Introduction to Fluid Machines and Compressible Flow

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Lecture - 23

Centrifugal Compressor Part III

Good morning and welcome you all to these session of the course. Today, we will discuss the compressor characteristics. Now, a compressor characteristics or performance characteristics of compressors are usually expressed in terms of the ratio of this stagnation pressure and temperature.

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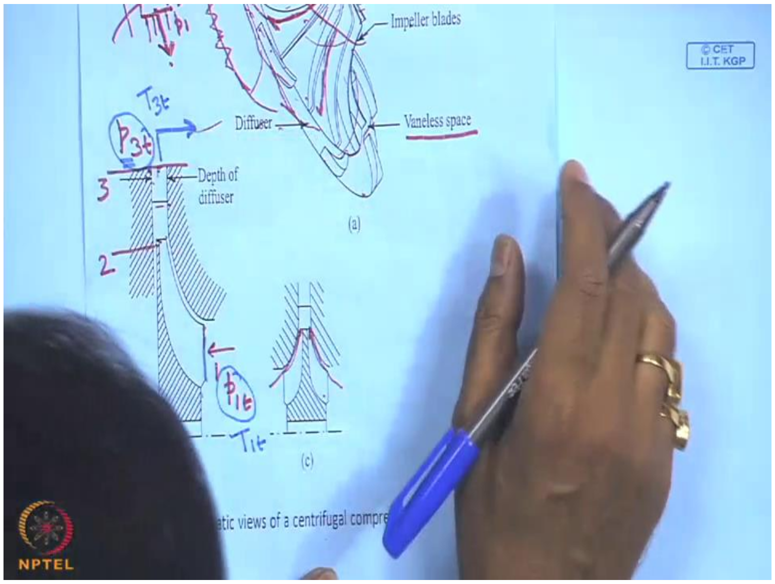

Now, let us first having a look that how do you make in naminculture. This is the stagnation pressure at inlet to the compressor and this is the outlet from the compressor, you will use the same naminculture that is the outlet of the diffuser. And similarly the stagnation of total temperature is T 1t and the T 3t is the outlet temperature, total temperature.

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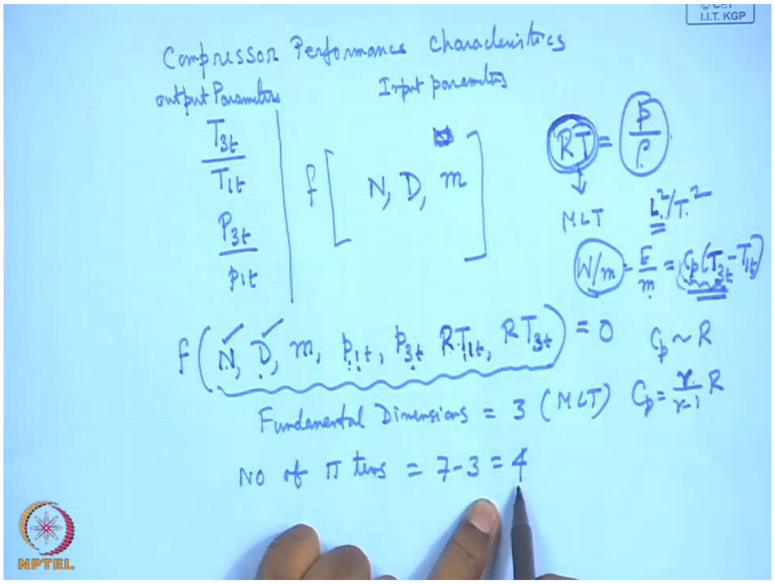

Compressor Performance Characteristics

output Parameters

Input parameters

$$\frac{T_{3t}}{T_{1t}} \quad \frac{p_{3t}}{p_{1t}} \quad f\left[N, D, m\right]$$

$$\boxed{RT} = \boxed{\frac{p}{\rho}}$$

MLT $\quad L^2/T^2$

$$\boxed{W/m} - \frac{E}{m} = C_p(T_{3t} - T_{1t})$$

$$f\left(N, D, m, p_{1t}, p_{3t}, RT_{1t}, RT_{3t}\right) = 0 \qquad C_p \sim R$$

Fundamental Dimensions = 3 (MLT) $\quad C_p = \frac{\gamma}{\gamma - 1} R$

No of π terms = 7 - 3 = 4

So we this naminculture the compressor performance characteristics, today we will discuss compressor performance characteristics. So, performance characteristics for a centrifugal compressor usually expressed in terms of the ratios of the total temperature or stagnation temperature. These are output parameters as a function of input, which is output parameters and this is input parameters usually this is done input parameters, this is output parameters ok. As a functions of N the rotational speed sorry, here I can write N the rotational speed, the size of the compressor D, the mass flow rate m, so this speed, damn size of the compressor and the mass flow rate. These are the parameters, input parameters and the ratios are expressed in terms of this parameter, the speed, the size and the mass flow rate.

So, now, you see that this can be expressed in a functional relationship like this, that the function of if you think in terms of the functional relationship, we can write this N D m, separately p 1 t, p 3 t, this case of this temperature we include R T 3 t. And these are the variables, which define the performance of a centrifugal compressor now, let me explain first. These are input parameters are I have told that this is the rotational speed, this is the size this is the overall diameter of impeller, this is the mass flow rate m, this is total pressure at the elect to the compressor this is the total pressure at the outlet of the compressor, outlet from the diffuser and this is the total temperature on inlet total temperature of outlet and they are multiplied with our they are because of the two things. It is multiplied with our T has a fundamental dimension temperature but, if you multiply with R T becomes equal to p by row and each dimension as a whole can be expressed in terms of M L T, because p by row is v square, L square you can very well know that p by row is L square v square. So, therefore if you find out the dimension of R T becomes v square L square the means p by row is v square the dimensions voice ok dimension voice this dimension is v square, L square by T square sorry, L square by T square. So, therefore it is L square by T square so, there could multiplying with R taking care of the R as a whole to reduce the fundamental dimension then at the sometime taking care of that physical concept that T 1 t and T 3 t are very important parameters describing the centrifugal pump performance.

Again another logic is there that you know that work done per unit mass or energy added per unit mass is giving by changing C p times this c p times T 3 t or T 2 t whatever you call, that means; it is the C p times the t and Cp is what? Cp is proportional to R that means; C p in case of specific heat at constant pressure, it is gamma by gamma minus 1 into R, gamma is specific heat ratio, that means it takes care of R that means; t alone has got no function, if you take multiplied with C p, C p T is the index of the energy, C p this T 3 t minus T 1 t is the work or energy input. On the other hand R T if we take together is reduces the fundamental dimension by one and things become little simple. So, therefore, if you do this way you can now explain that the entire fin, another thing very important that why you at not consider density? Because it is a compressible fallowness is from the beginning, I am telling that the densities very important in the compressible formation, but I am not including the density, density is implicitly included, because p is included R T is included so their ratio is the density. So, therefore, density is not included explicitly, it is implicit.

So, therefore, all the variables describing the centrifugal performance are there, how many one, two, three, four, five, six, seven and fundamental dimensions are how many? Fundamental dimensions are three, that is equal to 3, that is M L T, since we have considered the product R T. So, therefore, by Buckinghams pi theorem, number of pi terms will be 7 minus 3, 4, why you are doing this dimension analysis backing by applying Buckinghams pi theorem, because we want to express this relation in terms of non dimensional variables rather than dimensional variables, you should be reducing the number from 7 to 4. Now, by applying the standard proceed your of dimensional analysis what to we do it take three repeating variables, here what are the repeating variable? We take the repeating variables as D, we take the repeating variables as p 1 t and we take the repeating variable as R T 1 t, this D, p 1 t and R T 1 t are taking as the repeating variables, they are taken as let say that is repeating variables, they are taken as you can see repeating variables. And following the dimensional analysis with this three as the repeating variables, if we combined with N, then you gate a pi term that is your task you can do N D divided by root over R T 1 t time now writing this that means this takes the N. Now, the D is the repeating variable when it combines with m, this three repeating variables D, p 1 t, R 1 t then the second pi term comes as m root over R T 1 t divided by D square into p 1 t ok.

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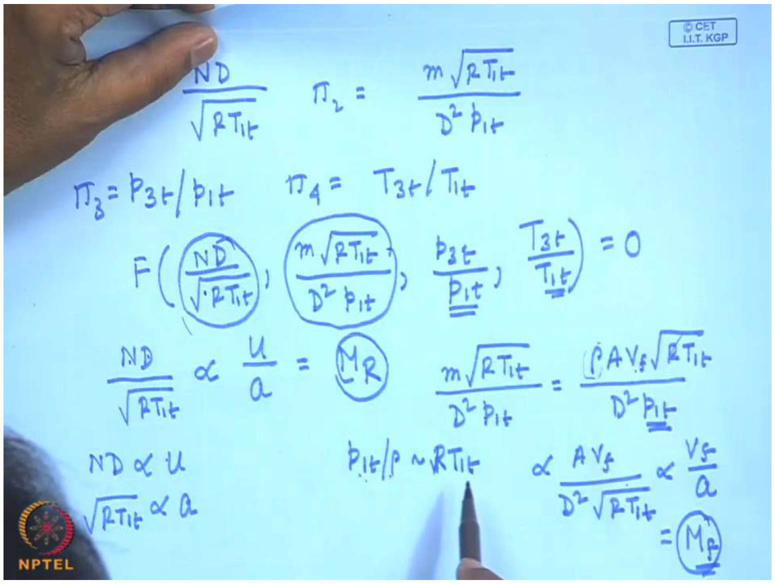

$$\frac{ND}{\sqrt{RT_{1t}}} \qquad \Pi_2 = \frac{m\sqrt{RT_{1t}}}{D^2 p_{1t}}$$

$$\Pi_3 = p_{3t}/p_{1t} \qquad \Pi_4 = T_{3t}/T_{1t}$$

$$F\left(\boxed{\frac{ND}{\sqrt{RT_{1t}}}}, \boxed{\frac{m\sqrt{RT_{1t}}}{D^2 p_{1t}}}, \frac{p_{3t}}{\underline{\underline{p_{1t}}}}, \frac{T_{3t}}{\underline{\underline{T_{1t}}}}\right) = 0$$

$$\frac{ND}{\sqrt{RT_{1t}}} \propto \frac{u}{a} = \boxed{M_R} \qquad \frac{m\sqrt{RT_{1t}}}{D^2 p_{1t}} = \frac{\rho A V_f \sqrt{RT_{1t}}}{D^2 \underline{\underline{p_{1t}}}}$$

$$ND \propto u \qquad p_{1t}/\rho \sim RT_{1t} \qquad \propto \frac{AV_f}{D^2\sqrt{RT_{1t}}} \propto \frac{V_f}{a}$$

$$\sqrt{RT_{1t}} \propto a \qquad = \boxed{M_f}$$

Then if you take this p 3 t is the 1 then automatically when obviously when one of the repeating variables at the same dimension it is a thumb rule and if you do it we will get that p 3 t by p 1 t is the pi 3 and similarly pi four, four pi term, which will be found out we what is that? p 3 t then T 3 t then we this we will get since T 3 t and T 1 t at the same dimension, we automatically gave T 3 t by T 1 t, this is by thumb rule it will always come, if you follow this and dimension analysis, we will find the same thing ok. Now, therefore, the equation can be retain as some functional relationship of the non dimensional term, that means; N D by root over R T 1 t, m root over R T 1 t divided by D square p 1 t, p 3 t by p 1 t, T 3 t the ratio of total pressures and the ratio of total temperature is equal to zero.

Before proceeding for that I like to tell you that these two are clear they are the ratio of total pressure and total temperature, these two have some physical significance for example, this pi 1 N D, what is the physical significance, R T 1 t, what is the physical significance? Now, N D is proportional to the tips speed of the impeller, the rotational speed into the impeller diameter and root over R T 1 t, I told you the D sound speed is giving by root over gamma t so its proportional to this sound speed, acoustic speed in the medium relating to the flow. So, therefore, this is proportional to u by a and which is known as mac number based on routers speed. So, therefore, this pi term signifies physically some sort of mac number based on rooters speed. Now, this pi 2, if he write here that pi 2 m root over R T 1 t divided by D square p 1 t now this can be retain it can be retain in terms of slow velocity and the density rho, the flow area A and the flow velocity V f, root over R T 1 t divided by D square p 1 t.

Now, one can write p 1 t by row is proportional to root over a proportional to R T 1 t, p by row is R T, so though this is total pressure and this is the density. So, therefore, it is R T 1 t so therefore these can be written as by cancelling that this is proportional to A V f then this will be cancelling out D square root over R T 1 t, because this square root and this is under root, this is not under root, so root over these. So, therefore, this tool be proportional to V f by again A that means; this is mac number based on flow velocity. This is known as flow mac number, this is known as rooters speed mac number that means; physically this pi term represents a mac number based on rooters speed velocity and this pi term represents a mac number based on flow velocity, this is just for your physical implication. Now, one thing that, if we express this relationship try to express for A particular machine then D is not necessary to be included, D is constant, you can drop D. And more over it is for a particular gas, for example the T R then R also can be on it here R is there, here R is there, so therefore, for a giving machine we take giving gas the same relationship, which is used non dimensionally this can be retain a some other function, for example the function, a function of N by root over T 1 t then these will be m root over T 1 t follow it clearly by p 1 t, p 3 t by p 1 t, T 3 t by T 1 t.

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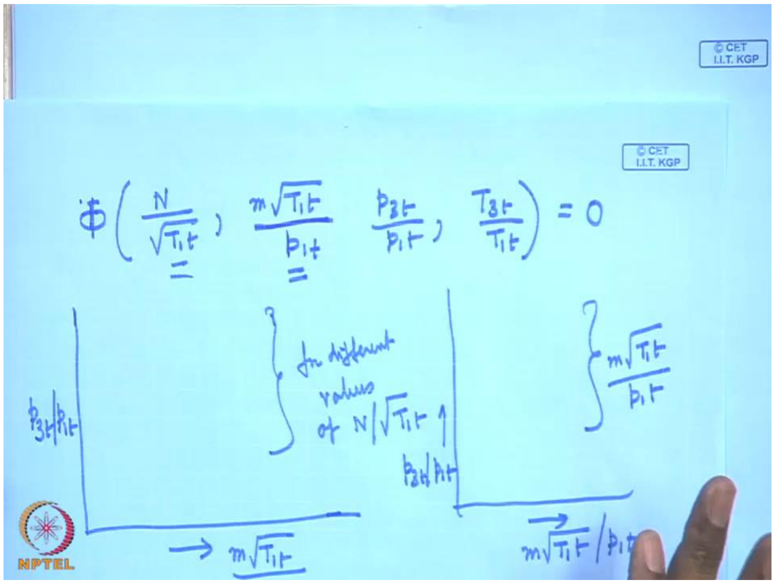

$$\Phi\left(\underline{\frac{N}{\sqrt{T_{1t}}}},\ \underline{\frac{m\sqrt{T_{1t}}}{p_{1t}}},\ \frac{p_{3t}}{p_{1t}},\ \frac{T_{3t}}{T_{1t}}\right) = 0$$

And for a giving machine for a giving gas the relationship can be expressed like that. Now, the see two I am not truly dimensionally so truly non dimensional, because who have drop the term R and D, but what happens is that even if their dimensional term but we take the help of the non dimensional this is to reduce the number of terms so the number of variables are reduced were some of the variables, which combines other primary variables need not be non dimensional but does not matter for giving size and for giving fluid, we can use these other functional relationship of the performance. So, usually what happens is that the performance now this express like this that a families of card is generated as the ratio of the pressure with the I will not tell this is non dimensional mass flow rate this is normalized mass flow rate the word normalized does not mean non dimensional different families of term I am not joining the card present different families of cards for different value, for different values of N by root over T 1 t one family, another family is the different value the same thing, the same thing this m root over T 1 t by p 1 t. And here is the ratio p 3 t by p 1 t with the different values of m root over T 1 t by p 1 t that means; two families of curves for both the ratio of total pressure then total temperatures for different parametric values for each family of the normalized rotational speed and normalized mass flow rate. This is basically the way the performance parameters or performance characteristics of a centrifugal palm is expressed.

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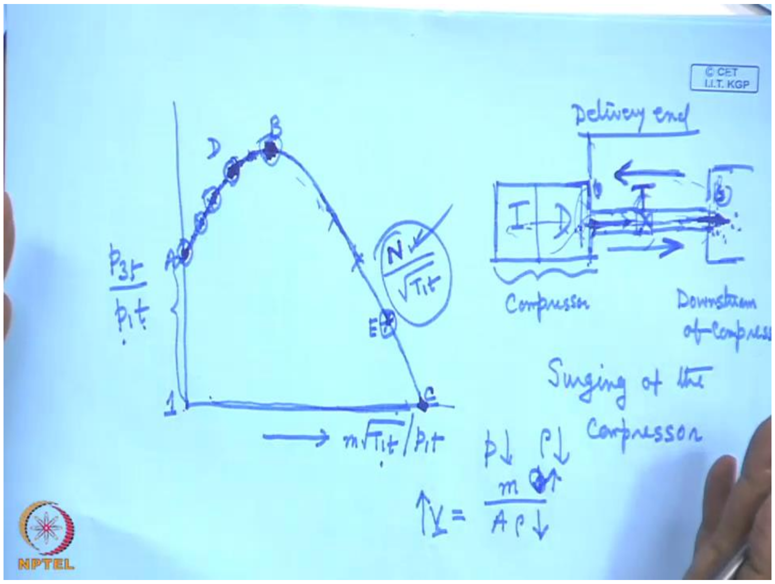

Now I will show you how does it look in case of a pressure ratio. Now, I will show you there the very important curve p 3 t that is the pressure ratio p 1 t versus the non dimensional mass flow normalized sorry, normalized mass flow p 1 t the curve looks like this I tell you the curve looks like this, the curve looks like this. I will explain let me first draw the label the curve now, I explain the three points that important point in understanding this. Now, what happens the if you make an experiment and draw the points we will get a curves like that initially increases with the positives slope which is a maximum then it has a negative slope continuously decrease and probably at high values mass flow rate it touches the axis of when the pressure ratio is one actually the pressure ratio starts from 1 ok. Now, try to understand physically the fact that when the mass flow rate is zero there is a pressure ratio, why? This is because in a centrifugal pump try to understand when the mass flow rate is zero, means; that you stop the delivery valve here then what happens? The impeller goes on rotating so therefore a centrifugal head or the energies important on the fluid in terms of a pressure raised. So, therefore, the pressure raise will take place in the impeller because of the centrifugal action, which we called as centrifugal ladies impressed on the fluid is imposed on the fluid, so fluid may not moved in the diffuser there will be a static fluid a static field is there pressure field and that pressure is due to the chanting action of the fluid in the impeller, which imposes a static pressurize, because of the centrifugal action, this will called as centrifugal head, that means; the centrifugal head, because of the rotation of the impeller is imposed on the fluid even if the valve is closed here so a pressure ratio will be developed. So, that is the pressure ratio by the action of the impeller rotation, which is show here at this pointing.

Now, when we slowly open the valve of the delivery line then what happens the flow commends is and when the flow commons is again you see that the flow takes place through the diffuser when so and also the vein less place now, when the diffusion process takes place to vein less place and the diffuser means; as a whole which is shown in the diffuser then what happened again pressurize takes place because of the diffusion process so therefore the raising pressure takes place as we increase the mass flow rate ok. As we increase the mass flow rate the raising pressure takes place this means the diffuser contributes its quota, diffuser contributes its quota to the pressurized, because of the diffusion process ok. So, because of that the pressurized increases and reaches a point maximum why beyond each, if you in increase the mass flow rate it will not be manifested in terms of the pressurized, this is because of the frictional losses. As I explained earlier in the last class that frictional losses composed of in frictional loss at the same time we loses due to separation ok. Along with that the incidence loses at there so all together the losses increases for which an increase in mass flow rate this not manifested we can increase in pressure ratio rather by a decrease in pressure ratio and this point corresponds to the maximum efficiency of the compressor. So, below which the compressor efficiency drastically falls ok, because of the losses and if we go and increasing the mass flow rate for a giving rotational speed. For example; here we are give a rotational speed, I tell you this is valid for one rotational normalized rotational speed one normalized rotational speed I show this particular curve for a giving rotational speed there may be a point, which may be or may not be obtain in practice, but they are may be a point for a giving rotational speed, if I go on I opening the valve wider and wider the mass flow rate may be saw, the pressure ratio may be unity, that means; there is no pressure raise the entire energy giving to the compressor to handle is been use to overcome the frictional law facing in handling the huge mass flow rate ok. So, that particular point may not be available for a given speed N, but it is theoretically incises can be incises. So, physically it is possible for a giving N, there may be a point, which gives a mass flow rate were pressure drop, pressure ratio is unity that means; the entire energies utilized to overcome the friction. So, therefore, A B C three points are important and this is a particular curve and these were we can generate a family of curve, we different rotational speed and similarly different normalized mass flow rate. So, the requestic curve is like that which has the positive slope maximum point corresponds to maximum efficiency then there is a negative slope, here the most important thing now I will discuss is the instability of this part of the characteristic curve.

Now, this part of the characteristic curve is having a positive slope and usually this part is arm stable and is very difficult to have this part of the carving practice rather this part, which is associated with the negative slope that pressurize and mass flow rate carve is stable, how? I am explaining.

Now, let us consider a compressor like this, let us consider the compressor like this, this is impeller, this is diffuser, this is totally the compressor try to understand this compressor ok. And what we do this is the delivery and we control a valve delivery valve here and this is further downstream of the compressor when the compressor is discharging here, this is the downstream the two thing we have to understand downstream of compressor and this is the compressor delivery this is the delivery end. Now, consider a case that the compressor is running with giving speed this valve is open partially at studies that some flow is there and compressor is discharging steadily and late the operational point is on the positive part of the curved let this point is D. Now, what happens by any chance if there is a reduction in flow in the compressor by any disturbance or any closer of the valve then what happens? This decrease in fluoride here, if you see is accompanied by decrease in pressure ratio, because this is in the positive slope the figure attain like that so therefore the delivery pressure will falling in the end.

Now, what happens you see again the D sport of the curves are initially it is very stream then finally it becomes flat as it happens for a curve which has a maximum then it reaches a maximum, because radiant touch to be zero here, so if this point is little bit on the stepper side of the car then these pressure falls rapidly, so the delivery end pressure falls rapidly. While due to the reduce mass flow rate the downstream side why this compressor is delivering air does not fall that rapid. So, therefore what happens as the result this pressure becomes higher than the delivery pressure that means a pressure radiant for flow is generated in the reverse direction this is the high pressure and this is the low pressure so, therefore the flow starts from the downstream end of the compressor to this delivery side that means to the compressor. You understand, because of this is does not fall rapidly with that this fall, what happens in this part that there may be a point and usually it happens so that this it is reduce more rapidly than that the downstream when the flow takes place like these then what happens the net flow through the compressor delivered by the compressor is reduce by the opposite flow. So, therefore, the flow rate is still reduced and the pressure is still reduced in tern it affects in reduction of delivery pressure again the reverse flow is increase and these to the what happens this makes the flow in the compressor total zero, there is no flow that means; compressor cannot deliver react anymore.

But still the delivery side there is a pressure, pressure ratio that I explain, because of the impeller action and by that time what happens is the mass flow delivery totally shutdown is reduced gradually gradually to zero then the delivery side pressure is reduced when the delivery side pressure is reduced at this condition A, then what happens this pressure becomes I and it takes up again repeat the flow in the positive direction. And therefore it starts repeating the cycle that means, it starts flowing in this direction again an instability in did using the flow it causes the flow reverse side. So, therefore what happens is a small disturbance in reducing the flow in these zone makes a repeating cycle that means the flow reverse side takes place again flow, the flow comes in again flow moves in this direction, comes this direction goes this direction, so this type of flow reverse will takes place going the operating point is on the stepper side of the positive slope part of the characteristic curve. And this is known as surging, surging of the compressor clear, this is very important thing. Now, we see these instability type of instability known as surging is not there in the negative slope part of the curve, because here what happened, if there is a decrease in mass flow rate this is associated with increase in pressure, decrease in mass flow rate increase in pressure. So, know a flow reversal that means from the downstream to the compressor side can takes place so there could these part is on stable and another thing I told you since the slope is stiffer initially and then flat so there is not necessarily that the point has to be immediately that downs off stream of the left side of the D, that means; there may be a point here even there may be a part of these positive slope were the surging will not occur that means surging may not starts going the operating point falls just left of B the maximum efficiency curve there may be some point have a some distance away from the maximum point were from the surging can start the onsite of the surging is there. So, this can be whole understood by this particular figure, well this can be well understood by this particular figure. Now, if I drop I show you this figure you see that as I told earlier that this is the curve the characteristics curve now, this point that means if I find out the search an set of the search point D for all curves of the family for different values of the parameter and if they are joined this is the surge line this the locus of the starting of the search point.

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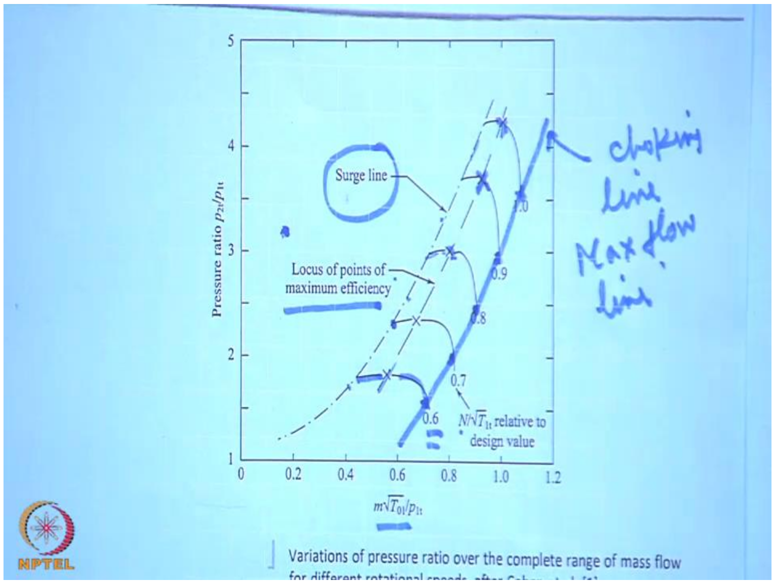

Variations of pressure ratio over the complete range of mass flow

That means, this part of the curve for a giving value point six of this N by root over T 1 t is this stable part, is this stable part, is this stable part, is this stable part and this cross point at the maximum efficiency that means; this line is the locus of points of maximum efficiency and this line is this surge line, so this part of the curve is the especially the curve is stable this is N root over T 1 t and this is with respect to mass ok, now, you understand, you have understood these things. Now, next what I like to tell you that ok, another important thing is there on this side of the curve that is in the negative slope there is another interesting point E, were you may stop, what is that? Now, you consider when the flow rate is increased the pressure drop decreases, pressure decreases, for example; if you make the valve avoid here, so what happens? The flow rate is increase, delivery pressure is decreased and it decrease in delivery pressure, decreases the row, now velocity of flow is proportional to mass flow rate divided by area into row. So, a decrease in the mass flow rate sorry, increase in the mass flow rate and the decrease in the density, because of decrease in the pressure, because you see increase in mass flow rate associated with the decrease in pressure the negative part of negative slope part, this part makes a huge increase in slow velocity and it may so happened that is also not always possible depending upon the value of N that a point may come when the sonic velocity may be attended some part of the compressor. So, when the sonic velocity is attend we cannot increase the flow any more by any change in the downstream this will be explained again in detail in your compressible flow class on that is known as chocking to the maximum flow condition chocking of flow.

When the flow at any part becomes sonic that means, the compressor will run very absolutely no problem but no further increasing mass flow rate possible that means, there is a point we have which will on the characteristic or which will indicate the limit of the maximum flow rate. So, therefore, I will show you here also along with the surge along with the constant along with the locus of this search point one set of search the maximum efficiency point they are may be another line, which is the joining of the E point there, which is the chocking of the maximum flowing chocking line or maximum flow line. So, therefore, the stable part of the characteristic line is bounded by left extreme by the locus of maximum by locus of this surge line one set of search surge line in the extreme right is the chocking line maximum flow line and in between the maximum efficiency line, ok, it is clear. So, this is as the whole is the, your characteristic curve ok, I think it is all right. Now, we will and stop the discussion and we will try to solve some problems ok, here it is ok.

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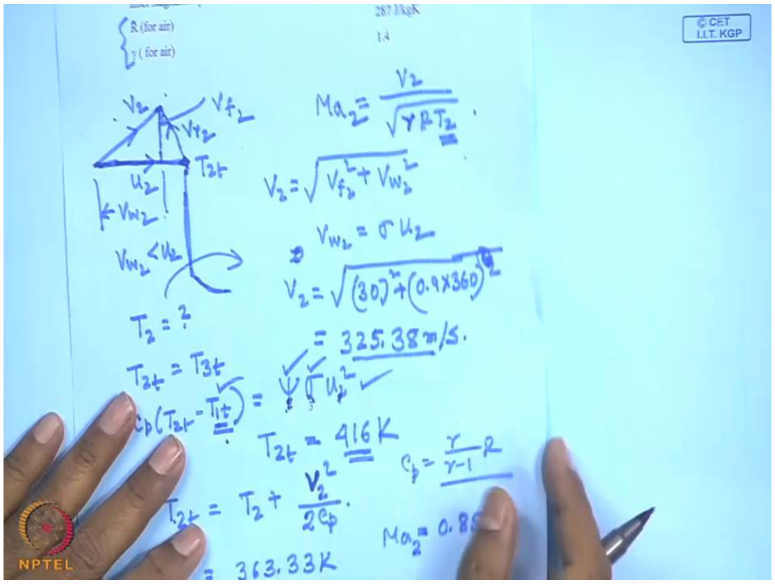

Now, let us see a problem ok, here also I did a mistake this will be not this will be a reputation of this, this will be N sorry, this will be N we are writing, N by root over T 1 t and this will be for T 3 t by T 1 t, this I did a mistake earlier, this would be like this, two curves ok. Now, let us solve the problem so we are discussed the principle of the characteristic curve, the concept of serge the serge line maximum efficiency line and the chocking line. Now, let us consider this problem a centrifugal compressor has an impeller tips with three sixty meters per second, this is the impeller tips meter, determine absolute mac number of flow leaving the radial vanes of the impeller and the mass flow rate, the following data are given, so data are given.

So, let us see that mac number of flow leaving the radial vanes, absolute mac number means this done absolute velocity, if we recall the vane now, let me recall the vane like this sorry, this is the vane, it is not so, then this is the vane ok, this is the vane. Now, if this is the vane then what is the diagram that the velocity diagram let me better show this things, which I earlier show request this is the thing, I think this will be better as to show like this, I can make themes like this can you see ok, this why I am writing that, why I am not doing the radial one here, one thing is important that slip factor is giving 0.9 that means here due to the slip what happens, we have a this is rotating in this direction due to, so V r 2 is not radial, because there is a slip. So, V w 2, this is V w 2 and V w 2 is less than u 2, but is u 2, u 2 is this one ok. This is this include so this is the absolute velocity V 2 and this velocity is V f 2, this is V f 2, so this is the diagram, because there is a slip so therefore this is the outlet channel. Now, what I will do? I will write what has to be found out that it has to be found out that mac number based on absolute velocity at the outlet of the impeller. So, therefore, I have to find out mac number V 2 by root over gamma R t 2 ok. Now, how to find out V 2? V 2 is the absolute velocity, now; V 2 is root over this is V f 2, V f 2 square plus this is V w 2, V w 2 square. Now, V w 2 is not u 2 ok, this is u 2, this is u 2 ok, so V w 2 is sigma u 2 ok. Now, u 2 is 360 meter per second, V f 2 is giving flow area, power input factor, impeller tip speed, flow area, mass flow rate, so, d f 2 is not giving, impeller tip speed is giving, radial component of flow velocity is giving V f 2 is 30 meter per second, you see that V f 2 is 30 meter per second, so therefore you get V 2 equal to 30 square plus 0.9 into 360 square, I will not do everything a calculations square.

And this square we will get a value of V 2 equals to what is the value are let me telling value of V 2 here, the value of V 2 is 325, you can check, 325.38 meter per second. Now, to find out the mac number you required this static temperature here in the formula it is gamma or T 2, T 2 is this static temperature at the outlet of the you know how to find out T 2, now our main object T 2, find out T 2, how to find out T 2? Now, let us first find out the total temperature T 2 t, you know that T 2 t here T 2 t is equal to T 3 t that means; outlet total temperature from the compressor that the outlet of the diffuser and we know that C p into T 2 t minus T 1 t that is the work done, that is equal to psi sigma the power input factor u 2 square divided this is ok. Now, here you see T 1 t is giving there, because the inlet stagnation temperature 300 k is given psi is giving power input factor is giving power input factor is giving 1. Now, slip is giving 0.9 sigma is giving 0.9 see psi 1 u 2 is giving 360 meter per second T 1 t is giving by inlet stagnation temperature is 300 k so everything is giving accept T 2 t from which we can find out T 2 t equals to what T 2 t becomes equal to ultimately if you calculate T 2 t will be 416 k. I am not writing every step, because everything I know mar input factor is given in the problem, sleep factor is given in the problem and T 1 t is given in the problem, u 2 is given in the problem.

Again I show you the problem that impeller tip speed is 360 per second, radio component of flow velocity 30 meters, slip factor is 0.9, flow area and impeller exit is 0.1 square it is not now required, power input factor is giving, isentropic efficiency giving, inlet stagnation temperature, inlet stagnation pressure then R and gamma. So, here what is required, power input factor, which is given as one the slip factor, which is giving as 0.9, u 2 is 360 meters per second and T 1 t is 300 q u get T 2 t.

Now, what happened? T 2 how to find out T 2? Now, T 2 has to be found out from the concept of the stagnation temperature, what is that T 2 plus the temperature velocity equivalence, temperature equivalence that is dynamic head that is V 2 square by 2 C p, that is the temperature equivalent of the kinetic energy V 2 square by 2 that is put this plus this is the total temperature from which we can find out T 2 is T 2 t is now, found out 416. Now, V 2 already you have found out 325.38 meter per second, the value of C p has to be found out from R and gamma, which you have read at school level that is specific that constant pressure is gamma by gamma minus 1 into R, so, you know this C p.

So, therefore, from here you can find out the static temperature as 363.33 k everything is known. So, you know this static temperature, when we substitute this static temperature here, we get the mac number equals to, the mac number 2 for example; the 2 suffix, I am using equals to 0.85 all right. Now, the next is the to find out the mass flow rate, how to find out the mass flow rate? You see the mass flow rate.

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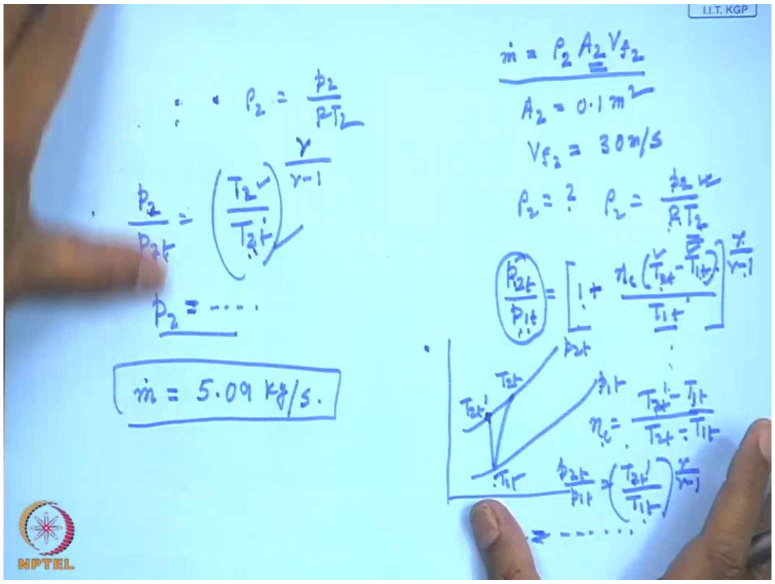

$\rho_2 = \frac{p_2}{RT_2}$

$\frac{p_2}{p_{2t}} = \left(\frac{T_2}{T_{2t}}\right)^{\frac{\gamma}{\gamma-1}}$

$p_2 = \dots$

$\boxed{\dot{m} = 5.09\ kg/s.}$

$\dot{m} = \rho_2 A_2 V_{f_2}$

$A_2 = 0.1\ m^2$

$V_{f_2} = 30\ m/s$

$\rho_2 = ?$ $\rho_2 = \frac{p_2}{RT_2}$

$\frac{p_{2t}}{p_{1t}} = \left[1 + \frac{\eta_c (T_{2t} - T_{1t})}{T_{1t}}\right]^{\frac{\gamma}{\gamma-1}}$

$\eta_c = \frac{T_{2t}' - T_{1t}}{T_{2t} - T_{1t}}$

$\frac{p_{2t}}{p_{1t}} = \left(\frac{T_{2t}'}{T_{1t}}\right)^{\frac{\gamma}{\gamma-1}}$

$= \dots$

Let me keep it here, so, that you can see the mass flow rate to find out, mass flow rate let us write the mass flow rate, mass flow rate is same thought out the machine. Let us write the mass flow rate base on the condition at the outlet of the diffuser, rho 2, the A 2 and the flow velocity. Now, here A 2 is the flow area at the outlet of the impeller, which is given you see here flow area is given radial component by flow velocity, the mass flow rate impeller tip speed flow area impeller exit that means; A 2 is already given 0.1 meter square. So, what is not given V f 2 is given, the radial flow velocity popular that impeller exit is given V f 2 is given, V f 2 is what? 30 meter per second is all right, 30 meter per second. What is not given? Rho 2, so how to calculate rho 2? Now, rho 2 is p 2 by R T 2. Now, T 2 I know this static temperature already T 2 is already calculated here, so, I do not know p 2, how to calculate p 2? Now, before calculating p 2, you have to calculate the stagnation pressure then, if you calculate this stagnation pressure then you can calculate the static pressure, so how to calculate this stagnation pressure? So stagnation pressure you can calculate p 2 by p 1 t from your the earlier formula 1 plus beta C into you can write like that T 2 t minus T 1 t divided by T 1 t to the power this has been told earlier gamma by gamma minus 1. So, remember this one that is the pressure this comes one her this comes from the isotropic relationship and then using the isotropic efficiency of the compressor, well so using this relationship, we can find out this relationship, if you remember this was derived in the class that means; I find out this way, that if these are the two pressure lines then what happens? This p 2 t, p 1 t then this is the thing, this is the T 2 t, T 1 t and this is the T 2 t dash. So, p 2 t by p 1 t is T 2 t dash by T 1 t to the power gamma by gamma minus 1. Now, this T 2 t dash T 1 t is found out by expressing this eta c is T 2 t dash minus T 1 t, I repeat again this was done earlier minus T 1 t. So, therefore T 2 t dash that means; p 2 t by p 1 t is T 2 t dash by T 1 t to the power this isotropic relationship p T relationship. So, these things is taken from here T 2 t dash T 1 t this has into this plus 1, it has into this by T 1 t plus 1 so therefore you can write this. This is the thing done earlier now T 2 t minus T 1 t you know already you know T 2 t you already know T 2 t, T 2 t is find out T 2 t is 416 k, you will know T 1 t, T 1 t is 300 k given, beta C is given in the problem beta C is what? Beta C is 0.9 so therefore T 1 t is known everything is known, you find out the p 2 t.

Now, after knowing the p 2 t, we have to know the p 2, because how? Because row 2 is p 2 by R T 2, I know it earlier R T 2, you have to know the static pressure so how to know the static pressure by the concept of stagnation pressure p 2 by p 2 t is T 2 by T 2 t to the power gamma by gamma minus 1, where from its comes, that means; the static pressure is changed to stagnation pressure that means when this is brought risk isentropically that means this process of changing from p 2 to p 2 t is obtain by bringing the fluid isentropically towards. That means; therefore the gaining temperature T 2 t from T 2 is maid isentropically so that the pressure ratio is will be related to the isentropic relation in the temperature ratio this is the relationship between total or stagnation pressure to the static pressure, so therefore T 2, T 2 t is known therefore we get p 2.

So, we can find out p 2 you understand, from here we find out p 2 t, p 2 t is known here we find out p 2 t, p 2 t is find out here and here we find out p 2, because p 2 t is known so finally p 2 is find. First we find out the ratio of the total pressure in terms of these you know everything p 1 t you know, you know p 2 t, when you know p 2 t you know p 2 from this equation, so p 2 is find out. So, going p 2 is found out then you can found out row 2 from p 2 by R T 2 and you can find out the mass flow rate. So I am not doing things by putting the numerical value but I tell you the A so these value is check 5.09 kg, because this will take more time all this putting this values that is why I am not doing so if you put this values you will get the result I think so there will be absolutely no problem now only substitute the numerical value and get the result and check the result ok. Now, next another problem I will discuss before I leave you, so another problem is this 1.

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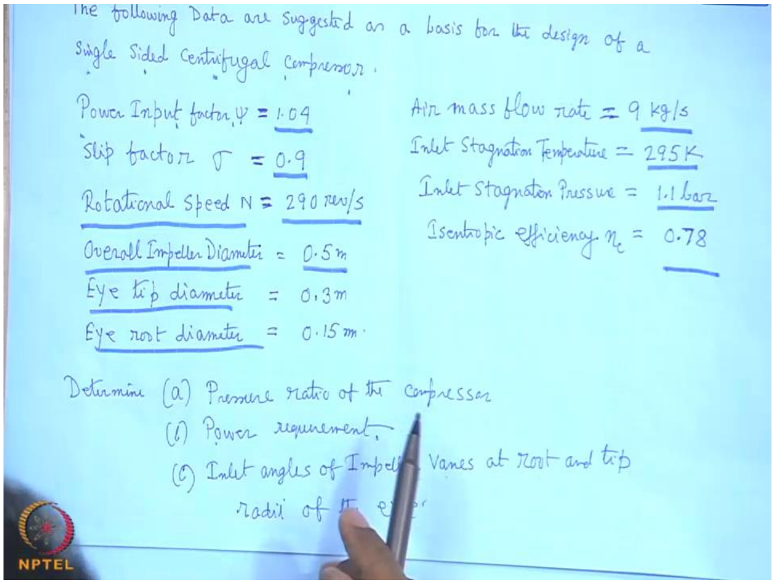

The following Data are suggested as a basis for the design of a Single Sided Centrifugal Compressor.

Power Input factor, ψ = 1.04

Slip factor σ = 0.9

Rotational Speed N = 290 rev/s

Overall Impeller Diameter = 0.5 m

Eye tip diameter = 0.3 m

Eye root diameter = 0.15 m.

Air mass flow rate = 9 kg/s

Inlet Stagnation Temperature = 295 K

Inlet Stagnation Pressure = 1.1 bar

Isentropic efficiency η_c = 0.78

Determine (a) Pressure ratio of the Compressor

(b) Power requirement.

(c) Inlet angles of Impeller Vanes at root and tip radii of the eye.

Let us consider a problem like this. The following data are suggested as a basic for a design of a single sided centrifugal compressor, single sided, power input factor is 1.04, slip factor 0.9 almost the similar problem, which we discussed earlier, rotational speed is to 290 revolution per second overall impeller diameter 0.5 meter, eye tip, eye root diameter, air mass flow rate is giving, inlets stagnation temperature is giving, inlets stagnation pressure, isentropic efficiency.

Now, what is to be found out determination pressure ratio of the compressor, power requirement, inlet angles of impeller vanes at rout and tip radii of the eye? Now, let us found out the most simple thing the pressure ratio, how to found out the pressure ratio, now pressure ratio to found out, what we have to do?

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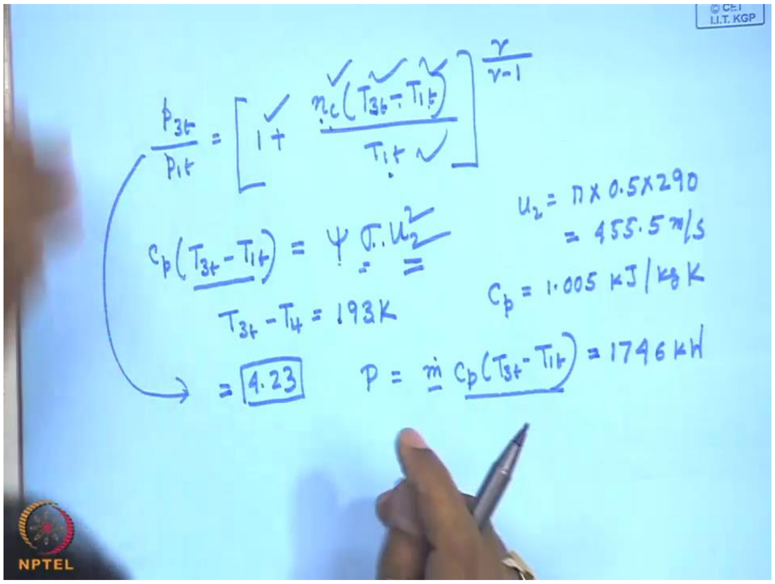

$$\frac{p_{3t}}{p_{1t}} = \left[1 + \frac{\eta_c (T_{3t} - T_{1t})}{T_{1t}}\right]^{\frac{\gamma}{\gamma - 1}}$$

$$c_p (T_{3t} - T_{1t}) = \psi \sigma . u_2^2$$

$$T_{3t} - T_{4} = 193 K$$

$$= \boxed{4.23}$$

$$u_2 = \pi \times 0.5 \times 290$$

$$= 455.5 m/s$$

$$C_p = 1.005 kJ/kg K$$

$$P = \dot{m} c_p (T_{3t} - T_{1t}) = 1746 kW$$

Let us again write the pressure ratio formula, if you write the pressure ratio. Now, I write p 3 t by p 1 t is equal to in terms of the stagnation temperature I, I think this you know again and again I am writing. Just now, I am discuss this that is stagnation or total pressure ratio here also the pressure ratio of the compressor total pressure ratio p 3 t by p 1 t has to be found out. Now, therefore, what we were required T 3 t minus T 1 t, how to find out T 3 t minus T 1 t? Again the same formula we know the work done to the fluid for unit mass and the energy edict to the fluid per unit mass it is shy par into sigma u 2 square.

Now, this thing can be found out provided shy sigma into u 2 square is given, what main in the problem let me see, this problem give the rotational speed and overall impeller diameter 290 revolution per second then overall impeller diameter 0.5. So, u 2 is equal to pie into the overall diameter 0.5 into 290 is equal to the rational speed that means; the dimensional speed is 455.5 meter per second, that means; u 2 is known sigma is giving in the problem, if you see the slip factor is 0.9 sigma is power input factor is 1.04 so if you put everything, you get the value of T 3 t minus T 1 t, which become to T 3 t minus T 1 t is 193 k here the value of c p is not giving in the problem, if the problem the value of c p is not given in any case you can used that for air the value of c p is 1.005 kilo joule, if the specific per kg k. So, therefore, you can use the value of c p that at T 3 t and minus T 1. It has the probably given in the where is it has seen, overall air mass isentropic efficiency 0.78 when you get the isentropic efficiency 0.78 this is this T 1 t is giving, T 1 t is the inlet stagnation temperature inlet stagnation or the total temperature whatever we call 295 k. So, everything is known and this p 3 t equals to, what is the value? Equals to 4.23 this you can check, so you can find out.

So, pressure ratio of the compressor is found, power requirement this mass flow rate into c p this work done per unit mass T 3 t minus T 1 t either these or these both the things are same so in you are find out the mass flow rate is given probably or power cannot be found out air mass flow rate 9 kg per second. So, work per unit mass into mass flow rate is the power you know the everything so power requirement in his now 1746 kilo watt alright. Now, the second part is the inlet angles of impeller veins of rout and t now at rout and t if we want to found inlet the impeller angles then what will happen?

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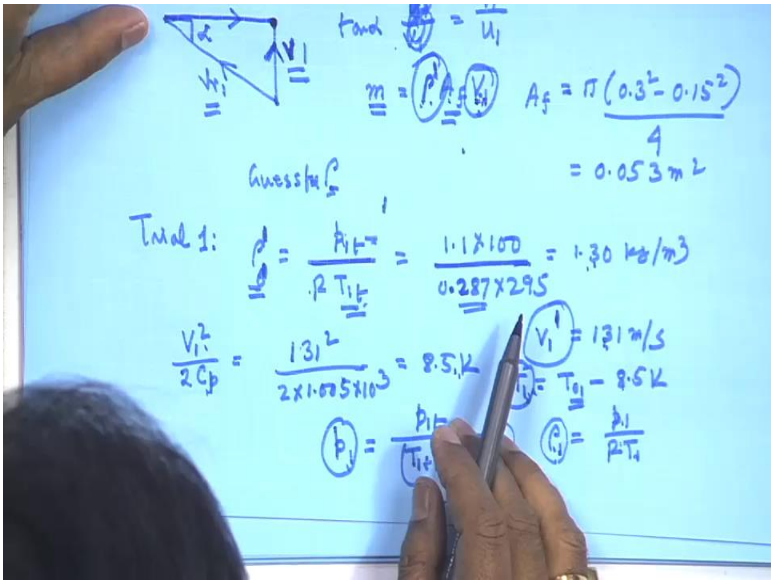

$\tan\alpha = \frac{}{u_1}$

$\dot{m} = \rho_1 A_f V_1$ $\quad A_f = \frac{\pi(0.3^2 - 0.15^2)}{4} = 0.053\ m^2$

Guess for ρ

Trial 1: $\rho = \frac{p_{1t}}{R T_{1t}} = \frac{1.1 \times 100}{0.287 \times 295} = 1.30\ kg/m^3$

$V_1 = 131\ m/s$

$\frac{V_1^2}{2C_p} = \frac{131^2}{2 \times 1.005 \times 10^3} = 8.5\ K$ $\quad T_1 = T_{01} - 8.5\ K$

$p_1 = \frac{p_{1t}}{T_{1t}}$ $\quad \rho_1 = \frac{p_1}{R T_1}$

Let us find out an route or tip anywhere at any representative section it may be route, it may be tip ok that means it may be rout or it may be tip that either it is this is the rout and this is the tip. So, rout and tip axial flow velocity is cost and the normal flow velocity is constant is giving their ok, you will assumed that however now what happens is that, if you know the u 1 the router speed and v 1 and v r 1, if this is this alpha just I wrote the I draw the diagram, so tan alpha u 1 by v 1. So, have to know low velocity or the relative velocity than alpha is o v 1 by sorry, v 1 by u 1. So, this is if I know the route this velocity they need to be route angle of the route at the tip it is angel of the tip now rout and tip you want to be found out best on the rout and tip diameter that I know because, I know the rotational speed, I know the eye tip diameter, I know the eye rout diameter but here I knew neither v 1 or v r 1 so, how can I found out. So, these type of problem is based on a trial, what trial, how to find out v 1? So, v 1 is not known now, if you see the mass flow rate basis, mass flow rated is given, row A f v 1 ok. Now, A f is giving, how A f is giving? A f is giving, because eye tip diameter and eye route diameter is their so one can find out A f as pi into 0.3 square minus 0.15 square by 4 and these become is equal to 0.053 square. So, I know if so I have to this two I know, what is that? I could to make a trial guess for row and find out v 1 how to guess for row, this row for a example, trial one this is a trial method, trial one, I guess row from the total pressure p 1 t, R T 1 t, at impeller I, I know t 1 t and I know p 1 t these on find out and these value is 1.1 bar into 100 by 0.287 into 295 this is the value giving here yes, 1.1 bar. So, these in terms of kilo joule kilo this a Newton for meter square converting this unit is there that's why 0.287, which is the values it should be 10 to the power 5 that another 10 to the power 3 then will be 287 types tic gas cost and you see that cost consistent you need it is returned and it becomes 1.30 kg per meter.

Now, if you know this row 1, fastile this row 1, row 1 trial 1 you put that thing row 1 and if you know the mass flow rate known, mass flow rate already giving in the problem, what is the mass flow rate? Mass flow rate is 9 kg per second. You get a value of v 1, now why you get the value of v 1, how to interate it should be a base on, which will interate v 1 get the value of v 1 you can calculate corresponding temperature dynamic temperature by v 1 c p, for example, 1 this density you get from this density you find out the v 1. So, from with this density, if you find out the v 1 then v 1 for one trial will be 131 meter per second.

So, when you know the v 1 then what you do with this v 1 you calculate the 131 second v 1 square by 2 into 1.005 into ten to the power 3 the corresponding temperature. So, if you know the temperature you can calculate the static temperature T 0 1 minus this 8.5 k, T 0 1 you know so, therefore you can find out the static temperature and at the same time we can find out this is a little laborious calculation I know but this is usually done in the design this static pressure as I told you formula earlier the static pressure and total pressure is related to the static temperature and total temperature to the isentropic relation so, I get p 1, when I get p 1 and T 1 I can find out row 1 as p 1 by R T 1.

That means; by guessing a row best on this stagnation can which I find out a v 1, when v is find out I find out the dynamic equivalent temperature and that temperature I can find out this static temperature and the static pressure by isentropic relation, when these two things know that is pressure and temperature then row 1 is p 1 by R T 1. That means row is getting corrected with these row I find out taking the corrected values, so these way about the row and v row 1 and v 1 1, v 1 1 and row 1 1, row 1 is getting corrected, so that we can get a converge values. When you have a converge value then we get v 1 value when we get the v 1, which is cost and the axial velocity of the flow thought out the impeller passages same, that means, it is same as the at the route and tip then what will you do and you will use the peripheral speed at the route then we will get the angel of the v 1 at the route and the tip then we get the angel let the tip.

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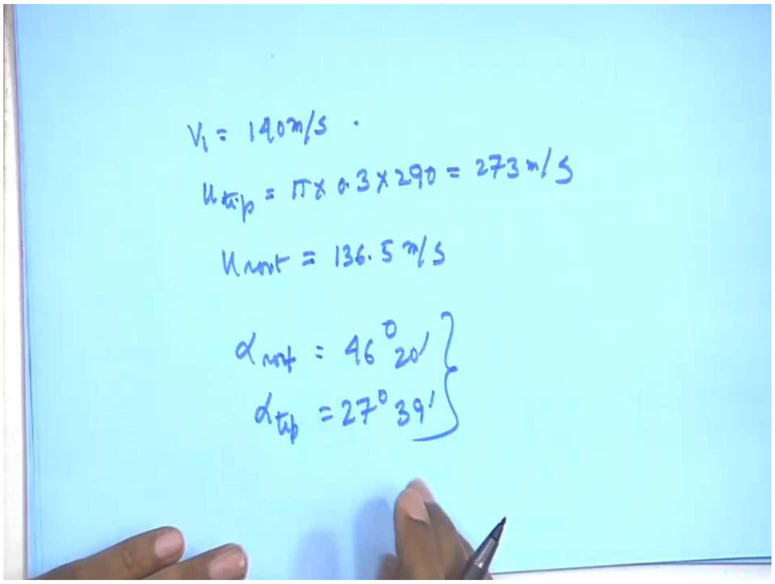

$V_1 = 140 \text{ m/s}$.

$u_{tip} = \pi \times 0.3 \times 290 = 273 \text{ m/s}$

$u_{root} = 136.5 \text{ m/s}$

$$\left.\begin{array}{l} \alpha_{root} = 46^{\circ} 20' \\ \alpha_{tip} = 27^{\circ} 39' \end{array}\right\}$$

And if you do so then you will get I am giving you the value v 1 converge value of v 1 comes out to be 140 meter per second and u tip find out pie into as the value is given you checked the rotational speed into tip diameter 273 meter per second, and u route is equal to just half of the diameter haft meter per seconds and alpha route that by 10 alpha that formula v 1 by u perpendicular to base by u it becomes 46 degree and alpha tip becomes 27 degree it is twenty minute in more from where I taken this. So, there is an iterative process by you have to do ok. Thank you, to day off to this.