Introduction to Fluid Machines, and Compressible Flow
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Lecture - 29
Disturbance Propagation, Stagnation, and Sonic Properties

Good morning I welcome you to this session. Last class we derived the speed of sound or the speed of propagation for an infinite small disturbance or a pressure wave through a compressible medium flowing with a velocity, and we have seen that the velocity of the disturbance wave or the pressure wave relative to a sound medium can be written as…

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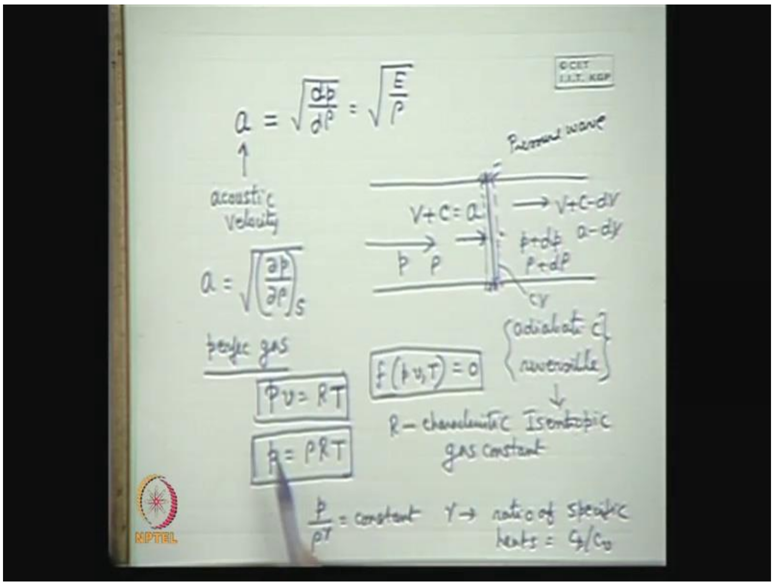

If I write it as a that is the relative velocity of the disturbance wave or the pressure wave is equal to root over d p by d rho or with the definition of bulk modulus of elasticity of the medium we can. So, this is the velocity of propagation of the disturbance wave or the pressure wave which is the acoustic velocity a sound velocity acoustic velocity relative to the speed of the fluid medium or relative to the flow of the flow velocity of the fluid medium.

Now, we have also recognized or we discussed in the last class that this value of d p by rho or e by rho, if you express in terms of the bulk modulus of elasticity of the medium can be evaluated as the can be evaluated explicitly in terms of the state variables that is pressure volume temperature or density provided we know the process constraint; that means, how the changes take place; that means, the value of d p by d rho under what process constrain, and also the equation of state.

Now, for this we have to depend on certain physical factors now let us see that this is the condition that the velocity of flow was like that if we consider these a frame of reference for our deduction with two kit attached to this pressure wave; that means, the pressure wave is fixed we see that this a is the speed of sound that is the acoustic velocity with which it is moving; that means, v plus c; that means, it is this equal to a; that means, with which the fluid flow to this pressure wave this is the pressure wave well, and ultimately here it comes with a velocities velocity is reduced v plus c by an amount d v that is a minus d v as a result its pressure is increased by p plus d p if the pressure is p, and density is rho, and density is changed.

So, acts as a compression wave; that means, the fluid which is flowing in this direction; that means, in the upstream fluid has in a higher velocity, and a lower pressure, and density where the fluid at downstream after the shock wave a sorry after the pressure wave sorry after the pressure wave its velocity is reduced, and pressure, and density is changed now if we consider this pressure wave of infinite small thin; that means, when we have described this control volume this control volume is very small. So, that the frictional effects are neglected, and at the same time, if we consider these changes to be very fast.

So, that heat transfer across this shock wave; that means, for this change; that means, if we consider this control volume we can consider the control volume using adiabatic condition; that means, the heat transfer does not take place in the short time, then we can tell the changes that occur due to the flow of the fluid through this control volume or across the pressure wave to be adiabatic, and also reversible this is, because when it is adiabatic there is no irreversibility due to heat transfer heat transfer is zero usually the irreversibility takes place due to heat transfer across a finite temperature difference which is absent there, and moreover if we consider this control volume to be very small considering this pressure wave to be very thin the dissipative effects due to friction can be considered as negligible.

So, therefore, we can considered the changes to be both adiabatic, and reversible the consequence of which together is the constant entropy; that means, isentropic; that means, the changes are isentropic. So, therefore, we can write the speed of this as a change in pressure with respect to the change in density we can write we should write it in partial differential nomenclature, because one of the state variable is constant; that means, that constant entropy. So, this should be the correct expression for the speed of sound or the speed of propagation of the disturbance or pressure wave with respect to the velocity of the medium.

So, now this value we can evaluate for a particular system, which if we take as a perfect gas now what is the definition of a perfect gas a perfect gas as you know is defined as from microscopic point of view as a gas or as a system where the intermolecular forces are assumed to be zero there is no intermolecular forces, and molecules are moving in a rectilinear path, but from the classical thermodynamics point of view we define the perfect gas are those gases whose equation of state bears a functional relationship like this p v is equal to r t the equation of state means the equation of state of any substance or any system relates this three variable in terms of a functional relation.

So, perfect gases are those systems which behaves whose equation of state is given by p v is equal to r t, where r is the characteristic gas constant characteristic characteristic gas constant which is a constant for a particular perfect gas, and it varies from gas to gas where p is the pressure v is the specific volume, and t is the temperature in absolute thermodynamic temperature scale. So, p v is equal to r t or we can write p is equal to rho r t this is the equation of state for a perfect gas.

So, if you consider this equation of state it can be proved or you have seen it earlier from other relationships for perfect gas that for an isentropic change of a perfect gas; that means, for a change with entropy constant the pressure, and density can be equated or related like this p by rho to the power gamma is equal to constant where gamma is the ratio of specific heats ratio of specific heats well ratio of specific heats; that means, it is the ratio of c p that specific heat at constant pressure divided by specific heat at constant volume we know this relation that p by rho to the power gamma is constant for an isentropic change or for an isentropic process executed by a perfect gas whose equation of state is given by p v is equal to r t or p is equal to rho r t.

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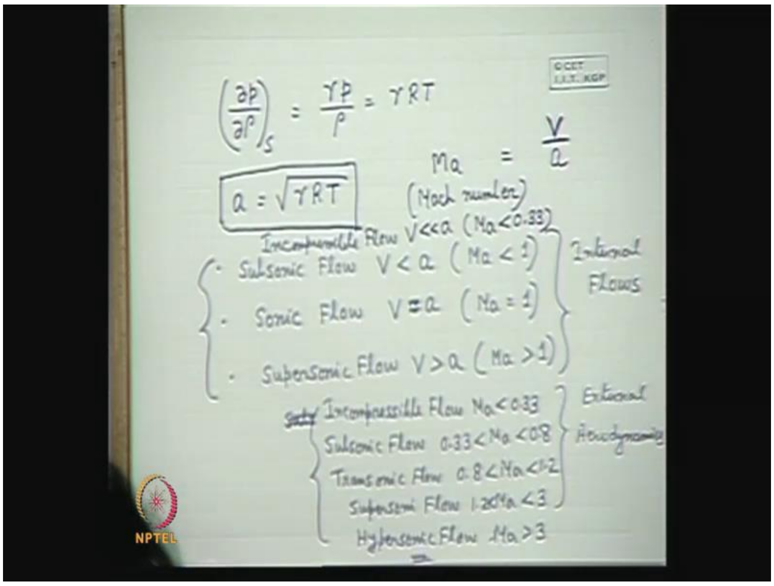

So, with the help of these two equations one can derive that del p by del rho the value of this derivative partial derivative at constant s comes to be gamma p by or gamma r t. So, therefore, it is a very simplified expression that a ultimately can be written as gamma r t or simply gamma p by rho. So, this is the final expression for the speed of sound relative to the velocity of the flowing medium as is equal to root over gamma r t it is directly proportional to the square root of the absolute temperature.

Now, after defining this speed of sound we define mainly three categories of flow one is subsonic flow subsonic flow subsonic flow is that where the flow velocity is less than the velocity of speed in that medium with respect to the flow medium that is a we have already recognized a dimensionless number known as mach number it was named after the scientist mach that is mach number which is equal to the ratio of the flow velocity to the acoustic velocity or speed of sound or velocity of sound in that flow or in that fluid at that particular condition. So, we can write in terms of the non dimensional number mach number. So, mach number less than one the flow is known as subsonic flow.

Then another condition is the sonic flow sonic flow where the flow velocity sorry is equal to exactly equal to the acoustic speed m a is equal to one, and another category of flow is supersonic flow supersonic flow where v is greater than a that is the flow velocity is more than the acoustic velocity, and m a is greater than one. So, it is not only the mathematical demarcations you will see afterwards there is a great change in the physical say physical processes of the system, and in the behavior of the hydrodynamic parameters in three's different regimes of flow.

Mainly the fluids are the the flows are divided into these three regimes for internal flows for all internal flows; that means, flow through a duct, but for external aerodynamics for external flows or external aerodynamics for external you write aerodynamics aerodynamics means dynamics of compressible flows usually air is taken. So, aerodynamics which means for external flows of compressible fluid more stringent definitions or divisions of flow are given from these three distinctions.

One is the subsonic flow one is the one is the incompressible flow first of all you here write one is this is always there incompressible flow one is incompressible flow when v very very less than equal to a or in terms of mach number it is equal to less than point three three now for external aerodynamics flow the incompressible flow remains as it is incompressible flow incompressible flow where mach number is now I am writing only in terms of mach number less than point three three there we call a flow as a subsonic flow subsonic flow when the mach number remain within this regime regime of mach number is point point three three to point eight in this range of mach number the flow density changes appreciably, but what happens is that no shock wave appears no shock wave appears no shock wave appears here, and flow becomes almost subsonic always subsonic almost no always subsonic.

But there is a region where the flow is told to be transonic transonic flow, when the mach number is within the region of one point two all these divisions have been made corresponding to certain classes of flow depending upon the certain physical differences in the physical changes or change in the trains of hydrodynamic parameters with the part in, and input variables. So, the divisions have come like that. So, it has been found for external aerodynamics within these range of mach number the flow behaves in a very mixed; that means, there is a mixed region of locally subsonic, and supersonic velocities supersonic flow, and the shock wave appears here what is that shock wave I will tell afterwards.

So, shock wave you can consider that is a wave which creates a sharp discontinuity in the flow field there is a severe discontinuity in the direction of the streamlines, and in other hydrodynamic properties of the fluid. So, this is the region where the flow is known as transonic flow another region is the supersonic flow supersonic flow where the mach number lies between one point less three here what happens the oblique shock waves takes place, and the density pressure temperature all varies sharply, and flow is totally supersonic; that means, the flow velocity is always more than the velocity of the sound another region is hypersonic which is definitely a supersonic flow, but mach number is greater than three, and in this regime of flow density pressure temperature all changes exposit

So, these are basically the different regimes of flow based on the relative values of the flow velocity with the relative values of flow velocity from the acoustic velocity or the velocity of sound in that flowing medium; that means, depending upon the relative value of the non dimensional or dimensionless number mach number we can divide the flow regimes.

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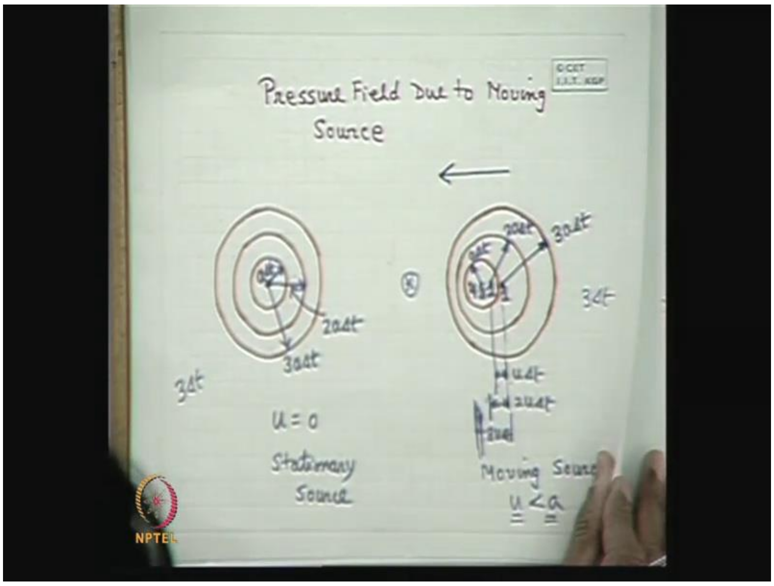

So, after this we will see how a disturbance created is how a pressure field created by a disturbance moves through a compressible medium, first of all you consider though it is written pressure field due to moving force first of all let us consider the pressure field due to a stationary source.

Let us consider a stationary source here which propagates pressure fields at the different times we have considered a time interval of three delta t after which we see how the pressure field moves, and what are the special locations. So, you see its moves in a spherical way; that means, the spherical field of propagation is generated; that means, a pressure wave which was sent here at the initial time t is equal to zero when we start our observation after a time of three delta t it has reached a point which can be found out by a circle or a sphere this is sphere two dimensional plane it shows that is circle it sees it is shown as a circle whose radius is three a delta t.

Similarly, which was emanated at the time of after delta t from the observation it has come here two a delta t, then again at after two delta t this is at d delta t; that means, this is propagating spherically like that when the source is at rest now if we consider when the source is moving; that means, a source emanating disturbances pressure disturbances also move in some direction let the direction of the movement in this way this is the direction of the motion of the source, then what happens if the moving source is such that its velocity is less than the acoustic velocity that is the velocity of the disturbance wave or the pressure wave that is emanating from the source, then what will happen this disturbance wave which is generated, and advance spherically these are always ahead of this source. So, source is at one. So, when after delta t it moves at two it emanates another wave at three it emanates another wave.

So, you see this way these moves from one to four this is the four. So, this is wrong this is the four. So, it moves from one to two this is u delta t its movement it is one to two two delta t it is one to three this point three delta t. So, therefore, you see one to two two one to three is two u delta t one to four is three u delta t when it moves this delta t, then it emanates a wave, then its two to three another disturbances. So, when it comes at four after a time interval of three delta t the pressure wave which was emanated from one it reaches three a delta t this is the outer sphere similarly when it comes at two when it came at two after delta t time, then the pressure wave which was given or which was coming which came from this point source at two this has come here; that means, two a delta t sphere with this as center.

Similarly, the pressure wave emanated from the point three at two delta t time when it came from one to two this has come here with a radius of a delta t when the source has come here. So, the movement of the source are encompass by the moving that the disturbance wave from. So, it cannot go ahead of that all right. So, therefore, one can say that an observer here will in this direction downstream for that source will receive the disturbance before he receives the source. So, he will receive the disturbance first.

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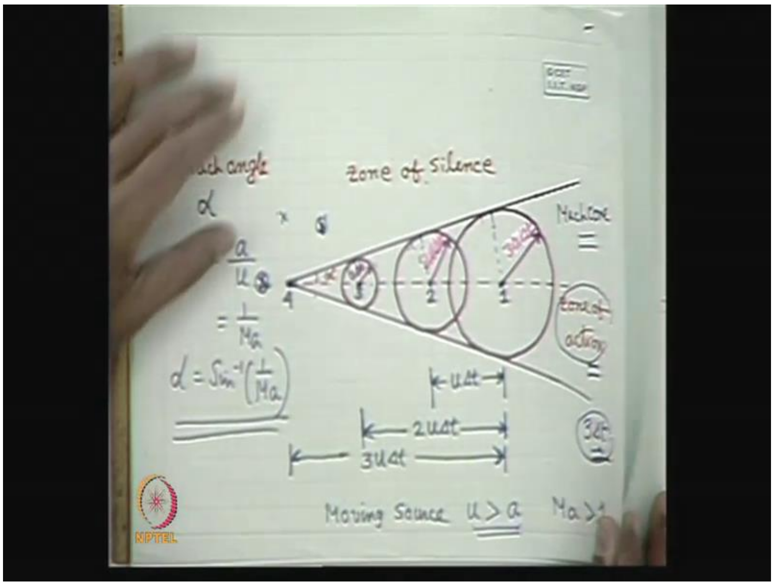

Now, here you see that what happens when the situation is that the moving source velocity is greater than the velocity of sound that is u is greater than a or mach number is greater than one in this case situation is like that let this is at one initially now at a time after a time t it has come to the point two which is u delta t after another delta t time it has come to a point three which is two u delta t, and u is greater than a, and after another delta t time it has come to point four we are always seeing the picture after a time interval of three delta t now you see the pressure wave which was emanated at the point one this has reached after three delta t like this the special location which is this has been covered by this spherical zone. So, which radius is three a delta t.

Similarly, the wave which was emanated from two has encompassed this zone two a delta t similarly at three it is a delta t, and here always you see that when it has move from one two three four. So, after a time of three delta t when it has just reached here just reached here just after three delta t all the disturbances which it emanated continuously discretized way, and we have seen at one two three these are like this. So, the point source has come ahead of the disturbance wave. So, at from point four we can draw a common tangent to all these spheres, and this angle alpha is known as the mach angle alpha the half of this angle, and this can be written as sine alpha you can see from here if you draw a perpendicular this is same for all that is the principle for which we can draw a common tangent that these becomes this divided by this this will be three a delta t, and this will be three u delta t or here this will be a delta t or u delta t; that means, sine of this angle will be a by u a by u which is equal to one by m a tan no sine perpendicular by hypotenuse this is the perpendicular this is the tangent this is the perpendicular. So, alpha the mach angle is given by sine inverse one by m a.

So, one interesting feature here is that you see at any point if a person stands here at any not at three delta t time he will not be aware of the disturbances created by this source; that means, the disturbance field will be within this mach cone. So, therefore, it is known as zone of action, and this cone is known as mach cone mach cone is the cone found like this with this point as the vertex at any instant the special location of the moving source as the vertex if we draw the common tangent this cone is known as mach cone, and this zone within the mach cone is known as zone of action, and zone of silence.

So, a observer will only have a feeling or will be aware of the disturbances created by the moving source when this point or the observer will be engulfed by the mach cone. So, this will depend upon the time, then it will move in this direction, and the disturbances will grow on this disturbance will be like this it will go for a delta t. So, that the mach cone will be expanded. So, until, and unless the point or an observer is taken within the mach cone the zone of action he will not be aware of that here if person or an observer standing here he will see the man first or he will see he will receive the moving source first before the disturbances are reached there that is the reason for which sometimes we see a supersonic moving object will seen first after we can receive first after this sound comes to this point that is the disturbance. So, disturbance field reaches this point ok.

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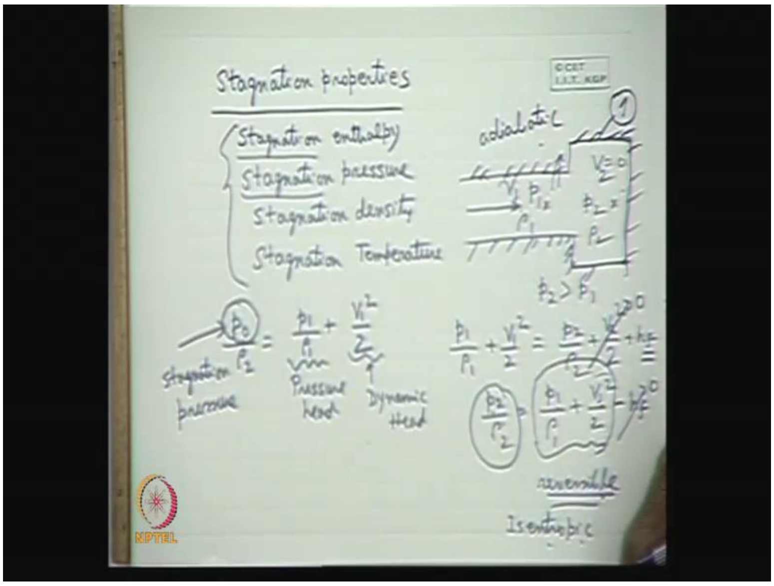

Next I will discuss the stagnation properties next I will discuss the stagnation properties what is what are stagnation properties it is very very important stagnation properties now we have already heard this word stagnation in your basic fluid mechanics classes what are first of all known as stagnation properties what is that stagnation pressure first is stagnation enthalpy stagnation enthalpy all stagnation properties stagnation pressure stagnation pressure stagnation temperature stagnation density you write density all with an adjective stagnation stagnation temperature stagnation temperature; that means, all with the adjective stagnation we know the word enthalpy pressure density what are or what is meant by the word stagnation stagnation means it is at rest.

Now, how do you define this stagnation properties in a fluid flow for example, if a fluid flowing with a uniform velocity across a section v its pressure is p density the rho now you see if is fluid is brought to rest; that means, it is coming through a long duct, then it is allowed to come to a closed chamber, then the fluid will be at rest. So, let us its velocity will be zero let this is the v one this is the p one rho one final velocity is v two is zero. So, there will be p two there will be rho two definitely if we measure the p two by a gauge we will see p two will be greater than p one is very simple from simple physics we can tell, because this velocity will be converted into pressure, because of the fact that the fluid has brought to rest.

But it is true that all the kinetic energy from this velocity v one will not be converted into pressured energy or will not be converted into pressure this is, because of the friction fluid friction or viscosity of the fluid that some of this mechanical energy corresponding to this velocity v will be dissipated or converted into intermolecular energy which will raise the temperature; that means, if we write the bernoulli's equation for example, the bernoulli's equation considering a point here, and considering a point here when the fluid has come to rest, then we can write p one by rho one plus v one square by two is equal to p two by rho we know that bernoulli's equation can be written for a viscous fluid with a consideration as this which is the loss of energy; that means, which is not appearing in the form of mechanical energy which is lost that some part of the mechanical energy is converted into intermolecular energy.

So, in this case v two is zero. So, therefore, simply we can write p two by rho is p one by rho plus v one square by two plus minus h f, but this we can write provided we consider the entire duct with this closed end is adiabatic definitely from the general energy point of view there is no energy coming in from outside or going from inside to outside simply the bernoulli's equation will give us like this. So, if is start with this stagnation pressure . Now, therefore, we see the pressure which is being built up rho two rho one rho two. So, the pressured energy it is not the sum of the total energy; that means, the sum of the total mechanical energy is this if there could not be any loss the sum of the total mechanical energy could be same. So, we could have told the entire kinetic energy is now converted into pressured energy the difference is accounted for this, but there is a loss. So, this is less than the total energy.

But if we consider the process to be frictionless; that means, reversible reversible reversible that is frictionless; that means, if we consider the fluid to be in visit consider the fluid to be in visit, then reversible adiabatic process means isentropic process isentropic process; that means, if we consider the flow to be isentropic, then what happens h f is zero in that case the pressure p two if it is denoted at p zero this is known as the stagnation pressure p zero by rho two is equal to p one by rho one plus v one square by two, and this is the definition of the pressure stag; that means, physically you will define stagnation pressure corresponding to any pressure, and velocity in a flow field is the pressure which could be generated or would be generated if the fluid is brought to rest isentropically; that means, if we imagine the fluid is brought to rest isentropically which cannot be cant done, because isentropic process is an hypothetical process, then the pressure which would be generated is the stagnation pressure.

So, therefore, this is known as pressure head that is the pressured energy for unit weight in a flowing fluid this is known as dynamic head that is the kinetic energy dynamic head per unit weight in the flowing fluid. So, the dynamic head is also converted into the pressure head. So, the entire dynamic head is converted into pressure head provided the flow is adiabatic, and reversible there is no conversion of the kinetic energy into intermolecular energy, because the agent which converts this that is the friction that is the fluid viscosity is absent. So, therefore, in this case the pressure is known as. So, do not tell that a stagnation pressure is the pressure when the fluid is brought to rest when the fluid is flowing, and it is brought to rest in practice the pressure which will be generated not refers to a stagnation pressure though calorically or barmally fluid is made to be stagnant.

But pressure when the fluid is made to be stagnant in practice is not the stagnation pressure stagnation pressure by definition refers to a theoretical situation when fluid is brought to rest isentropically, then that is the maximum limit that a pressure we can get from the conservation of energy the entire kinetic energy be converted into pressured energy. So, therefore, this is the definition of the stagnation pressure

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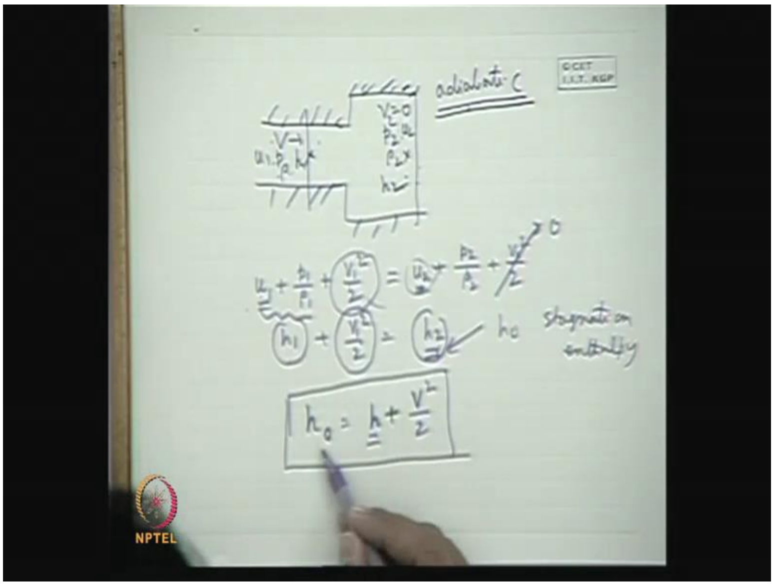

But the definition of stagnation enthalpy does not have a restriction of the for example, how this stagnation enthalpy will be fine does not have the restriction of this isentropic. Now, let the flow is adiabatic let the flow is adiabatic adiabatic what is leave you see that here the suction velocity is v pressure p density rho let the enthalpy is h zero velocity. So, v is equal to zero. So, let this is the p two v two is zero rho two, and let this is h two. So, now, we can write from the general equation that u one let this is u one that is u two u one plus p one by rho one plus v one square by two, if we neglect the changes in potential energy between these two sections one is at here another is at here, then we can write, and without any other energy interactions with the surroundings, because we have already considered adiabatic that is no heat interactions we consider the work interactions to zero, then u two plus p two by rho two plus v two square by two which is zero in this case.

So, this is h one is plus v one square by two is equal to h two now one beautiful thing is that. So, long it is adiabatic the enthalpy at this stage corresponds to the enthalpy plus the kinetic energy; that means, this kinetic energy part is completely converted into enthalpy whether friction is there or not difference is that if friction is there, then friction is there or not the difference is that that will make a distribution between u, and p by rho quantity you understand if the friction is not there, then u quantity will remain same for an I perfect gas, because you know for a perfect gas the internal energies are functions of temperature only there will be no rise in temperature, because of frictional dissipation from kinetic energy to intermolecular energy is not there. So, in turn it will be converted into pressure energy. So, how much will be converted into pressure energy, and how much will be converted to intermolecular energy from this part that is the business of the friction; that means, whether friction is present or not, but irrespective of this condition if the flow is adiabatic, then h one plus v one square by two is h two.

So, therefore, this is the stagnation enthalpy, and stag written as h zero stagnation. So, while defining this stagnation enthalpy we can tell corresponding to a particular corresponding to a particular condition of the flow at a particular location characterized by the flow velocity v pressure density enthalpy h we can tell this is the definition of stagnation enthalpy physically; that means, if the fluid is brought to rest adiabatically, then the enthalpy which would result from this is known as the stagnation enthalpy h zero.

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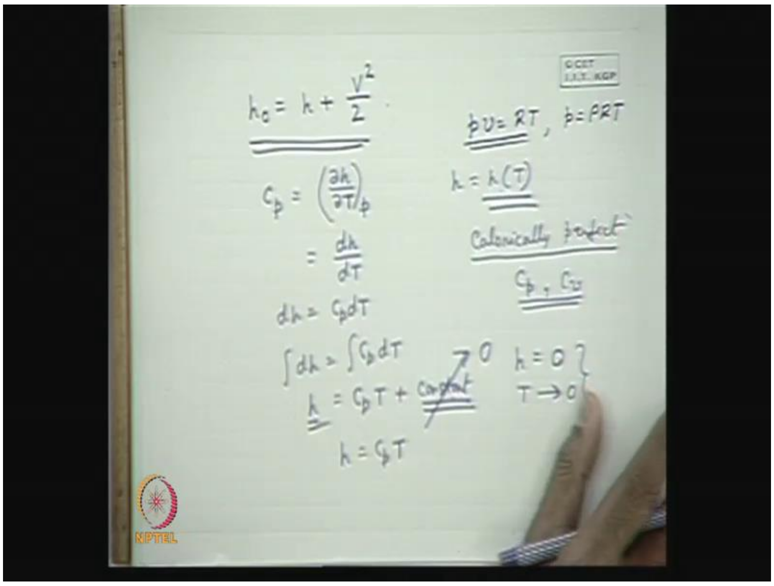

So, now we will derive certain important relation h zero is h plus v square by two now we know for an ideal gas what is the expression for h zero if you recall back in your to your thermodynamics you will see the specific heat at constant pressure is defined as del h del t at constant pressure del h del t at constant pressure we know for an perfect gas whose equation of state is given by p v is equal to r t or p is equal to rho r t enthalpy is a function of temperature.

So, therefore, we can write this instead of h we can write is d h d t, because the enthalpy is a function of temperature only. So, it is not a function of pressure. So, therefore, at constant pressure differentiation with temperature does not occur at all when we differentiate with temperature it is a ordinary differential total differential, because it is a function of temperature only pressure dependent is not there. So, one can write d h is c p d t another assumption is that if the specific heat at constant pressure is constant that is known as a calorically perfect gas calorically what is a calorically perfect gas that when ideal gas is calorically perfect which is not reacting, and whose specific heat at constant pressure, and specific heat at constant volumes are constants is not a function of any of the state variables these are known as calorically perfect gases.

So, for calorically perfect gases d h we can integrate upto this, this assumption is not required only the assumption was taken h as a function of temperature after that we can integrate it with this assumption of a calorically perfect gas where I take c p outside, and

I can tell that plus some constant arbitrary constant of integration this constant is usually not taken this is, because we are not interested in h we are interested in its change. So, therefore, this arbitrary constant does not come into the picture; however, one can also define that it is a reference datum that arbitrary constant is made forcefully zero if we considered the absolute enthalpy specific enthalpy or enthalpy whatever you call is zero when temperature approaches zero absolute temperature. So, this way also forcefully we can get rid of this.

So, simply that is why we can write h c p t we do not bother with the constant, because it will appear in the terms of the difference.

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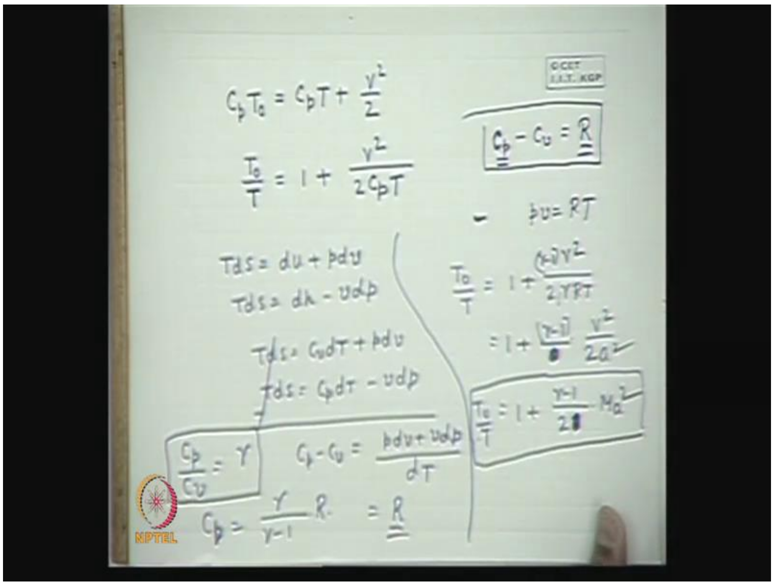

So, therefore, if we look this equation h zero is h plus v square by two we can simply write, then c p t zero for a perfect, and ideal calorically perfect gas c p t plus v square by two now we are interested in deriving the ratios between the stagnation properties, and local properties local properties means the value t which a fluid has at a particular point at a particular instant. So, this t zero by t therefore, comes out to be one plus v square by two c p one plus v square by two c p t zero by t one plus v square by.

C p t c p t.

C p t very good c p t, now what is c p we can find out that you know that the difference between the heat capacities or specific heat at constant pressure, and volume for an perfect gas is equal to its characteristic gas constant that you can find out from if you recall the here I can write the t d s equation which we developed last classes t d s is d u plus p d v general property relations, and t d s is equal to d h d h minus v d p now d u for a perfect gas we can write as c v d t plus p d v just recapitulating the old thing this we have already learnt at your school also c p d t minus v d p. So, if you subtract this, then you get this is zero, then you get c p minus c v is equal to what is that c p minus c v; that means, v d p plus p d v p d v plus v d p divided by d t.

And using this equation of state p v is equal to r t this can be expressed as c p minus c v is equal to r. So, therefore, if you use this, then we get t zero by t is equal to one plus v square by now c p minus c v is r, and if we c p by c v the ratio as gamma, then we can with the help of this equation, and with the help of this equation we can express c p is equal to gamma by gamma minus one into r. So, we write this two gamma by gamma minus one into r. So, v square that is gamma minus one two gamma r t c p gamma by gamma minus one r.

So, this can be written as one plus gamma minus one by gamma into v square what is this two gamma r t this is a square. So, one plus gamma minus one by two gamma into m a square. So, this is a very important relation that is the ratio of the stagnation temperature to the local temperature bears this ratio one plus gamma minus one by two gamma m a square all right.

Sir, one minute sir why gamma will not be there.

Gamma.

Sir, denominator will not be there gamma.

Gamma by gamma minus one no well it is gamma c p is gamma by gamma minus one this gamma will not be there. So, gamma minus one I am sorry, because gamma r t contains the gamma minus one by two m a square this is all right.

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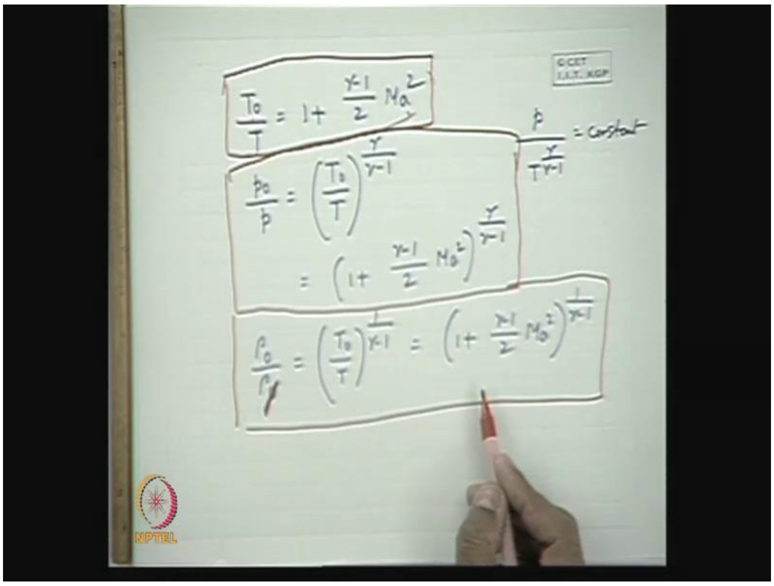

So, we get t zero by t as one plus gamma minus one by two m a square, then using the relationship that p zero by p for a perfect gas is t zero by t to the power gamma by gamma minus one that is the p t relationship p by t to the power gamma by gamma minus one is constant for a perfect gas.

So, we can write that p zero by p one plus gamma minus by two m a square to the power gamma by gamma minus one again using the relationship of rho versus t that t zero by t to the power one by gamma minus one we can write one plus gamma minus one by two m a square to the power one by gamma minus one. So, therefore, this is the relationship between the ratios of the stagnation pressure to local pressure stagnation temperature to local temperature, and stagnation density to local density t zero by t in terms of the mach number p zero by p, and rho zero by rho well when mach number equals to zero; that means, the fluid is brought to rest, then t zero by t one these are all derived considering the process to be isentropic. So, isentropicness is inherent to the definitions.

So, automatically t zero becomes t p zero become p, and rho zero sorry sorry rho zero becomes rho all right ok.

Yes sir.

Question thank you.

Sir what is sonic boom.

Sonic boom, yes sonic boom.