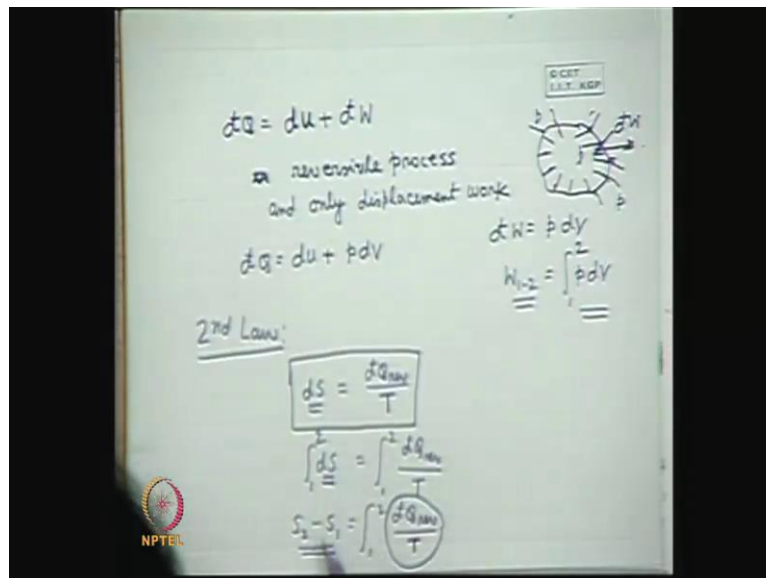


**Introduction to Fluid Machines, and Compressible Flow**  
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**Lecture - 28**  
**Thermodynamic Relations, and Speed of Sound**

Good afternoon, welcome to this session, last class we discussed the application of first law of thermodynamics to an open system or a flow process; earlier we discussed the application of first law to a close system. So, today we will be discussing first some important thermodynamic relations as an outcome of both first, and second laws.

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So, let me first tell you the first law. Now, if you see that first law applied to any process it can be written like that  $d q$  is equal to  $d u$  plus  $d w$ . So, we have already recognized this that the first law when applied to stationary system or a close system where internal energy is the only intermolecular energy  $u$ , then first law for infinite small process can be written as  $d q = d u + d w$  with a cut in  $d$  represents infinite small amount of it infinite small amount of work well. Now, for a special restriction of an irreversible process, and for an sorry for an reversible process reversible process, and only, and only displacement work displacement work only displacement work, and only displacement work we can write this equation as this  $d q = d u + p d v$ , where  $d w$  can be represented as  $p d v$ ; that means, the work done infinite small amount of work done can be written as  $p d v$ .

Or  $w$  for any finite process connecting its state points one, and two as you know basic thermodynamics can be written like that. So, until, and unless we know pressure as function of volume we cannot integrate this. So, for a reversible process, and only displacement work; that means, the work transfer between the system, and the surrounding take place to the displacement of the system boundary under reversible conditions; that means, always there will be an equilibrium of pressure; that means, this is the system. So, pressure exerted in the boundary will be same  $p$  uniform pressure.

And that has to be same with the pressure of the surrounding; that means, there will be always a pressure equilibrium that is the concept, and the process is infinite small quasi static process very long process very slow process, which is a reversible process where all dissipative effects are absent I am not going to tell you again about all the requirements for a reversible process. So, under the condition for a reversible process where only the work transfer is confined with the displacement of the system boundary system boundary either expands or collapses, then the type of work transfer takes place in either direction; that means, either work may come out or work may go under reversible condition these work transfer can be expressed as  $p \, dV$ .

So, therefore, under a reversible condition, and only displacement mode of displacement work mode is there for a closed stationary system. We can write, the first law as  $dq = du + p \, dV$  now, you see that before going further for thermodynamic relations. We must describe the second law what is second law of thermodynamics it is very important what is second law of thermodynamics can anybody tell a very simple manner what is second law of thermodynamics, while first law of thermodynamics is the quantitative conservation of energy what is second law which is another independent law of thermodynamics what is second law second law of thermodynamics puts.

Conversion of heat to work.

Yes conversion of heat to work this is true, but in general, because you see that second law of thermodynamic very broad discipline or very thing, because second law thermodynamics, if you ask a student of physics he will answer in a manner, if you ask in a student in chemistry student of chemistry will ask he will answer in a different manner if you ask a student in a material science he will ask in a different manner. If you ask student of a mechanical engineer he will interpret in a definite manner chemical engineer

metallurgical engineering, but the second law of thermodynamics as a whole is same its uses.

Or it can be look from different perspective. So, accordingly the definitions related to a specific field appears to be a different, but the basic second law is a law relating to the direction of processes, while the first law puts the constraint on the quantitative conservative of energy; that means, for any process dealing with the transformation of energy or transfer of energy energy quantity has to remain same. If mass to energy or energy mass conversion phenomena is kept as like this this is the simply first law of thermodynamic.

So, what are the types of energy with which you will be dealing the depends upon process to process mechanical engineer may be interested only with heat, and mechanical work chemical engineer may be interested with chemical energy with the heat energy, and other energy, but the total conservation has to be put total amount of energy will remains same. Similarly second law puts a restriction on the directions of a process that is why it is known as directional law; that means, all the process says or not do not occur in nature in all directions means there is there is there is a restriction for the direction of a process.

For an example I am telling second law is as simple as that the energy is flows from a high energy sorry the flux flows from high energy gradient to low energy gradient for example, heat flows from high temperature to low, temperature mass diffuses from high concentration to low concentration the fluid flows high energy to low energy. So, low energy to high energy the heat does not flow. So, this is also can be considered the second law of thermodynamic, which puts a restriction on the direction of the processes will see that there are certain processes which takes place in a particular direction.

Similarly, just you told that conversion if mechanical engineers are much more interested with the conversion of heat, and work continuous conversion of heat, and work, and it has been found that if you want to convert continuously heat in to work it is not possible to convert the equal amount of heat in to work. So, in conversions some amount of heat has to be always left as a residual; that means, heat cannot be completely converted into work continuously. While if you want to convert work continuously into heat you can do it; that means, one way you cannot convert the entire amount of heat into work

continuously in a cyclic process, because for a continuous conversion you require a cyclic process. While you can do it for work to heat conversion; that means, in a cyclic process you can continuously convert the equal amount of work in to heat.

So, this is a second this is also second law. So, second law gives us the restriction or puts an imposition or constraint impose on the direction of processes though the conversion of energy is valid conversion of energy may be, valid for both these directions, but one direction will not appear at all in nature; that means, for example, if heat if heat could have been converted inlet to work continuously the same amount there are no violation of conversion of energy, because nothing is destroyed nothing is created, but it cannot be done similarly some processes cannot be made to occur in certain directions nobody have seen in nature precisely this is the second law heat, flows from high temperature to low temperature. So, same amount of heat can flow low temperature to high temperature there is no violation of first law of thermodynamics, but this is directional law the heat always flows from high to low temperature.

So, these are the directions these are the outcome of natural processes which follows certain directions now, this can be put into more concise or compact in compact statement in the form of definition of an entropy; that means, it is very difficult to say that some, there are many processes which can occur in both the directions some processes may occur in both the directions for example, to heat body heating of a body the body can be cooled cooling is also possible some processes may not occur in reverse direction for example, if you convert work into heat adiabatic conversion of work into heat or intermolecular energy.

For example there is a moving body for example, there is fluid you start it; that means, you make the work transfer the fluid becomes hot; that means, the work is being converted into intermolecular energy will not tell heat better in sections a intermolecular energy insulated vessel heat is the energy in transit that appears, because of the temperature gradient later rather we will call will tell in a strict sense that work is being converted adiabatically intermolecular energy, but the reverse is not happening; that means, if you cool the liquid or we cool the fluid the work may not be come out.

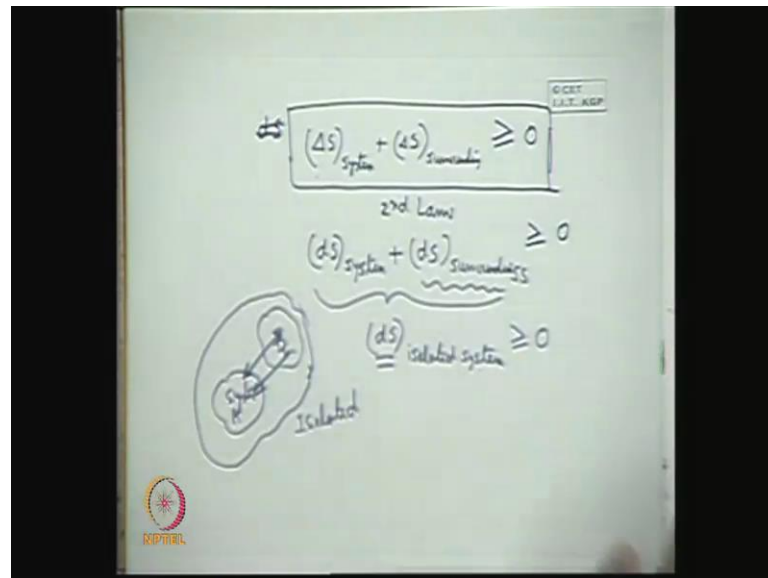
So, there are some processes that distinctly occurring one direction reverse is not possible understand, but there are processes which can occur in both direction. So, how

do you put a directional constant in those processes for example, you can heat a body you can cool a body you can compress a gas expand a gas; that means, you can make transfer to a close system in you can take work out from a close system. So, to make a general restriction or a constraint on the direction of all the processes it is made in a more compact, and console statement through the definition of entropy through the definition of entropy. So, entropy in a very limited sense of definition for our use we will define like that entropy is a property of a system.

And which is defined like that a change in entropy differential form is defined as  $dq_{\text{reversible}}/T$ ; that means, if for an infinite small process executed by a system in a reversible manner. If  $dq_{\text{reversible}}$  is the heat transfer in the process, then this divided by the temperature at which the heat was transferred represents the change in the entropy, this is the first line of definition of entropy in the classical thermodynamics with the through the definition of heat  $dq_{\text{reversible}}/T$ ; that means, if you want to define the change of entropy between two points, then we have to integrate it one by one  $\int_1^2 dq_{\text{reversible}}/T$ , and entropy being a point function can be written as  $s_2 - s_1$ .

So, we see that one interesting thing that, this heat transfer  $dq_{\text{reversible}}$  it is a path function it does not depend on point on state points, but if it is divided by  $T$  it becomes a exact differential which is equal to  $ds$ . Which can be integral from one to two. So, therefore,  $1/T$  acts an an integrating factor type of thing which is being multiplied with the  $q$  gives you the definition of a point function, which is precisely in the definition of entropy well. So, therefore, we get the definition of entropy like this, and the second notice the directional of law with.

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This definition of entropy that all process takes place, in such way that entropy change for example, if I write a finite entropy change entropy change of system. And surrounding together, and surrounding together algebraic sum algebraic sum should be greater than is equal to 0 the very important, principle delta s system plus surrounding; that means, whenever a system interacts with other systems the other system become surrounding what the definition of system, and surrounding system we define a certain mass certain quantity of mass within certain boundary the definition of system, and surrounding is everything is external to the system that is surrounding, but in proper sense we means surrounding those systems or those parts external to that system on which we have concentrating our attention with which the system is interactive that means.

If there are two systems interacting with each other, if one is system other is surrounding if there are four five systems a number of systems interacting with each other if we consider one as a system other with which particular system. Which we have identified interacting is unknown as surrounding. So, therefore, if we identify one system, and all other interacting system as surrounding is a arbitrary definition, then we tell any change delta s of this system, and delta s of the surrounding with which it is interacting; that means, algebraic some of the entropy change of all interacting system you understand that is known as delta a system plus delta s surrounding.

So, there is no restriction which one will be system which one will be surrounding for example, system a interacting system b. So, if system a is define system system b is the surrounding for system a similarly if you consider system b at the system system a will be surrounding a system b; that means,  $\Delta s_{\text{system}} + \Delta s_{\text{surroundings}}$  means the algebraic sum of the change always greater than or equal to 0. that is the directional law that is process should takes place; that means, a particular system will execute a process with interacting with another system, such that the algebraic sum of the entropy change between the two systems will be always greater than or equal to 0. it can never be less than 0 this is the pure directional constraint.

And the second law of thermo dynamics defined from the principle of entropy, and this statement is known as the principle of a increase of entropy; that means, the change of entropy algebraic sum of the change of entropy have all the interacting system as to be greater than or equal to zero; that means, if a body is heated; that means, another body is cooled wherefrom the heat is coming. So, therefore, when the body is heated if you call this as a system the body from, where it as taken the heat you call as the surrounding. So, if you calculate the entropy raise of the body which is heated, and at the same you calculate the entropy decrease of the body which has been cooled.

And algebraically at the raise of the entropy for this, and decrease of the entropy for this in this you will see the sum of two will either greater than or is equal to 0 in which case it will be 0 when the interactions will be reversible, but when the interaction will be irreversible all natural process. that are irreversible processes, then this will always greater than zero; that means, all natural interactions all natural processes will take place in such a way that the consequence of the process is a net change of entropy to have a non 0 value which is unknown as the production of entropy.

Because if the algebraic sum of the entropy change is greater than zero; that means, before this start of the process, if we consider a entropy reservoir some absolute value of entropy was there, and, because of the execution of any natural process the algebraic sum of the change of the entropy for all systems interacting with each other; that means, for system surrounding grater than zero; that means, it is adding to the entropy reservoir which means a production of entropy; that means, any natural process in the universe always increases the entropy the bank increases its storage that entropy is monotonically increasing function of time.

So, whenever there is a process entropy is always increasing, but for reversible process the limiting case its hypothetical case theoretical case a reversible process your quasi static process free from all dissipative effects, and for all potential lack of potential gradients the entropy change is 0 reversible process, and hypothetical process as you know from your thermodynamics. The basic definition of a reversible process is a process where the basic potential gradient which causes the process is zero; that means, a reversible is a heat transfer at 0 temperature gradient a heat transfer at 0 temperature gradient cannot occur you understand. So, therefore, all reversible process are process what the potential gradient is zero.

So, therefore, reversible process is not a process say its prelist static condition. So, therefore, a limiting case of reversible process is conceived impact is by creating infinite small potential gradient. So, that a very slow process will take place. So, for a finite amount of energy interactions its takes a long time that is why infinitely long process or a quasi quasi means almost that a quasi static process reversible process. So, if the interactions are reversible, then the entropy change algebraic sum of the entropy change of all the interacting system that is system plus surrounding will be 0 otherwise, for all natural process is reversible process which are finite processes, which takes finite time to complete a process these are all irreversible processes these are associated with dissipative effects, these are associated with the potential gradients causing the processor process.

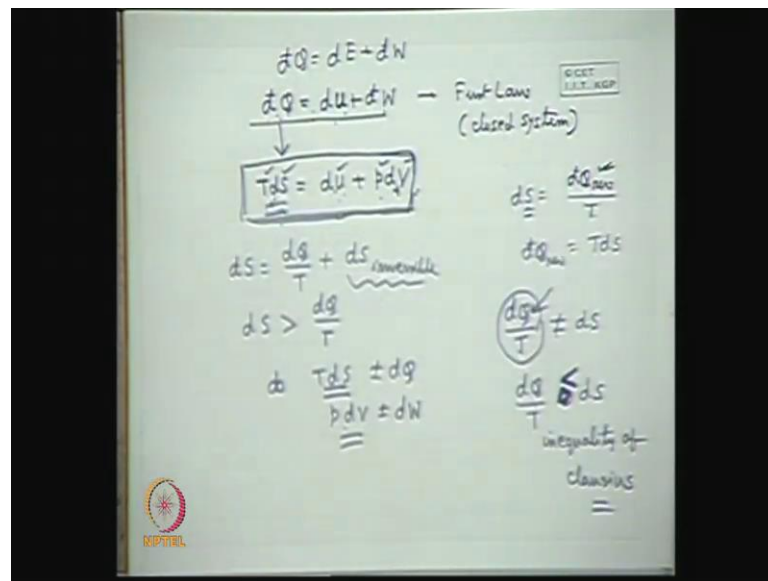
So, for them the entropy change of the interacting systems, the algebraic sum of the entropy change of the interacting systems will be greater than 0. Which increases the entropy. So, therefore, this is the basic second law second law which can be written that  $d s$  in differential form  $d s_{\text{system}} + d s_{\text{surrounding}}$  means if I identify one has the systems all other, interacting system is the surrounding is greater than zero, and sum of this two as a whole is known as  $d s_{\text{isolated}}$ ; that means, isolated system  $d s_{\text{isolated}}$  is greater than, or is equal to 0  $d s_{\text{isolated}}$  is greater than or is equal to 0 what is this isolated system; that means, a system, and the surroundings.

For example a system a interact with system b only system b, it is surrounding. So, system, and surrounding combinely make an isolated system isolated system is system as you know which is total isolated from other part of the surrounding; that means, the



interaction taking place between system a, and system b. So, no other body is coming into picture as far the interactions are considered. So, therefore, system a, and system b consider to be isolated system. So, therefore, system, and surroundings surrounding means all other interacting system comprises an isolated system. So, therefore, this is known as  $dS$  of an isolated system is greater than or is equal to 0 this is the second law of thermodynamics in brief I tell you.

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Now, with definition I tell you one very important thing, you must remember at the that again I start that  $dQ$  is equal to  $dU$  plus  $dW$  this is the first law this is the first law there is no doubt first law apply to a close system, close system if I wrote it  $dQ = dU + dW$  this is for any system for any system when I write this is a stationary your close system this is, because for a close system. We neglect the kinetic energy, and other potential energies only the intermolecular energy has the internal energy now, when we consider the interactions in a close system, as a reversible, and we consider the displacement work is the only form of work, then we can write in place of  $dQ$  is  $TdS$ , and this  $PdV$ .

Now, you see I take the help of a reversible process to modify the first law or rewrite the first law in the form that  $dQ$  is replaced by  $TdS$ , because you know, if we consider a reversible process we know the definition that change of entropy is defined like that for a reversible process  $dQ_{reverse}$ ; that means, the  $dQ$  that is the amount of heat transfer in a

reversible process is  $tds$ . So, therefore, we can write  $tds$  is  $du$ , and we know that if we consider only the displacement work as the only mode of work transfer by the close system, then under reversible condition this  $dw$  the work transfer is  $pVd$ .

Now, this equation if we look you see, that this contains all the property values. So, therefore, from the view point of mathematics. It is seen that this relationship should be valid for any process related to this properties, because the property entropy is valid for any process whether its reversible or irreversible, but its definition is made through the heat transfer of a reversible process for an example. Let me tell you that if there is a process which is irreversible, and there is a value of  $dq$  for a small process infinite small process where the value of a transfer is  $dq$ , and at a temperature  $t$  for example, then this  $dq$  by  $t$  is never equal to  $ds$  of the process,  $ds$  is equal to  $dq$  reversible by  $t$ ; that means, if the process could have been made in a reversible process manner, then the value of  $dq$  reversible process by  $t$  could have been  $ds$ .

So, in that case we know that  $dq$  by  $t$  is greater than  $ds$  what is this principle this is inequality of clausius. This is inequality of clausius  $dq$  by  $t$  is greater than  $ds$  usually this is a this is expressed like this  $ds$  is equal to for any process  $dq$  by  $t$  plus  $ds$  irreversible; that means, this is the component for irreversibility of the process which is 0 for a reversible process; that means, for a reversible process  $ds$  is always greater than the  $dq$  by  $t$ ; that means, for any process there must be a value  $dq$  by  $t$ , because  $dq$  is defined there is a heat transfer to a process you understand, but the  $dq$  by  $t$  is not the entropy change for that particular process. So, entropy change is defined by.

Sir  $dq$  by  $t$  less than  $ds$ .

$Dq$  by  $ds$  is greater than  $ds$ .

Sir here it has.

Ah sorry sorry I am extremely sorry  $dq$  by  $t$  less than  $ds$  I am sorry.

Sir otherwise  $ds$  is greater than  $dq$  by  $t$ .

Yes yes  $dq$  by  $t$  all right all right all right  $dq$  by  $t$  is less than  $ds$  or I am sorry this is a mistake  $dq$  by  $t$  is less than  $ds$  this is the inequality of clausius; that means,  $ds$  is more than  $dq$  by  $t$  clear now, it is clear. Now, it is very important concept that though here we

have taken the help of a reversible process to derive this from the first law, but this law is valid for any process. So, far as the property values are concerned which means that  $tds$  is equal to  $du + pdV$  is valid for any process, but difference is that for any process  $tds$  is not equal to  $dq$ , and  $pdV$  is not equal to  $dw$   $pdV = pdV + tds$  this  $tds$ ; that means, only for a reversible process this equation, and first law are cinnamons they are not independent equation.

But for all over process this, and first law of thermodynamics are cinnamons first law of thermodynamics is like this, but this is a property relations this means, that  $tds$  is not equal to  $dq$ , and  $pdV$  is not equal to  $dw$  this you must bear in your mind that  $tds = du + pdV$  is a property relation for all processes whereas,  $tds = dw + pdV$  is a first law of thermodynamics for only reversible processes, because  $tds$  can be substitute at by  $dq$ , and  $pdV$  can be substituted by  $dw$  which cannot be done for all other processes. So, therefore, this will remain as a property relations for any process.

But for a reversible process this, will be the first law of thermodynamics clear I think you have understood this, because here, lies the major confusions that though we derive  $tds = du + pdV$  from the first law with the help of reversible process executed by a close system with displacement mode of work, but when you derive this process this constant is dealing, then it acts as a property relationship for all processes; that means,  $tds$  of all process is equal to  $du + pdV$  have all processes, but whether  $pdV$  have all processes will be equal to  $dw$  or  $tds$  of all process will be  $dq$  that have to be a examine whether the process is reversible or irreversible here lies the confuse; that means, this equation property relations equation, and the first law of thermodynamic are cinnamons for reversible process for all irreversible process they are not cinnamons well.

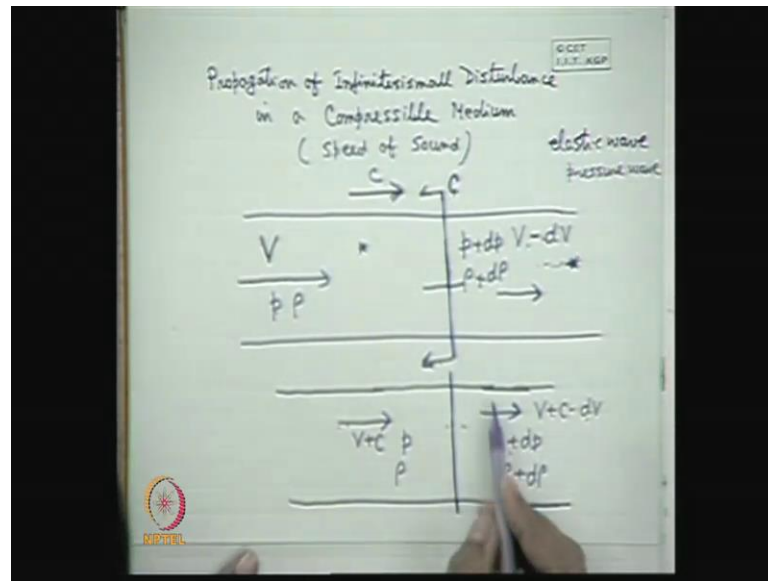
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The image shows a whiteboard with handwritten mathematical derivations. At the top, the equation  $Tds = du + pdv$  is boxed. Below it,  $H = u + pV$  is written, followed by  $dH = du + pdv + vdp$  and  $= Tds + vdp$ . The next line shows  $Tds = dH - vdp$  boxed. At the bottom, two more boxed equations are shown:  $Tds = du + pdv$  and  $Tds = dh - vdp$ . A small logo is visible in the bottom left corner of the whiteboard.

So, therefore, this is a very important property relations,  $t d s$  is  $d u$  plus  $p d V$  another important property relations comes from the definition, of  $h$  you know that  $h$  is  $u$  plus  $p V$  well. So, we can write  $d h$  as a simple differentiation  $d u$  plus  $p d V$  plus  $V d p$  well now, we know that  $d u$  plus  $p d V$  is  $t d s$ . So,  $t d s$  plus  $V d p$ . So, therefore, we can write  $t d s$  is equal to  $d h$  minus  $V d p$ .

So, these two are very important property relations we can write it in terms of intrinsic property; that means, specific value; that means,  $t$  small  $s$ ; that means, specific entropy  $d$  small  $u$  plus  $p$  small  $V$  similarly  $t d s$  is  $d h$  minus  $V d p$ ; that means, this will follow. So, these two equations are very important property relations always will use  $t d s$  is  $d u$  plus  $p d V$   $t d s$  is  $d h$  minus  $V d p$  all right. So, with this property relation this will come afterwards at several vocations in deducing many formulae for compressible flow.

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Now, we will start the propagation speed of sound in a compressible flow, propagation of infinitesimal disturbance in a compressible medium in a compressible medium that is speed of sound why the speed of sound comes I will tell you. Let us consider a duct where, there is a flow of a compressible fluid with any velocity  $V$  now, at any point here on the ground stream. Now, if you create an infinitesimal disturbance here, then what will happen this disturbance will propagate through this medium, now, usually what happens if it is rigid body an infinitesimal disturbance. If you create here the disturbance will be immediately sent at the point on the upstream; that means, all the points in the medium will be simultaneously picking up this displacement.

Or this disturbance if this disturbance causes a displacement of a particle adjacent to this point it will instantaneously propagate the disturbance to all other particles, this happens also in an incompressible fluid or a liquid. So, that it is almost, then instantaneously process; that means, particle is displaced here this displacement will be sent instantaneously almost instantaneously to all points; that means, if you consider in the direction of the upstream it will be propagated instantaneously, but if the medium is compressible whose coefficient of compressibility is very high it does not happen.

So; that means, if you create a small disturbance what will it do it will displace the immediate adjacent points, and will increase its density reduce volume, and it will do in

tern the next neighboring points, and that next neighboring points, and this way this disturbance will propagate during a finite time within a finite time in the upstream direction; that means, it will not be sensed instantaneously that any point in the off stream. So, this will be gradually propagating through the displacement, and increasing the density of at adjacent points consecutive neighboring points in a case of a compressible medium.

So, therefore, we see that if we create a disturbance this disturbance ultimately what happens it propagates in the form of a pressure wave in the form of a wave form an elastic wave; that means, elastic wave as you moves which changes the condition here after it moves through certain distance in the upstream direction, then slowly slowly it propagates in the upstream direction, and changes the condition. So, it moves in the form of an elastic wave or pressure wave elastic wave or pressure wave the sound energy exactly moves through a compressible medium in by the same mechanism. So, that we considered as the propagation of the sound; that means, you create a sound energy here.

Or which can be thought of an infinitesimal disturbance, how does it propagates. Let us consider at any instant the wave this elastic wave or a pressure wave as come to this point with a velocity  $c$  let us consider the velocity that is the pressure wave or the elastic wave is moving with a velocity  $c$  in the upstream direction let us consider the velocity of the compressible fluid or the fluid, which flowing with a velocity be its pressure is  $p$ , and its density is  $\rho$ . Now, when the pressure or elastic wave as generated in they are come through this part of the fluid, it has some imparted some velocity to the fluid in the directions of its motion. How can you conceit it the fluid could have been stagnate then.

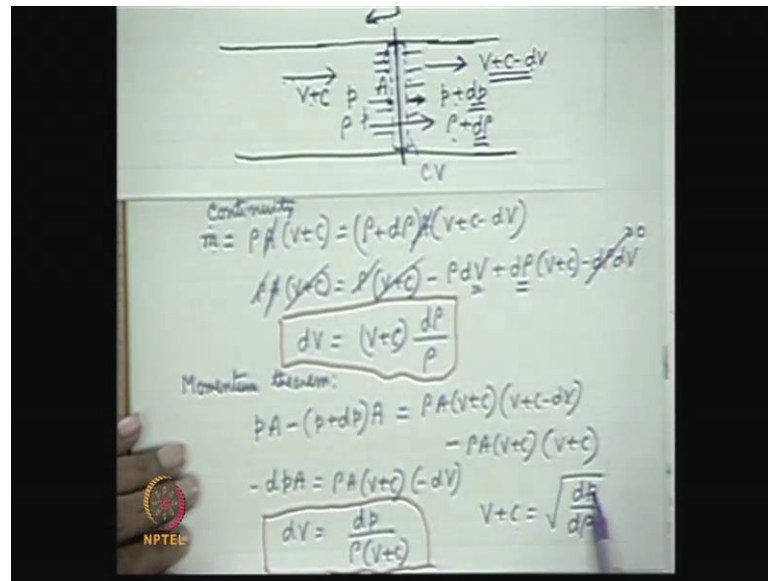
If a pressure plus is generated here or a disturbance is generated here, then the disturbance while, it is moving with some velocity  $c$  after it passes through this portion it has it will create by the action of the impulse some small velocity in the direction in this direction to the fluid element. So, therefore, it is in this case of a stagnate fluid. So, for a flowing medium we can consider the velocity of the fluid may be in this direction if this velocity is very high, then this will be this velocity will be very small, which will be generated due to the passing of this pressure wave or elastic wave. So, therefore,, then velocity will be reduced by an amount  $c$  minus  $d v$ , and let, because of this the pressure is changed from  $p$  plus  $d p$ .

Lets the density changes from  $\rho + d\rho$  well. Now, you see how to analyze this situation now, this situation is an unsteady situation with respect to the flow of the fluid why, because that any point with time the things are not steady, because depends upon is arrival for example, at this instant when the pressure wave or elastic wave is at this position the velocity is  $v$ , and  $\rho$  is undisturbed, but when it will cross this position, then the pressure density will change velocity will change. So, therefore, this is a situation where the flow is turned to be appears to be unsteady.

So, therefore, if we consider a situation where we give an opposite direction velocity; that means, we choose a frame of reference, where this elastic wave appears to be steady the situation will be, and steady flow situation. Let us make it here itself you can understand; that means, lets consider a situation where we consider these to be steady elastic pressure wave to be steady; that means, this frame of reference; that means, we fix the coordinate axis with this pressure wave with respect to that coordinates axis the pressure wave remains to be stationary that is means to this situation, that we add a velocity  $c$  in this direction; that means, with respect to this frame of reference the wave will appears to be stationary while the flow will be in this direction with a velocity  $V$  plus  $p$ , and  $\rho$  density, and when it comes here its velocity is used  $V$  plus  $c$  minus  $d v$ , and pressure will be  $p$  plus  $d p$ , and  $\rho$  plus  $d \rho$ .

So, you see with respect to a stationary frame of reference this wave pressure act as a compression wave why the velocity the fluid is coming with an uniform steady velocity approach, this stationary pressure wave, and across this wave that when it passes through this wave its velocity is reduced, and pressure, and intensity is increased now, this is a steady situation, and we can analyze this problem. We can analyze this problem with the help of conservation of mass, and momentum what we do we consider a control volume circumscribing this special wave.

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Let this is a control volume Now, if I write the equation of conservation of mass, then we can write the mass flow rate  $m$  dot at this section plate the cross sectional area of the duct or the pressure wave they are same  $a$ , then we can write  $\rho$  area into the velocity  $V$  plus  $c$ , and under steady situation this will be same; that means, it cross a wave, but there is no generation or no depletion of mass within this control volume. So, the control volume represents the steady state control. So, mass flow in is equal to mass flow out. So, we can write this with respect to this situation, where we consider a change in density as  $\rho$  plus  $d\rho$ , and a sorry a  $V$  plus  $c$  minus  $dV$  well.

So, now, if we equate this two we get  $\rho a (V+c)$  is equal to  $(\rho+d\rho) A (V+c-dv)$ . So,  $\rho a (V+c)$  is equal to  $\rho A (V+c) - \rho dV + d\rho A (V+c) - d\rho dV$ . Now, we consider this wave to be infinitesimal, and the strength is also very small. So, that this  $d\rho$ , and  $dV$  are very small in a way that their product can be neglected as compared to the value of  $d\rho$ , and  $dV$ . So, this second order terms are neglected now,  $\rho A (V+c)$   $\rho A (V+c)$  cancels. So, this minus this is 0 from which we can write is equal to  $\rho A (V+c) (-dV)$  by  $\rho A (V+c)$  well, now, if we apply the momentum theorem this is the continuity that is conservation of mass continuity.

Now, if we write the momentum theorem momentum theorem, if we write the momentum theorem the what do you get the momentum theorem applied to the control volume at steady state. You know the momentum theorem at steady state tells that is the



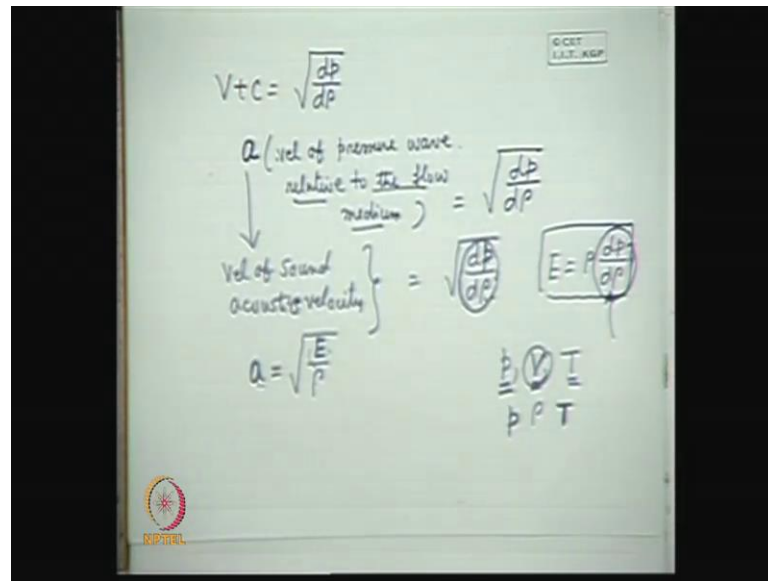
net force acting in any direction. Let us example, we consider this direction the direction of flow of the fluid is equal to the net rate of momentum a flux from the control volume in that direction; that means, a flux minus in flux since there is no generation that is there is no change in the momentum within the control volume. So, if you apply the momentum theorem, let us write the net force acting in this direction in the control volume.

So, net force are the force rates which is due to the pressure  $p$ , and  $p$  plus  $d p$  we discard the frictional forces this is, because that this pressure wave is very thin. So, the control volume circumscribing this pressure wave enveloping this pressure wave will be very thin. So, that this areas are very small. So, the surface area. So, small the frictional forces are negligible compare to pressure forces. So, therefore, the forces in this direction direction of flow will be  $p$  into  $a$  minus this side  $p$  plus  $d p$   $p$  plus  $d p$  into  $a$ , and that is the net force acting in this direction of flow all right that must be equal to the net rate of momentum a flux in this direction.

What is momentum a flux in this direction that is the mass flux  $\rho a V$  plus  $c$  times the flux velocity that is  $V$  plus  $c$  minus  $d v$ ; that means, the fluid is coming out from the control volume with this velocity plus  $c$  minus  $d V$  what is the influx of momentum which has to be subtracted that is the mass remains the same, because this, and these are same, and this is multiplied with this inflow velocity to the control volume that  $V$  plus  $c$ . So, if you calculate it you will get  $d p$  into  $a$  minus  $d p$  into  $a$ . So,  $\rho a V$  plus  $c$ . So,  $V$  plus  $c$   $d V$  minus  $V$  plus  $c$ ; that means, simple minus  $d v$ ; that means, we can write  $d V$  is equal to minus minus cancels.

So,  $d V$  is equal to what is that  $d p$  by  $\rho$  into  $V$  plus  $c$ . So, now, we equate  $d V$  from this two  $d V$  from these two this is the final step for continuity equation, and this is the expression of  $d V$  as the finals step from the momentum equation, if we equate this two, then you will get  $d V$  if we equate  $d v$ ; that means,  $d V$  is eliminated you will get  $V$  plus  $c$  is equal to root over  $d p$  by  $d \rho$ . You see that root over  $d p$  by  $d \rho$ . So,  $V$  plus  $c$  we get as root over  $d p$  by  $d \rho$  that is increase in pressure divided by increase in density change of pressure with respect to density  $d p$   $d \rho$  all right.

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So, therefore, we see that we can write  $V$  plus  $c$  as  $\sqrt{dp/d\rho}$ . What is  $V$  plus  $c$ ? What is  $V$  plus  $c$ ?  $V$  was this speed of the elastic wave of pressure wave when  $V$  was the velocity of the fluid in this direction, and this is moving in the off stream direction, because it was created this pressure pulse or the disturbance was created at the downstream. So, it is moving in the upstream direction. So,  $V$  plus  $c$  is the relative velocity; that means, it is the you write it as  $a$  which is the velocity of pressure pulse pressure wave relative to the relative to the fluid medium relative to the flow medium relative to the flow medium; that means, flow medium as a velocity is equal to  $\sqrt{dp/d\rho}$  let if it is a stagnation flow.

So, the velocity of the pressure pulse in the direction; that means, there is no question of asking upstream downstream it is a stagnate; that means, we are from it was originated from there it flow in this direction with a velocity  $\sqrt{dp/d\rho}$ . So, in case of a flowing medium we can tell that the velocity will represents by  $a$  now, the velocity of pressure wave will. Let to the flow medium is  $\sqrt{dp/d\rho}$  since the sound energy moves in a compressible medium in the similar fashion this is the speed or velocity of sound velocity of sound or acoustic velocity.

So, therefore, we see the acoustic sorry acoustic velocity in a compressible medium relative to the flow, medium relative to the flow medium that is relative to the velocity of that medium is giving by  $\sqrt{dp/d\rho}$  as you know that definition of  $e$  that

elasticity of modulus, what is the definition of  $e$  is equal to  $\rho \frac{d p}{d \rho}$  we have seen it earlier. So, therefore, we can replace it, and we can write, root over  $e$  by  $\rho$ . Now, it is clear that all medium are compressible medium, but liquids are incompressible; that means, whose change of pressure with respect to density is very high; that means, the value of elasticity is very high; that means, it requires a very high pressure for change in density well.

So, therefore, this speed sound in a medium, which is more or less incompressible its very high; that means, the speed of sound is related to the modulus of elasticity in this way that if modulus of elasticity is more; that means, fluid is more in less compressible, then the speed of sound is very high  $e$  by  $\rho$ . Now, next is this can be represented in a different that Now, you see this  $\frac{d p}{d \rho}$  that is the change of pressure with the change of density the value of this depends up on how you change make the change of pressure, because we know that the change of pressure can be made in with various process constants.

For example an isothermal change of pressure, may be made where the volume may change, because out of  $p, V, t$ . If you think this three properties we can keep  $t$  constant, and we can vary  $p$ , and  $v$ ; that means, you can change pressure, and we can see the variation in the  $V$  for example,  $p, \rho, V, t$ . Similarly other process constants kept fix where the temperature may be allowed to vary along with the variation of  $p$ . So, therefore, the values of  $\frac{d p}{d \rho}$  the exact value of  $\frac{d p}{d \rho}$  can be found out provided we put a process constant until, and unless, it is very difficult to be described even if we define in terms of the bulk modulus of elasticity the bulk modulus elasticity from its basic definition cannot be evaluated until.

And unless we put a process constant to evaluate the  $\frac{d p}{d \rho}$ , because it is very difficult to know the  $\frac{d p}{d \rho}$ , because  $p$  can be vary to change the  $\rho$  under various process constant we can keep the temperature constant, we cannot may not keep the temperature constant, and if you do not keep the temperature constant we may keep some other things some other restrictions. There may be a number of restrictions or temperature may vary. So, specifically will have to know the restrictions, and will have to know the explicit relationship between pressure density, and temperature which is known as the equation of state for that medium or for that substance or for that system

only when we can find out or evaluate the value of  $\frac{dp}{d\rho}$  or as derive in the value of bulk modulus of the elasticity do not think either.

It is  $\sqrt{\frac{e}{\rho}}$  or  $\sqrt{\frac{dp}{d\rho}}$  is value is known until, and unless we put the process constant. So, therefore, we have to find out from our physical understanding what should be constraint process constraint or what situations, the pressure pulse is propagating. So, that we can put from there a process constant to evaluate the value of  $\frac{dp}{d\rho}$  or the value of  $p$ . So, that we can finally, find out the numerical value of the speed of the sound in a compressible flow, that I will discuss in the next class well please any question this you have to understand there is a lot of concept rather than mathematics well.

Thank you.