of the system. Now, therefore, we see that if we replace this as the enthalpy you know from the definition of enthalpy, it is the internal energy plus the product of pressure as specific volume. So, you can write h 1 plus v 1 square by 2 plus g z 1 is equal to h 2 plus v 2 square by 2 plus g z 1 plus w.

Now under all usual operating conditions of engineering systems, we have found that the change in kinetic energy and the potential energies are much small as compared to the changes in the enthalpy in such system. So, therefore, we can neglect the changes in kinetic and potential energies as compared to the change in enthalpy. And we can write under such condition W is simply equal to h 1 minus h 2 that means, the change in the enthalpy from the inlet to outlet is giving as the work transfer, provided the heat transfer between the open system and the surrounding is neglected; that means, the system is properly insulated. So, that the heat interactions with the surrounding is prevented. So, nowadays we know that for any open system it is the enthalpy difference which gives the work done . Now, therefore, the enthalpy difference we get as the work transfer. Now if we consider the process to be reversible, reversible means without friction and any other dissipative effect, and no heat transfer is there.

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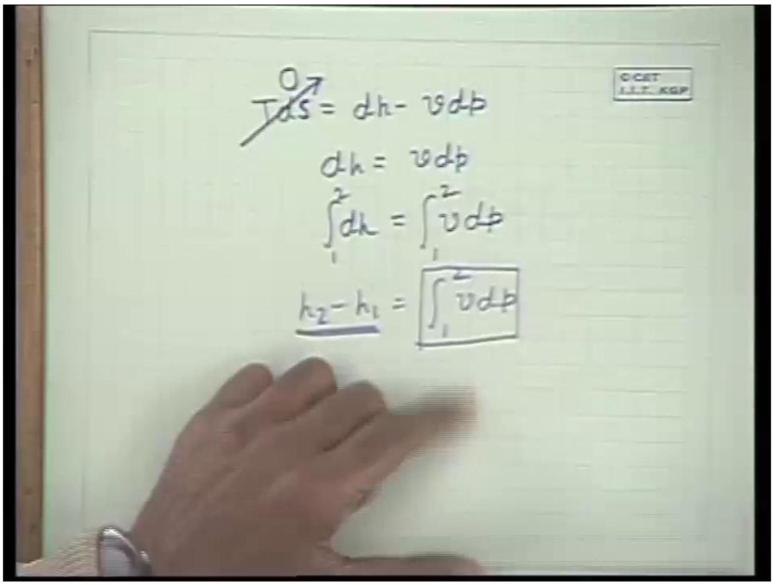

Then we can write from thermodynamic equation, first of all you know the general thermodynamic equation T d s is d h minus v d t, where this s is the specific entropy h is the specific enthalpy, and v is the specific volume this a generalized thermodynamic relations. Under reversible adiabatic condition, which is isentropic condition, we can write d as zero. So, therefore, it is clear that d h is nothing but v d p or integral of d h is equal to integral of v d p, for between any two sections one and two, which can be written as h 2 minus h 1 is integral of v d p. So, therefore, we see this integral of v d p refers to this enthalpy differences which equals to the work transfer, which equals to the work transfer as we have discussed in an open system. This v is the specific volume of this system.

So, therefore, if we come to our earlier slides then we see that this left hand side is d p by rho which is nothing but the flow work, because 1 by rho is v. So, flow work means that is the work transfer between a open system, here the turbo machines or the fluid machines with the surrounding; and this flow work becomes equal to half u 2 square minus u 1 square. So, therefore, we see some work is being done on the fluid system, if it changes is radial location from one to two, where at two the radial position or the value of the radial location is more than that at one. That means, if it goes further from the center of rotation then this is positive some work is being done on the fluid element. And vice verse takes place that when the fluid changes its radial position in a way that it comes nearer to the axis of rotation then this is negative then work is being release by the fluid.

So, therefore, if we look to this diagram well then we can realize. Therefore, from this equation that whenever a fluid moves from this position to this position; that means, from an upstream position to this downstream position away from the axis of rotation wok is being imparted on the fluid, and this work is stored in the fluid as energy. And this energy is mainly in the form of pressured energy why because it is very simple because of this radial pressure gradient the fluid when moves from this place to a place where at two where the radius from the axis of rotation is more than the fluid attains a higher pressure that means, fluid possess a higher pressure. And therefore, the work done on the fluid is stored in the fluid as pressured energy.

So, this way we can tell that moment of a fluid element with a rotational motion from one radial location to other radial location physically implies that head is either given to the fluid and it stores it or head is being extracted from the fluid from its stored head. So, this is the physical implication of this term that in a fluid machine as the fluid flows in a rotational flow field from one radial location to other radial location. So, either the head is gained by the fluid or head is developed by the fluid. So, this is the contribution and that head gained or developed is in the form of the pressure head. So, that is why this gain in head or loss in head is known as static head here the way it is written this head is the head developed by the fluid.

So, therefore, you see the positive value of it indicates that fluid comes from a higher radius to a lower radius from the center of rotation. So, u 1 is more than u two; that means, it releases some of its static head or mainly the pressure head, contributes this thing to the turbo machines to be developed. So, the contribution of the second term is like that. What is the contribution of the third term, which is very important and also very interesting physical significance. Just it is obvious from mathematical expression, it is the change of relative velocity, and one interesting thing is that where everything is that the first one is the inlet, and second one is the outlet here, it is just the opposite. That means, it is the change of velocity head relative based on relative velocity from the outlet to inlet. What does it mean?

Now, what is the concept of relative velocity. Let us think in this fashion .The relative velocity is the velocity with respect to the moving vane; that means, if the vane could have been fixed, the inlet velocity could have been v r 1 and outlet velocity could have been v r 2. Now you consider a fixed vane where the inlet velocity is v r 1 and outlet velocity is v r 2, then what are the possibilities are under which there can be a change in the velocity number one is the friction on the vane that due to the friction on the vane vane is at rest. So, the velocity may change, where the outlet velocity will be lower than the inlet velocity will be reduced, because of the friction.

Another opportunity is there what is that, if it is not a single vane open to atmosphere, if there are number of vanes in a closed casing. And if the flow takes place to the passage of two vanes gliding over one vane, then even if the vane is at rest consider series of vanes at rest, then the whether the fluid velocity will change or not depend upon the flow cross sectional area. That means, you simply consider a flow through a fix duct, simply flow through a fix duct, when a flow takes place through a fix duct, the flow velocity changes from upstream to downstream system under two conditions. One is the friction with the wall and another is that if the flow area changes, where the pressure changes and velocity changes, this is the consequence of continuity and Bernoulli's theorem.

There due to the change in the flow area the velocity changes because of the continuity for example, if the flow area is converging the velocity increases; and due to this change in the momentum, there is a change in the pressure that is even for an ideal fluid according to Euler's equation or Bernoulli's equation. So, because of the change in velocity due to its flow through a varying area passage, even a static duct is there, either it is found by two vanes or it is formed or it is made by walls just like a duct. So, velocity of fluid can change.

Here also now you see that relative velocity change can take place in the similar fashion; that means, if the area flow area between the vanes changes then there can be a change in the relative velocity. Otherwise the change in the relative velocity is not there, only there will be a little change due to friction. And if the vane surface is very smooth this change due to friction is not much. Change due to fraction always makes the outlet velocity slightly lower than the inlet velocity, but the relative magnitude of the outlet relative velocity with respect to the inlet relative velocity due to the change in the cross sectional area in the vane passage will depend upon the fact whether the area gradually decreases or converges in the direction of flow or diverges in the direction of flow.

If we allow a converging area in the direction of flow then v r 2 will be more than v r 1 you understand? So, in that case what will happen if v r 2 is more than v r 1, consider a fixed vane v r 2 is more than v r 1 means pressure will be less at outlet than the inlet. That means, fluid releases its pressure or contributes or gives away its pressured energy. If the vane is at rest means what practically we will have to give some support to keep them at rest. So, if it is it moves in developing work the release in the pressure of the fluid gives some energy that is fluid releases the pressured energy.

In opposite case, when we allow a diverging passage then what happens the v r 2 decreases or according to Bernoulli's theorem pressure increases; that means, when v r 2 is less than v r 1 because of a diverging passage then pressure is increased; that means, fluid gains the pressured energy. So, therefore, you see that the change in the relative velocity means either fluid gives away its pressured energy to the turbo machines the rotor of the turbo machines or it gains pressure energy from the rotor of the turbo machine, which is manifested in terms of the increase or decrease in pressure.

So, therefore, here you come in this expression v r 2 minus v r 1 square says that is v r 2 is more than v r 1, which is possible only if the flow through the vane passage. Flow variant of the vane passage is converging in the direction of flow then v r 2 is more than the v r 1 then this quantity implies a change in the pressured energy. Because if you write the Bernoulli's equation then we will see that p one by rho plus v r 1 square by rho; that means, considering the vane to be in a static condition then p two by rho plus v r 2 square by rho.

So, therefore, you see that v r 2 square minus v r 1 square, now you see v r 2 square minus v r 1 square by two g is nothing but p 1 minus p 2 by rho g. So, this is simply change in the pressured energy, which is equal to that due to the change in the dynamic head based on the relative velocities. Since this is manifested in terms of the pressure head, this we will not tell as the dynamic head transfer. Now we can tell that both of these terms two terms contribute in the energy transfer of the fluid machines in terms of this static head; that means the pressure energy of the fluid. So, therefore, they are told or they are called as change in static head.

So, therefore, today we conclude that the general expression for the head transferred to the turbo machines by the fluid is given by 1 by g v w 1 u 1 minus v w 2 u 2 this is known as Euler's equation. Where v w 1 and v w 2 are the whirling or tangential component of velocities of the fluids at inlet and outlet, and u 1 and u 2 are the tangential velocities of the rotor at inlet and outlet. And with the help of velocity triangles at inlet and outlet, we can express this into three components, which are very interesting from their physical point of view, where we can get an idea that this total head which is being transmitted to the turbo machines by the fluid is contributed by the dynamic head of the fluid and static head of the fluid. So, this part is contributed by the dynamic head of the fluid, where the absolute velocity of the fluid is changed if it is lower then it gives the head; otherwise, it gains the head.

Similarly, the change in the rotor velocity as the fluid passes from one radial location to other radial location and along with the change in the relative velocities of the fluid from its inlet to outlet provided a varying area passage is given for the flow of fluid contributes to the change in the static head of the fluid. That means, either the fluid loses its static head, what is that static head that is the pressured head; loses its pressured energy, and it is giving to the turbo machines or it extracts from the turbo machine the pressured energy or the static energy in head. So, this is the three components for the head.

So, today I will end here.

Thank you.

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So, book well.

Editor Comments: No audio from 30:26 min to 49:49 min, only slides are shown.