

Introduction to Fluid Machines and Compressible Flow

Prof. S.K Som

Department of Mechanical Engineering

Indian Institute of Technology, Kharagpur

Lecture No. # 24

Axial Flow Compressor Part I

Good morning and welcome you all to this session of the course. Today, we will be discussing the axial flow compressor, last class we have more or less completed a brief discussion on this centrifugal compressors and now, we will be discussing axial flow compressor. The basic principle is mostly the same as that of a centrifugal compressor, what is the basic principle that by observing the mechanical energy from outside the working fluid, which is usually air gains its static pressure that is the basic purpose of a compressor whether it is a centrifugal or an axial flow compressor.

An axial flow compressor like a centrifugal compressor also consists of a rotor and a stator. Rotor is the rotating part of the compressor, where the mechanical energy is being supplied and it consists of a number of blades, which rotate and the mechanical energy is supplied in the form of an external torque under which there is a rotational motion of the rotating blades there are the blades, which rotate.

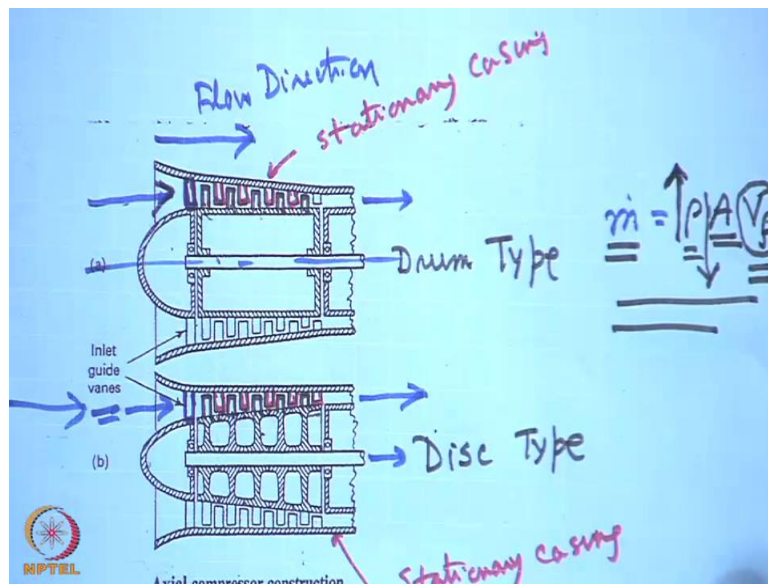
And the air acquires the energy and ultimately what happens its velocity and static pressure is increased and the stator part of the compressor, which is known as diffuser, the function of which is that here we gain the static pressure from the velocity of the air by allowing the air to have a decelerating flow in this passage it is known as diffusion process, the diffusion takes place, where we get more static pressure by virtue of the or of the expense of kinetic energy and finally at the outlet we get air at high pressure but at low velocity.

So, the basic principle is same but the major difference between the axial and the centrifugal compressor is that the flow takes place in the axial direction, whereas in the centrifugal compressor we have seen that flow takes place in the radially outward direction. So, the inlet and outlet of the flow whereas in the radial location and whereas in

the peripherals speed of the rotor, but here what happens the flow takes place in the axial direction. So, therefore, the inlet and outlet of the flow takes place at a particular radial location and all such radial locations the flow inlet is there and flow outlet is there that will be explained when I will explain through the diagram.

So, the major difference is the entire flow takes place in the axial direction and there are different number of stages, each stage consists of a rotor blade the rotor and a stator consisting of stator blades and this way a stage is considered that a rotor and stator and number of stages are there. This is the overall structure of an axial flow compressor that differs from a centrifugal flow compressor, which is a radially outward flow type it is an axial flow type through number of stages. At the same time you should know that these axial flow compressor the advantage your centrifugal compressor is that it can handle a large amount of at the flow rate is more as compared to the centrifugal compressor it can run more efficiently with high flow.

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So, therefore, at the high flow is required the axial compressor is more axial compressor is important high flow and also the weight is low ok. Now, let me explain the overall structure of an axial flow compressor, you see that this is an axial flow compressor, axial flow compressor. Now, you see an axial flow compressor, how does it look here. There are two types of axial flow compressor; one is the drum type, this is drum type, I tell you,

drum type, what is the difference I tell you, this is the disc type. Now, basic structure consists like that there is either a drum rotating drum this is the shaft or a disc on which this router blades are being mounted this is the router blades and these router blades this is the drum type is mounted on the rotating drum and with the rotation of the drum. For disc, in a disc type this is the router blades, this is the router blade so router blades rotate, so these router blades and number of blades are there along the periphery of the drum. So, at any axial location you see there are a number of blades, if you see this view front view, you see one blade there are number of blades in the periphery, which had being attached to either drum or disc known as disc type.

And there is a stationary casing this is the stationary casing I write, this is the stationary casing, here also this is the stationary casing and these stator blades are screwed or attached to this stationary casing these are the stator blades. And one router and one stator, one router and one stator comprises one stage so, here also the stator blades are fixed to the casing. And there is a inlet row of guide, when upstream of the first router this is actually not this is stator this is also a stator static part attached to that and these are known as inlet guide vanes, which first direct the air properly in the axial direction.

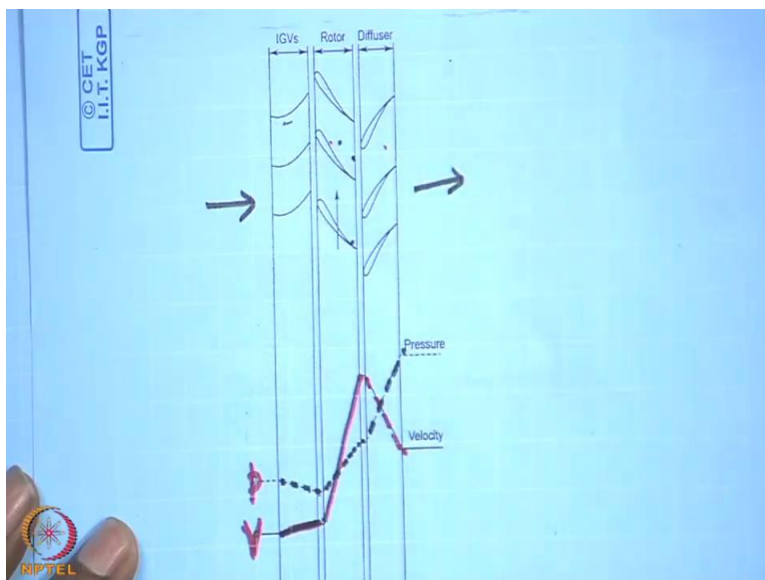
So, this is the flow, flow direction, this is the air flow like that, flow direction so this is the flow direction and this is the direction of the axis, this is the direction of the axis, the flow takes place in the axial direction. First it comes to the inlet guide vanes then through router and stator then router and stator a number of router blades number of stator blades and number of router blades number of stator blades and a combination of router and stator comprises the stage. This is an overall view of the axial flow compressor the air enters in this way; the air enters in this way, so therefore air flows through this annulus area formed by the blade passages. So, this is an overall ah diagram of an axial flow compressor air goes out.

So, axial flow compressors there are other things that I will tell you that while designing this annulus area it is made a convergent type that means the annulus area goes on decreasing in that direction of the flow, why? This is because, you know from continuity mass flow rate is written as mass flow rate can be written as ρ times the flow area times the velocity of flow if here. And this we will consider an average flow velocity over a

cross section and this is the annulus area the cross section the area provided to the flow and this is the average density at that section. Now, as the flow takes place here by the action of the rotor and stator the pressure increases this is the increasing direction of the pressure so density increases. So, for a given mass flow rate, because at steady state from continuity the mass flow rate will be same throughout the machine so, if you want to make the axial flow velocity constant we have to decrease the cross sectional area.

And the main motto is to make the axial flow velocity constant despite the increase of ρ for which we have to decrease the annulus flow area same for drum type or a disc type. This is one of the major considerations in the geometrical design of the compressors, the blade hide and other things are will come after wards, but this is one of the main considerations. So, this gives you a overall picture of an axial flow compressor, how does it look like, what is the basic direction of flow the flow takes place in the axial direction. Now, if we see the number of vanes now, before that we consider now, if you see from the top view then we will see a particular rho is like this including the guide vane, its look like this, you see.

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When you would look from the top ok, over the periphery then you will see that this is the direction that is these are the rotor, this is the rotor vane, this is the diffuser and this is the inlet guide vanes. Rather I think you can, yes, see this way, this will be the way

actually yes. So, therefore this at the number of blades that you can see or better you see this way know this way it is better. So, that these goes from these in this direction why they have made a flow direction like that I do not understand this direction will be here, I think the flow this is a peripheral direction the numbers of blades are there so this will go like this and will come like this, I think this is the direction, not this is the direction.

So, the flow takes place like that there are inlet guide vanes, which directs the flow in such a way ultimately it goes through the router series of router blades and the diffuser this is the diffuser where the velocity is decelerated and the pressure is increased. If you see this diagram you see that pressure in the inlet guide vanes decreases slightly, because of the skin friction, because the inlet vanes there is no energy interactions be the static vane so pressure lot takes place, because of the friction. Then in the router the energy interaction takes place it moves so therefore router imparts mechanical energy to the fluid by virtue of which its pressure its velocity and similarly, if you see the velocity, velocity should remain almost constant, but these inlet guide vanes cross sectional area is slightly converging type so that the velocity slightly increases.

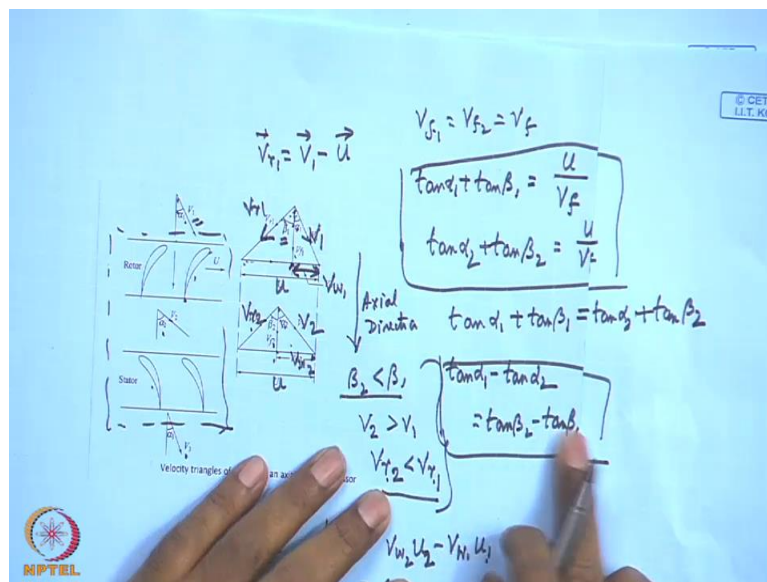
So, this is the velocity part, I just doing it by rate velocity then what happens by virtue of these mechanical energy given to it the both velocity and pressure increases this is the increase in the velocity, this is an increase in the velocity in the router, because it gains energy there and this is the increase in the pressure. And this is the static pressure, the increase in static pressure takes place, because there is an divergence in the area this flow passage area is made in such a way there is an increase in the static pressure, while it flows through that this is being reflected or manifested by a change in the relative velocity that I will tell after wards. Then it comes to a series of stator blades known as diffuser what happens in the diffuser? We get a rise in pressure that is known as difficult diffusion process in respect to this deceleration of flow that means the flow velocity is reduced.

So, this is a typical pressure velocity diagram, this is the pressure, this is the velocity diagram, while flowing through the router and diffuser blades and upstream this you consider the first row of router blade so the upstream of that there are inlet guide vanes ok. Now, if you consider you come to again this diagram that flow takes place at all radial locations now, if we consider the flow velocity V_f is uniform over all radial locations and

if we consider the flow at any mean height that means, at a mean height of the blade that a mean radius from the center, yet the flow is basically two dimensional, the flow has got an axial component and a tangential component, the radial component is relatively less. So, if the radial component is less, we can consider that two in a components of flow, mainly in the axial another is the tangential, which is perpendicular to this plane of the figure ok.

And at the same time, if you consider the weight at any cross section along the blade height ok, at different radial location, if you measure from the central line is constant that means, the uniform velocity distribution, which is also constant in the axial direction for which the annulus area is reduced that is different, but radially if we consider it is uniform and two component of velocity then it is almost a two dimensional. So, this is a two dimensional representation of a three dimensional flow.

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So, with this representation now, you can have a look of the velocity triangle. Now, if you see that the same thing that we already shown that the a blade passage formed by the rotor and the stator you see these rotor and the stator that means, this part if we draw like this that means, if we see from the top this side know oh, oh sorry, sorry, now, it is ok, now it is, if we see from the top the two rotors earlier things were visible know ok. So, two rotor blades and two stator blades just a representative figure, I show you this is what a representative blade passage found by two rotor blades and two stator blades of a stage.

Now, this is a stage now, what happens? The stage at the inlet to the stage the air comes with some velocity V_1 with an angle α_1 , which is nothing but the velocity coming out of the stator of the previous stage with some angle α_1 and this angle is made in the axial direction, this is the axial direction you understand this is the axial direction. So, this is the well axial direction ok.

Now, what happens? The rotor blade shape is like that and if you see the velocity triangle you see the velocity triangle is that here we can write that V_{r1} is the relative velocity as you know that is V_1 minus oh, not visible that will be a problem with this thing, they have not this is not actually made in this proper way V_{r1} is V_1 minus u . Now, another difference is this from the centrifugal compressor here the, if you see this again that the flow takes place axially, if we this diagram is drawn at a section taken at the mean radius mean height of the blade. So, therefore here what happens the inlet and outlet takes place at the same radial, we can take it at different radial locations. So, therefore when we see the picture from the top, we see that the inlet and outlet of the air flow takes place at a given radius. So, the peripheral speed of the rotor is same depending upon the radius at that point.

So, therefore u remains the same, you see the inlet velocity triangle is like that, this is the absolute velocity with which the air strikes the rotor blade with an angle α_1 with the axial direction. Then if you make the vector diagram, vector triangle diagram that velocity triangle then this is your V_{r1} , this is the V_{r1} , think you can see this is V_1 , this is V_{r1} and this is the u and this is the V_w1 this part, this is the whirl component or tangential component of velocity V_1 this is your velocity triangle. And this velocity, because of smooth flow without incident loss this β_1 should glide that means, should match the angle of the blade at the inlet and this is the β_1 , so, β_1 is the relative velocity angle, which is the angle of the blade at the inlet.

Similarly, if you see the outlet diagram, the outlet is made like this. So that the flow area increases in the direction of flow, this is the flow direction ok. So, this is the flow velocity V_{f1} , so at the inlet. Now, at the outlet, if you see the velocity diagram this is the relative velocity V_{r2} , this is V_2 , this is the same u and this is V_w2 ok, this is α_2 and β_2 . Now, β_2 is less than β_1 , this β_2 is less than β_1 fluid is directed more

towards axial direction this happens, because of the camber of the blade in a way that the annulus area increases ok. The annulus area that means these area sorry increases sorry decreases no sorry, annulus area increases, because of this increase in the velocity V_{r2} . So, what happens here you see that V_1 and V_2 that another important thing is that V_{r2} then V_2 , v_2 is more than V_1 , V_2 has to be more than V_1 , because in router the fluid gains the energy, but V_{r2} , V_{r2} is less than V_{r1} why? This is because there is an increase in static pressure, if V_{r2} has to be less than V_{r1} so this area has to be more. So, make the annulus area more the camber of the blade has to such that β_2 is less than β_1 , these are the important considerations of this velocity triangle.

Now, when it comes to the stator blades then what happens, this is the absolute velocity V_2 we take it comes makes an angle α_2 in the axial direction so stator blade does not move so there is no velocity triangle simply the velocity direction is change as per as the camber of the stator blade of the curvature of the stator blade. Here also the stator blade velocity is changed in such a way that α_3 is reduced from α_2 it is directed more towards axial direction and the design is made such way that α_3 becomes again equals to α_1 so that it can smoothly glide or enter to the router blade of the next stage so, we make this α_3 is equal to α_1 these are the important considerations. So, this show you can draw the velocity triangle at inlet and the velocity triangle at outlet of the router blades and this is the velocity how does it change in the stator blade ok. Now, we can write certain formula from this, but problem is that this cannot be shown here at the same diagram ok.

Let me adjust it, if you can see that thing this is the state now, if you can say, if you can see now, this blades you may not you may this cut so that you can hold it here. So, now, it will see this triangle what we can write from this now V_{f1} as I have told in the design is made V_{f2} and V_f that means, this axial flow velocity. Now, from the geometry of this $\tan \alpha_1$ is what this base divided by V_{f1} $\tan \beta_1$ is this base divided by V_f . So, if you add this two things then we get $\tan \alpha_1 + \tan \beta_1$ is equal to this plus this is the u , u by V_f because V_{f1} is V_{f2} is V_f , so this is I am writing V_f .

Similarly, from the outlet velocity triangle $\tan \alpha_2$, if you make the $\tan \alpha_2$ from the simple geometry $\tan \beta_2$ is equal to u by V_f ok, u by V_f . Now, if you equal make

this two equal then you can write $\tan \alpha_1 + \tan \beta_1$ is equal to $\tan \alpha_2 + \tan \beta_2$ ok, $\tan \alpha_2 + \tan \beta_2$ ok, that we can write or we can write $\tan \alpha_1 - \tan \alpha_2$ is equal to $\tan \beta_2 - \tan \beta_1$, this is one very important relationship, this two this relationship and the geometrical relationship, this two relationship we can get from the two velocity triangles ok. Now, if we come to the work here itself I can write now, work if you write here now, work per unit mass W by m work per unit mass is nothing but what is work per unit mass we know from the Eulers equation that $V w_2 - u_2$ that is $V w_1 - u_1$ alright, here u_2 is u_1 and that is equal to u . So, therefore it becomes equal to $V w_2 - V w_1$ into u , why? Because u_1 is equal to u_2 is equal to u ok.

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The image shows a handwritten derivation on a blue background. At the top right, there is a small logo for '© CET IIT, KGP'. The derivation starts with the equation for work per unit mass:

$$\frac{E}{m} = \frac{W}{m} = u V_f (\tan \alpha_2 - \tan \alpha_1)$$

$$= u V_f (\tan \beta_1 - \tan \beta_2)$$

Below this, the temperature rise is given by:

$$C_p \Delta T_{st} = u V_f (\tan \alpha_2 - \tan \alpha_1)$$

Then, the temperature rise is expressed as:

$$\Delta T_{st} = \frac{u V_f (\tan \alpha_2 - \tan \alpha_1)}{C_p}$$

Finally, it is written as:

$$= \lambda \frac{u V_f (\tan \alpha_2 - \tan \alpha_1)}{C_p} \quad \lambda < 1$$

Below the final equation, there is a note: $\lambda \rightarrow$ Work done factor.

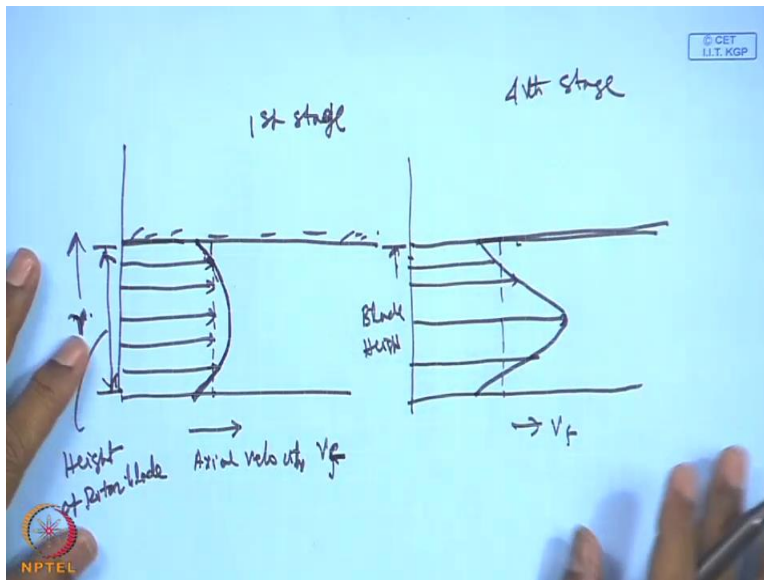
Now, we see that, if this is true then we can write W is equal to what is $V w_1 - V w_2$, let me see again now, $V w_1 - V w_2$ is, what is $V w_1 - V w_2$ from this diagram? Now, $V w_1$ can be written as the this one $V w_1$, $V f_1 \tan \alpha_1$ similarly, $V w_2$ is $V f_2 \tan \alpha_2$, $V f_1$, $V f_2$ is $V f$. So, therefore this equals to $V f \tan \alpha_1 - V w_2$ and $V w_2 = V f \tan \alpha_2$. So, if you follow this velocity triangle then you can write this is equal to $u V f (\tan \alpha_2 - \tan \alpha_1)$ again from the equality as I am done already that $\tan \alpha_2 - \tan \alpha_1 = \tan \beta_1 - \tan \beta_2$, so we can write $u V f (\tan \beta_1 - \tan \beta_2)$ ok, where from because, we had this thing earlier shown that, that earlier we shown this thing that actually this is a

problem here this is $\tan \alpha_2 - \tan \alpha_1$ minus $\tan \alpha_2$ is $\tan \beta_2 - \tan \beta_1$ ok.

So, therefore $\tan \beta_1 - \tan \beta_2$ tan other this is the work done per unit mass sorry, or energy added per unit mass whatever you can write it is your concept energy here. Now, if you consider that the change in the stagnation temperature, which we called it a total temperature, I give the normal temperature Δt that means change in the stagnation temperature of the compressor total change in that C_p is the total energy added to the air of the working in fluid per unit mass that we will equate this one $V_f \tan \alpha_2 - \tan \alpha_1$, I can write $\tan \beta_1 - \tan \beta_2$ tan does not matter either of these two I can write they two are equal. So, therefore the increase in the stagnation temperature is given by this formula, which is very important that increase in the $\tan \alpha_2 - \tan \alpha_1$ divided by C_p . Now, what happens is that, the actual stagnation temperature rise is less than this and this is taken care of by a coefficient λ first of all let me write that, which is less than λ , this is $\lambda < 1$, $\lambda < 1$ by this coefficient λ , which is known as the work done factor this is the λ , which is known as the work done factor, λ is work done factor.

Now, this is mathematical what is the physical meaning of that and this λ equals to one that means that the theoretically the energy per unit mass or work done per unit mass, which you calculated is not actually important to the liquid sorry, to the fluid to the air here a less amount is being imparted, because of this λ factor, which is less than 1, which is known as work done factor this is because of the fact that we assumed at the beginning this axial flow is uniform over the entire blade height this is not so and that creates a problem.

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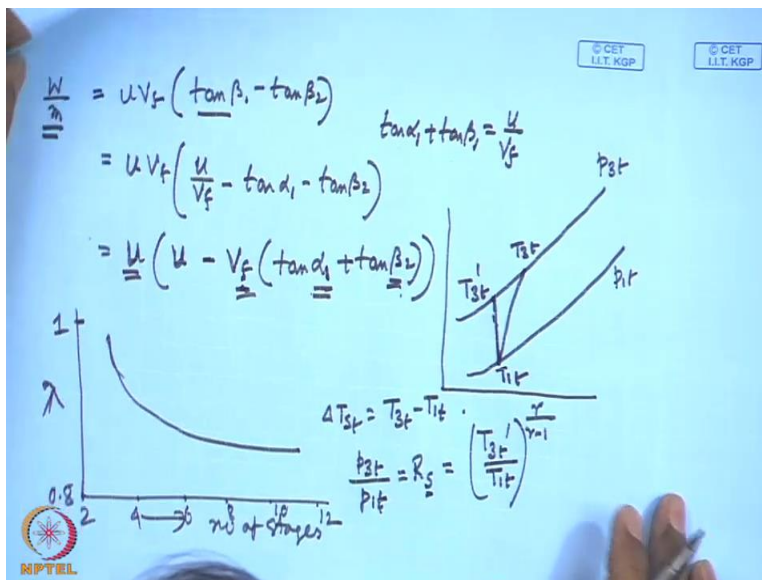
Now, let us see that thing that, if we consider the this way, if we consider the axial velocity that is V_f and if we consider this is the radial direction, this is the radial direction r and if we consider this as the blade height, rotor blade height this is the blade height, this is the height of rotor blade, height of rotor blade. Let us consider the first stage, now according to our two dimensional assumptions, we have considered that the this is the dotted line I am showing this is the axial velocity distribution, this is the axial velocity distribution over the blade height, this is the blade height here I cannot write, because this space is not there, so here it could have written the alternate is the height of the that is the radial direction and this is the height of the rotor blade, I just make a hatch like that for your understanding.

So, uniform distribution, but what happens in practice, because of the three dimensional effect the velocity distribution is not uniform, the flow velocity distribution this is the axial flow velocity V_f so, distribution become like this some peak be at the center and then again it reduces. So, this becomes the distribution of the axial velocity actual distribution becomes like that instead of this uniform one.

And if you go to further stage for example allow for example a fourth stage number of stages at there are fifth stage some represented blade height also will change as you go on that there is a decrease in the area however I show you with the same height fourth stage

for your understanding. So, if this is the V f, this is the blade height, the same diagram, but its look like that this is the uniform one the axial velocity, this becomes more peaky little this side and there. And after certain stage this diagram this velocity distribution, which is queued with a peak becomes almost same like the fully developed flow type, but because of this distribution not being uniformed what happens the work done per unit mass is changed, which is taken care of by the lambda. Now, after this I will show you that, you can have some idea that, if I express the work done per unit mass W by V f.

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Now, W per unit mass is now we can write W per unit masses u V f into tan beta 1 minus as I have done earlier tan beta 2 this I can write. Now, we know that tan alpha 1 plus tan alpha 2 is tan beta 1 plus tan beta 2 that we know or we can replace tan beta 1 another result I know that what is this, the tan alpha 1 again we know that tan alpha 1 plus tan beta 1 is u by V f that I know. Now, I can replace tan beta 1 in terms of tan alpha 1 and can get this expression u into now V f tan beta 1 I can write tan u by V f minus tan alpha 1 minus tan beta 2, this is the way I can write, why we will understand now, if you take u here then you can write u minus this V f cancels V f into tan now this tan alpha 1 and tan beta 2 this

α_1 and β_2 this is fixed this is the outlet angle of the router blades and this is the α_1 that means, this is the outlet angle of the stator blades or the angle of the absolute velocity at the inlet of the that is the inlet of the router blade ok. So, if these are fixed then it is seen that for a given peripherals b at any height the work done per unit mass depends upon V_f . so when there is a reduction in the flow velocity work done per unit mass is increased when there is an increase in flow velocity it is decreased. So, therefore with this, if you compare this you will see that it changes with the radial locations here in this radial locations here the work done per unit mass is decreased, why? in the work done per unit mass is increased, because of a reduction in V_f near the tip and the root, but the overall effect of this reduction of the work done per unit mass that already a locations where it is more than the axial velocity, because we have made the calculation based on the uniform axial velocity is count it counter ways the gain at the t root and the tip, because of the deduction in flow velocity V_f by this formula.

So, what happens? Finally the work done per unit mass based on the uniform flow velocity is being reduced by a factor known as work done factor clear. Now, this work done factor ultimately decreases with increase in number stages this work done factor a typical graph is like that, usually this value rise from 1, 0.8 to 1 and with number of stages let I start with stages 2 4 6 8 10 12 like this. The work done factor goes like that this for your idea this is the work done factor λ so more number of stages λ is less and less number of stages λ is high ok.

Now, we come to the pressure rise, pressure rise is very simple again in this page itself I will do. Now, if we consider that two pressures just like we did earlier that p_3 t and p_1 t, if we consider three at our this thing then what happens that this is our sorry, this is dotted this should be this is our isentropic one this is our actual one that means, if this is T_1 t this is T_3 t dash and this is T_3 t. So, our ΔT_s t is T_3 t minus T_1 t, but our pressure rise p_3 t by p_1 t, which usually we write here is the R_s here s suffix is given per stage, the per stage the pressure rises the pressure ratio is p_3 t not rises p_1 t that is related to this T_3 t dash by T_1 t to the power γ by $\gamma - 1$.

And again we know that again we know that θ_C ok, we know that the isentropic efficiency again it is stage efficiency, which is defined as T_3 t dash minus same thing I

am repeating again and again $T_3 - T_1$, which gives that $T_3 - T_1$ from here is $1 + \eta_s$ into $T_3 - T_1$ divided by T_1 . So, therefore we have to place it here so that we get R_s is equal to the same expression $1 + \eta_s$ and this is defined as ΔT_s so that it is expressed in term of this in terms of this thing T_1 to the power $\gamma - 1$ alright ok. Now, with this I tell you the overall principle of action of a stage of an axial flow compressor ok, what happens in the rotor and the stator.

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The image shows handwritten mathematical derivations on a blue background. At the top, there is a small diagram of a stage with a rotor and a stator. The first equation defines stage efficiency η as the ratio of rotor enthalpy rise to the total enthalpy rise (rotor plus stator). The second equation shows $\eta = \frac{(\Delta T)_A}{(\Delta T)_A + (\Delta T)_B}$ with a legend: A → rotor, B → stator. The third equation shows the work input $\frac{W}{m} = c_p \Delta T_{st} = u v_f (\tan \alpha_2 - \tan \alpha_1)$, where $\Delta T_{st} = \Delta T_s$. An NPTEL logo is visible in the bottom left corner.

$$\eta = \frac{(\Delta h)_{\text{rotor}}}{(\Delta h)_{\text{rotor}} + (\Delta h)_{\text{stator}}} = \frac{(\Delta T)_{\text{rotor}}}{(\Delta T)_{\text{rotor}} + (\Delta T)_{\text{stator}}}$$

$$\eta = \frac{(\Delta T)_A}{(\Delta T)_A + (\Delta T)_B}$$

A → rotor
B → stator

$$\frac{W}{m} = c_p \Delta T_{st} \quad \Delta T_{st} = \Delta T_s$$

$$= c_p \Delta T_s = u v_f (\tan \alpha_2 - \tan \alpha_1)$$

Now, we come to a thing, which is known as degree of reaction, which is very important degree of reaction, what is meant by degree of reaction? Now, try to understand one thing again I will repeat the thing which I told you earlier in the fluid machines class when I discussed the hydraulic machines the machines using water. Now, one has to understand that any fluid machines has a rotor and a stator ok, whether it is turbine or it is a compressor or pump ok. So, the basic purpose of the stator or the diffuser in a pump or compressor is to change the velocity to static pressure, but the question comes whether in a rotor the static pressure will change or not, first of all try to understand in terms of pressure, static pressure will change or not. So, static pressure will change in the rotor depending upon the rotor construction, if it is a radial flow machines, machine type rather I will tell machine type, that means; the radial flow machine automatically the pressure changes, because of the centrifugal action of the centrifugal head that is centrifugal force

when the radial location changes the peripherals its speed is changing. So, therefore this is manifested in terms of the increase in a static pressure, I explain so many times that a radial pressure gets imposed when the fluid has a tangential velocity I mean it changes its radial location, but in an axial flow machines, when there is no change in the tangential flow from inlet to outlet not necessarily there will be a change in the static pressure.

The change in the static pressure will depend upon the router design and construction that means, if you have to change the flow area, if you change the velocity relative to the router in flow course of flow to the router passage then only there will be change in the static pressure. And that is the measure of reaction in a reaction turbine that whether there is a change in the static pressure in the router itself or not, for example; in impulse turbine if you remember water turbine, the pelton turbine the water jet is striking the router that is the pelton wheel at atmospheric pressure, throughout pressure is same there is no change in the pressure this is known as impulse turbine, where as impulse turbine when it goes through the runner blade there is a change in the pressure.

Similarly, in the centrifugal pump always there will be a change in pressure in the impeller this is because of the radial flow in a rotating or tangential flow field similar is the case of centrifugal compressor, but in axial compressor the question comes whether the flow passage changes in course of flow through the router so that the relative velocity changes or not. I told you that, if the relative velocity changes in a sense that, if V_{r2} is less than V_{r1} then we can consider that there in we think that there is a change in the static pressure and when there is a change in the static pressure there will be change in the static temperature also. So, therefore whether there is a change in the static pressure or not that we will call the machine is reaction type or not usually there is a change in static pressure and the static temperature while flowing to the router V_{r2} is made less than V_{r1} . And therefore the question of reaction comes and the degree of reaction in this context is defined as this way the in terms of enthalpy it is defined the change in enthalpy.

Let us consider per unit mass in the router divided by the change in the stage Δh_{router} plus Δh_{stator} this is known as Δh_{stage} . Since the enthalpy change for an ideal gas is equal to this C_p into change in the temperature and since C_p is constant here we have considered the ideal gas it is independent of the temperature we can cancel it out and

reality also the C_p does not vary much with the range of temperature this can be written in terms of ΔT the static temperature change in the rotor divided by static temperature change in the stage that means in rotor plus ΔT stator.

Now, this is the degree of reaction when there is no change in static temperature or static pressure degree of reaction is zero. so degree of reaction is zero for a reaction machine there will always a degree of reaction since ΔT rotor has got a value more than zero so this is the measure of the extent by which the total the fraction by which the total change that is total change in the static temperature of this stage is taking place to the rotor ok. Now, with this definition let us now see the how we can find out an expression for this.

Now, let us consider now again the definition since we give this s for stage and if rotor and stator this things we will not write so simply we tell that A stands for stator for our convenience and B stands for rotor and we write this definition as ΔT this is a for example, you write sorry, this you write rotor, this will be better and this you write stator then you write $\Delta T_A + \Delta T_B$ ok.

Now, you see that work done per unit mass is nothing but C_p into ΔT stagnations now, if we make V_1 is V_3 now you see this diagram, we told that α_3 is α_1 , but in our design, if V_3 is made V_1 now see that it approaches with some V_1 from the earlier stage then while it passes through the rotor it gains energy and this V is increase V_2 is greater than V_1 . Then these V_2 is again by the diffusion process that means, these is decelerated to get a rise in static pressure is that means V_3 is less than V_2 is less than that. And it is also displaced more towards axial direction this deceleration is made in such way V_3 comes back again to the original V_1 , if we make it designed like that. That the absolute velocity at the discharge of the or at the outlet of one stage becomes equal to that of its inlet velocity at this stage you understand. So, that we can write V_3 is V_1 in that case we can write ΔT that means, what is T total t stagnation temperature at 3 for example; $T_3 t$ now I write the total temperature T here you see then $T_1 t$ is T_1 static plus V_1 square by $2 C_p$ similarly here you see $T_3 t$ the total temperature is T_3 plus V_3 square by $2 C_p$.

So, if V_1 is V_3 that means the dynamic equivalent temperature ok that means, the

velocity equivalent temperature has same, that means the difference in the total or stagnation temperature is difference in their static temperature. So, that ΔT_s that is the stagnation temperature is ΔT_{static} and here s is written that is per stage that means ΔT_{stage} that means this is ΔT_{stage} that is static temperature per stage of the stage is equal to the stagnation temperature that means this can be written as ΔT_s and this is nothing but the work done formula that means, if you remember that this what is this formula $u V_f \tan \alpha_2 - \tan \alpha_1$. So, therefore $C_p \Delta T_s$ is given by this correct, bracket is there, there will be a bracket now, this is correct. Now, how to find out this for ΔT_A ? So, therefore now, I find out the value for ΔT_A .

(Refer Slide Time: 45:25)

The image shows a whiteboard with handwritten mathematical derivations. The top equation is boxed: $\frac{W}{m} = u V_f (\tan \alpha_2 - \tan \alpha_1) = C_p \Delta T_s$. To the right, it says $\sec^2 \alpha - \tan^2 \alpha = 1$. Below the boxed equation, the derivation continues: $u V_f (\tan \alpha_2 - \tan \alpha_1) = C_p \Delta T_s + \frac{1}{2} (V_2^2 - V_1^2)$. Then, $C_p \Delta T_s = u V_f (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} (V_2^2 - V_1^2)$. Finally, it uses the identity $\sec^2 \alpha - \tan^2 \alpha = 1$ to get $C_p \Delta T_s = u V_f (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} V_f^2 (\tan^2 \alpha_2 - \tan^2 \alpha_1)$. There are logos for CET IIT KGP and NITRR in the corners.

Now, router the energy is actually given in the router so W by m is written in terms of the total one that means, $u V_f$ again I am writing $\tan \alpha_2 - \tan \alpha_1$ for your convenience $\tan \alpha_2 - \tan \alpha_1$ and this becomes is equal $C_p \Delta T_s$ static temperature rise per stage, because the static temperature rise equal to the stagnation temperature. Now, if I write this W by m that is $u V_f \tan \alpha_2 - \tan \alpha_1$ in terms of the static temperature change of the stator I am not permitted, because stator static temperature so, entire energy used not to increase static temperature, but at the same time to increase its velocity V_2^2 by V_1^2 . So, an energy balance in the router gives that router energy takes and the air static temperature is increased this is the energy change due to increase in static

temperature that is the static enthalpy rise plus the kinetic energy change. So, two summation of this two from a study flow energy equation is the energy input per unit mass bases so this is that. So, therefore we can write $C_p \Delta T_A$ is equal to $u V_f \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} V_2^2 - V_1^2$.

Now, you see $V_2 - V_1$ from the velocity triangle now from the velocity triangle, if you recall that V_1 is what? If you see this triangle so this is α_1 in terms of α_1 the cosine of α_1 is which one V_f by V_1 . So, V_1 is V_f by $\cos \alpha_1$ and V_2 is V_f , V_f same as I have told earlier by $\cos \alpha_2$ that means, this is V_f by $\cos \alpha_2$. So, with this thing I can write this equal to $u V_f \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} V_f^2$ I take V_f common V_f^2 into 1 by $\cos \alpha_2$ that means $\sec^2 \alpha_2 - \sec^2 \alpha_1$, I think I am correct $\sec^2 \alpha_2 - \tan^2 \alpha_2 = 1$, that relationship at the school level you know so therefore, if you use that these becomes $u V_f \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} V_f^2$ then this is also $\tan^2 \alpha_2 - \tan^2 \alpha_1$. Now, I use these expression for $C_p \Delta T_s$ that is stage and these expression $C_p \Delta T_A$ for the rotor the stage means $\Delta T_A + \Delta T$.

(Refer Slide Time: 49:06)

Handwritten derivation on a blue background:

$$\omega = \frac{(\Delta T)_A}{(\Delta T)_S}$$

$$= \frac{u V_f (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} V_f^2 (\tan^2 \alpha_2 - \tan^2 \alpha_1)}{u V_f (\tan \alpha_2 - \tan \alpha_1)}$$

$\tan \alpha_2 = \tan \alpha_1$

$$= 1 - \frac{V_f}{2u} (\tan \alpha_2 + \tan \alpha_1)$$

$\frac{u}{V_f} = \tan \alpha_1 + \tan \beta_1$

$\frac{u}{V_f} = \tan \alpha_2 + \tan \beta_2$

$$\frac{2u}{V_f} = \tan \alpha_1 + \tan \alpha_2 + \tan \beta_1 + \tan \beta_2$$

$$\omega = \frac{V_f}{2u} (\tan \beta_1 + \tan \beta_2)$$

Then I write can write the omega, which is equal to ΔT_A by ΔT_s as c_p, c_p

cancels so therefore this becomes equal to $u \sqrt{v} \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} \sqrt{v} \tan^2 \alpha_2 - \tan^2 \alpha_1$ divided by the total stage, which we derived earlier $u \sqrt{v} \tan \alpha_2 - \tan \alpha_1$ this can be written as $1 - \frac{1}{2} \sqrt{v}$ then $\sqrt{v} \sqrt{v}$ will cancel that is \sqrt{v} by $2u$ and this will be $\tan \alpha_2 + \tan \alpha_1$ ok, now, this is the definition ok.

Now, this $\tan \alpha_2 + \tan \alpha_1$ so this is the definition of final definition of the one, but now, if we change it to $\tan \beta_1$ then what we call that we already have found out these expression, if we see this expression at the beginning, which we found out that $\tan \alpha_1 + \tan \alpha_2 - \tan \alpha_1$ is equal to $u \sqrt{v}$ ok that what we did just a minute u by \sqrt{v} by u is what is that one the very beginning, which we did if you see that thing at the beginning u by \sqrt{v} , $u \sqrt{v}$ is $\tan \alpha_2 - \tan \alpha_1$ and $\tan \alpha_2 - \tan \beta_1$. So, if $\tan \alpha_2 + \tan \alpha_1$, if you remember this u by \sqrt{v} again I am writing $\tan \alpha_1 + \tan \beta_1$ from the geometry that is from this, this one, we initially write yes, yes this one, if you add this two u twice u by \sqrt{v} is $\tan \alpha_1 + \tan \alpha_2 + \tan \beta_1 + \tan \beta_2$. So, again I am going to write that for your benefit $\tan \alpha_1 + \tan \beta_2$ so therefore we write twice u by \sqrt{v} from here is $\tan \alpha_1 + \tan \alpha_2 + \tan \beta_1 + \tan \beta_2$.

Now, if $\tan \alpha_1 + \tan \alpha_2$ is substituted or is just eliminated in terms of $\tan \beta_1 + \tan \beta_2$ from this that means $\tan \alpha_1 + \tan \alpha_2$ is written twice u by \sqrt{v} minus this, if you write here then finally you get an expression for the degree of reaction, you get finally an expression if do it. It is very simple there is nothing difficult you get an expression is equal to \sqrt{v} by $2u$ into $\tan \alpha_1 + \tan \beta_2$ that means, simply you make $\tan \alpha_1 + \tan \alpha_2$ u by \sqrt{v} minus $\tan \beta_1 - \tan \beta_2$ and then make it clear $1 - 1$ will cancel so you will see a resultant like that this is the final expression for the degree of reaction. Now, let us make an case study that, where we get a 50 percent degree of reaction.

(Refer Slide Time: 52:59)

$$\frac{u}{V_f} = \tan \beta_1 + \tan \beta_2$$
$$\frac{u}{V_f} = \tan \alpha_1 + \tan \beta_1$$
$$\frac{u}{V_f} = \tan \alpha_2 + \tan \beta_2$$
$$\alpha_1 = \beta_2$$
$$\alpha_2 = \beta_1$$

That means 50 percent or half of the total enthalpy or static temperature rise of the stage takes place in router in that case we get a very good result that u by V_f is equal to $\tan \beta_1$ plus $\tan \beta_2$ now this is one very very important result. Now, if you compare this result with this results u by V_f is $\tan \alpha_1$ now you compare this with the result earlier that means this one u by V_f is $\tan \alpha_1$ u by V_f is $\tan \alpha_1$ plus $\tan \beta_1$ and that just now I wrote earlier also I wrote this is simply from the geometry. So, with this three relationship you get very beautiful result, when you equate this with this that means we get α_1 is β_2 and if we equate this with this then we get α_2 is equal to β_1 and as we told earlier that α_1 is α_3 that α_1 is α_3 then that becomes equal to α_3 . So, this is one very important result, what does it keep this important result gives, if I write here that this results if I write here α_1 is β_2 is α_3 and α_2 is β_1 then you see what we get? α_1 is β_2 , β_2 is the outlet angle of the blade rotating blade router blade and that becomes equal to α_1 , what is α_1 ? α_1 is this one, which is α_3 that means outlet angle of the stator blade that means, outlet angle of the router blade equals to the outlet angle of the stator blade. Again β_1 β_1 is the inlet angle of the stator blade, which becomes is equal to α_2 , α_2 is the absolute velocity angle at the outlet of the router, which exactly equals to the inlet angle of the stator blade that means the inlet angle of the router blade is equal to the inlet angle of the stator blade. So, therefore inlet angle of this router blade is equal to the inlet

angle of the stator blade, outlet angle of the rotor blade is equal to the outlet angle of the stator blade. So, they are cambered in opposite direction, but their inlet and outlet angles are the same. The inlet angle of one blade is equal to that of the other and the outlet angle of one rotor blade is equal to that of the stator blade, this type of design of blade is known as symmetrical blading and it is easy for construction. So, therefore we see for a 50 percent degree of reaction 0.5 we get a symmetrical blading, that means, the inlet angle of the rotor and stator blades are the same similarly the outlet angle of the stator and rotor blades are the same.

Thank you, today up to this.