

**Introduction to Fluid Machines, and Compressible Flow**  
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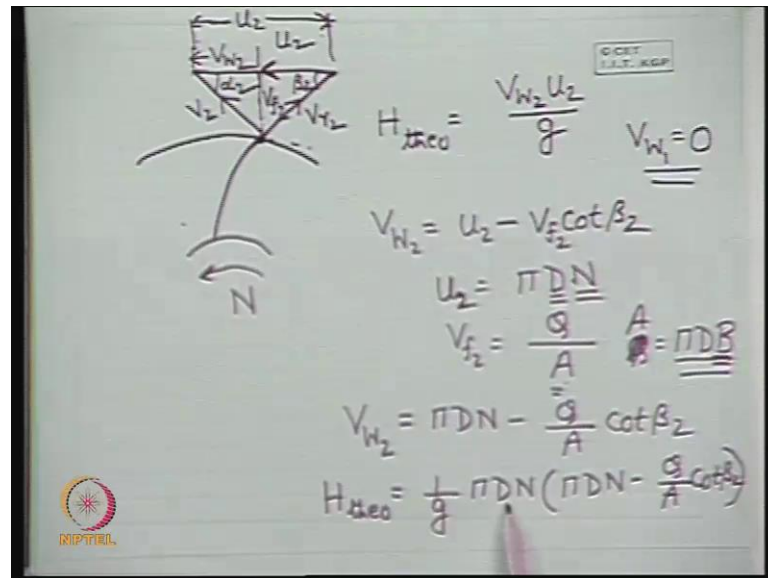
**Lecture - 15**  
**Characteristics of a Centrifugal Pump**

Good afternoon I welcome you to this session. Today we will be discussing the characteristics of a centrifugal pump last class we discussed the head developed by a centrifugal pump, and also discussed the phenomena slip in a centrifugal pump, now last class you asked a particular question at the end that whether by increasing the number of bins we can reduce the phenomena slip yes its very correct, but by increasing the number of bins, and by reducing the passage area blade passage area we can definitely reduce the phenomena slip, but the number of, but on the other hand what happens is that with an increase in number of bins the frictional losses that is the friction between the fluid, and the blade surface increases.

So, therefore, the number of bins are decided in the compromise of the slip, and the frictional losses, because both of them are detrimental from the head developed point of view, because both slip, and the frictional losses reduce the head developed by the pump. So, today we will be discussing the characteristics of a centrifugal pump what is meant by characteristics or performance characteristics that is the performance of a pump now the performance of a pump or the characteristics of a pump are described or usually described by its relation between the head developed, and the flow rate at a given rotational speed as you have noted earlier that the specific speed of a pump is characterized by the rotational speed flow rate, and the head developed  $n$ ,  $q$ , and  $h$ .

So, therefore, these three quantities are very important performance parameters, and the relationship between head developed with the flow rate at a given rotational speed is usually referred to as a characteristic curve or the characteristic of an centrifugal pump. So, today we will discuss that, let us first see that.

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If you recall the diagram the velocity triangle for a pump impeller as we have seen that that this is moving like this, this is typical blade. If we recall this diagram well, if you recall this diagram that this that the blade outlet this is the  $v_r$  two this is the the blade speed at the outlet, and this is the absolute velocity. So, this angle is the blade angle with the tangent; that means, this angle with the tangent this is beta two well, and this is alpha two that is the angle the absolute velocity makes with the tangential direction. So, this velocity component is  $v_f$  two flow velocity that is the radial velocity at the outlet.

Now, we see that the head theoretical head developed theoretical head developed is  $v_w$  two  $u$  two this we have proved this is, because the  $v_w$  one is zero  $v_w$  one is zero the inlet velocity triangle is such that it gives zero wheeling velocity zero wheel velocity. So, that this is the net head imparted by the rotor to the fluid, and under theoretical conditions; that means, if we consider the fluid to be in visit, and in the absence of slip this is the head developed by the pump; that means, this is the theoretical head; that means, fluid is considered to be in visit along with that the phenomena's of slip is absent well.

So, now from the trigonometric relationship from this outlet velocity triangle we can write which one is, then  $v_w$  two  $v_w$  two this is  $v_w$  two, and this is  $u$  two this, this is  $u$  two this is  $u$  two this is the symbol  $u$  two in this direction velocity triangle this is  $u$  two this is  $v_w$  two. So, we can write  $v_w$  two is  $u$  two minus this. So, from simple trigonometric

relation we can write this is  $v_f^2$   $v_f^2$  is the flow velocity that is the radial component of velocity in this case this is we can write as  $v_f^2 \cot \beta^2$  in terms of  $v_f^2$  this one  $v_f^2 \cot \beta^2$ ; that means, this part is  $v_f^2 \cot \beta^2$  which is subtracted from  $u^2$  we get the  $v_w^2$ ; that means, the wheeling component of velocity at the outlet.

Now, pump for a pump moving with a rotational speed in we can write this  $u^2$  is  $\pi d n$  in the relationship between the linear speed, and the rotational speed where  $n$  is the revolutionary speed rotational speed with the same unit of time for example, it is revolution per second. So,  $u$  will be per second meter per second  $d$  is the diameter of the impeller which is the outlet diameter; that means, the diameter of the impeller at the tip of the blade which is usually referred to as the diameter of the impeller.

So, henceforth whenever we will come across with this terminology diameter of the impeller we will mean that it is the diameter at the blade tip that is the outlet diameter  $\pi d n$  now  $v_f^2$  that flow velocity can be expressed in terms of the volumetric flow rate through the impeller divided by the cross sectional area  $a$  well this cross sectional area is the area normal to this flow velocity. In fact, this is equal to  $\pi d$  into  $b$  where  $b$  is the width of the impeller at the outlet width; that means, perpendicular direction this direction the width  $\pi d$  into width is the perpendicular area; that means, the area perpendicular to the flow velocity cross sectional area we simply sorry I am sorry  $a$  is equal to we simply denote it by  $a$ . So, this is the cross sectional area of flow  $q$  by  $a$  is  $v_f^2$  all right. So, if you substitute this we will get  $v_w^2$  is what  $\pi d n$  minus  $q$  by  $a \cot \beta^2$  well.

Therefore we get  $h_{\text{theoretical}}$  is equal to  $v_w^2 u^2$  by  $g$ ; that means, we can write one by  $g v_w^2 u^2$   $u^2$  is  $\pi d n$ ; that means,  $\pi d n$  into  $\pi d n$  minus  $q$  by  $a \cot \beta^2$  all right  $v_w^2 u^2$  by  $g$ .

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$$H_{theo} = \frac{\pi^2 D^2 N^2}{g} - \frac{\pi D N}{g A} \cot \beta_2 Q$$

$$H_{theo} = K_1 - K_2 Q$$

$$K_1 = \frac{\pi^2 D^2 N^2}{g}$$

$$K_2 = \frac{\pi D N}{g A} \cot \beta_2$$

$$H_{theo} = K_1 - K_2 Q$$

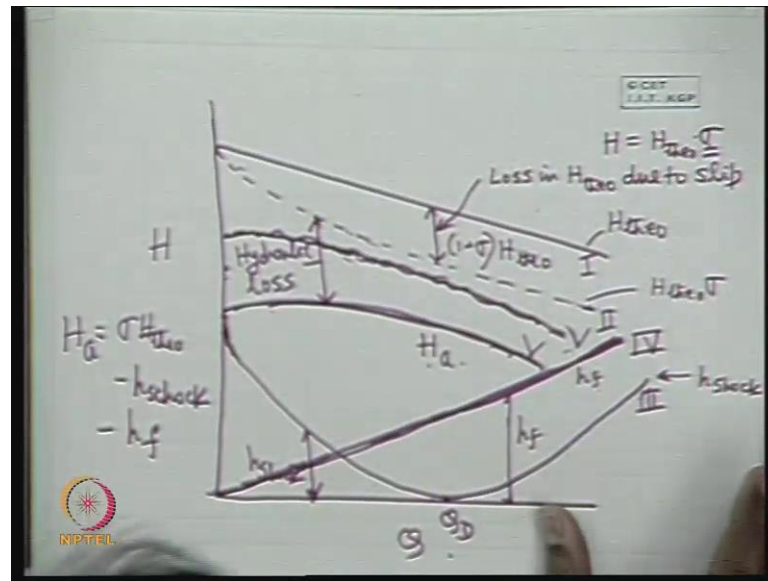
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So, this can be written as same expression here I write h theoretical is equal to pi square d square n square by g minus q into pi d n divided by g into a cot beta two rather this q I write here this into q ok.

Now, for a pump of a particular design, and moving with a rotational speed n fixed r p m n revolutionary speed n this quantity is constant pi d square n square g pi d n g a are constant, because their dimensions are fixed the diameter of the impeller the cross sectional area of the flow at outlet the angle of the blade at outlet all are fixed. So, it can be written as k one minus. So, this can be expressed as this where this is k one k one is equal to this quantity pi square d square n square by g, and k two is equal to this quantity pi d n by which means this includes the geometrical parameters, and the rotational speed of the impeller.

So, we see that the head discharge relationship can be expressed as this the head developed theoretical head developed is some constant that is the linear relationship k two k one minus k two q.

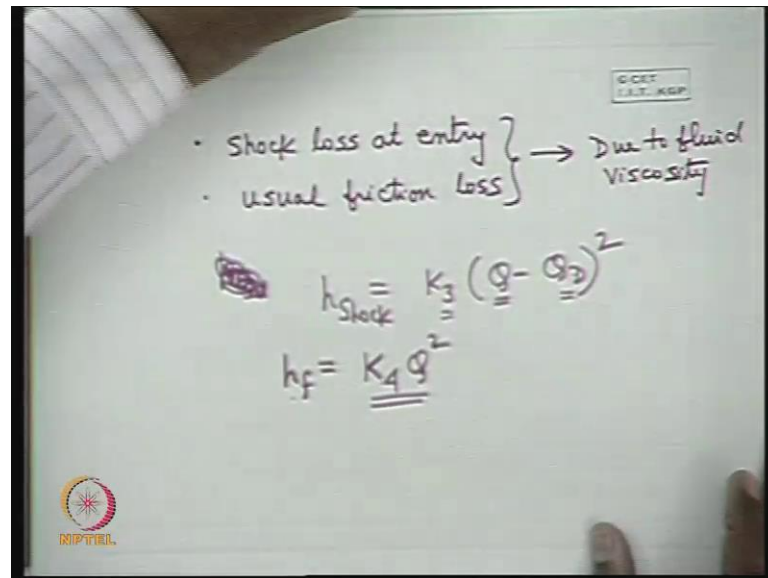
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So, this can be expressed in a figure like this, this is  $q$ , and this is  $h$ . So, this can be expressed like this theoretical head that is  $h$  theoretical; that means,  $k_1$  minus  $k_2$   $q$  this expression  $k_1$  minus  $k_2$  now the actual head developed now due to slip this head will be reduced by the factor  $\sigma$  into  $h$  theoretical that if you take care of the slip, then  $h$  will be  $h$  theoretical into the slip factor  $\sigma$  if we take care of the slip only; that means, you consider the in visit fluid, but at the same time we consider the slip phenomena which is also taking place for in visit fluid.

And this value of this slip factor changes with flow and. In fact, this decreases with increase in flow rate; that means, the phenomena of slip becomes more prominent with increase in flow if we multiply these, and find the head developed or we correct the head in consideration of the slip phenomena I just draw it with this dotted one this is the curve which is  $h$  is equal to  $h$  theoretical times  $\sigma$  which means that this represents the one minus  $\sigma$  into  $h$  theoretical; that means, this is the loss in the head developed due to loss in  $h$  that is head developed loss in theoretical head developed due to slip due to slip this is this. So, you can draw this curve slip factor decreases with increase in flow.

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Now, we have to take care of frictional losses, and other losses. So, there are two types of major losses that take place this can be written like that one is the shock loss shock loss at entry another is the usual friction loss usual friction loss both the things are due to the viscosity of the fluid due to fluid viscosity which means that if the fluid viscosity is zero both the losses cannot take place, but the cause of the shock loss at entry is differing from that due to frictional loss.

Now, we have seen that the fluid the design of the blade is made in such a way that at design condition the fluid glides the blade at the inlet; that means, the angle of the fluid at the inlet relative to the blade becomes equal to the angle of the fluid at the inlet relative to the blade becomes equal to the angle of the blade at the inlet with respect to any specified direction the direction is tangent direction along the tangent at that point; that means, the blade angle at the inlet equals to the angle of the relative velocity of the fluid at the inlet which means that fluid smoothly glides over the blade.

But when pump works at a condition rather than the design conditions or, because of some altered conditions of flow it may. So, happen that the fluid may not strike or may not start the blade at the inlet with the angle of the blade at the inlet; that means, the inlet angle of the relative velocity of the fluid will be different from that of the blade angle at the inlet which means the fluid obliquely hits the blade for which there are losses these losses takes place, because of formation of eddies the change in the direction of the flow

velocity takes place for which the eddies are formed, and these eddies cause a loss in mechanical energy or a conversion of mechanical energy into the intermolecular energy.

So, fluid viscosity is the agent of curtailing the mechanical energy due to this phenomena the loss taking place a loss taking place, because of a change in the direction of the relative velocity from the angle at the inlet from the angle of the blade at the inlet, but this cause of change in the work head or conversion of a part of the work head into the intermolecular energy which we think as a loss is due to the fluid viscosity. So, therefore, fluid viscosity is responsible for this loss; that means, for a real fluid if the entry angle of the fluid; that means, the fluid with respect to the blade the (( )) relative velocity of the fluid differs from that at the inlet angle of the blade fluid cannot enter smoothly along the blade cannot glide along the blade for which this type of losses take place this is known as shock loss.

Another loss is the usual friction loss now this shock loss can be expressed as this if we write the shock loss that is the loss of head rather I write  $h_s$  the shock  $h_s$  is the shock loss or is full shock this is expressed as some constant  $k_3$  can be expressed as  $q$  minus  $q_d$  whole square now why it is expressed like this where  $k_3$  is a constant  $q$  is the flow rate through the impeller, and  $q_d$  is the flow rate at the design condition; that means, at the design condition the fluid enters with the same angle of the blade at the inlet the glides along the blade for which the shock loss is zero.

So, when  $q$  is equal to  $q_d$  this is zero, but on either side of the  $q_d$ ; that means, when the flow is lower than the flow at design point design condition or even greater than that the shock losses increases. So, therefore, this is expressed as in index two as a  $q$  minus  $q_d$  to the power two this type of functional relationship occur. Now friction loss now this we can show like this in this graph this curve this curve represents therefore, let this curve be give the name three let this curve be give name two let this. So, this is the  $h_f$  shock. So, this is the magnitude for a given flow rate of the shock head loss due to  $h_s$  shock this is zero when  $q$  is equal to  $q_d$  on either side of this  $q_d$  this increases.

Now, come to the frictional loss well what is frictional loss frictional loss is the usual viscosity phenomena between the fluid friction at the solid surface as you know for which even for the flow of a fluid through a fixed duct we get the pressure loss. So, this is simply the frictional loss this is due to the viscosity of the fluid friction between the

fluid layer, and the solid surface, and between layer to layer of the fluid. So, this is usual friction loss as. So, we did the flow of fluid through a solid duct which can be expressed in terms of the square of the flow rate this is an usual information that is known to you that the loss in energy due to fluid viscosity is proportional to the square of the velocity when the flow is in the turbulent region you have seen that where the pressure drop is proportional to the square of the velocity.

So, frictional loss in head due to loss in head due to friction is proportional to the flow velocity or the proportional to the flow rate square proportional to the square of the flow velocity or the square of the flow rate in the turbulent region of flow, and in all fluid machines the flow is in the turbulent region. So, it can be expressed as a constant  $k$  four times the square of the flow rate. So, this can be shown like this, this curve is let four curve four curve four is  $h_f$ .

It is a (( )) straight line parabolic sir.

It is parabolic it is not straight not at all straight I am very sorry just my drawing says like that it is definitely parabolic parabola whose vertex is at the origin very good it is not at all straight it looks like that it is a parabola; that means, at any  $q$  this is the value of  $h_f$  good it is parabola definitely  $h_f$  is  $k$  four  $q$  square  $y$  is  $k$   $x$  square where  $y$  is the ordinate, and  $x$  is the typical parabola very good.

Now, if we deduct these two losses; that means,  $h_f$  plus  $h_{\text{shock}}$  from this head; that means, from  $h$  theory into this we get a curve if you just deduct this we get a curve like this this is zero we get a curve this curve is the sorry this curve is the this this is the actual curve sorry this will be here, because this will touch I am sorry. So, this will be this this will be little bit this part you take a this is the curve five. So, this is the actual head  $h_a$ . So, this is the hydraulic loss. So, this is the hydraulic loss hydraulic loss this is the hydraulic loss.

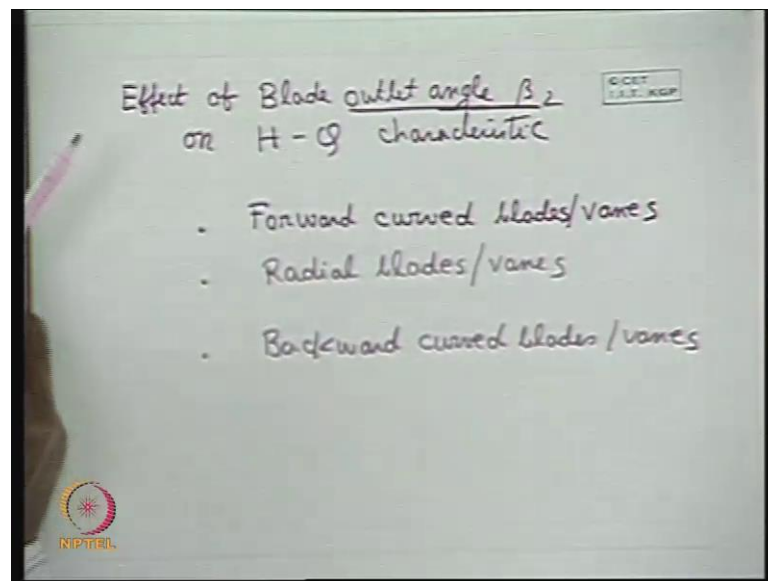
Sir at origin both of them touch.

Yes, no it may not touch it may not touch here it looks like this it may not touch. So, this what is done is that sum of this plus this is deducted from this point deducted from this curve that the ordinate of the four curve four again I am telling, and curve three the sum of the ordinates of curve three, and curve four is deducted from the ordinate of curve



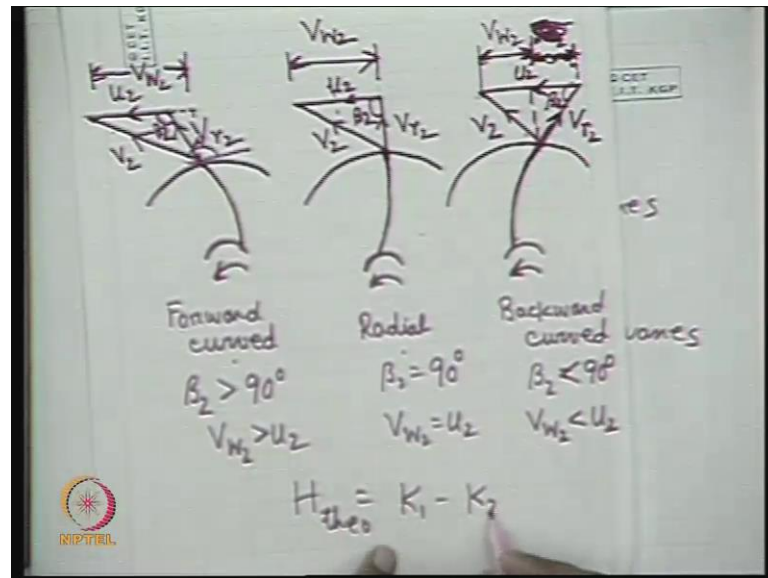
two, then we construct this curve; that means,  $h_a$  is I write this thing this will be clear  $h_a$  is  $\sigma h$  theoretical which is the ordinate of the curve to at any point minus  $h_{\text{shock}}$  well minus  $h_f$  it looks like that it is, because if I draw it here, then I cannot show the hydraulic loss that is why I made it like that it is it appears to be at the same point not necessarily it depends upon the relative magnitude of all this thing. So, ultimately this is converted to a curve like that which keep the actual head versus discharge curve this is the qualitative train these are all qualitative trains all right.

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Now, next I will discuss the effects of effects of blade velocity on  $h$   $q$  characteristics effects of effect of blade velocity no I am sorry blade outlet angle effect of blade outlet angle  $\beta_2$  on  $h$   $q$  characteristic  $h$  here means the actual head characteristic  $h$   $q$  characteristic. Now let us see this diagram, if you can see it I think this is done in an exaggerated way before that I tell you that depending upon the outlet angle of the blade the blades settings are categorized in three distinct way one is forward curved. These are the terminologies you must know forward curved blade or vane blades or vanes another is the radial radial blades or vanes another is backward backward curved blades or vanes depending upon the outlet angle the blades can be a setting of the blades can be categorized into three categories.

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One is the forward curved blades radial blades, and backward curved blades you see that what are meant by that now in the setting of forward curved blades forward curved blades are those where this is forward curved forward curved forward curved; that means, the curvature is in the direction of the rotation; that means, the blade curvature is in the direction of the rotation this is radial; that means, the blade becomes radial almost initially there is little curvature, but finally, towards the outlet blade is radial, and this is the backward curved blade backward curved blade vanes or blades one curve one blade or one vane is shown where the curvature is in the opposite direction to the direction of the rotation.

This is forward curved blade forward curved blade setting this is the radial blade setting this is the backward curved. So, difference is there only in the velocity triangles shape of the velocity triangles in this case the velocity triangle takes this shape you can see this thing this is the  $v_r$  two which under design condition should match the blade outlet angle this is  $\beta_2$  two this is  $\beta_2$  two this is the absolute velocity  $v$  two this is  $u$  two in this case the tangential component of velocity at the outlet is more than the blade velocity at the outlet, and this angle  $\beta_2$  two this is the same angle with the tangent; that means, this angle is greater than ninety degree; that means, obtuse angle  $\beta_2$  two is greater than ninety degree here  $\beta_2$  two is ninety degree; obviously, at design condition the relative velocity angle with the tangent direction of the tangent is same as that of the blade at outlet blade is radial; that means, the  $\beta_2$  two is ninety degree. So,  $v_r$  two makes ninety

degree with  $u^2$ . So, you get a right angle triangle right angle triangle as the velocity triangle at the outlet.

In this case which is the usual case which we have already discussed this velocity triangle we drew earlier with reference to impeller blade where  $\beta_2$  is less than ninety degree; that means, is a backward curved vane the curvature of the vane or blade is in the opposite direction to the direction of the rotation. So, this is  $v_r^2$  this is  $u^2$  this is  $v^2$  here the tangential component of velocity at the outlet is same as that of the blade velocity at the outlet in this case it is more than the blade velocity of the outlet in case of radial blade it is equal to the blade velocity of the outlet, and it is less than the blade velocity at the outlet this is  $u^2$  this is  $v$ .

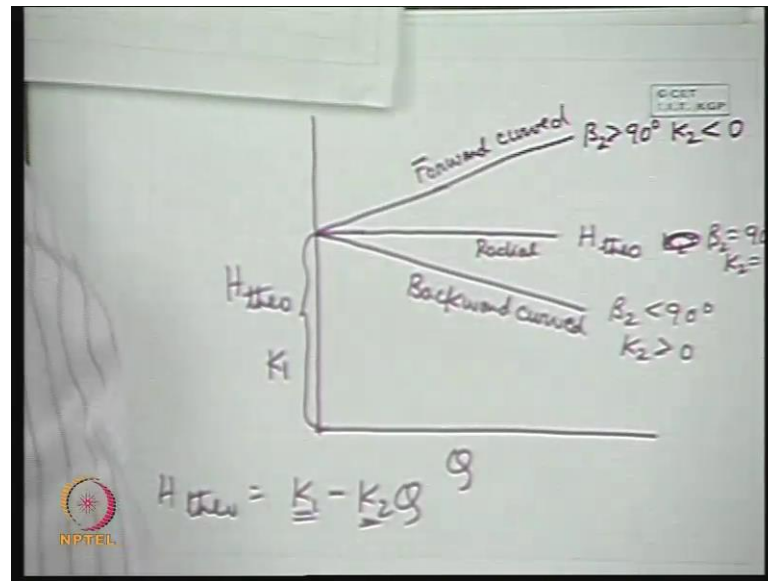
$V_w^2$  will be that this part.

Sir, backward curved.

I am sorry hurriedly I did it you are very correct very good this is a drawback doing it earlier  $v_w^2$  fine. So, in this case  $v_w^2$  is greater than  $u^2$  in this case  $v_w^2$  is  $u^2$  very good in this case  $v_w^2$  is less than  $u^2$  very good I thought that something is going to happen that your face tells like that correct  $v_w^2$ . So, therefore, if you see the general relationship that h theoretical where we developed this expression this  $k$  one minus if I express in this way  $k^2$  into  $q$  what is  $k^2$  if you just go through that this  $k^2$  was this quantity  $\cot \beta_2$ .

So, when  $\beta_2$  is less than ninety it is positive when  $\beta_2$  is greater than ninety this  $k^2$  is negative automatically depending upon the sign of  $\cot \beta_2$ . So, therefore, depending upon whether the  $\beta_2$  is more than ninety or less than ninety. So, therefore, in this case  $k^2$  is less than zero. So, automatically this will be plus in this case  $k^2$  is zero, and in this case  $k^2$  is greater than zero. So, therefore, it is for the backward curved vane, we get a linear curve with a negative slope sloping downwards.

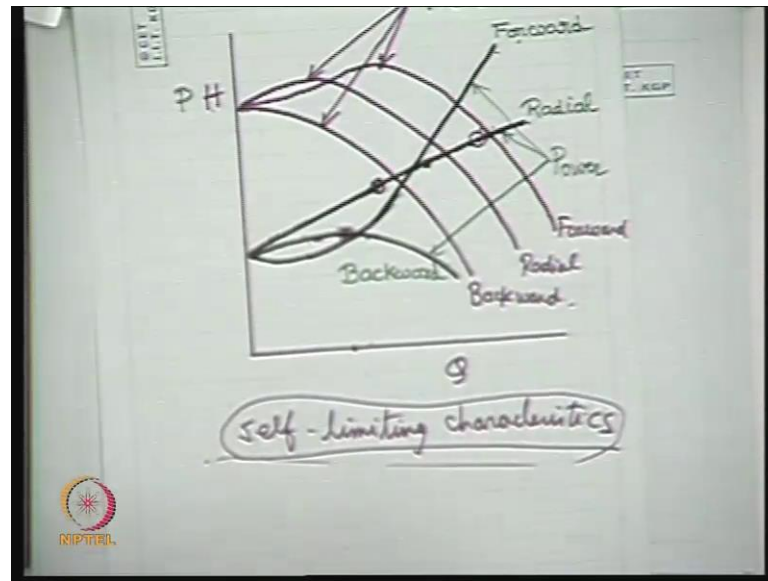
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If we now draw this, the theoretical head simply only the theoretical head versus the  $q$  we will see that this is for radial blade radial blade  $k_2$  is zero  $k_2$  is zero all right. So, this is this is  $k_1$ . So, radial blade  $h_{theoretical}$  which is independent of the flow rate this increases with the flow rate with a positive slope this is there  $k_2$  is zero rather first I write  $\beta_2$  is ninety degree, and  $k_2$  is zero this is this is radial this is forward forward curved in this case this formula this  $k_2$  is negative; that means,  $\beta_2$  is greater than ninety degree, and  $k_2$  is less than zero, because in the actual equation  $h_{theoretical}$  is  $k_1$  minus if I describe this as  $k_1$  minus  $k_2$  here  $k_2$  is less than zero automatically it becomes plus, and this is the backward curved vane backward curved in which case  $\beta_2$  is less than ninety degree, and  $k_2$  is greater than zero. So, automatically it is giving a negative slope.

Now, you if the losses slip, and other losses that is the shock loss, and the frictional loss are taken into account these curves also rework to their actual counterparts that I just show you this is the basic thing.

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That theoretical curve will take this shape this will ultimately come to this shape see that this is the these three the pink color for this this is for h, and these are for power that I will come afterward now this is for backward this one is for radial, and there is the little change like this one is called forward. So, forward curved blade there is the little increase with the h q characteristics initially, then it is followed by a negative slope same is that for a radial, but for a backward curved vane its always with a negative slope.

Now, similarly if we plot the power versus q that is the power requirement, but this depends upon the overall efficiency of the pump. So, it has been found from the tests on pump the power versus flow rate curves are like this for a forward curved blade the power goes on increasing monotonically initially the rate of increase is slow, then it goes on increasing for a radial one it is almost linear; that means, it is gradual continuous monotonic, and gradual increase with the increase in flow rate while the most interesting part is that the power attains a maximum value and; that means, that it increases with an increase in flow rate and, then decreases; that means, at a given flow rate it attains the maximum value.

And unusually it has been found that this maximum power; that means, this point corresponding to this flow is associated with the design point, and the maximum efficiency in case of backward curved vane. So, this maximum power point is associated with the maximum efficiency of the backward curved vane. Now in this relation I like to

tell you a very important thing which sometimes you may be asked that in case of backward curved vane we see that when the flow rate changes from its design value the power required becomes lower.

Therefore if a motor is used to drive the pump at any condition other than the design condition; that means, if it drives the pump at part load, and what will happen that it can be if it is rated at the design condition, then that it can be safely used that for the part load the power is decreased; that means, a motor rated for the design condition can be used for this backward curved vane, because at part load the power is automatically decreasing on both the sides this is known as self limiting characteristics very important self limiting characteristics therefore, the backward curved vane, but yes; that means, if you use a motor for backward curved vane centrifugal pump which is rated for the design condition, but if the motor if the pump works at altered condition from the design condition if by chance the flow rate is increased or decreased; that means, that part load when this pump is working at part load, then the motor can be used with a lower rating.

But when the pump develops this maximum power at the design condition the same pump can be used more safely, because this is rated at the design condition you understand that is a pump which is rated at design condition when it is when the pump is can drive the pump safely at part load, because the power is less than, and automatically it can take care of the maximum power at the design condition this is known as self limiting characteristics; that means, the pump can be safely used under altered conditions from the design condition.

But what happens for other cases forward, and radial vane for example, in the radial case the power goes on increasing monotonically with the flow rate. So, if a pump we select for a maximum power for example, we do not know we know the design condition corresponds to this point on the power curve just an example. So, when by chance the flow rate increases from that. So, the power requirement will be more. So, a motor rated at the design condition will be overloaded, and the motor will fail, but if we take a motor you understand if we take a motor which is rated for the maximum power, then what will happen the motor will be always under utilize, because pump will not be operating at the maximum power. So, we will have to pay for the extra rating.

But if you select a motor which will be of smaller size, and rated at a smaller power for example, the power may be this is may be the point where the design condition is there; that means, the motor pump efficiency is maximum here, and we expect the pump will be running under this condition only by chance the flow rate increases. So, power will increase. So, motor will fail, but if you take a motor whose rating which is rated at this very high power; that means, you take a factor of safety, then we will see under almost all conditions the motor will be under rated.

So, therefore, it is very difficult to choose to have a choice or to choose on the motors from economy side in case of radial, and forward curved vanes this is, because in these two cases the self limiting characteristics is not there this is, because of this typical trend of variation of the power with the  $q$  which is there in case of backward curved vanes. So, therefore, we choose always a backward curved vane as the impeller vanes of a centrifugal pump all right ok. So, today I think upto this next time, we will proceed to pump, and system characteristics.

Thank you any query, please any query you may ask please any query.

Thank you