

Introduction to Fluid Machines and Compressible Flow
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Lecture - 25
Axial Flow Compressor Part II

Good morning and welcome you all to this session of the course. Now, today will be solving some problems relating to axial compressor, last class we will discussed in brief the principle of operation and the degree of reaction of an axial compressor. Now, today we will see that how we can solve problems this use an understanding becomes much better. Now, let us concentrate on one problem.

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entry (ii) stage pressure ratio, and (iii) the power required to drive the stage (for air, $R=287 \text{ J/kgK}$, $\gamma=1.4$).

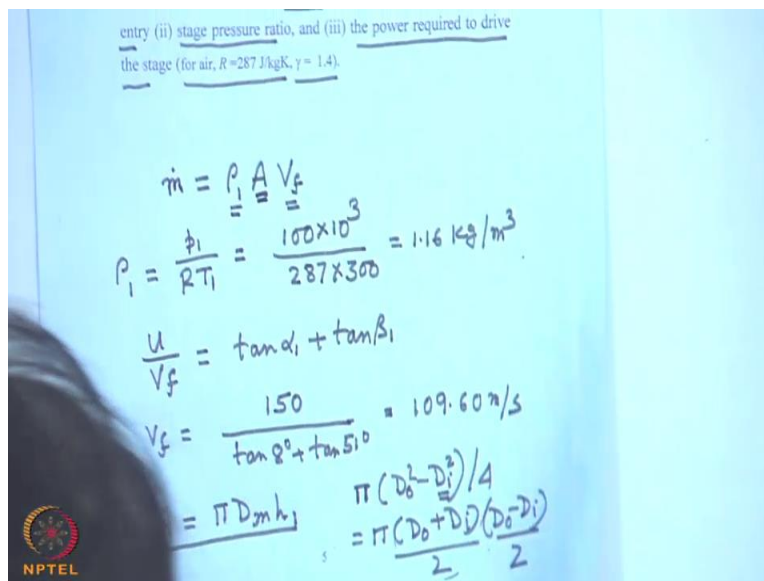
$$m = \rho_1 A V_f$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{100 \times 10^3}{287 \times 300} = 1.16 \text{ kg/m}^3$$

$$\frac{u}{V_f} = \tan \alpha_1 + \tan \beta_1$$

$$V_f = \frac{150}{\tan 8^\circ + \tan 51^\circ} = 109.60 \text{ m/s}$$

$$= \pi D_m h$$

$$= \frac{\pi (D_o^2 - D_i^2) / 4}{2} = \frac{\pi (D_o + D_i)(D_o - D_i)}{2}$$


This is the problem number one, which will be solving the conditions of air, let me read out the problem first, the conditions of the air at the entry of an axial flow compressor stage are p_1 giving the inlet pressure and temperature is given. The air angles are β_1 , so these nomenclature is known to us this is the blade angle at inlet and β_2 the blade angle at the outlet that is the root of blade angles at the inlet and outlet that is 51° both are 51° . And α_1 is α_2 is 8° this is giving the α_1 at the angle the absolute velocity at the inlet and the α_2 is the angle of the absolute velocity at the outlet of the rotor blade. The mean diameter and peripheral speed are 0.5 meter, this is the mean diameter and this is the

peripheral speed respectively, that means; the peripheral speed at the mean diameter. The mass flow rate through this stage is 30 kg per second, the work done factor is 0.95 all these things are known to us mechanical efficiency is 90 percent.

Assuming an isentropic stage efficiency of 85 percent, what have to be found out, we have to be find out blade height at entry stage pressure ratio and the power required to drive the stage this is given for air the characteristic gas constant 287 joule per kg k and the ratio of specific heat 1.4. Now, sorry, to find out the blade height at entry first of all we have to understand that it is the geometric blade height, which is basically related to the area at the inlet. So, therefore, if we write the equation of mass flow rate that think we can we have to select these equation to find this, we can right that ρ_1 inlet density and the area at inlet and the velocity of flow, which is same throughout.

Now, here first of all we have to find out $A \rho_1$, now eric with in A that is the anoles area the blade height is there, the blade height can be found out from the anoles area A. Now, let us find out ρ_1 , so ρ_1 at the inlet can be found out from the equation of state as p_1 by $R T_1$ well, so p_1 is what? p_1 is given as 100 kilo Newton per meter square. So, therefore 100 into 10 Newton per meter square 100 kilo Newton into 10 to the power of 3. And if you write the value of R 287 and T is 300, we get the value of ρ_1 and ρ_1 comes out to be 1.16 kg per meter cube, alright, we get ρ_1 . How to get v_f , the flow velocity is it given assuming in isentropic state air the flow velocity is not given but we can find out the flow velocity from the expression, we developed earlier that u by v_f is what? $\tan \alpha_1$ plus because, we choose this expression flow velocity is not given but β_1 and α_1 is given so, that we can find out the flow velocity and u , what is u ? u is given 150 meter per second so 150 divided by $\tan \alpha_1$ plus $\tan \beta_1$, that means; \tan of 8 degree plus \tan of 51 degree and if you solve for it, you will get a velocity 109.60 meter per second.

So, therefore, you can get the anoles area, but before that you put anoles area in terms of this blade height, anoles area is π into the mean diameter into blade height true, where from does it come, it comes that anoles area is π into outer diameter square minus inner diameter. If you consider the inner diameter as the diameter of the dram on disk and the outer diameter is the diameter of the casi then this is the anoles area this can be written as π into D_0 plus D_i sorry, by 4, D_0 plus D_i by 2 and D_0 minus D_i by 2. Now, it is this is the mean diameter D_0 plus D_i

by 2 and this is the blade height. So, therefore, this can be expressed in terms of the mean diameter and blade height that area.

So, therefore, now, if I substitute here, we can write \dot{m} , \dot{m} is given 30 kg per second is equal to $\rho \cdot V$, first I write V , because I have found out V is 109.60 into π , what is the D ? D is given as 0.5 meter ok, 0.5 meter it is given. So, therefore, π into 0.5 so only unknown is here blade height h , which gives h equals to rather h is the blade height at the inlet so I write h is blade height at the inlet so, h is equals to 0.15 meter so, the first, but is over the h , blade height at entry. How to find out stage pressure ratio? Stage pressure ratio is simple to find out stage pressure ratio to find out, what you have to do? We have to find out this static temperature, the stagnation temperature ratio or this stagnation temperature difference otherwise you cannot find out the static pressure ratio.

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$C_p \Delta T_{st} = \lambda u V_f (\tan \beta_1 - \tan \beta_2)$
 $\Delta T_{st} = \frac{0.95 \times 150 \times 190.60 (\tan 51^\circ - \tan 10^\circ)}{C_p}$
 $C_p = \frac{\gamma}{\gamma - 1} R = \frac{1.4}{0.4} \times 287 \text{ J/kgK} = 1005.5 \text{ J/kgK}$
 $\Delta T_{st} = 16.37^\circ \text{C}$
 $R_s = \left[1 + \frac{\eta_s \Delta T_{st}}{T_{1t}} \right]^{\frac{\gamma}{\gamma - 1}}$
 $= \left[1 + \frac{0.85 \times 16.37}{300} \right]^{\frac{1.4}{0.4}}$
 $= 1.17$
 $P = \dot{m} (C_p \Delta T_{st})$
 $= 548.39 \text{ kW}$

So, which one is known, now let us find out write this formula that C_p into ΔT_{st} that means; the stagnation temperature rise is equal to today last class we have discussed that it is the work done factor into $u \cdot V_f$ by C_p is always there, $u \cdot V_f$ into $\tan \beta_1 - \tan \beta_2$, where from it comes, that this is the work done per unit mass, this equal to $C_p \Delta T_{st}$. So, therefore, this is multiply with a work done factor, so here what we had? We have u , we have V_f , we have β_1 , we have β_2 .

So, we can find out ΔT_{st} , we can write this, what is the value of λ here, λ is 0.95 it is given well, then the u is 150 meter per second, v_f we have calculated just now 190 I am sorry, v_f is 109 know, v_f is come out as 109 yes yes, v_f is 109.60. So, this is λu into v_f , v_f is 190.60 that is the v_f we are calculated and \tan of into \tan of 51 degree minus \tan of 10 degree divided by C_p . Now, C_p value you can calculate here, here C_p is not given even if it is not given you can take that in this problem γ and characteristic gas constant is given, C_p can be found out as γ by γ minus 1, that means; that 1.4 divided by 0.4 into 287 that is joule per kg k, that means; if you find out C_p it will be 1005 joule per kg k, this is the standard value of C_p for air.

So, C_p then you can find out ΔT_{st} and that becomes ΔT_{st} becomes equals to if you calculate it, I tell you the value 16.37 degree Celsius. Then you can find out the pressure ratio or as I told that nomenclature and this is the formula again and again we tell this will be one plus the isentropic stage efficiency that is θ_s with the isentropic stage efficiency ΔT_{st} this formula I have told in let temperature divided by γ by γ minus 1 ok. So, this can be written as 1 plus this substituted the value substitute, which you can substitute the value ΔT_{st} 16.37 divided by 300 and if you put the value of γ then you get the value of the stage pressure ratio as 1.17.

Third one is that, what is the third one? The power required to drive the stage, power required is very simple, because this is the $C_p \Delta T_{st}$, C_p power required is what? Power per unit mass into the mass flow rate, mass flow rate into $C_p \Delta T_{st}$. So, ΔT_{st} you know this already we have found out multiply with the C_p that means this part, that means actually this part, so this part we know and mass flow rate already we know 30 kg given in the problem, 30 kg per second so therefore, if you put the value C_p this one ΔT_{st} 16.37 degree Celsius and mass flow rate 30 kg per second, you get a value of 548.39 kilo vat, if you consistent in the unit final you will get this result this is the power, so this is one example.

That we can work with next another problem I will tell you, where you have some idea about the degree of reaction ok.

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Blade speed (U) 300 m/s

Assuming constant axial velocity across the stage and equal absolute velocities at inlet and outlet, determine the blade angles of the rotor for a shock free flow (c_p for air = 1005 J/kgK).

$$\frac{W}{m} = c_p \Delta T_{st} = 1005 \times 30 \quad \beta_1 = ?$$

$$\beta_2 = ?$$

$$\frac{W}{m} = (V_{w2} - V_{w1}) U$$

$$= U V_f (\tan \beta_2 - \tan \beta_1)$$

$U = 300 \text{ m/s}$ $\tan \beta_2 - \tan \beta_1 = 0.67$

$V_f = 300 \times 0.5 = 150 \text{ m/s}$

$$A = \frac{V_f}{2U} (\tan \beta_1 + \tan \beta_2)$$

$$0.6 = \frac{0.5}{2} (\tan \beta_1 + \tan \beta_2)$$

$$\tan \beta_1 + \tan \beta_2 = 2.4$$

$$\beta_1 = 56.92^\circ$$

$$\beta_2 = 40.86^\circ$$

So, this problem is this, the preliminary design of an axial flow compressor is to be based upon a simplified consideration of the mean diameter conditions. Suppose that the characteristics of a repeating stage of such a design are as follows, that means; a particular stage these are repeated for different stage. Stagnation temperature rises, that means; indirectly the work done on the fluid is given, degree of reaction is given, flow efficiency is given 0.5, but this is actually not flow efficiency, I think it will be better, if you write it has flow number v_f by u is actually flow number sometimes they write at flow efficiency however flow number blade speed that means, it is the ratio of flow velocity to the peripheral speed. Peripheral speed is also separately given 300 that means, we get v_f multiply 0.5 with them. Then what is the problem, assuming constant axial velocity across the stage and equal absolute velocities at inlet and outlet that means, v_1 is equal to v_3 that we have already done. Determines the blade angles of the rotor for a shock free flow that means, there is no incident loss so, how to find I work out this problem. Now, this problem, if you work out, you see that the degree of reaction is given now; first of all we know the stagnation temperature rise. So, first of all we find the work per unit mass is equal to is known, that is what, that is C_p into ΔT_{st} and what is C_p ? C_p is 1005 joule per kg k and that is 30 and finally this becomes equal to your work per unit mass.

Now, how to find out β_1 and β_2 ? So, what we have to find out, we have to find out β_1 , we have to find out β_2 , the two angles ok, you remember that β_1 , β_2 so, I tell you

again that this one we have to find out, we have to find out β_1 β_2 , so what we will do? We will take some of this relationship, $\tan \beta_1 - \tan \beta_2$ is equal to work done, where you will get that formula that v_f ok, I am coming to it again, it was done earlier, if you remember that $W_{by\ m}$ is $V_2 - V_1$ into u and that becomes is equal to u into v_f into $\tan \beta_2 - \tan \beta_1$. Now, we know u , we know v_f , v_f is 300 into 0.5, u is 300, u is 300 meter per second well.

And you can see, I think you can see very well and v_f is 300 into 0.5 that is 150 meter per second, $\tan \beta_2 - \tan \beta_1$ and β_1 , we have to find out. $W_{by\ m}$ we know so therefore, if we put this here and this value here we get a relationship $\tan \beta_2 - \tan \beta_1$ equals to what $\tan \beta_2 - 0$ point you get everything is known. Now, we have to get another relationship relating β_2 and β_1 that will come from the relationship of degree of reaction. Now, degree of reaction, if you remember is given by u by v_f degree of relation is v_f by u , v_f by $2u$ if you remember that today we have done it or last class we have done it $\tan \beta_2 - \tan \beta_1 + \beta_2$. So, we have done these things so here also we know v_f by u is 0.5 that means 0.6 that means rather I write here, I write here it will be better 0.6, 0.5 v_f by u 0.5, which is given in the problem by 2 into $\tan \beta_1$ so it is a simple problem, which gives relationship $\tan \beta_1 + \tan \beta_2$ is it is what 1.2 to 2.4 it is very simple.

So, this is one relation and this is one relation so as simple as this and finally if you solve for $\tan \beta_1$ and $\tan \beta_2$ finally you get β_1 is 56.92 degree and β_2 is 40.86 degree so these are answers you can check. So, therefore, with a degree of reaction you can that is only that we have to deal with the equations, which equal sense will be depending upon the parameter given so these are the problems, which clear makes your concept more clear.

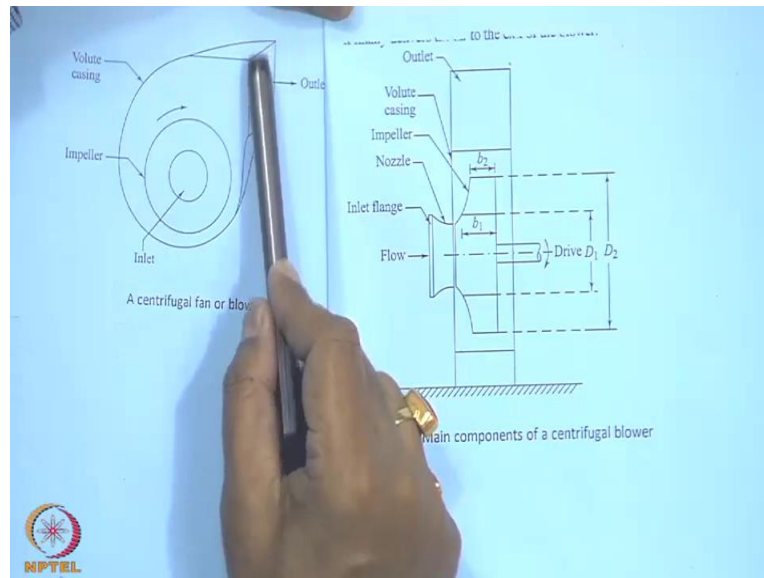
Now, after this I will start the fans and blowers. So, fans and blowers there is nothing much different from that of compressor the different is like that as I told earlier at the beginning of my fluid machine class that the when the output for a compressible flow for example that I have handling air at the outlet of a machine, where energy is given and we get the fluid at the outlet of the machine having higher energy acquired stored energy, if that is mostly in the form of high pressure static pressure, but less velocity you call it has complicit, whether it is centrifugal it is axial, but when the basic purpose is to have fluid with very high velocity, where we have to deliver high rate of flow with high velocity of flow. Therefore they are the machine must deliver

the fluid with higher energy by observing energy from outside mostly in the form of the kinetic energy with high velocity but relatively much less pressure static pressure those machines are known as fans and blowers. So, fans and blowers and compressor of same kind in a sense that they take energy from outside and the fluid going through it gains its internal energy but for fans and blowers this energy mostly in the form of kinetic energy rather than pressure static pressure is more of velocity then that means static pressure.

Now, among fans and blowers, fans are those machines, where the static pressure at the outlet of the machine is few millimeter of water gage, you can understand that atmosphere pressure is state meter of water gain it is so less few millimeter of water gage, where as in case of blower it is something more than thousand millimeter of water gage still is very less ten meter of water gage is the atmosphere pressure. So, this is the pressure getch pressure that means avoid the atmosphere at the outlet why this pressure is solo, why the pressure is required, first of all pressure is required, because this machines deliver air for some purpose, that means, air has to flow it may be required that to make to supply the air through a deck. So, therefore, we have to overcome the friction of the deck, that means; this flow has to take place through a deck, it has to overcome the frictional lence not only through a deck through a room. So, therefore, downstream resistance has to be frictional resistance has to be overcome to deliver that flow so this machines develop that pressure sufficient to overcome the resistance.

Now, in case of blower it handles more amount of air when less amount of air is handled and the circulated or been sent through a deck, we employee fans for which the frictional resistance is layer. So, therefore, relatively less static pressure is required at the delivery end of the machines whereas in case of blowers, which handles more air and at a high velocity it may have to be convey that transport at through a long deck the frictional resistance is much more that means; it has to overcome more frictional resistance so, that the pressure drop is more. As a consequence this static pressure at the outlet of the machines has to be higher this is the reason for which blower for blower static pressure is higher, because it handles more amount of air. So, this is the difference between blower and a fan, so fan and blowers are basically the compressor, but in this case the static pressure at the end of the machine is much lower as compare to his velocity.

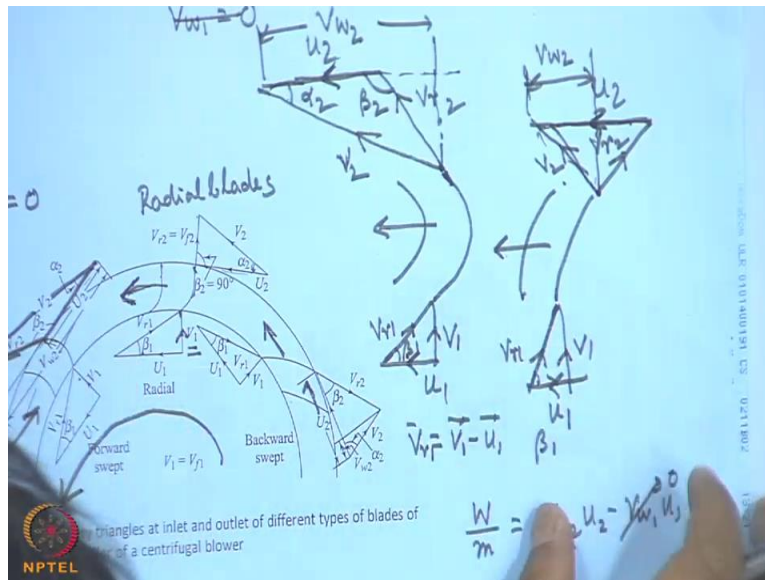
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So, you see a typical centrifugal fan or blower here, I will show you simultaneously I do not know it may be a little difficult ok, I will try it yes, this is the thing. So, let me show you this is the thing, let me just a minute, let me make this things, let me show you like this, let this is a typical centrifugal fan or blower. Now, centrifugal fan air blower consist of an inlet this is the impeller the same principle impeller, which imparts the energy to the fluid then after the impeller there is a final casing knows has volute chamber, spiral or scroll casing this is the volute, where this gain the energy in term of both velocity and pressure rise at the end of the impeller high velocity and high pressure some of the velocities are then converted in the volute chamber to static pressure depending upon the requirement at the outlet of the machine.

So, this is the picture this can be shown, if we take a section like this then it is this, this is the impeller you see, so this is the inlet the flow at the inlet takes place through a nozzle that means; there is little acceleration of the flow before it enters into the impeller. Now, impeller it goes readily with same way that it happens in a centrifugal comprises that means; it is shuck or it is induced the axial direction then the flow changing the radial direction. So this is the readily outward flow then it comes out this is the volute case and this is the outlet so this is the main component of a centrifugal fan or blower.

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So, after this I will tell you about the as the typical type of blades in a blower or fan. Now, blower or fan both are the same machines as I have told only the static pressure rise is different for blower and fan and depending upon the fluid required the fan blade shape is different. Now, three types of blade shapes from the fluid mechanical principles are used, I tell you. One is known as forward swept blade, you see this left on what is that if you consider the motion in this direction that means; this is the direction of rotation that means; the peripheral speed is in this direction the tangential. So, curvature of the blade is in the direction of the motion this is known as forward swept. Another type of blade is which is known as radial blade, this is known as, this is known as, this is forward swept blade, so this type of blade is known as radial blade what is the radial blade? Radial blade their outlet that means; at the outlet the blade is radial, but at inlet there is a curvatures and this curvature is forward swept, why? Because this is the direction of the motion the curvature in the direction of the motion, so radial blades are radial outwards plat, but this is car at the inlet which is forward. Another is the backward swept, that means is curvature this is the direction of the motion, its curvature is in the opposite direction of the motion. So, velocity triangles will defiantly change that I will tell you, you can understand, because this direction of velocity and the curvature of the blades are in the opposite direction and relative directions are changed.

Now, before that I tell you the forward swept blade are used, where large fluorides are required

relatively large fluorides and higher pressure rises required as compare to backward swept. Whereas radial blades are preferred, where the fluid that is air use contest more impurities and dust, this is because of the fact that these are less turn blockage and they work more efficiently with the dust laden gas, so therefore this radial blades are preferred for that. Now, if we see the velocity triangle, so velocity triangle at the let for example in the radial you see this is same for all the blade, what is that? There is the inlet velocity is axial, absolute velocity direction this is the axial direction, axial this has got no component in the tangential direction that is V_{w1} component is 0, V_{w1} is zero everything that is the problem here, V_{w1} is 0 ok, V_{w1} is zero see that can be same V_{w1} zero can be same its same V_{w1} is 0.

So, radial blades entry for all blades are same that is V_{w1} is axial this is the relative velocity, which matches the angle of the blade this is the peripheral velocity this makes the vector diagram that is the velocity triangle. Now, at the outlet also you see this is the peripheral p this is the direction of the flow direction of the rotation speed these velocity that is the relative velocity matches the blade angle that means it is radially outward and this is the absolute velocity α_2 is the absolute velocity angle, β_1 is the angle of the blade at the inlet and this is α this is 90 degree, this is the β_2 , which is radial this is α_2 angle of the absolute velocity.

Now, let us see the velocity triangle for the forward swept blade the velocity triangle for the forward swept blade is this one, this is the it is visible it is visible velocity diagram, let me draw the velocity diagram here, I think for you it will be forward swept I write. So, this is the inlet diagram it is already same as that, that means; there is no tangential component it is axial and this is the now here forward swept this is the u , the direction of u is this.

So, therefore, the direction of u this is V_{r1} ok and this is the u and this is the V_{r1} this is the β_1 ok, V_{r1} is equal to $V_1 u_1$ minus u_1 very good, because they differ it is a radial formation radial outward. Now, at this end what will happen this will be my radial flow so the relative velocity direction and these will be the the diagram will look like this so this will be obviously from the V_{r2} this will be u_2 , which will be higher than u_1 and this will be the V_2 .

So, this is the simple diagram, because the velocity is in this direction so it has to be like that. In this case this is defined as the β_2 that means in the positive sense u_2 is this direction with this is an optives angle this is the angle β_2 with the tangent and this angle is α_2 so this is

the velocity triangle, which is shown here like this. I think you can see, but this diagram I think is very difficult to see these things are not properly shown not legible. However now, for a backward swept this can be visible, backward swept one I again draw it here for the backward swept what will be that so these curvature is in the opposite direction that means; this is like this, this is in the opposite direction, but the direction of peripheral velocity is like this.

So, here also so this is the same thing that this is the inlet velocity triangle ok, this is the inlet velocity triangle, this is V_{r1} , this is V_1 let this is v_1 sorry, u_1 and this angle is beta one. So, the outlet angle for example here or here I can draw the velocity this relative velocity is this then what is this velocity, relative velocity this direction ok, that is V_{r2} then these velocity is in this direction, this velocity is in this direction. So, therefore the absolute velocity will be so these looks very odd, so little more the relative this is u_2 this will be V_2 .

So, this will be now, if you draw it in case for a given value of u_2 at the outlet and for a fixed value of v_a we will see that for forward swept blade the component that V_2 is more and the component that V_{w2} that this one V_{w2} is much more compare to these, this is V_2 this V_{w2} . So, therefore, you see V_{w1} is 0 therefore work done per unit mass is again $V_{w2} u_2$ since $V_{w1} u_1$ V_{w1} is 0. So, therefore in this case this is ok, understandable so in this case V_{w2} is more than this and that is the reason that forward swept blade is used, where we require more work and more flow will be available and more static pressure rise will be there ok.

So, more work will be imparted to the can be imparted to the fluid flowing through it. So, three types of possible blade configuration forward swept, radial blade and backward swept blade are possible. So, with this now, other treatments I tell you in the axial flow fans and comprises are almost identical to that of the centrifugal comprises, which I have told in case of a centrifugal blowers and centrifugal fans.

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A centrifugal fan running at 1500 rpm has inner and outer diameter of the impeller as 0.2 m and 0.24 m. The absolute and relative velocities of air at entry are 21 m/s and 20 m/s respectively and those at exit are 25 m/s and 18 m/s respectively. The flow rate is 0.6 kg/s and the motor efficiency is 80%. Determine (i) the stage pressure rise, (ii) degree of reaction and (iii) the power required to drive the fan. Assuming the flow to be incompressible with density of air as 1.2 kg/m³.

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$$\text{Work done / mass} = \frac{V_{w2}u_2 - V_{w1}u_1}{m}$$

$$\frac{L}{m} = \frac{V_{w2}u_2 - V_{w1}u_1}{m} = \left[\frac{(V_2^2 - V_1^2)}{2} + \frac{(u_2^2 - u_1^2)}{2} + \frac{(V_{r1}^2 - V_{r2}^2)}{2} \right]$$

K.E entry
Dynamic Head
Static Head
Static energy



Now, with this I will go on solving a problem. Let us see this problem, a centrifugal fan ok a centrifugal fan running at 1500, that is fifteen hundred r p m, as inner and outer diameter of the impeller as 0.2 meter and 0.24 meter that is the inner diameter of the impeller, outer diameter of the impeller.

The absolute and relative velocity of air at entry are twenty so absolute and relative velocity of air at entry are these respectively and those that mean exit entry and exit absolute and relative velocities are given, the flow rate is 0.6 kg per second, the motor efficiency is 80 percent, determine the stage pressure rise degree of reaction and the power required to drive the fan, assuming flow to be incompressible with density of air as 1.2 kg per meter. Now, this particular problem I tell you, we will make here total understanding of the fan or a blower clear so let us first see that what this problem tells. Now, this problem tells that velocity absolute relative is given.

Now, if you recall this type of problem what you will do first you have to find out the stage pressure rise so how to find out the stage pressure rise so we have to find out first of all you find out the stage pressure rise by which expression we have to find out the stage pressure rise stage pressure rise means what the total pressure rise in the stage ok. So, total pressure rise in the stage depends upon the work done, total work done, if you first find out the work done per unit mass

what is it work done per unit mass? Now, if you remember the work done per unit mass is $V_2 u_2$ minus $V_1 u_1$ without going for a change in temperature stagnation temperature so far we did this is, because this problem is told at assuming the flow to be incompressible.

So, for an incompressible flow better, we do this type of analysis that this is true for any fluid incompressible or compressible. If you remember that this were done, but $V_1 u_1$ that is work done that is W by m can be written as different components, if you remember that is V_2^2 square that is work done on the fluid minus V_1^2 square by 2 plus u_2^2 square minus u_1^2 square by 2 ok, plus V_{r1}^2 square minus V_{r2}^2 square by this, I explained in details in one earlier class of our fluid machines that this can be split like this from the geometry of the velocity triangles, why it is required, because this gives very clear understanding.

This is the gain in kinetic energy, where the absolute velocity here in terms of compressor, in terms of pump, in terms of fans blowers, where the fluid gains energy V_2 is always greater than V_1 . So, this is the gain in kinetic energy, this is the gain in the kinetic energy or dynamic, we sometimes tell as dynamic head, it is per unit mass, usually energy per unit mass or unit way you call it as head. And these two is the gain in the static head, why it is called static head or static energy, which is not very much used that means, this is manifested in terms of the increase in the pressure of this way. This is because of the change in this is the change in peripheral velocity at outer radius and inner radius and the fluid therefore when it reaches that outer radius from inner radius gains in static pressure and that change in the pressure energy is given by u_2^2 square minus u_1^2 square by 2.

Similarly, the change in static pressure, because of the change in the relative velocity, this is again another part of the diffusion that means, change in the relative velocity takes place, where V_{r1} is more than V_{r2} that means, V_{r2} is less than V_{r1} . So, passages made in such a way that V_{r2} this is a diverging passage for which the pressure is gain so this pressure is gained out of the momentum change or the change in the kinetic energy related to the blade passage. So, these two this is due to the centrifugal action the radial pressure gradient is imposed, I told many time and this is because of the diffusion from the change in the kinetic energy V_{r1} is higher V_{r2} is lower. So, therefore, pressure is lower and pressure is higher at the outlet so the combination of these two is the static head static energy and in these connection I like to tell you again this is a very important concept for all of you that in a radial flow machine whether it is outward or

inward even if the passage is uniform there is no change in the relative velocity still there is a change in the static pressure rate, because of the change in the u the peripheral velocity in case of a turbine there is a loss in pressure in case of a pump or compressor there is a rise in pressure. So, therefore, any radial flow machine outward or inward has to have some pressure change in the rotor at least by this one but along with that, if you make a diffuse in passage in case of compressor there is an additional pressure rise ok.

So, these two sum up view this static head or static energy this is very important I want to repeat it again, so here what happens I know V_2 , I know V_1 , I know V_{r1} , I know V_{r2} , because all the relative velocity at inlet and outlet is given. And again I know u_2 u_1 , because the impeller diameters are given and the speeds are given. I am not solving this problem I just write u_2 is $\pi D_2 n$ and u_1 is $\pi D_1 n$, n is given rotational speed 1500 one fifteen hundred rpm. So, that we can fifteen hundred rpm so that we can do this thing better I write here, so that u_1 u_2 I found out, u_1 this way, u_2 this way we can find out and then what we do, we can find out the work done.

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inner and outer diameter of the impeller and relative velocities of air at inlet and exit are 25 m/s and 18 m/s respectively. The motor efficiency is 80%. Assume the flow to be incompressible.

$w_2 u_2 - v w_1 u_1$
 $+ \frac{(u_2^2 - u_1^2)}{2} + \frac{(v_2^2 - v_1^2)}{2}$
 Static Head
 Static energy

$\frac{W}{m} = 184.26 \text{ J/kg}$
 $P = \frac{\dot{m} \left(\frac{W}{m} \right)}{\eta_m}$
 $= \frac{0.6 \times 184.26}{0.8}$
 $= 138.19 \text{ W}$

$(\Delta p)_{\text{stage}} = \rho \left(\frac{W}{m} \right)$
 $(\Delta p)_{\text{static in rotor}} = \rho X$
 $(\Delta p)_{\text{stage}} = 221.11 \text{ N/m}^2$
 $(\Delta p)_{\text{static}} = 110.71 \text{ N/m}^2$
 $\Omega = \frac{(\Delta p)_{\text{static}}}{(\Delta p)_{\text{stage}}}$
 $= \frac{110.71}{221.11} = 0.5$

Now, if we know this work done for an incompressible situation, we can write that Δp stage is nothing but ρ times this work done per unit mass, that means; you just multiply with ρ , with ρ you get the Δp per stage, total in the stage you find out the Δp ok fine. Now, static pressure rise is this one, so Δp static in the rotor, because this is in the rotor, Δp static in rotor is equal to ρ into let this part I denoted as X , so this part is denoting as X . So, if you find out this by putting this value, you get Δp the total stage that means, impeller and the diffuser total Δp stage, because the work is done only in the impeller Δp total is 221.11 ok Newton per meter square and Δp in rotor or static pressure rise static ok, so that is static pressure across the stage is 110.71 Newton per meter square. Now, here this Δp static divided by total one is known as, this is the static pressure rise in the rotor and this is the total pressure rise this is the total energy given ok. So, therefore the degree of reaction in this case we define as Δp stage sorry, Δp static this already I did earlier, Δp stage now, one thing just 221.11 divided by 110.71 oh sorry, it is just reverse 110.71 by 221.11, this will roughly 0.5.

Now, here this is very simple this is numerical but thing is that what is the concept earlier also, if you recall the earlier discussion that the degree of reaction was defined in terms of the total pressure change in the machine that is change in the total pressure in the machine divided by the change sorry, change in the static head or the static pressure in the machine divided by the total

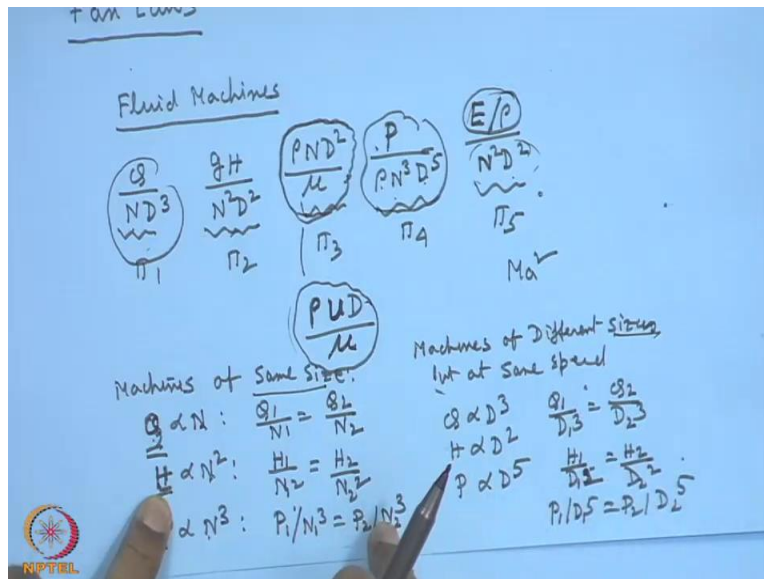
head or the total energy given in the machines. And this definition holds true for every machine in case of centrifugal compressor and axial flow compressor this was manifested in terms of the enthalpy change, where you consider along with the temperature change ok, but here what happens when you consider the flow to be incompressible with the constant density of air and there is no change in temperature. So, therefore this degree of reaction is better to be estimated by that definition, which you discussed in case of hydraulic machine that Δp is static divided by Δp stage ok,

So, this is the same so therefore we find this now next is the power required, what is the next one? The and the stage pressure rise degree of reaction and the power required to drive the fan, how to find out the power required? Power required is very simple, power required you can write here, power required is the mass flow rate into work per unit mass, that means; work per unit mass is this one and you can find out the mass flow rate, what is the mass flow rate? The flow rate is 0.6 kg per second so therefore, 0.6 kg per second, you know mass flow rate W by m that divided by the mechanical efficiency probably there is a motor efficiency of 80 percent, that means; you write the motor rating speed that means; you write the motor efficiency in the denominator, that means; you there the mass flow rate is given as in this problem 0.6 kg per second.

Now, W by m , which is calculated I have only calculated here, I have told you the value of Δp stage naturally W by m here, W by m calculated will be 184.26 joule per kg this is so therefore this would be 0.6 into 184.26 divided by 0.8, which is ultimately in 138.19 what, ok. So this is the thing, that means; this problem is looked from a different angle that work done per unit mass is true that is the Euler equation this can be split in this term for in an incompressible flow with a cost and density it is better to looked upon this way that this is the static head rise and this is the kinetic energy, sum of all these three is the energy in put to the machine and that is the total head that is raised in the fluid. So, this is the denominator their per stage and this is the numerator ok. So, therefore, the degree of reaction is this divided by this it was discussed earlier when I discussed the hydraulic machines in general this is the degree of reaction so this is brought out to be this numerically Δp stage, which is found out ρ times work done per unit mass that is the total pressure change in the machine and static pressure change in the machine is ρ times this part this is x again I repeat this part the static head that means, this is the energy per

unit mass this into rho is the delta p static so delta p static by delta p stage is this one is 0.5 clear. Now, after this I will tell you something else that what is fan laws.

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Because, sometimes you will see that people tell about fan laws, what is fan laws, this is the terminology actually. Now, in general for any fluid machines, when I discuss the similarity principles one of the earlier lectures in fluid mechanics from all the variables involved in a fluid machines in the physics of fluid machines rather I will tell, we derived by application of the Buckingham pi theorem different pi terms as the criteria of similarity parameters.

If you recall that see my earlier lecture note then you will find this terms are like this Q by one term is like this N D cube another term is g H by there are five terms N square D square another term is rho N D square by mu another term is p by rho N cube D five another term is E by rho divided by N square D square. So, these are all pi terms, this is let pi 1 this is pi 2 this was told at length in one of the earlier class there are pi 4 and 5. Now, when we derive the pi terms for centrifugal comprise as we had four pi term this is because this term we neglected effect of viscosity we did not take in fact the viscosity as less effecting fluid machines, why I will tell you now, earlier also I told, but these four terms we got, but not in this fashion in a different fashion I told you in earlier in last classes this is an essence or corollary of pi theorem that depends upon the choice of the repeating variable you add a number of terms at semi where some pi terms,

which may not match exactly the pi terms you want or the ρu express that the result then a combination of the pi term is also a pi term. So, different combinations can be made by making the numbers same so that you arrive at different pi term so therefore the pi terms of centrifugal complex as which you derived can be recoupled or rearranged to get the similar type of thing. So, this is a thing where earlier explained I am not going into detailed of it but this term I tell you as you know this term represent a sort of Reynolds numbers, because $\rho N D$ is the velocity $u D$ by μ that means, u is the peripheral speed that means it is the Reynolds number based on peripheral speed. So, it can be change to Reynolds number based on other flow velocity also physically it represent a sort of Reynolds number.

Now, in fluid machines the flow is highly turbulent, so in those turbulent flow the Reynolds number has got very less influence on the flow parameter, mainly in the special laws or any other parameter the Reynolds number does not have much influence. So, therefore, in centrifugal machines also we have not included the property μ to find out the dimensionless step. So, therefore, in the fluid machine earlier I also told this term is not of that relevant so only these terms are relevance. Now, if you take this term now, you see which is not only for fan for any fluid machines that, if you now this term is a non dimensional power ρ into D^5 this term you know that E by ρ is the square of the acoustic speed or speed of sound in the fluid medium related to the fluid and this is the square of the peripheral velocity.

So, therefore, this is some sort of square of the mach number, mach number square type of this. So, therefore, you see that now, if you see all this pi terms now you see that for any fluid machines, if we make the fluid machines same machines of same size then we can tell that Q is proportional to N , because D is same from this pi term, which gives $Q \propto N^1$ is $Q \propto N^2$ this is school level things, from the second pi term we see that head, because flow and the head is very important, what is head this is $g H$ that means; energy per unit mass head means energy per unit mass that means this is the energy either gained by the fluid or developed by the fluid that given air by the fluid depending upon whether is a compressor pump or turbaned. H is proportional to N^2 that means, $H \propto N^1$ square $H \propto N^2$ square. Similarly, now, if we take this term then we can write power is proportional to N^3 , which means that $p \propto N^1$ by N that means; this is the skilling long, $p \propto N^2$ by that means, we are interested in three quantity Q , H and p . And multiplication of this two is this one, so if it is proportional to N , this is N^2 square, it

has to be N^3 , that means; power is proportional to N^3 , head is proportional to N^2 , Q is proportional to N for same size and machines of different sizes, but at same speed, different sizes, but at same speed one can derive from this formula that Q is proportional to D^3 , H is proportional to D^2 , obviously, if you know these two you do not have to see the π term otherwise π term is wrong, that means; it has to be D^5 that means, Q_1 by D_1^3 is Q_2 by D_2^3 , H_1 by D_1^2 is H_2 by D_2^2 .

And similarly, p_1 by D_1^5 is p_2 by D_2^5 that means, this scaling loss with speed for the same size and with size, which is represented by the impeller diameter as a representative characteristics size for the same speed is known as this is this scaling based on the similarity parameters of fluid machines and valid for all fluid machines, but usually because this grow like this that initially fans and blowers and they were designs to make the proto type they use this type of scale so till date this scaling laws are known as fan laws in designing the fans. So, therefore, while starting the fans we must know what is fan law, fan law is nothing but this scaling law for the volumetric flow rate the head that means, energy per unit or energy per unit mass you can take gH does not matter gH is constant and the power with the speed for the same size and size for the same speed is known as fan laws.

So, I think today we will stop here and we have I like to close the lecture on this fans blowers and last class we discuss the axial flow compressor before that we discuss the centrifugal compressor and including all today I will stop or I will close my lecture series on fluid machines initially hydraulic machine basic principles of fluid machines hydraulic machines and then centrifugal compressor axial flow compressor and the fans and blower. So, next class we will start the compressor fluid.

Thank you.