**Introduction to Fluid Machines, and Compressible Flow**

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**Lecture - 27**
**Introduction to Compressible Flow Part – II**

Good morning I welcome you all to this session last class, we have discussed the first law of thermodynamics we started discussing the first law of thermodynamics, and then we define a property enthalpy h which is the combination of intermolecular energy a part of the internal energy or in a stationery or closed system. That is the internal energy plus the product of pressure, and volume, and we also recognize that this internal this enthalpy is a property of this system bears is physical significance relating to energy transfer an open system or a flow process. So, today therefore, we will apply this first law for a flow system or a flow process or an open system, and we will see how does enthalpy bear, a physical significance with the energy transfer across the boundary of such an open system.

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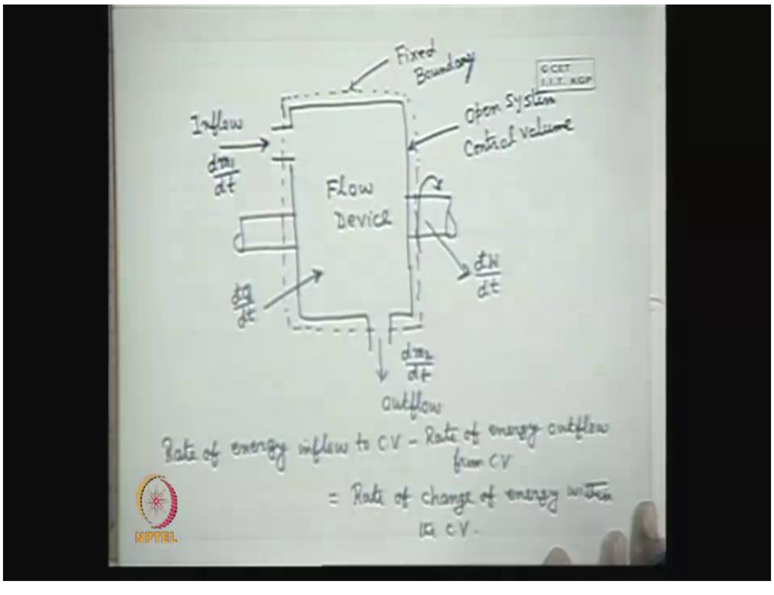


So, for that what we do? We just consider a flow system, like this let us let us consider a device a flow device this is known as a flow device flow device well which has an inflow where there is continuously an inflow of fluid, and continuously an outflow of fluid inflow, and outflow of fluid, and this flow device is connected with a shaft. So, that a shaft work is coming out in the form of the rotation of the shaft let us consider an infinite small amount of work d w d cut, w as I explained earlier is coming over a time d t s that we can represent that d w d t as the rate at which the work is coming out from a flow device in the form of shaft work.

And also let us consider, similar way infinite small amount of it d cut q is added continuously over an interval of time d t. So, that we can represent that d q d t as a continuous rate of heat flowing or heat transfer into this flow device. Now this flow device where there is a continuous flow of fluid at 1 section, and continuous flow of fluid out in other section it is flow in, and flow out, and developing both work, and heat; that means, transferring in terms of work, and heat in the surrounding is termed as an open system or a controlled volume. In fact, we consider a close a boundary circumscribing this flow device this way if we consider a boundary this way.

This this system within this fixed boundary is known as an open system or a control volume or a control volume which characterized by this fixed boundary as I have already told you that a an open system is a control volume system or simply a control volume, where the boundary where the system or the control volume is characterized by the fixed boundary, and across which the mass transfer takes place there is continuous inflow of fluid, and there is continuous outflow of fluid now, if we want to make the conservation of energy statement for this open system or control volume. It is very simple that we can write it like this the rate of rate of energy in flow to the control volume I write simply c v.

Minus the rate of energy well outflow from, c v must be equal to the rate of change of rate of change of energy within the control volume or the open system in in consideration within the c v this is the very simple, and a preliminary statement of conservation of energy that the rate of energy in flow to control volume minus the out flow rate of energy, outflow from control volume is equal to the rate of change of energy with the control volume now, you see therefore, we have find out what are the inflow energy inflow quantities into the control volume, and what are the outflow quantities.

In the very first case we see that we have considered, the work is coming out in the form of the shaft work from the control volume this is 1 way the energy is coming out in the form of work, and similarly the energy in the form of heat is going into the control volume, because we have considered these heat energy to be going into the control volume for the general deduction apart from this the energy quantities coming to the control volume due to the inflow of fluid mass across this section. Similarly energy which is going out of the control volume are associated with the f flux o fluid mass from the control volume across this boundary.

]so therefore, we have to now, recognize to know the different energies, which are coming in to the control volume or coming out of the control volume, what are the energy? Quantities that are associated with the flow of a fluid stream. So, we have to first recognize that. So, what are the different quantities or different energies different forms of energies that are associated with the flow of a fluid stream with the flow of a fluid stream the energy quantities or the different energies, those are associated are first is intermolecular energy that is the kinetic, and potential energy of the molecule intermolecular energy exist in all substances under all conditions if it is at a temperature above that the absolute 0.

So, this intermolecular energy number 2 is the kinetic energy this is, because of the flow velocity though a fluid flow or fluid stream lowing stream possesses kinetic energy another energy is a very well known energy is the potential energy. This is by virtue of its placement or virtue of its position in the conservative force field if there is no such external conservative force field. So, gravitational force field is there. So, gravitation potential energy is there apart from that there is another flow energy, which is there in a flow of streams or fluid is the pressure energy that you have read in your fundamental fluid mechanics class.

What is that pressure energy or it may be termed as flow work this is, because of the existence of pressure at a particular section in the flowing fluid as you know that it is, because of the pressure a section a fluid layer at a section is capable of pushing the neighbouring layer to make its flow through the . So, therefore, to maintain the flow the layer at any section the fluid layer at any section has to continuously push the neighbouring layer, and by virtue of its pressure t can do. So, and it makes its way through. So, therefore, we see to maintain a flow each, and every section the fluid at each, and every section does work on the neighbouring layer.

And that work is known as flow work whose magnitude is pressure times, this specific volume per unit mass well that is the work per unit mass, and I we look from this angle that this capability of doing this work by a particular layer of fluid at any section on its neighbouring layer to make its way, through the flow is known as the energy which is inherent in the fluid or stored in the fluid layer, and it is stored in the fluid layer in the from of pressure energy we tell. It in the as pressure energy sometimes we refer it to as pressure energy or sometimes we refer it to as low work.

So, therefore, we see what are the energy quantities, that are associated with a flow of fluid stream 1 is the internal energy by virtue of its state thermodynamics state another is the kinetic energy by virtue of the flow of velocity another 1 is the potential energy by virtue of its position in a conservative force field, and last 1 is the pressured energy by virtue of the pressure in the flow field. So, I we recognize these energy quantities ten we can now, come we can now, recognize the energies which are coming into a into the control volume, and going out of the control volume.

So, therefore, if we again see this let us consider the rate of inflow of mass, as d m on d t, and let the outflow of mass is d m to d t the rate of the outflow this is the rate of inflow of mass this is the rate of outflow of mass. Now, to identify those energy quantities we will have to fix the state of the fluid at inlet let the inlet pressure is given by p 1 the corresponding specific volume is given by v one. Let the in let velocity is given by v one, and let from any arbitrary referenced atom the elevation the vertical height at the inlet section is given by z 1 the corresponding quantity at out. Let is given by p 2 with a subscript to v 2 that is the specific volume v 2 is the pressure velocity v two, and also the elevation from a referenced atom vertical height as z two.

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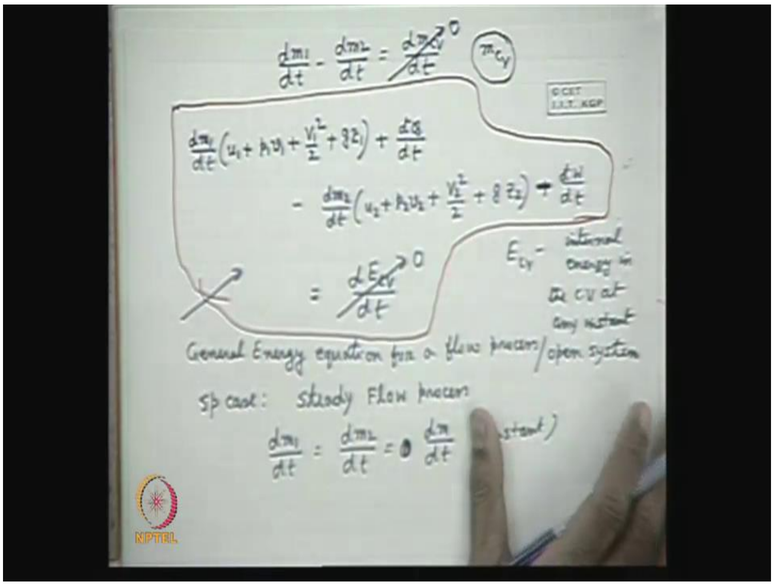


Therefore with this nomenclature we can now, write the rate of energy coming in to the control volume associated with the inflow of fluid, as d m 1 d t that is the rate times this specific that is the per unit mass internal energy plus the flow work we write first that the pressure energy plus the kinetic energy per unit mass plus the potential energy you consider only. The gravitational potential energies present; that means, no other external conservative force field is there. So, this is the rate of energy coming into the control volume through the inflow of fluid mass well, plus as we have already considered in this figure for this control volume.

This heat is being added at the arte of d q d t. So, we write d q d t. So, therefore, this is the in flow of energy to the control volume similarly, what is the outflow of energy from the control volume minus that is the energy quantities which are going out with the fluid stream at the outlet; that means, the outflow of fluid that becomes u two, that is the internal energy at the outlet p 2 v 2 pressured energy per unit mass at the outlet fluid stream plus v 2 square by 2 plus g z 2 plus the work quantity, because we have considered the for the control volume the shaft work is coming out.

That means controlled volume or the flow device or the open system develops work in the form that work is coming out in the form of the shat work, coming out to the surrounding. So, therefore, we can write it this is equal to d w d t, and difference.

Minus.

D t minus.

Sir d w by d t.

Very good minus, and this will be equal to the rate of change of energy within the control volume. Where e c v is the energy in the control volume internal energy in he control volume at any instant internal energy in the control volume in the control volume in this c v at any instant this internal energy at any instant comprises all the energies in the control volume, that is the intermolecular energy the kinetic energy due to the motions of fluid particles within the control volume plus the potential energy. So, this is the statement of the conservation of energy.

So, now, under a special an this you can consider as the general steady flow in a general sorry not steady general flow, general energy equations for a flow process we can consider this as the general energy equation general you can write it general energy equation general energy equation for a flow process for a flow process for a flow process or open system for an open or for an open system. Now, for a special case when the flow process is steady as a special case for steady flow, process for a steady flow process when the flow process is steady.

That means we know the definition of a steady process or a steady flow fill; that means, each, and every point in this system the properties are in variant with time; that means, as a whole the properties within this control volume or the open system would be invariant with time this will happen when the mass flow rate at inlet, and mass flow rate at outlet will be same. So, there will not be any mass accumulation within the control volume, and at the same time the energy inflow to the control volume must be equal to the energy outflow from the control volume. So, that there is not change of energy within the control volume with time.

So, that the energy quantity remain same the mass remain same. So, all the thermodynamic properties at each, and every point within the control volume will remain same. So, this situation refers to a steady flow process. So, therefore, we can write in those process first of all from mass point of view d m 1 d t is equal to d m 2 d t is equal to zero; that means, the control volume mass, will not increase sorry sorry sorry sorry I am sorry its constant let d m d t I am sorry this is constant otherwise for an general case we can write, similar to energy conservation d m 1 d 2 minus d m 2 d 2 very correct d m c v.

Which means the general conservation of mass the rate of mass inflow to the control volume minus rate of mass outflow from the control volume is equal to the rate of change of mass in the control volume. So, in n c v is the mass of the control volume at any instant. So, for a steady flow this becomes 0 this is equal to d m d t similarly this becomes zero. So, if we now, this is the general equation for a steady flow process, but conventionally this is written in a form like this if we divide the terms each, and every term by the d m 1 d t or d m to d t which is equal to each other at let it be d m d t, and divide each, and every term by d m d t, then we get an equation if we divide the terms on the the left hand side all the terms by d m d t.

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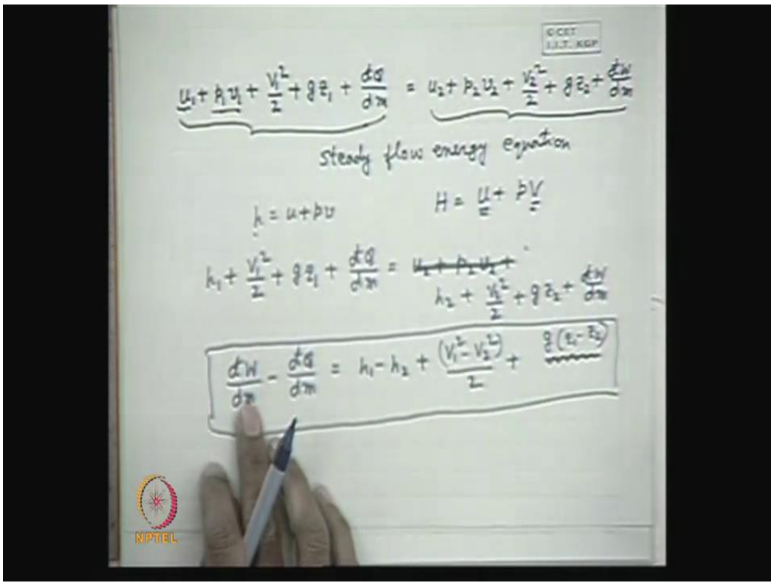


We get an equation in this form u 1 plus p 1 v 1 plus v 1 square by 2 plus g z 1 plus d cube d m well is equal to what we can write, u 2 plus p 2 v 2 plus v 2 squarer by 2 plus g j 2 plus d w d difference is that here, we what is what is implied by this equation the rate of energy inflow is equal to the rate of outflow from the control volume each, and every term represents the energy per unit mass. It is more conventional to represents the energy quantities per unit mass, this is the heat flowing to the control volume per unit mass this is the work flow out of the control volume per unit mass.

In this equation these were the energy quantities on time bases the rate of energy, coming in rate of heat coming in. So, rate of energy quantity going out rate of heat going rate o work coming out or going out, but in general the for a steady flow process. It is conventional to express this quantities in terms of mass bases. So, therefore, each, and every quantity is the energy per unit mass. So, energy coming in per unit mass is equal to energy going out per unit mass in the control volume, and it is out of the control volume. So, this is known as steady flow energy equation the special case steady flow energy equation.

Now, you see that we have defined this, summation of u plus p v as the enthalpy. So, specific internal energy plus pressure times specific volume can be defined as specific enthalpy since the enthalpy is defined as u plus p v you known u is an extensive property volume is an extensive property. So, if you define the corresponding intensive property by dividing by the mass of the system. We can define the specific enthalpy the sum of specific internal energy plus the product of pressure, and specific volume. So, therefore, we can, write the equation in terms of the enthalpy quantities v 1 square by 2 plus g z 1 plus d q d m is equal to u 2 plus p 2 v 2 plus sorry we are writing h 2 plus v 2 square by 2 plus g z 2 plus d w d x.

This can be manipulated algebraically in this fashion that you can write this way that d q rather here you can write this way, d w d m minus d q d m is equal to h 1 minus h 2 plus v 1 square minus v; that means, we are taking this here, and we are writing this in the left hand side a simple algebraic manipulation h 1 minus h 2 d w m all right g z 1 minus z 2 divided by oh this is g z 1 minus z two. So, you see that this is the final outcome of the steady flow energy equation, but incidentally it happens that in all engineering devices all these things that heat transfer work transfer do not take place simultaneously. And moreover the changes in potential energies, and kinetic energies are sometimes neglected for example, let us consider only the devices which are only work interactions which are only work interactions without any heat interactions.

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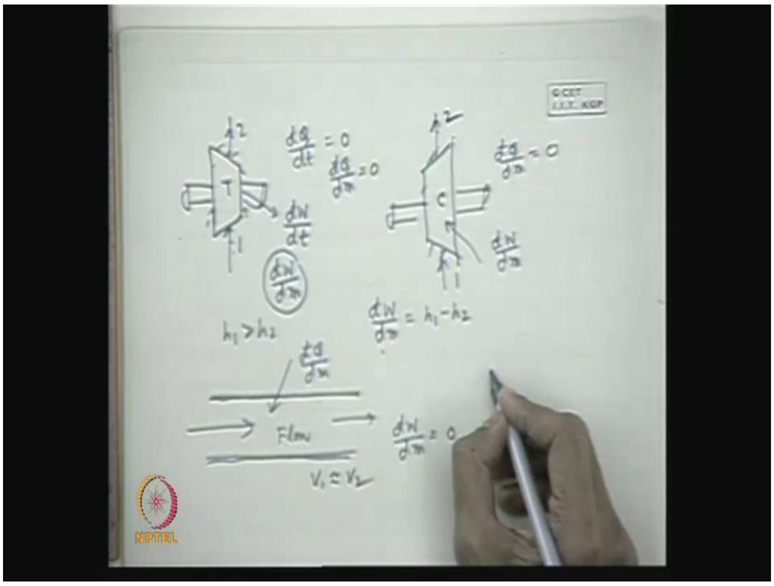


Let us consider turbine or compressor, let us consider a turbine for example, the simple case the turbine you known the turbine this is insulated. Let this is 1 this is 2 let this is insulated. So, in this case d w d t is there which is coming out whereas, d q d t is 0 or you can tell d w d m is there, and d q d m is zero. And if we neglect the change in the kinetic energy at the inlet, and outlet of this turbine, and also the changes in the potential energy, then what we get we get from this equation, you see that this becomes 0 this becomes zero. So, simply, and also this is zero; that means, the work coming out from the turbine in that case is h 1 minus h two, and therefore, we see that for only work interacting devices or example in a turbine. When work is coming out where the changes in kinetic, and potential energies are negligible compared to the changes in the enthalpy.

It is the property enthalpy whose difference between the inlet, and outlet state straightaway give the work interaction quantity. Similarly in case of compressor if you see that the reverse in case of a compressor, if you see in case of a compressor c let us in case of a compressor if you see it is the same thing that if it is insulated let this is one, and this is two, and if it is insulated similarly way, d q d m is zero, but  it absorbs energy; that means, shaft work is filled into the compressor; that means, it has d w d m,,, but  in this direction.

In that case also we can express the same equation that is in this case, also d w d m is h 1 minus h the same equation here h 1 is greater than h 2 always. So, that this quantity is positive work is coming out in this case h 1 is less than h two; that means, the enthalpy of the outlet in is outlet section is more than that at inlet. So, that d w d m is negative, which implies physically the work is gong in similarly there are devices like heat exchange are where there is only heat input to a system for example, a fluid flowing through a pipe flow, and there is no work transfer; that means, d w d m is zero.

And if the flow area is uniform we can neglect the kinetic energy change; that means, v 1 is almost equal to v two, and if we change the potential energies neglect the changes in the potential energy here also we can write from this equation, this same this equation again we can write here making this 0 minus d q d m is h 1 minus h 2 or d q d m is h 2 minus h one; that means, it is the difference of enthalpy which gives this heat transfer; that means, this is positive; that means, heat is added when h 2 is more than h 1 if h 2 is less than h 1 the heat is rejected.

So, therefore, we see now, physical significance of enthalpy is such that in flow devices steady flow devices the difference between the enthalpy at inlet, and outlet section gives the net work, and heat interactions by the device with the surroundings. So, for only work interacting devices. It is say the work transfer is simply coming out as the difference in enthalpy provided the change in kinetic, and potential energies are neglected, and in all practical applications for those devices which interact in terms of either work or heat the changes in kinetic, and potential energies are usually very small as compared to the changes in enthalpy.

So, therefore, it is the change in the enthalpy quantity, which is straightaway giving the work. So, here you see the enthalpy in such system bears more or less the synonymous role as the internal energy as if it is, because of the release of enthalpy or absorption of enthalpy the work transfer takes place in a turbine the work is done, because of the change in enthalpy. So, enthalpy bears the same physical significance or same physical entity as the internal energy does for a closed systems, similarly in a compressor the work is absorbed by virtue of which its enthalpy is raised, because it is the difference of enthalpy which is given the work quantity.

Similarly, in case of heat interacting devices where only heat interaction takes place it is by virtue of the change in enthalpy the heat energy is coming out this equated with the change in enthalpy. So, therefore, in this way we can tell that enthalpy bears a similar thing that is the internal as internal, but truly speaking it is not the internal energy it is a property of this system whose definition comes only by the mathematical expression h is equal to u plus p v. Now, we see some more examples which may come in your compressible flow calculations that there is no heat, and work interaction.

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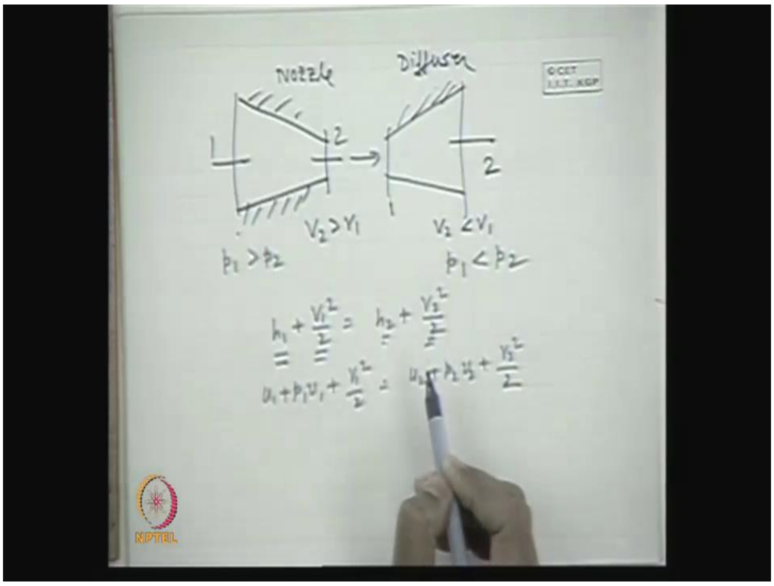


That means the flow takes place through a nozzle, but there is a change in velocity for example, the flow through a nozzle or a diffuser flow through a nozzle or a diffuser this is a nozzle this is a nozzle one, and two, and this is a diffuser flow through a nozzle, and diffuser here what happens v 2 is greater than v one, and here v 2 is less than v 1 that is one, and two, similarly the p 1 here is greater than p 2 here here v 2 is more less than v 1 p is less than p two.

So, pressure increases velocity decreases pressure decreases velocity increases in this case of you see the energy equation, here again the common these 2 terms are zero. So, this terms we will take care of and if we neglect the potential energy changes to be 0, because the there is no change or the change in the vertical heights at the inlet, and outlet are small, then here the equation tells like that, h 1 plus v 1 square by 2 is equal to h 2 plus v 2 square by two. Now, if you explicitly see u 1 plus p 1 v 1 plus v 1 square by two; that means, this is the conservation of energy when heat, and work quantities are not there the flow takes place in such a way the total energy comprising the enthalpy, and the kinetic energies remain same.

That means, here the kinetic energy is created out of the enthalpies similarly here the kinetic energies are destroyed or kinetic energies are reduced at the pressured energy the enthalpies are increased by virtue of the kinetic energy, you see that this is the expression. So, this expression holds good for this type of now, I will deduce the Bernoulli's equation which we have already read at you fluid mechanics the well known, Bernoulli's equation from this principle or from this equation of energy equation for a steady flow process for this bernoulli's equation. If you recall bernoulli's equation is basically the equation for conservation of energy.

This is a mathematical statement for the conservation of energy for the flow of fluid, but this is not the Bernoulli's equation perfectly, this is the broad statement now, what type of energy conservation in Bernoulli's equation we consider only the mechanical energy. We see how the total mechanical energy is conserved that is why it is known, as mechanical energy equation this is the conservation of mechanical energy, and we a simplified form of energy in certain special cases of fluid flow; that means, in case of flow of an inviscid steady flow of fluid inviscid steady flow of fluid.

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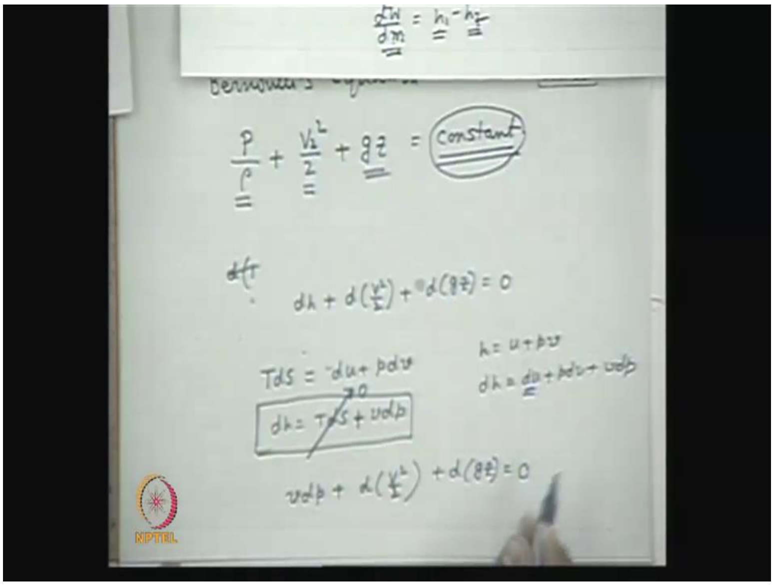


What we know that in Bernoulli's equation, if you recapitulate the your Bernoulli's equation Bernoulli's equation Bernoulli's equation we know that the pressured energy p by rho sometimes, we can write it in terms of this specific volume or 1 by rho these are synonymous, but usual convention this that in fluid mechanics we use is as 1 by rho we use density rather instead of specific volume, but in thermodynamics we prefer this a convention the specific volume. So, p by rho plus v square by 2 we know these expression g z is equal to constant what does it mean.

So, this quantity as we have seen is the pressured energy or the flow work per unit mass this quantity is the kinetic energy per unit mass in a flow of fluid, and this quantity is the potential energy. So, sum of these three quantities give the total mechanical energy per unit mass in the flow of the fluid which are associated with the fluid stream, and Bernoulli's equation tell, that for an inviscid incompressible 1 of the conditions inviscid incompressible steady flow of a fluid these three things are constant, and this is constant only along a stream line if the fluid is rotational.

And if the fluid is irrotational constant is constant throughout the flow field this very important again I tell you this is the recapitulation in many times, this question is asked in an interview or in viva quize what is Bernoulli's equation, and what are kits restriction. So, if you tell the Bernoulli's equation it is basically the conservation of mechanical energy. So, that those mechanical energies associated with the flow of a fluid stream. This is the pressure energy of the flow work kinetic energy, and the potential energy usually we will consider the gravitational potential energy. So, sum of these three energy will be constant provided there is no energy interactions from outside.

So, sum of these three energy will be constant provided there is no energy interactions from outside. So, first condition there is no heat, and work interactions in the fluid flow from outside number 2 there is no change from mechanical energy to other form of energy or intermolecular energy, which we physically call it as dissipation of energy or degradation energy of energy from the second law of thermodynamics, because energy is converted from a higher grade to a lower grade. So, there is no such conversion which is only possible if the friction of the fluid is as same; that means, fluid is non viscous or inviscid.

So, inviscid fluid, and fluid is incompressible pressure remains same sorry I am sorry density remains same density does not change in the flow. So, for an incompressible inviscid, and steady flow fluid without any work, and heat interactions these three quantities p by rho represents the pressured energy per unit mass, the v square by 2 represents the kinetic energy per unit mass, and g z represents the gravitational potential energy per unit mass sum of these three quantities remain constant, but this constant value remains same only along the stream line only along the stream line.

But for a rotational flow, but  for any rotational flow this is constant throughout now, I would like to tell you I do not know whether you known, these things from basic fluid mechanics class that why this constant along stream line why in a rotational flow what isa the difference between rotational irrotational flow from the energy from the energy view point from the view point of energy transfer you know the rotational, and irrotational flow definition is that a flow is said to be irrotational when the rotation is 0 rotation of fluid element that is the curve of the velocity vector is 0 how do you define the rotation of a fluid element. It is the average angular velocity of the 2 adjacent linear sides of a fluid element which were initially perpendicular.

And this is given by the curl of the velocity vector three rotational component in three coordinate direction. So, if the rotation is 0 flow is irrotational,,, but  if the rotation is not 0 the call of the velocity vector is not 0, then the flow is rotational from the energy point of view in a rotational flow field. There is always a work interactions work is either given to the fluid from outside or work is extracted from the fluid form outside from the inside to the outside from the inside to the outside. So, because of this work giving to the fluid or taking from the fluid the constant varies from stream line to stream line,,, but  if there is more interaction along with no heat interaction, then the fluid Behaviour is the rotation.

So, it is both in visit, and irritation, but it may be an in visit flow, but  rotational flow, but  the work interaction may take place from outside to the fluid or inside that mean between the flow of fluid with the surrounding this is very important. So, if you discuss the work interaction; that means, flow of an in visit fluid without any work interactions from the surrounding also no heat interactions from the surrounding, then the fluid behaves as an irrotational fluid in visit irrotational fluid, and visit irrotational flow is synonymous to isentropic flow in thermodynamics isentropic flow without work interactions.

You can write, isentropic flow without work interaction. Now, let us immediately recapitulate hurriedly how we can derive, this equations now, from the steady flow energy equations now, you see this is steady flow energy equation general steady flow energy equations. Now, if we apply this to a fluid visit irrotational flow of fluid visit irrotational in visit steady flow of fluid. So, irrotational flow means there is no work or heat interaction this is 0. So, therefore, we can write that this steady flow energy equation takes the from h 1 plus v 1 square by 2 plus g z 1 is h 2 plus v 2 square by 2 plus g j 2.

This it is very simple, we can write from a differential in a differential form as d of we can write h rather we write d h plus d v square by 2 plus d of g j is equal to 0, because if we integrate it between section one, and 2 you get h 2 minus h 1 plus v 2 square by 2 minus v 1 square by 2 plus g j minus 0 you can write this, now, if you recollect the thermodynamic property relation which I will discuss again in this class that t d s is equal to d u plus p d v, and we can write h is equal to u plus p v.

So, d h is equal to d u plus p d v plus p d v sorry. So, if you replace the d u in terms of d h we can write d h is t d s minus v d this is the very useful relationship, which we get I will explain this in the next class also d h is t d s plus v d p yes d h is t d s plus v d p all right. So, if you substitute this. So, for an isentropic flow. So, I have told you that in visit irrotational flow is synonymous to isentropic flow without work interaction. So, if I replace it we get v d p plus d v square by 2 plus d of g z is.

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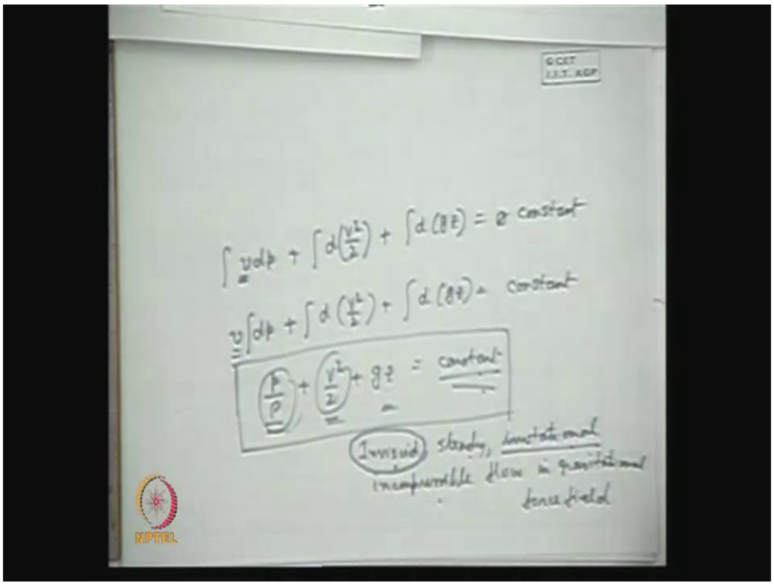


So, I we integrate this equation well, if we integrate this equation we get integration of v d p plus integration of t v square by 2 plus integration of d g z is equal to zero. So, far we have put the constant of isentropicness of the flow isentropic that is the flow is isentropic that is in visit without any work interaction, but incompressibility has not put into consideration. If we put the incompressibility into fact that this v comes out from the integration plus sorry it is constant after integration it is constant very good d v square by 2 plus integration of d d z is equal to constant ok.

So, therefore, we can write after integration p by rho v can be written as 1 upon rho plus v square by 2 plus g z is constant, and this is constant throughout the flow field, and this is simple the mechanical energy equation for an in visit for an in visit invisuid steady irrotational irrotational. These are the conditions in compressible flow in gravitational field in gravitation force field; that means, if a in visit steady irrotational in compressible flow in gravitational force field takes place the irrotational flow means there is from the energy point o view no energy interaction neither it nor work total mechanical energy that is the pressure energy kinetic energy, and this thing constant.

But if we draw this of conservation of energy for a real fluid we use a term as loss which is meant as the loss in mechanical energy. So, some form of mechanical energy is lost; that means, has disappeared, but it has taken place in the other form as the intermolecular energy. So, this conversion we term is as a loss in the mechanical energy equation, because this is a loss from the sense of total mechanical energy; that means, the fluid flow is unable to conserve its mechanical energy as a whole well this is, because of the friction. So, this is all about the principle of application of first law of conservation of energy to an open system any question.

Thank you.