

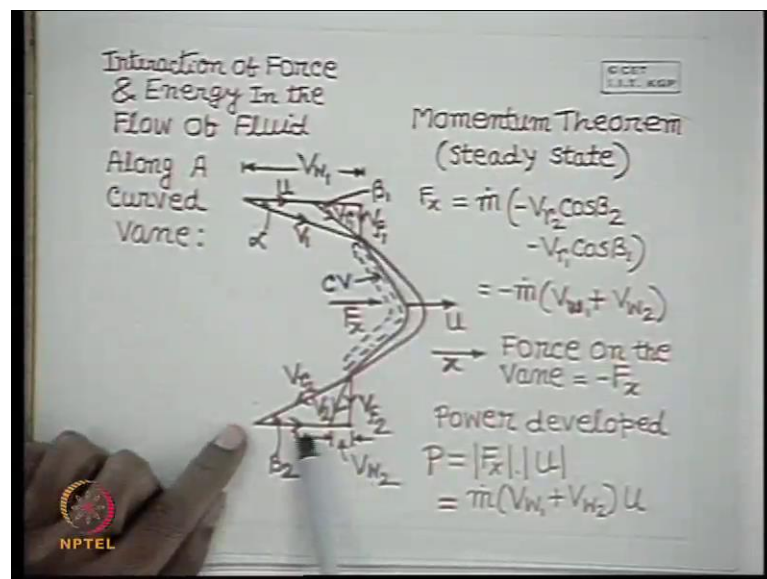
Introduction to Fluid Machines and Compressible Flow
Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 2
Energy Transfer in Fluid Machines Part – I

Well, now, today we will start the description of rotodynamic machines. As I have told earlier, the rotodynamic machines are those machines where there is a continuous motion of fluid and a part of the machine, known as rotor. And because of this continuous relative motions between the fluid and the rotor of the machine, it is possible for an energy transfer to take place between the fluid and the rotor.

So, therefore the basic principal of this machine is based on the free dynamic principles, free dynamic principles, which is basically the utilization of useful work due to the force exerted by a fluid striking on a series of curved vane, which is mounted on the periphery of a disk that is rotating the periphery of a disk that is attached to a rotating shaft. So, therefore, to understand the basic principle of a rotodynamic machines we should understand clearly the force interaction and the energy transfer that takes place while a stream of fluid passes through a curved vane.

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So, this is a little recapitulation of what you have already studied at your basic fluid mechanics course that we studied here, the interaction of force and energy in the flow of

fluid along a curved vane. Now, see here, this is a curved vane, which is moving with a velocity u and a jet of fluid is striking the vane with a velocity, v_1 is the velocity, absolute velocity with which the fluid strikes the vane. And the fluid, after flowing through this vane, comes out with a velocity v_2 . This is the velocity v_2 .

Now, since the vane is moving with a velocity u , so the jet appears to strike the vane. That means, with respect to the vane, the jet strikes it with a velocity v_{r1} , which is the velocity of the jet relative to the vane. Similarly, it is going out with velocity v_{r2} , that is, the relative velocity of the fluid with respect to the vane. Now, these relative velocities at inlet and outlet are determined just by vectorial subtraction, from v_1 the velocity u of the vane and from v_2 , the velocity u of the vane.

So, this vectorial subtraction is shown in terms of the velocity triangle, as you have already read, at the inlet and outlet. Now, let the suffix 1 refer to inlet condition and suffix 2 refer to outlet condition. Now, we see in this triangle, this is v_1 , that inlet velocity of the fluid. This is the u , the vane velocity and this is the v_{r1} , that is, the relative velocity of fluid with respect to vane at inlet. This component, this perpendicular component to the motion of the vane is denoted as v_{f1} and is usually known as flow velocity. Similarly, the component of the fluid velocity, absolute fluid, we are saying, the direction of the vane motion is conventionally symbolized as v_w , the suffix 1 is at the inlet.

So, this v_w , I tell you, this is a conventional symbol. w stands for whirling component. This is because, in actual case, this velocity of the vane is in the tangential direction because the motion of the vane mounted on the periphery is a rotating motion. So, therefore the linear velocity of the vane is in the tangential direction and that is why, this component is known as tangential component or whirling component for which a conventional symbol w is given in the suffix.

Similar is the case in case of outlet velocity triangle. This is the vane velocity, this is the relative velocity of the fluid with respect to vane and this component is the whirling component or the component of the flow velocity, that direction of the vane velocity and this is the flow velocity, that is, the direction of the, that is the, sorry, the component of the fluid velocity in the direction perpendicular to the vane velocity.

Now, our basic purpose in this case is to analyze what is the force exerted by the fluid on the vane or vice versa, vane on the fluid and by virtue of the vane motion, which is the, what is the amount of energy that is being transferred or developed due to this force, due to this action of the fluid on the vane. So, to analyze this, as you know, we apply the momentum theorem.

Now, to apply the momentum theorem we have to take a control volume of the fluid like this, which is just adjacent to the vane. Now, you see, that this type of analysis can be done on the basis of both, system approach and the control volume approach. In a system approach what is done? The Newton's law is applied in a sense, that you consider a particular mass of fluid, consider its change of momentum as it flows along the vane, find out the change of momentum in a specific direction. And in control volume same thing is done, but the version is different. We find the momentum aflux, net momentum aflux in a particular direction and equate this with the force in that particular direction.

So, if you if we apply this theorem for a steady state situation, the situation is steady, then we find, that if F_x is the force acting on the control volume in the direction x , then it will be the net rate of momentum aflux from the control volume in that direction x because we are interested in the direction x , that is, the direction of the vane velocity, the force in that direction. So, the expression on the right hand side is either the net rate of momentum aflux x , momentum aflux from the control volume or from a system approach it is the change of momentum, change of momentum in the x direction of a fluid mass taken as a system. In either way you can see it and that becomes equal to the force, is equal to the change of momentum times the mass flow.

Now, you see the velocity at the outlet is $v r 2$. Now, here we have to consider the relative velocities because in this case, the control volume is moving with a velocity since the vane is moving with the velocity u . This is an inertial control volume. So, the coordinate axis will be fixed to this control volume. So, therefore the velocities, which we have to take are the relative velocity.

So, you see, the component of the velocity in the direction of vane velocity, here the β is the angle made by $v r 2$ with the direction of vane velocity. It will be $\cos \beta$ because this direction is opposite to that of the vane velocity or to that of the positive direction of the specified axis x . This is the momentum aflux minus the

momentum influx, that means, $v_r 1 \cos \beta_1$. β_1 is the angle made by the relative velocity with the vane direction. So, this component is in the direction of the vane velocity or in the positive direction of x . So, minus sign is that because it is the aflux minus influx. So, both the terms are with a minus sign, so it comes out, so minus $m \dot{}$.

Now, this $v_r 1 \cos \beta_1$ or $v_r 2 \cos \beta_2$, if you see from this triangle, so this comes out to be $v_w 1$ and $v_w 2$. So, therefore we say, that force on the vane is equal to minus F_x . That means, this is the force that is being acted on work that is being acted on the control volume. So, the force acting on the vane is in the opposite direction, that means, if this is the F_x , it is the in opposite direction of F_x , minus F_x .

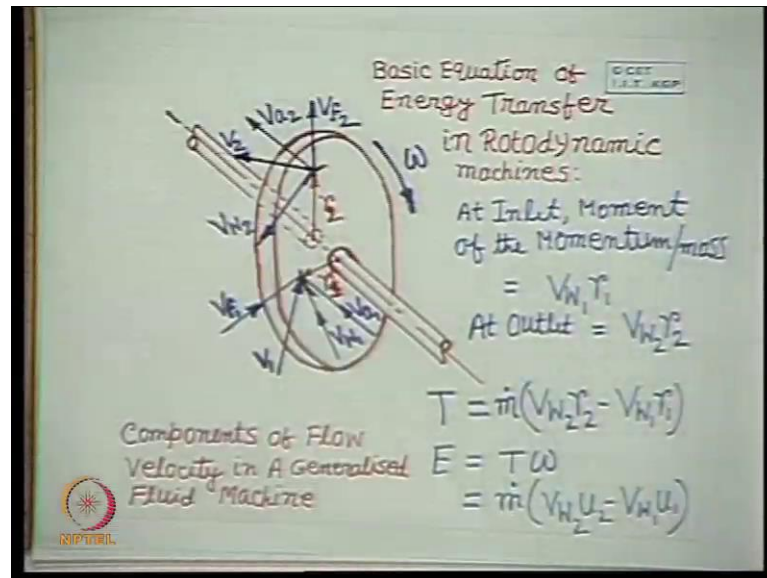
Now, power developed due to the motion of the vane is then, force into the velocity, that is, $m \dot{v}_w 1$ plus $v_w 2$ into u ; u is the vane velocity. So, this way we can develop an expression for the power developed due to the action of the fluid passing over a curved vane. I think you have understood this, alright.

So, from this two triangles you can get from here triangular relationships geometry, that it is $v_w 1$ plus $v_w 2$ and this minus sign is because the force, which is acting on the fluid element or the control volume is in the opposite direction to the specified axis, that means, in a direction opposite to the vane motion. So, therefore the force on the vane is in the direction of the vane motion. However, the expression for power developed is written as the multiplication of F_x and u , they are in the same direction. So, their absolute values are taken. Well, ok, now, yes please.

Student: Sir, how $v_r 2$, $v_r 2 \cos \beta_2$ is not $v_w 2$?

Professor: $v_r 2 \cos \beta_2$ is not $v_w 2$ plus u and here, it is $v_w 1$ minus u , that cancels out actually. So, ultimately you get $v_w 1$ plus $v_w 2$. Yes, correct, $v_r 2 \cos \beta_2$ is not $v_w 2$, it is $v_w 2$ plus u . On the other end, $v_r 1 \cos \beta_1$ is also not $v_w 1$, it is $v_w 1$ minus u . If you substitute that, automatically it cancels out and becomes, because I felt, that you have already done with that in your basic fluid mechanics course. So, this thing you know well. Very good, I am happy that you are asking questions.

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Now, we come to the basic equation of energy transfer in rotodynamic machines. Now, in a rotodynamic machines what happens is, that the rotor of the machine is a rotating wheel on which the vanes are mounted and the wheel is mounted on a shaft where the rotation is imparted. So, in this case, the same principle is applied here and we analyze this in the similar fashion with the help of a diagram here, which is the general representation of a rotor or the representation of a rotor of a generalized fluid machine.

Now, components of flow velocity in a generalized fluid machine, here in a most general sense we consider the rotor where the fluid enters at a velocity v_1 at any point whose radius of rotation from the axis is r_1 .

Now, before that I like to mention, that there are few assumptions for the analysis. One assumption is, that the flow is steady, so there is no mass accumulation, no mass depletion anywhere in the system. And number two assumption is, that flow is uniform over any cross-section normal to the flow velocity, which is very important and that means, that the velocity vector at a point is representative of the flow over a finite area. That means, we analyze with respect to a velocity vector at a point and we assume, that this is uniform over the entire flow area, that is, an area normal to the flow velocity, so that this is the representative of the entire flow through the fluid machines, well.

So, with these assumptions now we consider, that at any point the velocity is v_1 , that is, the inlet point, a very general case whose radius of rotation from the axis of rotation is r_1 .

1. Similarly, the fluid goes out or discharges at a point from the rotor whose radius of rotation from the, whose radius, sorry, whose radius from the axis of rotation is r_2 .

Now, the velocities v_1 and v_2 can be resolved into three components, that may be an arbitrary angle at which the velocity, flow velocities strikes the rotor, which can be resolved into three reference directions: one is in the direction of tangential, one is in directional, that tangential direction, which is the tangent to the rotor at that point; another is the direction, which is the axial direction, that means, it is parallel to the axis of the shaft; and another is the radial direction, which is perpendicular to the axial direction.

So, these three mutually perpendicular directions, the velocities are reserved, one is the tangential direction, another is the axial direction, another is the radial direction. So, these three perpendicular directions are accordingly symbolized as V_W . One is the suffix at inlet, that is, the tangential component, whirling component, that is why, the suffix W is used. The suffix V_a is the axial component, that is, component parallel to the axis of the rotation. And as I have told earlier, this symbol F is used, V_F1 for the inlet, that is, the flow velocity that is in the radial direction. Similar way, the velocities are dissolved in tangential direction as V_W2 at the outlet, the axial direction V_a2 and the flow direction V_F2 . Now, the rotor is moving with an angular velocity ω , which is a constant angular velocity. This is the steady state problem.

Well, now let us apply the momentum theorem or the Newton's laws of motion, either with respect to a system or a control volume here. Now, here the momentum, which will be considered is the angular momentum. This is because here the work transfer takes place due to the rotation of the shaft. So, we will be considering the angular momentum or moment of the momentum. It is very simple. If we consider a system approach our version will be, that considering a fluid mass as it passes from the inlet to outlet what is its change in angular momentum, or if we consider a control volume of a fluid, then what will be the net rate of afflux of the angular momentum from the control volume.

Now, here one thing is very important. We are not bothered about the path in the rotor. It is only the inlet and outlet, that decides the change because if the inlet and outlet conditions are fixed, kinematic conditions are fixed and the mass flow rate is steady, so

the change in momentum or the moment of the momentum, whatever you say, depends upon the inlet and outlet conditions, well.

Now, if we write the momentum, moment of the momentum at the inlet, for a unit mass at inlet, what will be its value at inlet, at inlet, ok. At inlet moment of the momentum, moment of the momentum is equal to, that is, the moment of the tangential momentum. That means, $V W_1$ times the r_1 radius from the axis of rotation, it is per unit mass, per unit mass. Similarly, the same thing at outlet, at outlet, the same thing at outlet is equal to $V W_2$ into r_2 .

So, therefore, by unit mass the change in the moment of the momentum of a fluid mass or the net rate of aflux of the moment of the momentum per unit mass from a control volume will be $V W_2 r_2$ minus $V W_1 r_1$ and that multiplied by the mass flow rates. That means, this will be the net rate of angular momentum aflux or the rate of change of angular momentum, rate of angular momentum, net rate of angular momentum aflux when we refer it to a control volume, that is, a control volume approach, control volume of the fluid or it is the net rate of change of angular momentum for a system as it passes from inlet to outlet.

So, in both the cases, that equals to the torque, that is, the angular momentum theorem, that is, the angular momentum theory applied to a system or to a control volume, the torque is equal to the rate of change of angular momentum of a system or torque is equal to the net rate of angular momentum aflux from a control volume at steady state. So, that is equal to the torque that is being imparted on the fluid by the rotating disc.

Now, the energy, rate of energy that is being imparted to the fluid will be nothing but the torque into the angular velocity ω . And that if we multiply the angular velocity and recognize, that ωr_1 is the velocity of the linear velocity, that tangential velocity of the rotor at inlet and ωr_2 is the linear or tangential velocity of the rotor at outlet. And denoting them by the symbol u , we can write $V W_2 u_2$ minus $V W_1 u_1$.

So, therefore we see, that energy transfer by unit time, the rate of energy transfer in the fluid as it passes from inlet to outlet becomes equal to the mass flow rate, $m \dot{\text{flowing}}$ times $V W_2 u_2$ minus $V W_1 u_1$, where u_2 and u_1 are the tangential velocity, that is, the linear velocity of the rotor at the outlet point and u_1 is that at the inlet point because in a generalized case, we have to consider that inlet and outlet are not in the same radius

from the axis of rotation. There is not at the same radial plane. So, this is the general equation and here in this equation we see, that this is the energy, rate of energy fluid.

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$$\dot{E} = \dot{m}(V_{W1}u_1 - V_{W2}u_2)$$

$$\frac{\dot{E}}{\dot{m}} = (V_{W1}u_1 - V_{W2}u_2) \quad \text{Euler's Equn}$$

$$\frac{\dot{E}}{\dot{m}gK} = \frac{H}{g}(V_{W1}u_1 - V_{W2}u_2)$$

when $V_{W1}u_1 > V_{W2}u_2$
when $V_{W2}u_2 > V_{W1}u_1$

Now, if we write the rate of as E dot, then we can write this is just the negative of that, m dot into V W 1 u 1 minus V W 2 u 2 where we tell, that here E dot is the rate of energy that is being supplied to the rotor by the fluid. Conventionally, we take the positive sign of this energy transfer when it is being transferred by the fluid to the rotor or the machine. So, therefore, we always express in this fashion. This can be expressed as per unit mass basis that means, the energy transfer per unit mass E dot by m dot is equal to V W 1 u 1 minus V W 2 u 2.

As you know, in fluid flow we define a term head, what is head? What is the definition of head? Sometimes we call that is the pressure head, is a velocity head, potential head, total head, what is the definition of head?

Student: Height.

Professor: Height, yes energy can be expressed in terms of the height, height of the fluid column because head is expressed in terms of the linear dimensions, but what is the definition? It implies the concept of energy. What is that the height of the fluid implies? The concept of pressure, sometimes it concept of energy, it implies the concept of energy also, it is the energy per unit weight for a fluid flow. It is the energy per unit weight. So,

if you see the dimensions, you see the dimension of energy per unit weight is the height. For example, the height of a liquid column exerts the pressure at the base and at the same time it has due to that pressure as a pressure energy and pressure energy per unit weight is that height, ok. So, therefore, it is the energy per unit weight is the height. So, therefore, we express it as the energy per unit weight that means, weight at g and the expression comes like this, $g V W 1 u 1$ minus...

So, therefore these three equations, they are dependent equations, any one of them is called as Euler's equation. These are known as Euler's equations according to the name of this scientist, this is known as Euler's equation. This should not be confused with Euler's equation of motion. This is the Euler's equation for fluid machines. This comes from the equation of motion.

But another Euler's equation you have already read in your fluid mechanics class, that is, the equation of motion for an in-visit fluid is a generalized equation of motion for a flow of an in-visit fluid. But this Euler's equation is the equation for the energy transfer between the fluid and a rotodynamic machine. This is the form where the rate of energy transfer is giving, that is, the energy transfer per unit mass and this expression is the energy transfer per unit weight or a head, sometimes we call it as head. Here you can write head h , that is, the head transfer.

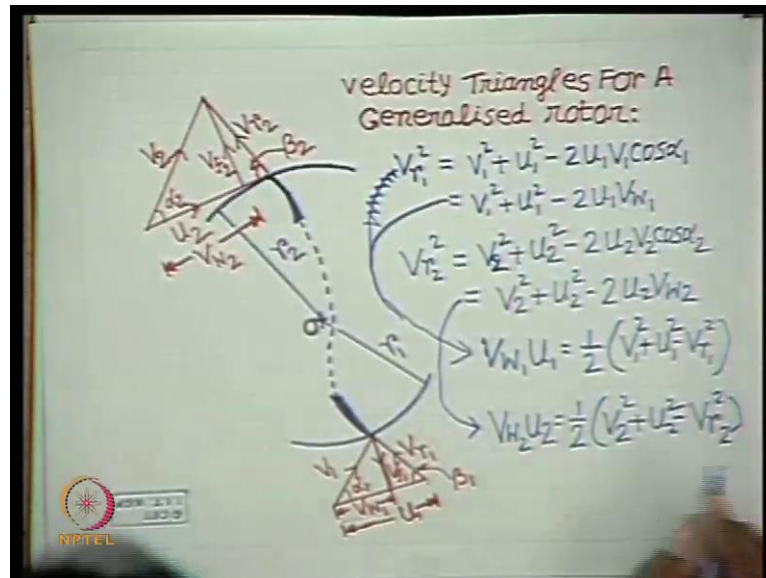
Now, it is very simple. You can understand, that when, when $V W 1 u 1$ is greater than $V W 2 u 2$, this quantity is more than this this. $E \dot{}$ is positive, h is positive, that means, energy is being transferred to the rotor. We get mechanical energy from the fluid, from the stored energy in the fluid while $V W$. When it is the reverse, that is, $V W 2$ is greater than $V W 2 u 2$, I am sorry, $V W 1 u 1$, that means, this is greater than this, this is greater than this, then this is negative, means, the energy is transferred from the rotor to the fluid as it passes from its inlet to outlet.

So, if a turbine, as we know, that classifications depending upon the direction of energy transfer, this is always greater than this. The $V W 1 u 1$ is always greater than $V W 2 u 2$. And in case of compressors where the mechanical energy is being imparted to the fluid by the rotor of the machine, $V W 2 u 2$ is greater than $V W 1 u 1$.

Now, I will come to the different components of energy transfer. Different components of energy transfer means, that how we can express the basic Euler's equations in a

different fashion where we can recognize, that the energy transfer between the fluid and the rotor consists of different components, which are, which have their very interesting physical implications.

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To analyze that let us consider the velocity triangles for a generalized rotor. Now, this is a rotor, a generalized rotor where we show the inlet and outlet part of the vane, rotating vane with the radius of rotation at r_1 and r_2 , that is nothing different from the earlier picture, but with the velocity triangles drawn in inlet and outlet. Here, you see, the inlet velocity is v_1 and the relative velocity of the fluid is v_{r1} . This is the whirling component of the inlet velocity, this is the velocity of the rotor at the inlet, linear velocity or tangential velocity.

Now, in this context I like to tell you, that probably you know, that the relative velocity should be such that it should approach the vane without shock, that for a shock and smooth approach of the fluid to the vane, so that fluid can glide along the vane. The relative velocity should make an angle with the direction of the vane motion, which is same as the angle of the vane at its inlet. So, therefore the vane angle at the inlet is same as the angle of the relative velocity β_1 . So, therefore, we will consider henceforth the angle of the relative velocity with any direction. For example, here in the direction of the vane velocity is equal to the vane angle at that point.

Similarly, here you see, the outlet velocity triangle is shown where v_2 is the velocity of discharge of the fluid and v_{r2} is the relative velocity of the fluid at discharge whose angle is β_2 with the tangential direction, that is, the same angle the vane makes at the outlet with the tangential direction, that is, the outlet vane angle. So, this β_1 is the inlet vane angle. Now, this u_2 is the tangential velocity or the linear velocity, that is, in the tangential direction of the vane at the outlet.

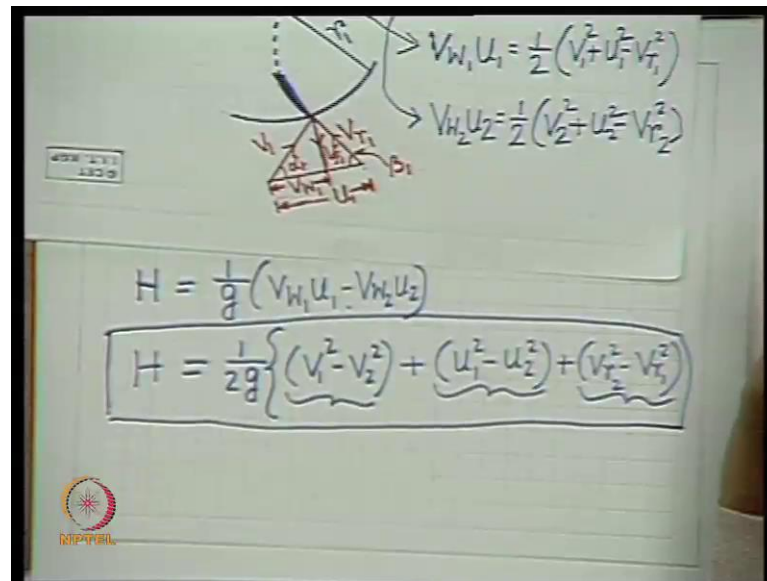
Now, α_1 is the angle with, angle, which the velocity v_1 at the inlet makes. Similarly, α_2 is the angle, which the velocity v_2 at the outlet makes with the tangential direction. Now, these velocity triangles gives the picture of the relative velocities and the absolute velocities at the inlet and outlet of a generalized rotor. The v_{F2} and v_{F1} are the flow velocities at the outlet and inlet.

Now, if we apply certain trigonometric formula for the triangles we see, at the inlet triangle we can write, I think you can see it, that v_{r1}^2 is equal to v_1^2 plus u_1^2 minus twice $u_1 v_1 \cos \alpha_1$ or equal to, we can write v_1^2 plus u_1^2 square. Now, $v_1 \cos \alpha_1$ is the whirling component or tangential component of the velocity at the inlet $u_1 v_w$, ok. Can follow?

Similarly, from the outlet triangle we can develop the same relationship, v_{r2}^2 square is equal to v_2^2 square. That means, v_2^2 square plus u_2^2 square plus u_2^2 square, sorry, this 2 will be big, minus twice $u_2 v_2 \cos \alpha_2$, or we can write v_2^2 square plus u_2^2 square minus twice $u_2 v_w$, well, from the two velocity triangles, alright.

Now, from here, therefore we can write, that not from here, sorry, this is from this step, not from this step. From this step we can write, that $v_w u_1$ is equal to half v_1^2 square plus u_1^2 square minus v_{r1}^2 square. And from this step we can write $v_w u_2$ is equal to half. v_2^2 square plus u_2^2 square minus this thing, v_{r2}^2 square. Now, if you see this expression with this expression and if you recollect the Euler's equation, Euler's equation was that energy transfer or head.

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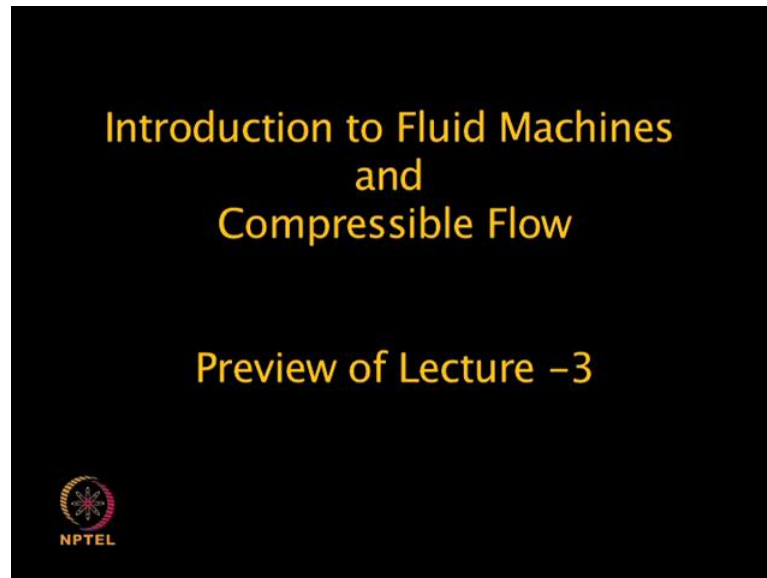
Let us consider the head that is being given to the machine by the fluid. If you recollect, it was $\frac{1}{g}(V_{w1}u_1 - V_{w2}u_2)$, that is, the amount of energy per unit weight that is being transferred from the fluid to the rotor.

So, just by mere substitution, very simple mathematics, I am telling in fluid machines, mathematics are not as complicated as you have in fluid dynamics, no partial differential equations, ordinary differential equations. This is very simple because in uniform flow we only deal with some algebraic states. So, if we just substitute this, we will get $\frac{1}{2g}$, three distinct terms we will get, one is $V_1^2 - V_2^2$. Another is $u_1^2 - u_2^2$ and another is $V_{r2}^2 - V_{r1}^2$.

So, this is another form of the energy equation. It is not an independent equation. Just by exploiting the relationship between the velocities from the velocity triangles at inlet and outlet, we have just expressed this expression into a different form showing distinctly the three components as the energy transfer components in case of fluid machines. And next part will be the discussion or the physical implication of this three components.

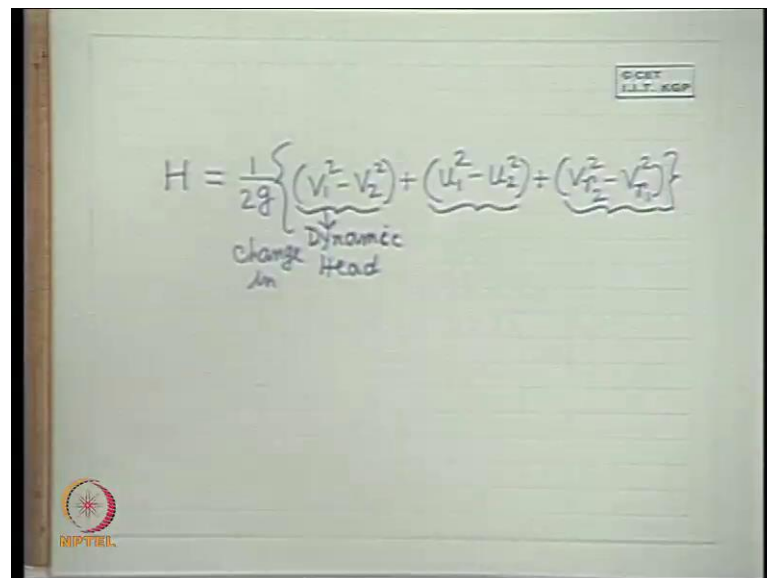
Thank you, today I will stop here, that I will discuss in the next class.

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Good morning welcome you to this session. We will discuss today the energy transfer in fluid machines part two in continuation of earlier discussion.

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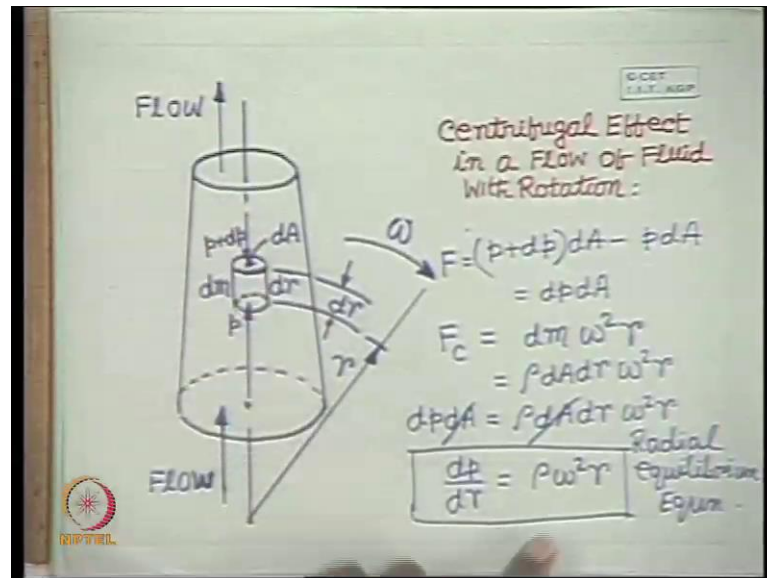
Now, last, in last discussion we have recognized, that the energy transfer to the rotor of the machine by the fluid in terms of the energy per unit weight, which is known as head can be expressed as, just let me write, $\frac{1}{2g} V_1^2 - \frac{1}{2g} V_2^2 + \frac{1}{2g} u_1^2 - \frac{1}{2g} u_2^2 + \frac{1}{2g} V_{r2}^2 - \frac{1}{2g} V_{r1}^2$.

So, in last, last session we have recognized, that the energy per unit weight, that is, the head transfer to the fluid rotor can be split into three distinct components where the nomenclatures are like this. You can have a recapitulation, that where V_1 , V_2 , u_1 , u_2 , V_{r2} , V_{r1} are like this, that V_1 and V_2 are the absolute velocities of the fluid, that inlet and outlet of the rotor u_1 and u_2 are the tangential velocities of the rotor at inlet and outlet. These are the rotor velocities, whirling velocity of the rotor at inlet and outlet and V_{r1} and V_{r2} are respectively the relative velocity of the fluid with respect to the rotor at inlet and outlet.

So, if V_1 , V_2 , u_1 , u_2 , V_{r2} , V_{r1} are defined like this, we can express the head that is transferred to the machine by the fluid as it flows through the rotor vanes can be written like this. Now, you see, that these three terms have got there different physical implications. Now, let us see first what is the term $V_1^2 - V_2^2$. This implies a change in the velocity head of the fluid or in change in the kinetic head, kinetic energy per unit weight of the fluid or simply, it can be told as dynamic head, dynamic head, you can write a change in dynamic head, change in, change in dynamic head. This, this one, change in dynamic head.

So, therefore, due to the change in the dynamic head of the fluid, that means, it is the change in absolute velocities, as it flows pass the vanes the work is being transferred or energy is being transferred to the machine. Similarly, this term represents a change in the head due to the change in its position, radial position with respect to the axis of the rotation when a fluid has got a rotational velocity, and it changes its radial position with respect to the axis of rotation. There occurs a change in the head or energy in the fluid.

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Now, this term can be better understood if we see this one. Let us consider a container where the fluid is flowing in this direction and container is given an angular rotation ω like this. So, basic objective is to show, that when a fluid element under a rotational velocity changes its position in radial coordinates with respect to the axis of rotation. Let this is the axis of rotation at this point perpendicular to this plane of the figure about which the container is rotating, then we can show, that the work is either being done on the fluid element or work is being extracted from the fluid element how we can show.

Now, let us consider a fluid element at a radius r of thickness dr and area dA . Now, you know, that whenever there is a rotational flow field, it induces a pressure gradient, a pressure variation in the flow in the direction of the flow exists for which the pressure in the positive direction. This direction of the r is higher than that at this upstream plane. So, therefore if we take the force balance of the fluid element we see, that net force acting on the fluid element in the radially inward direction, is can be written as p plus dp into dA where dA is the cross-sectional area of the fluid element minus p dA , well, which can be written as $dpdA$. So, what is $dpdA$ is the net force in the radial inward direction. Let me denote it by F , that is equal to dp into dA .

Now, this radial inward force balances this centrifugal force due to the rotational motion of the fluid element. So, this radial inward force balances the centrifugal force of the

fluid element under rotational velocity. So, what is the centrifugal force? What is the centrifugal force? Let F_c for the fluid element, it is the elemental mass dm times the linear velocity due to this rotation, that is, the tangential velocity V^2 by the radius or the radial location from the axis of rotation. This can be written in terms of the angular velocity as $dm \omega^2 r$. This is the usual expression of the centrifugal force, which is acted on this fluid element.

Now, if we substitute the mass in terms of the area and the other geometrical dimensions and the density of the fluid element, we can write it $\rho dA dr$. So, $\rho dA dr$ is the mass of the fluid element, so this is the angular velocity square into r . Now, at equilibrium these two are equal that means, the fluid motion is possible in this direction provided there is a balance between the centrifugal force and the inward radial pressure force. So, if we write this, we get the expression dp into dA is equal to $\rho dA dr \omega^2$ into r . So, dA cancels out. Well, we can write then dp/dr is equal to $\rho \omega^2 r$. This equation is a very well known equation in the fluid flow with rotational velocity and is known as radial equilibrium equation. This is known as radial equilibrium equation.

I can write it, that radial, radial equilibrium equation, this equation simply implies, that when there is a rotational velocity in a flow field and fluid flows in the radial direction, then inward radial pressure gradient is imposed on the flow field, which provides the necessary pressure forces to be balanced with the centrifugal force. You know, that in any rotational motion of, in your solid body there is, there are two forces are in balance with each other, one is the centrifugal force, which tends to make it flying away from the path and another is a centripetal force, which is a force, which makes the possible to have the rotational motion, which is inward towards the center of rotation. So, this centripetal force is provided by the pressure gradient through these pressure forces. This is the well-known radial equilibrium equation.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, the differential equation is written as $\frac{dp}{\rho} = \omega^2 r dr$. Below this, the equation is integrated from point 1 to point 2: $\int_1^2 \frac{dp}{\rho} = \int_1^2 \omega^2 r dr$. This is then simplified to $= \frac{1}{2} (\omega^2 r_2^2 - \omega^2 r_1^2)$. An arrow points from this result to the equation $\text{Flow Work} = \frac{1}{2} (u_2^2 - u_1^2)$. At the bottom, a diagram of a closed system is shown with work W being done on it, and the equation $W = \int_1^2 p dV$ is written next to it. The whiteboard also features a small logo in the bottom left corner that says 'NPTEL' and a stamp in the top right corner that says 'CCET I.I.T. KGP'.

Now, if I write this in a little different form, the same equation can be written as dp by ρ is equal to, dp by ρ is equal to ω square r dr . Now, if I integrate this equation dp by ρ , integrate this equation ω square r dr between two points 1 and 2, between two points 1 and 2, which physically indicates the two points.

Let 1 is at the inlet and 2 is at the outlet. It may be any two points in the flow field, 1 is an upstream point and 2 is a downstream point, which may be at the inlet and outlet in a flow passage, well, then we can write this 1 by 2 dp by $d\rho$ is equal to half ω square r 2 square minus ω square r 1 square, which is nothing but half the linear velocity or the tangential velocity due to the rotation. At the point 2, the section 2 minus u 1 square. What is the meaning of this? Now, what is this dp by ρ form 1 to 2 integral? This is the flow work, flow work.

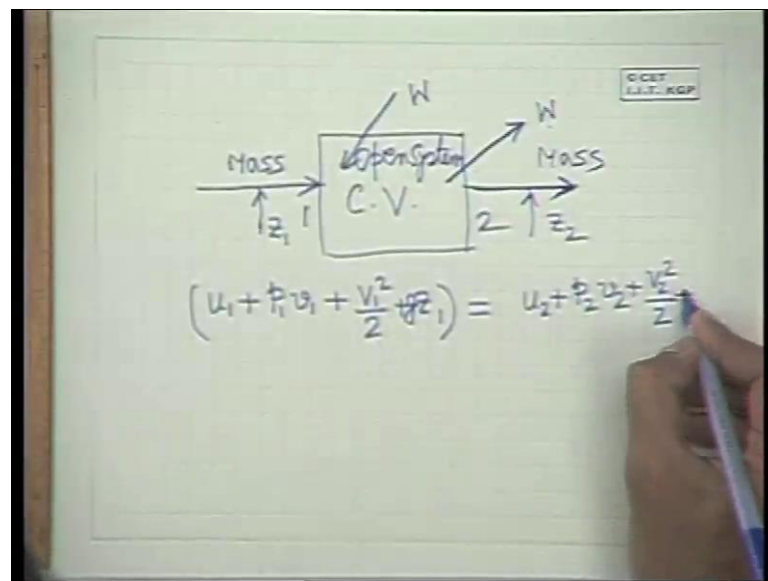
Now, I come to the concept of flow work now. If you recollect thermodynamic general energy equation, you know what is flow work, let us recapitulate little bit of thermodynamic concept. You know when you have a closed system, when you have a closed system and it interacts with the surrounding in terms of work, either work is being developed by the system to the surrounding or is absorbed from the surrounding to the system.

Mechanical work if you consider, the most usual form is by the displacement of the system boundary, by the displacement of the system boundary for a closed system

because the mass within that system is fixed. And under reversible condition, this work transfer is written as $p dv$ where dv is the v cut, dv is the change in the volume. So, the integral is made between the two state points, 1 and 2, well.

But what happens wherein the system is a, is an open system? That means, in thermodynamics we know there are two types of system, one is the closed system and the mass is fixed with the same identity that is known as control mass system, usually we tell as system. Another system is there where the mass is not fixed with the identity. There is a continuous flow of mass in and flow of mass out where the volume is controlled, volume is fixed, known as control volume system and usually, we tell as open system or a control volume. So, in case of an open system or a control volume, that means, an open system, open system or a control volume, there is a continuous influx of mass and energy continuous mass coming and mass going out.

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Similarly, in this control volume if it interacts with surrounding in the form of work, that means, if it develops work or it absorbs work, which comes in our case of fluid machines. A fluid machine is an open system, continuously the fluid comes into the machine at one part and it goes out of the machine by virtue of which the machine develops work to the surrounding in the form of shaft work. In some machines it is being developed to the surrounding. The shaft work is being obtained by us and in some cases, the machine absorbs the work from the surrounding. That means, the work is being put to

the shaft in the form of the shaft work. This is the case of compressors and pump while the work is obtained in case of turbine.

So, how to find out this work? In this case, in this cases we write the steady flow energy equation. This is a little recapitulation of your thermodynamic concepts, so that you can recognize or appreciate the term dp by $d\rho$ as the flow work. So, what is that? If we write the thermodynamic equation, general energy equation of thermodynamics at section 1 and 2, then we can write, that the internal energy at 1, u_1 associated with the mass flux plus the pressure energy, which is retained in thermodynamics in terms of the specific volume rather than the density plus, we write the kinetic energy v_1^2 square by 2 per unit mass basis, if we write.

So, if we consider the potential energies at this and this sections are given like this, if we denote z_1 and z_2 are the elevations from reference datum, so this quantity represents the amount of energy influx per unit mass with the mass flow coming into the control volume. Similarly, the amount of energy going out from the control volume associated with the mass flux out of the control volume per unit mass will be the same energy quantities with their values at the outlet section denoted by the suffix 2. I am sorry, per unit mass means, this will be gz_1 , so this will be gz_2 .