Introduction to Fluid Machines, and Compressible Flow
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Lecture - 26
Introduction to Compressible Flow

Good morning I welcome you to this session of fluid machines, we have completed the discussions on fluid machines, and now we will switch over to a new topic introduction to compressible flow. So, at the outset I must start with the definition of a compressible flow what is meant by compressible flow. So, as you know the compressibility is a property of fluid, and it is characterized by a parameter known as bulk modulus of elasticity, and physically the compressibility the property of the fluid is a measure of its change in volume or density with respect to the pressure.

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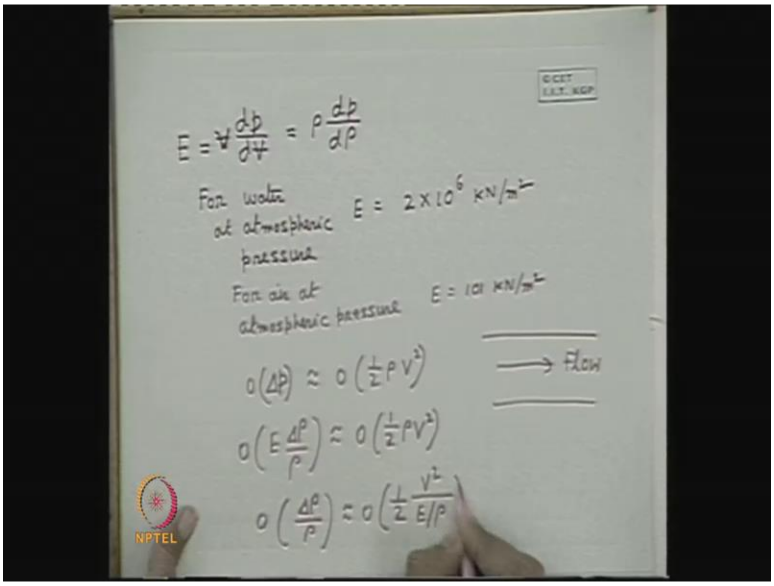

Now, if we look to the definition of elasticity the characteristic parameter for the compressibility of a fluid. We will see the weight is defined the bulk modulus of elasticity is defined like this it is d p into V by d V where the V with a cut I use to represent the volume to distinguish, it from the velocity or is equal to rho d p by d rho. So, therefore, you see the bulk modulus of elasticity is defined this way. Now a large value of bulk modulus of elasticity represents a large change in pressure required to cause a definite change in volume or density.

And for fluids whose bulk modulus of elasticity is very large are usually termed as incompressible fluid, because a change in volume or density is very low as compared to the change in pressure. Similarly for the fluids whose bulk modulus of elasticity is relatively very low; that means, which suffer a considerable change in volume or density for a given change in pressure are termed as compressible fluids.

For an example I can tell you that for water at atmospheric pressure, for water at atmospheric pressure for water at atmospheric pressure the value of e is equal to 2 into ten to the power six kilonewton per meter, square as compared to that for air at atmospheric pressure for air at atmospheric for air at atmospheric pressure e is equal to hundred 1 kilonewton per meter square. So, you can very well see that water is almost in compressible practically, because a such value of e indicates a very large change in pressure is required to cause a little change in volume or density as compared to that at that of air.

Now, question comes this is the characteristic property of a fluid, but what is a compressible flow is it true that compressible flow means the flow of compressible fluids whose elasticity or coefficient of bulk modulus of elasticity is very low, and flow of all incompressible fluids are incompressible flow it is not exactly. So, compressible fluids are defined in this way, that the if the change in density brought about by the change in pressure due to the flow is very less those flows we treat as compressible flow. Now the concept comes like that even if the fluid itself is compressible for example, air whose bulk modulus of elasticity is very low.

If it flows in such conditions that the pressure differences the maximum value of the pressure difference due to the flow in such that, it cannot change the density or volume in the flow very much, then the flow can be treated as incompressible. So, therefore, a flow is whether incompressible or compressible depends upon whether the change in volume or density encountered in the flow is small or large. So, therefore, it is very much tag with the flow condition, because the change in pressure is not an arbitrary one.

So, if the change in pressure is very low in the flow. So, that the change in volume, and density is low those flows can be considered as incompressible. So, to have a criteria for an incompressible or compressible flow for the fluids, we should confine ourself with these directions we see that a rough order of magnitude we can find out in this way that in any flow of fluid the pressure difference delta p the order of pressure difference can be written as like this, it is in the order of the dynamic head where V is the velocity of fluid we consider any flow through a duct any flow. We can consider like that the delta p the maximum pressure difference or the order of the pressure difference in the flow which will be encountered will be in the order of the dynamic head half rho V square is true.

Now, it is very simple manipulation. Now if we express this delta p in terms of the coefficient of modulus of elasticity from this expression. You see that which we can rho we can write like, this is e delta rho by rho instead of delta p is in the order of half rho V square or we can write the order of del rho by rho is in the order of half V square by e by rho well. Now after this therefore, I write it again that order of so, we see that the order of del rho by rho is equal to the order of half V square e by rho.

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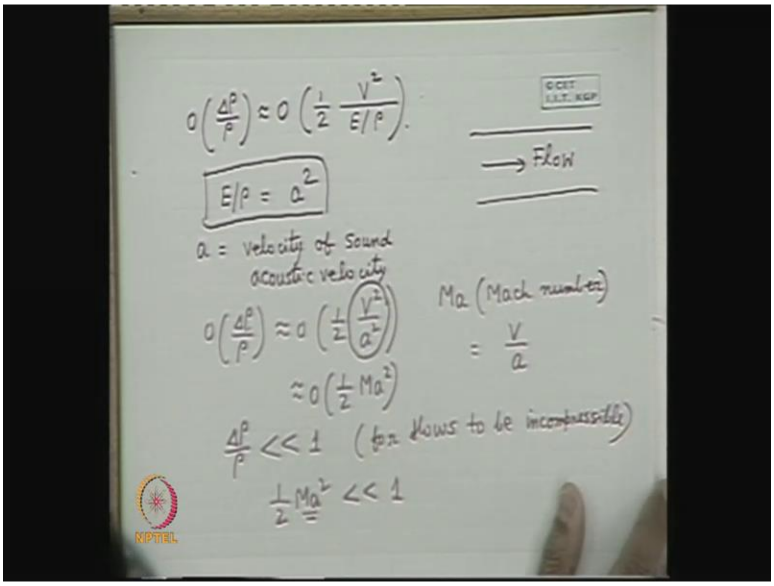

Now, in any flow this in any flow situation through a duct, it may be through a duct or it may be a flow over a body. Now this value e by rho represents the square of the velocity of sound in that flow at that condition, where a is this velocity of sound velocity of sound velocity of sound or acoustic velocity velocity of sound or another name is acoustic velocity acoustic velocity in the fluid at that particular condition.

So, this is the definition which probably you know is already derived in classical physics preliminary physics e by rho a square. So, therefore, if I use this definition we see that the order of the change in density to the density the ratio of change in density to the instantaneous density or the initial density whatever you call is half V square by a square. Now this ratio of V square by a square this is the ratio of the square of the velocity of flow to the square of the velocity of sound in that fluid at that condition. So, there is non dimensional number known as mach number. It is after the scientist mach who first discovered it mach number or introduced it is defined by V by a.

So, therefore, mach number is a dimensionless number, which represents the ratio of the velocity of fluid at any conditions to the velocity of sound in the fluid medium at that condition, this ratio of V by a is known as mach number. So, therefore, we can write in terms of a dimensionless number half m a square. So, we see the change in density as a fraction of the density itself rho is in the order of half m a square. So, e power criteria for incompressible flow is delta rho by rho is very less than 1 for incompressible flow for flows to be for flows to be incompressible, for flows to be incompressible for flows to be incompressible delta rho by rho should very very less than one. So, therefore, the criteria is that half m a square should be very very less than one; that means, the mach number of flow should be such that half m a square should be very very less than one. So, that to make the delta rho by rho that is change in density with respect to the density itself is very less.

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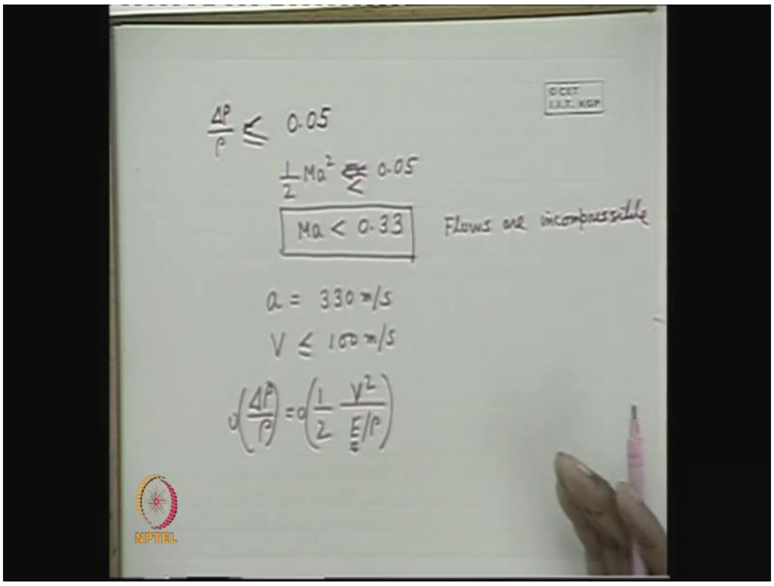

Now, to have a definite quantitative criteria, we set this delta rho by rho as or like this less than equal to 0.05, which means that we can neglect a density variation of 5 percent of the initial one.So, a change in density of 5 percent a change in density of 5 percent or less than 5 percent can be ignored, and the flow can be consider to be incompressible if it is. So, flow, then a quantitative criteria can be defined that half m a square should be very less than point 0.05 or should be simply here simply less than 0.05 from which we can derive that m a should be less than is equal to point three three.

So, this is the very important conclusion you have to remember throughout your life whenever you deal with flows of fluids that, when the mach number of flow is less than 0.33 the variation in density is 5 percent that 5 percent of the initial density or below the 5 percent at 0.33 it becomes 5 percent. So, mach number is equals or less than 0.33 the change in density equals to or less than 5 percent of the initial density, and the flow can be considered to be incompressible flows are incompressible.

So, therefore, we see whether a flow, will be compressible or incompressible will depend upon this dimensionless parameter mach number just an example. I am telling that flow of air, and normal pressure, and temperature you know that the speed of sound at that condition at n t p through air is three thirty meter per second. So, we this criteria we can say that the velocity of air at this normal condition temperature, and pressure if it is less than equal to 100 meter per second this is a thumb rule we tell that the flow of air is incompressible; that means, in a situation where there is a flow of air is 50 meter per second we can tell the flow is incompressible flow.

So, in that situation the pressure difference associated with that flow, that is a flow of air at 50 meter per seconds at the atmospheric condition cannot bring about a change in volume or density, which is more than 5 percent, and we can neglect that change in volume, and and change in density in the flow, and we can treat the flow to be incompressible all right. Now, you see another interesting feature is that we are found out that del rho by rho del rho by rho is the order wise is in the order of half V square divided by e by rho.

Now the bulk modulus of elasticity for incompressible flows are very large very large; that means, for all liquids, which are treated as incompressible fluids, because they are bulk modulus of elasticity is very large otherwise the velocity of sound through that medium is very large. So, usually even for a very small velocity we get the value of delta rho by rho is very high.

So, therefore, a sorry very low sorry very low. So, therefore, flows of all incompressible fluids are usually incompressible, because even with very high velocity encountered in practice they cannot bring about a delta rho by rho more than 5 percent this is, because of their very large values in e it is not practicable theoretically you can consider infinitely high velocity which can make,, but it is not practicable.

So, under all practical conditions flow of all incompressible fluids or flow of liquids are incompressible why the reverse is not the true; that means, flow of compressible fluids that is the flow of gases may be incompressible provided its velocity is low, and that is not in absolute velocity. It is related to the velocity of sound, and the criteria is the dimensionless parameter mach number, that if the velocity is such that it corresponds to a mach number of flow less than 0.33, then the density change or volume change is lower than the 5 percent. So, therefore, the flow can be considered as incompressible flow all right.

Now, before going to the next chapter we should recapitulate little bit of thermodynamics, because the knowledge of thermodynamics, and the property relations derived from thermodynamics will be very much applicable in the directions of compressible flows. So, first in first 1 or 2 lectures, we will be recapitulating the basic laws of thermodynamics first, and second law thermodynamics, and important property relations. So, therefore, we must first start with the first law of thermodynamics. So, what is first law of thermodynamics first law of thermodynamics is basically the law of conservation energy as you know the first law of thermodynamics is basically the law of conservation of energy.

Now, if we keep aside the physical phenomena of conversion of mass to energy, and energy to mass, we can tell that the conservation of energy is that energy is neither create nor destroyed that we know since our childhood. This is the conservation of energy; that means, if energy is transformed from 1 form to other or if energy is transferred from 1 system to other system in the same form in both the cases energy total energy remains constant it is neither created nor destroyed; that means, if the energy disappears in 1 form it appears in other form this is simply the conservation of the energy as simple as that, and first law of thermodynamics is nothing, but synonymous to this principle of conservation of energy.

But in the applications of fluid flow, and classical thermodynamics as applied to mechanical engineers or other engineering, disciplines the same principle of conservation of energy. We look from a view point where the heat is being converted into work or work is being converted into heat, because heat, and work these 2 types of energies are first described by classical thermodynamics at the energy in transit energy in transit; that means, the energy quantities, which transfer from 1 system to other system are either in the form of heat or in the form of work. So, therefore, we are interested to define or reshape the conservation of energy while applied to a system or applied to process where the heat, and work energies are appearing as the energies in transit.

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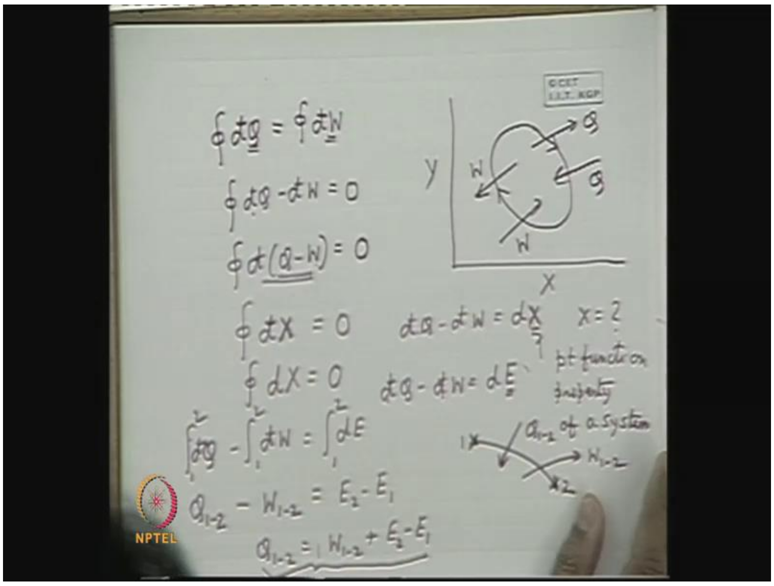

So, therefore, if you recapitulate these you know the first law of thermodynamics is written like that in any cyclic process executed by a system. So, this d q the cyclic integral of heat transfer is equal to the cyclic integral of work transfer, here this cut I mean maintain to distinguish this d from the exact differential, because as you know, this q is a path function heat flow, and work flow or work transfer is also a path function. So, d cut. Where this d cut q I simply will pronounce it is a d q is the infinite small heat transfer d w represents the infinite small work transfer hence forth.

So, cyclic integral of d q is d w; that means, in any cyclic processes; that means, if a system executes in a thermodynamic cycles executes a processes in a thermodynamic cycle; that means, in any thermodynamic property diagram. There will be a close loop the total heat transfer during the cycle; that means, heat may be coming out heat may be given in in some processes, there is no restriction in the direction work may come out in some processes may go in. So, as the whole the if we make the accountability of the energy will see the sum of all the heat transfer process in a cycle must equal to the sum of all the work transfer process in a cycle this is a mere recapitulation of your basic thing.

So, if we write it in a different that way d q minus d w is zero; that means, we can write cyclic integral d of q minus w is 0 this gives a very interesting thing that though the q, and w are the path functions, but their difference becomes a point functions, because this cyclic integral of their difference is zero; that means, if we represent this d q minus d w as some d x d cut x is zero, then we can tell that cut is not required for that, because perfect differential of any quantity integrated over a cycle must be zero, that is the basic definition from mathematics.

So, you see the difference between q, and w over the cycle is 0 which means that q minus w can be expressed by a point function x. So, therefore, we can write that d cut q minus d cut w can be expressed as a perfect differential of a point function x what is that x this comes straight from, the mathematical concept that this minus this over this cyclic integral is zero; that means, d q minus d w can be expressed as a change of a point function where x is a point function, and this point function, and the property of a system you know, that any point function is known as property of a system property of a system, and this way the birth of internal energy comes. So, this is the definition of internal energy.

So, therefore, we can write d q minus d w can be expressed as a change of a property which is a point function known as internal energy. So, the in physical implication of this mathematical statement comes like this that, if we consider a process from 1 to two, then this equation implying a infinite small process can be written like, this if we integrate this d q from 1 to 2 minus d w from, 1 to 2 is equal to d e from 1 to 2 as you know this q, and w are the path functions, and they cannot be integrated like this.

So, therefore, we have represented this d with a cut this is not a exact differential. So, therefore, this is written as q 1 two; that means, the heat transfer in this process depends upon the path of the process does not depend only the state points, similarly the d w 1 to

2 to be represented as the work transferred during the process. 1 2 is usually written as w 1 2 which depends upon path of the process whereas, e being a point function which is the internal energy by definition of the system it can be written as e 2 minus e one.

So, simply the first law can be written as q 1 2 is equal to w 1 2 plus e 2 minus e 1 this is also the conservation of energy applied to a system, that if we considered the direction in this way. That the heat added is positive simultaneously you will have to take that work out as the positive, then we can interpret this physical the amount of it added to the system during its change from a state 1 to state 2 by a process 1 2 is equal to the work delivered by the system plus, the change in its internal energy. So, this is precisely the first law which is written for a system.

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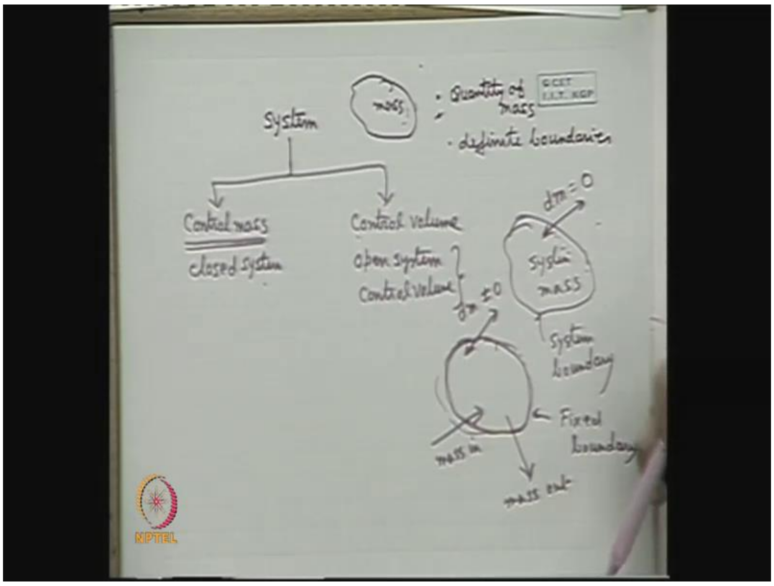

Now, this can be again written in a differential form rather, I will tell this is written in a d q is d e in an infinite small process differential form many people tell you the differential form, but I will tell for an infinite small process, because q, and w cannot be expressed as any differential these are the path functions; that means, either in a differential form will automatically mean in that case that d e is the differential of internal energy, but q, and w are the infinite small amount of work, and heat that takes place for an infinite small process.

So, this is the outcome of the first law of thermodynamics as applied to a process executed by a system involving heat, and work transfer now we come to a definition of a property enthalpy. Which is very important enthalpy enthalpy how do you defined enthalpy what is the definition of enthalpy please what is the enthalpy how it is defined h.

Property.

Good h enthalpy is a property which is defined as u plus p into V good. So, the very first line of definition of enthalpy is like, this the first line of the very first definition of enthalpy comes from its mathematical statement, that h is equal to u plus p V what is u u is the intermolecular energy. Now before that I tell you that this is the total internal energy total internal energy. So, f I write the internal energy of any system it comprises several types of energies, that can be stored in the system internal energy of a system is the energy that is stored in the system, and it is a point of a function it depends upon the state of the system. So, it is a point function.

So, therefore, internal energy are those energy, which can be stored in the system at a given state. So, it comprises first the intermolecular energy which is the kinetic energy, and potential energy of the molecules, which depends upon the state of this system precisely the temperature. Similarly this system itself may have velocities; that means, the macroscopic particles of the system may move within the system even, if the system is a closed system. There may be a substantial motion of the system that is the different particles of the system may be in motion.

So, therefore, the kinetic energy is an energy. Which may be contained by the system, and another type of energy may be stored or contained by the system, that is known as potential energy what is that energy? this is the energy by virtue of the stay of the system or the position of system in a conservative force field. So, there may, be number of conservative force fields magnetic force field electrical force field in which the system is exposed the system is placed. If all conservative force fields are relief the gravitational force field is there show at least, there will be gravitational potential energy or simply potential energy. So, this kinetic energy of the particles of this system potential energy, and the intermolecular energy are the total are the contributions are compressing the total internal energy.

So, if I write the internal energy general symbol u is the intermolecular energy. So, the kinetic energy of the particles plus the, let us consider only gravitational force field that the potential energy. Let us write the mass of the m g z the total potential energy total kinetic energy m, and the total internal energy. So, this is the internal energy total internal energy now in a closed system in equilibrium the kinetic energies are not appearing, because the system is at rest the particles is at rest, and if you neglect the potential energy not, because of its absolute value, because you know the absolute value of potential energy to ascribe the absolute value of potential energy is very difficult we also we always measure it in terms of its change.

So, if you neglect the change in potential energy of the systems between different states we can neglect this m g z the potential energy part. So, we can tell the internal energy for a close system or a stationary system simply comprises the internal molecular energy. So, u is the intermolecular energy. So, therefore, this typical combinations of u p, and V where p is the pressure, and V is the volume defines the term enthalpy. Now you see u is the point function p is the point function V is a point function. So, therefore, enthalpy is a property, and it is a point function another interesting thing is that the dimension of enthalpy is the dimension of energy, because u is the internal energy its dimension is energy the product of p, and V this dimension is energy.

So, in enthalpy is a property, and is dimension is energy. So, it is something similar to energy. So, very first line of definition of h does not give by any physical concept, but immediately the query comes why such a combination is defined as a property. So, you know you start. It many properties first we start with measurable properties first we start with observable properties that, we can see the mass we can fill the temperature, then comes with the measurable properties the volume cannot see, but we can measure pressure we cannot see, but we can measure.

So, therefore, you see that after wards in thermodynamics, we mix several combinations out of this preliminary properties or primary properties to define other properties, but why such definition is required, and why such a particular combination is made. So, that query is satisfied. If we go little further to see the physical significance of such a combination to yield the definition of another property for example, this enthalpy if we see the physical significance of this enthalpy parameter or this enthalpy this property enthalpy we have to extend our first law to a to an open system or a steady flow system.

So, let us do that we consider a steady flow system or a open system. So, before that I think I should tell you the system different types of system. So, how do you define a system, and there are 2 types of systems 1 is the control mass system another is the control volume system, how do you define a system system is a definite quantity of mass within a fixed boundary; that means, the 2 very interesting characteristic of system a definite quantity of mass, and a the content or separated from the outside by a define boundary.

So, quantity of mass quantity of mass quantity of mass quantity of mass, and definite boundaries definite boundaries are the 2 characteristic feature of a system. So, system is defined just like that it is some quantity of mass. a definite quantity of mass at any instant, and bounded by definite boundaries. Now the system basically is divided into 2 ways 2 categories 1 is control mass system another is control volume system what is a control mass system in a control mass system the mass, which is contained within the system by its system boundaries. So, therefore, we can see a system is a definite quantity of mass, and this is the system bounded.

So, if the if the mass identity remains the same within the system boundary; that means, there is no flow of mass either in or out; that means, d m z is 0 from the system boundary; that means, the same mass not only the amount, but with the identity remains the same, then we call the system as the control mass system which is usually told as closed system. So, the characteristics of closed system the additional characteristics apart from the definite quantity of mass within definite boundaries that the identity of the mass remains same; that means, in other way there is no mass flow in or out from the system.

So, if you take some mass flow out, and make an inflow of the same amount to make the mass of the system remains same that will not satisfy the characteristics of a control mass system or closed system, because in that case though the mass remains same, but the identity of the mass changes; that means, the same identity has to be there while on the other hand the control volume these the system. Where we allow the mass flow; that means, the d m not is equal to 0 that may be mass in there may be mass out there may be mass in there may be mass out.

But the restriction is that there is the volume of the system remains same; that means, the boundary is fixed fixed boundary. Now you can ask me sir, then what is the difference here the difference for the closed system that system boundary may move system boundary is not fixed at any instant there should be a boundary of the system, but there is no restriction that the boundary of the system has to be fixed; that means, the volume of the system may change while the mass, and the identity will remain the same that is why its control mass system whereas, in the control volume system the fix boundary the boundary will not move, there is no displacement in the boundary the volume of the system is controlled in a control mass system the boundary remain fix the boundary may move, but control volume system the boundary remain the volume remain fixed the boundary will be fixed the volume remains same.

So, this is known as control volume system, it is simply known as control volume or open system. this is known as open system or simply control volume the system we do not use. So, we see there are 2 types of system 1 is the closed system another is the open system or control volume an example, of closed system is your reciprocating pump we have seen that 1 of the boundaries that is the piston which is moving. So, at any instant the boundary is defined, but instant to instant 1 of the boundaries the piston which is moving; that means, in a closed system there is a there may be a displacement of the boundary say boundary may expand or boundary may contract. So, that the volume of the system may change whereas, a control volume system is a system what the boundary is rigid; that means, the volume is same,, but through the boundary the fluid can go out or the sorry the mass system mass can go out can come in. So, this is the control volume system.

So, now 1 difference is that if you see in a control volume system if the same amount of mass comes in, and the same amount of mass goes out; that means, the net rate of mass inflow or mass outflow is zero, then the total mass remains same. So, in which way it differs from that of a closed system is that the mass identity is changed. So, therefore, at steady state a control volume system differs from, that of a closed system is that though the mass remain same in both cases the identity of the mass is same in closed system whereas, the identity of the mass is not same in the control volume system. So, control volume system we simply tell us control volume, and an open system whereas, the control mass system we call as close system or simply system when we call system this is henceforth, you know that system means is the closed system or control mass system sometimes these are not used.

So, system implies the control mass system or the closed systems, and with all its characteristics. Similarly a control volume means a control volume system or another name is the open system; that means, where the mass flow out or mass flow in is applicable or allow; that means, mass flux across the system may bounded that is possible, but the restriction is that the boundary should be rigid; that means, the boundary should be stiff; that means, there is volume boundary should not move or should not displace; that means, the volume of the system should remains same that is the control volume or open system. So, if you see the application of first law to an open system, then you will come to the physical significance of enthalpy I think the time is up. So, next class I will discuss any query please.

Thank you.