Introduction to Fluid Machines, and Compressible Flow

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Lecture - 10
Analysis of Force on Francis Runner, and Power Generation

Good morning, I welcome you to this session. Today we will discuss the force exerted, and the power generation the force exerted on the runner of a francis turbine, and the power generation. Last class we discussed the head across the runner or across a francis turbine what is meant by net head across a francis turbine or the head or the work developed by francis turbine, and the variation in the head during the flow in the francis turbine today we will discuss the force exerted by the water on the francis runner, and the corresponding power generation.

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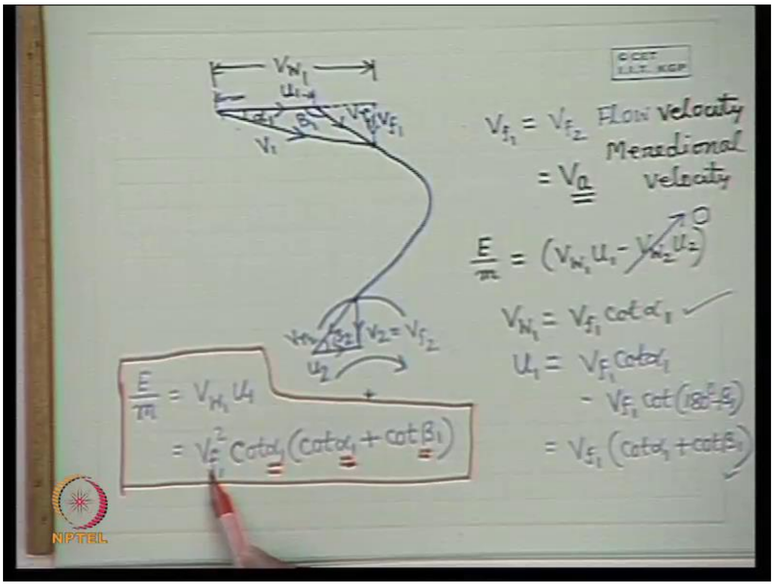

Let us see a typical francis runner first analyze let us see how a francis runner blade if you look a sectional view is like this this is the inlet, if I draw the inlet velocity triangle it will be like this let me draw the inlet velocity triangle. Now this francis runner is moving this rotating let this is the this is the inlet this is the outlet this rotates is the r p m. So, this is the direction of the velocity at the outlet, also we can draw the diagram like this. So, this is the rotor velocity at the inlet u one, this is the relative velocity of the liquid with respect to the rotor v r one.

And this is the absolute velocity v one the outlet velocity triangles look like this this is the velocity v two, this is the outlet relative velocity are the outlet v r two, and this is the rotor velocity u two. Now if we take a particular vane this is like this the section of the vane is like that through which the water flows between two vanes. So, we can draw the velocity vector diagram or velocity triangles that inlet, and outlet like this now the function of the guide vane is to direct the velocity, in such a way that the relative velocity always makes same angle as that of the angle of the vane at in the inlet; that means, this angle.

If we denote at beta one is the angle, which the vane at its inlet makes with the tangential direction u one is the rotor velocity in the tangential direction the linear rotor velocity which depends up on the r p m of the rotor, and the radial distance at this inlet point from the axis of rotation. Let this is the axis of rotation when the fluid flows through the runner vane, and comes out the runner vane is designed in such a way that fluid comes to without any tangential component it is perpendicular to the tangential direction.

So, therefore, this is v two, and this is the rotor velocity or the tangential velocity of the rotor at the outlet which depends upon the rotational speed, and the radius radius at the outlet for the axis of rotation, and this is the corresponding relative velocity of the liquid with respect to the rotor, and this also should match with the vane angle at the outlet; that means, if I denote this as beta two this is the vane outlet angle now flow through runner you have to understand the main direction of the flow through the runner is radial, and tangential at the inlet radial, and tangential at the inlet.

But while it flows out of the runner the tangential velocity is almost diminished there is no tangential velocity. So, the flow becomes radial or little axial this is very important; that means, if you look a turbine for example, the turbine runner in a horizontal plane with the shaft at the shaft being the vertical. So, the inlet is in a horizontal plane in a directions such that it has got both tangential component, and the radial component while flow through the rotor it comes out mostly in a different plane with a radial flow, and an axial flow, and ultimately it is turn completely in your axial direction which is vertical in downward in case of vertical shaft that is along the draft you.

So, you have to understand this way that enters in a direction which is combination of radial, and tangential this is known as a mixed flow radial, and tangential. So, the flow through the runner is usually termed as mixed flow which is which has got both radial component, and the tangential component while it flows out of the runner the tangential component is reduced almost to zero it is discharged mostly in the radial direction, and axial direction little axial component is there; that means, when it comes out of the runner it has got almost radial component without any tangential component which is immediately turn by the pipe into axial direction at the inlet to the draft.

So, this you will have to understand this is the nature of the flow now if you look into this diagram we see that therefore, this is your let this is v r one. So, this is your v f one as we know that is the flow velocity; that means, this is the velocity in the direction perpendicular to the tangential direction; that means, it is radial direction similar here the flow velocity itself is the absolute velocity, because there is no tangential component of the flow at the outlet. So, v two is equal to v f two now one of the main design constraints of the flow is v f one is equal to v f two which is the floe velocity flow.

Flow velocity this component is some time refer to as meridional velocity mere dional; that means, the velocity component perpendicular to the tangential direction the design is made in such a way that the meridional component or the flow velocity component at inlet becomes equal to that at outlet, and ultimately this becomes equal to the axial velocity v a at the inlet to the draft now let us see that the typical velocity from the typical velocity triangle what is the power that is being developed that is being giving by the fluid to the runner vane.

As we know that the energy per unit mass is given by the expression from the Euler's equation v w one u one minus v w two into well what is v w one in this case this is our v w one this is our v w one which we can write in our case v w one from this triangle v w one can be let this angle is alpha one where alpha one is the angle of the guide vanes at its exit, because the incoming velocity the direction of the incoming velocity in the direction of the angle of the guide blade guide vanes are fixed.

So, therefore, when fluid coming out from the guide vanes its direction of the velocity coming from the guide vane; that means, there is no relative velocity relative velocity is the actual velocity or the absolute velocity, because guide vanes are fixed is equal to the angle of the guide vane that is outlet which is the angle of the inlet velocity absolute velocity with the tangential direction that is alpha one. So, we can wrote v w one from this triangle inlet velocity triangle as v f one cot alpha one now our main aim will be to express the energy per unit mass developed in terms of the angles of the vanes at inlet, and outlet v f one cot alpha.

We can write similarly what is the value of u one u one u one is this one this one is u one we can write v f one cot alpha one; that means, this one minus this part minus v f one in terms of v f one if I write cot one eighty degree minus beta one cot of one eighty degree minus beta one if I define beta one with this angle if I define this angle by beta one. So, this becomes v f one cot of alpha one this is minus beta one. So, this becomes plus cot beta one now in this case we see the design is made in such a way that the fluid as zero tangential motion tangential zero component in the tangential direction zero velocity component of velocity the tangential direction which means hurling component of the velocity at outlet is zero; that means, this term is zero.

Now, you can understand very well if this term is made zero we get the maximum work or the maximum power developed by the turbine, because this becomes zero. So, this expression becomes maximum. So, in this case we can write e by m is simply v w one u one, and if we substitute this value of v w one, and this value of u one this becomes equal to v f one square cot alpha one to cot alpha one plus cot beta one well. So, this is the expression for the power generation or the energy per unit mass rather we can tell this is the energy per unit mass. So, energy per unit mass that is being released by the fluid as t flows through the vane or runner of the francis turbine is given by v f one square cot alpha one cot alpha one plus cot beta one.

So, it becomes a function of the inlet angles of the absolute velocity or the exit angles of the guide vanes, and the inlet angle of the runner blade along with the flow velocity which determines the rate flow through the runner well now we like to find out what is the efficiency eta.

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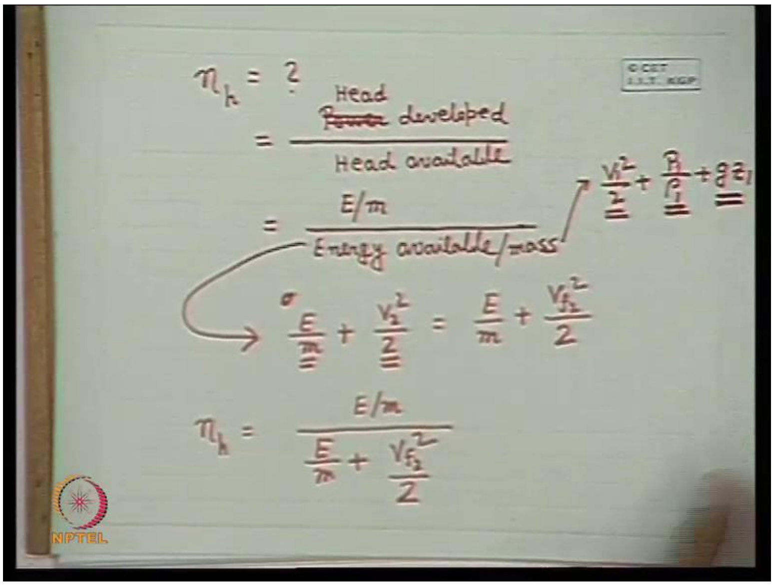

What is the efficiency hydraulic efficiency now what is hydraulic efficiency as you know the hydraulic efficiency the nominator is the power developed power developed power developed or rather I can write the head developed rather I can write the head that is the energy per unit weight head developed divided by the head available; that means, the energy available energy available at the nozzle entrance now power developed we know that is e by m if I express this per unit mass.

So, this is the energy develop per unit mass. So, what is the energy available per unit mass what is this energy available per unit mass energy available please tell me just I will ask you energy available per unit mass we have to find out now if we consider that friction to be zero, then the energy available at the inlet to the runner becomes exactly equal to the energy available at the guide vanes, because if we neglect an frictional laws the total energy remains same. So, what is that energy at the inlet to the guide vane or at the inlet to the runner.

How do you find out please can you tell this b square v one square by two just like the pelton wheel pelton wheel we told that it is v one square by two, but in this case if I have to find out the energy available at the runner inlet what is the energy available at the runner inlet how can I find out.

P 1 by rho g.

Yes, very good it is p one by rho g plus v one square by two plus the data might if at all any that depends upon our choice of reference that term; that means, it compresses kinetic energy pressure energy, and the potential energy very good, but how to find out it is true that this quantity corresponds to v one square by two pus p one by rho one per unit mass basis, and g z one. So, this quantities very absent in case of pelton wheel this is, because the entire energy was in the form of the kinetic energy, but here at the entrance to the runner the pressure energies, and the potential energies there.

So, to find this we have to write in this fashion you can find this in this fashion this energy available per unit mass can be written as the energy developed plus the energy lost; that means, v two square by two; that means, e by m plus v f two, because v two is v f two v f two square by two this is this is, because the this is the energy which is going out of the runner. So, therefore, this is the energy which is expected by the runner. So, if you discard the friction we can tell this plus this this is a total amount of energy which the runner receive that its inlet; that means, this is the energy which compresses the kinetic energy pressure energy, and the potential energy you understand.

So, therefore, tactfully we find this without going for a evaluating what is the pressure energy or what is the stamp p one by rho one we just simply add the amount of energy with which the turbine is going. So, therefore, v two square by two e by m. So, if I add this thing. So, we can get this this is the amount of energy which was at its inlet, because this amount of energy, and the energy developed equals to the total amount of energy that the turbine received or the runner received. So, therefore, I can write tactfully eta h is equal to what is that e by m divided by e by m plus v f two square well by two.

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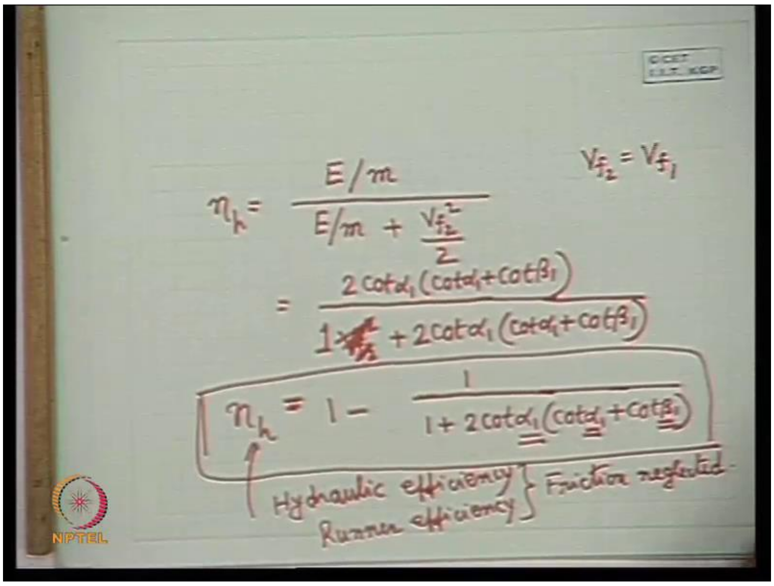

Now, if I substitute again if I write this thing well if I write this eta h is equal to e by m e by m well divided by e by m plus v f two square by two, then it is. So, you all like the concept now is algebraic steps now if I substitute this expression for e by m, and consider that v f two is equal to v f one. So, what I will get v f one will cancels. So, I get two cot alpha can you see this this line two cot alpha one into cot alpha one plus cots beta one; that means, e by m is v f one square cot alpha one into cot alpha one plus cot beta one that I am substituting in place of e by m, then therefore, what I will be getting two; that means, v f two square plus two of this quantity again cot alpha one.

1 plus.

1 plus.

So, this has canceled.

Yes sir.

Good, good, good, because v f two is equal to v f one which is canceled very good one plus very good cot alpha one plus cot beta one this is, because v f one square cancels. So, this can be written in a manner one minus one by; that means, if you add one, and subtract in the nominator one plus two cot alpha one into cot alpha one plus cot beta one. So, this is the expression conventional this is the most useful, and popular expression of the hydraulic efficiency or the runners efficiency in terms of the angels of the vane at inlet, and the guide vane angle at the outlet; that means, alpha one is the angle which the absolute velocity makes with the tangential direction.

And this is the angle with the relative velocity makes with the tangential direction which means alpha one is the angle of the glued vanes at the inlet at the at this outlet. So, all the angles are referred with respect to tangential direction, and beta one is the angle of the runner at the inlets. So, this is the useful expression for hydraulic efficiency or runner efficiency this is the hydraulic efficiency hydraulic efficiency or runner efficiency provided or runner efficiency well or runner efficiency provided the friction is neglected provided the friction is neglected; that means, the runner efficiency becomes the hydraulic efficiency friction neglected in both the guide vanes, and in the runner blades.

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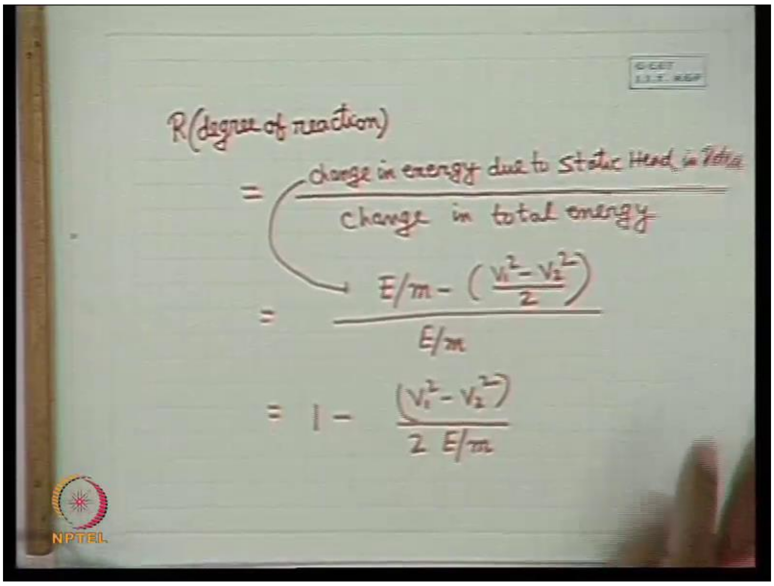

Now, next we come to the expression of degree of reaction well r degree of reaction degree of reaction how did you define degree of reaction in a turbo machines or hydraulic machines please tell me the degree of reaction is the change in energy due to please tell me change in energy change in energy in exactly change in energy due to static head in the rotor that is most important static head in rotor divided by the...

Total change in…

Total change in change in total energy; that means, change in total energy is the power developed change in total energy; that means, it is true that change in total energy is e by m that is the energy per unit mass which we as power that is the change in total energy of the fluid, but what fraction of it is change due to static head; that means, due to the change in the relative velocity, and change in the centrifugal head that means. In fact, it is the change in the pressure of the liquid pressure energy of the liquid, and the change in the pressure energy of the liquid takes place.

Because of the two things we have already recognized earlier this very important that why the pressure pressure energy will change in a liquid, and the liquid flows if there is no swirling or tangential velocities. So, pressure changes only, because of the change in if velocity of the flow velocity of the liquid, and this is with respect to a fixed duct. So, therefore, when the duct is moving or the liquid flows through a passage where the solid as a motion; that means, varies a slip velocity between the liquid, and the solid. So, therefore, in that case what happens that it is the relative velocity whose change will cause a change in the pressure number two.

If along with that liquid posses a swirling motion or the motion in the tangential direction which comes, because of a rotation in the passage rotation of the duct I gave one example, then what happens if the liquid flows in such a way that there is a change in its radial location from the axis of rotation in course of its flow, then a centrifugal head is either released in this or imposed on the fluid for which there will be a change in its special energy or in change in static pressure. So, therefore, change in the pressure energy either release or gain depends upon two factors change in the relative velocity, and change in the tangent centrifugal centrifugal head; that means, change in the tangential velocity.

So, when the relative velocity in decreases, then there is a increase in the pressure when the relative velocity increases, then there is a decreasing pressure in a turbine in the relative velocity increases. So, there is decrease in pressure or decrease in pressure energy similarly in course of flow the centrifugal head is also decreased, because the fluid comes from a higher radial location it is a radial inward flow higher radial location from the axis of rotation to a lower radial location. So, this two quantity constitutes the change in its static head now it is difficult to evaluate the change in the relative velocity that you know that v r two square minus v r one square, and also the change in the centrifugal head that is u one square minus u two square, of course this part is simple.

But without doing that what we do change in energy due to static head can be found out if we deduct from the total change in the energy the change in the dynamic head which seem to that we means we can write this as e by m minus that is the change per unit mass minus this quantity its simple the change in the dynamic head; that means, this part is the energy change due to static head; that means, this compresses the change in the pressure head due to both the change in the relative velocity, and change in the centrifugal head, because of the change in the rotor velocity from inlet to outlet. So, now, it becomes very simple that we can write this it in a fashion one minus v one square minus v two square well by two into e by m.

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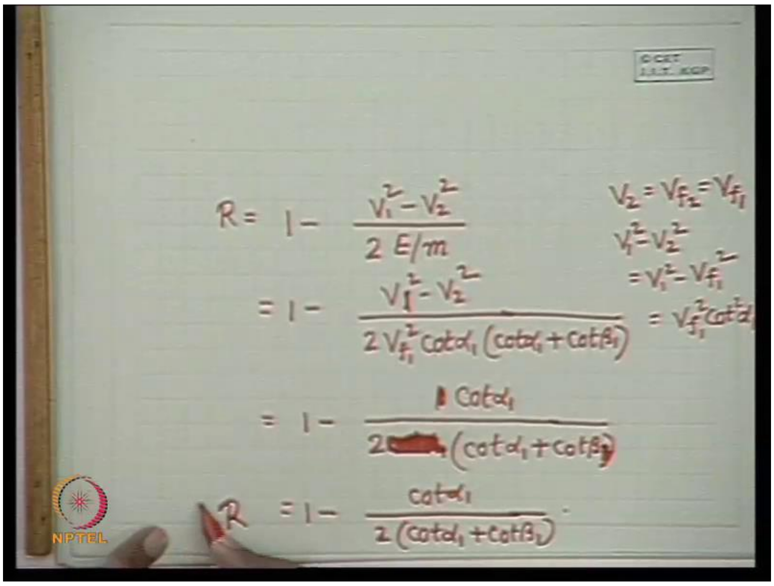

Now, we substitute the value of e by m again. So, let me write r is equal to one minus v one square minus v two square well two e by m well if we again see the value of e by m if we again see the value of e by m is v f one square cot alpha one into cot alpha one plus cot beta one, then what we can write one minus v one, sorry v one square minus v two square by two v f one square cot alpha one into cot alpha one plus well cot beta one now v two is equal to v f two, and that is equal to v f one why f you see the velocity triangle that you see the design constant is that v f one is v f two. So, v f two is equal to v f one now if it is…

So, then this quantity v one square minus v two square therefore, I can write here v one square minus v two square can be written as v one square minus v f one square which can be written as v f one square cot square alpha one why, because from this diagram v one square minus v f one square is v w one square this one, and v w one is v f one cot alpha well. So, therefore, we can write that v w one square is v f one square cot square alpha one. So, v f one square cots square alpha. So, simply we can write one minus therefore, v f one square will cancel out one by two cot alpha one well into this simple algebraic steps only concepts the their when we write this term numerical nominator of this term degree of reaction.

Cot alpha.

Sir nominator cot alpha.

Plus cot well.

Cot alpha divided by.

Nominator.

Sorry very good cot alpha one by two cot very good, because v f one square will cancels. So, cot alpha one will not be there in the denominator at all very good happy. So, if you write this v f one square will cancel, and cot square alpha one will be cancel to be cot alpha one. So, two I am sorry. So, two cot alpha one plus cot beta one say again let me write the expression one minus cot alpha one divided by two cot beta one I think ok.

Yes sir.

This is the degree of reaction r now we come with the deduction of specific speed.

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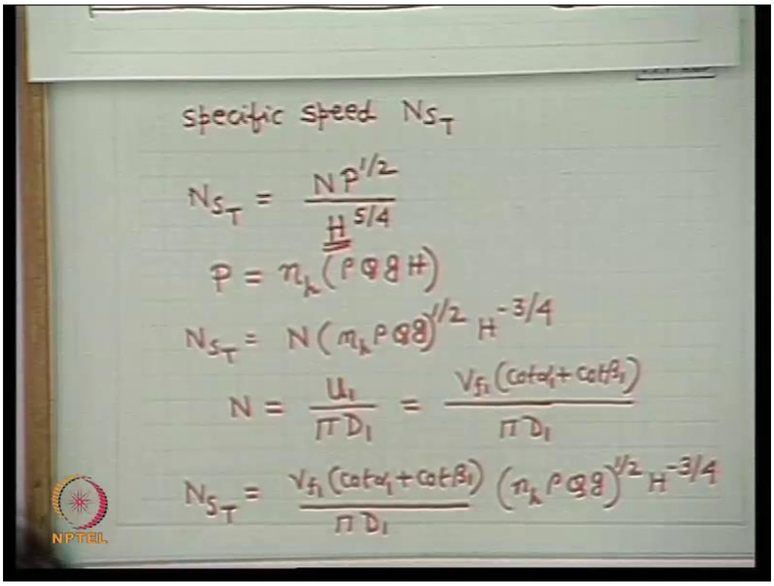

Specific speed well specific speed specific speed n s t if you recall the definition of specific speed for a turbine what its value n s t whenever you will talk about specific speed you will always refer to the dimensional specific speed until, and unless it is told as not dimensionless or non dimensionless specific speed we will always refer to as a dimensionless specific speed. So, what is the value for a turbine n.

N p to the power half.

N p to the power half very good h to the power five by four in the similarly way as we did for pelton wheel we will find out first we will find out power in terms of the hydraulic efficiency well, and the head available rho q q is the volumetric flow g h here by definition h is the head available at the inlet to the turbine; that means, this is the head available to the turbine. So, by definition h is the like that here I will write in terms of definition itself rho q g h eta h well. So, if I substitute this values, then I will get n s t is equal to n p to the power half means eta h rho q g to the power half h I will take out, because h to the power five by four is there in the dominator.

So, therefore, I will write h to the power minus three by four you manipulation that h to the power half will come out, and h to the power five by four which will give a h to the power minus three by four, then again with the same technique you can write we can write the n as the rotational speed u one divided by pi d one; that means, by equating the tangential velocity of the rotator at its inlet with the rotational speed I can write u one by pi d one it can be written at the outlet also u two by pi d two, but I write at the inlet from which I get n is equal to u one by pi d one, and u one.

If you recall the value I show you that we got the value of u one earlier u one is v f one cot alpha plus cot beta one this is from the velocity triangle this is u one. So, this u one is v one v f one cot alpha one minus v f two v f one sorry cot alpha one cot beta one. So, therefore, v f one cot alpha one plus cot beta one this we already derived. So, you just substitute this value of u one as v f one cot alpha one plus cot beta one divided by pi d one now you substitute this value in this n s t that is the specific speed. So, this becomes equal to u one that is v f one v f one cot alpha one plus cot beta one divided by pi d one into eta h rho q g to the power half, and h to the power minus three by four.

Now, this h we can write as what that is the head available if you remember we found this head available as now if we see this this is the head available I do not write again, because this is the work developed; that means, this is the change in the energy in the turbine runner, and this is the energy with which it is going out. So, this is the dominator of this expression of the hydraulic efficiency that is the head developed; that means, e by m plus v f square by two.

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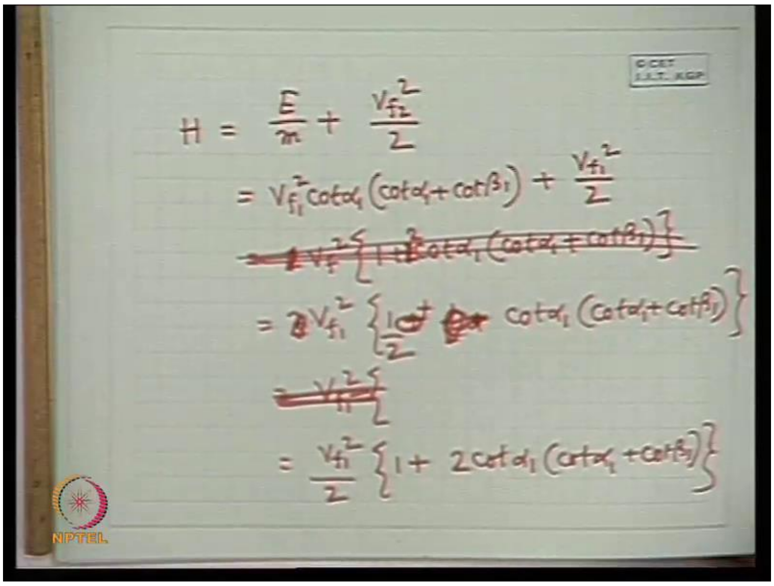

So, which is again if we write, then we get that h is equal to by the definition h is the develop which is e by m we recognized earlier plus v f two square by two what is e by m we derived earlier please.

That is the head available.

Head available that is the head available yes head available.

Head available.

Head this is head available this in this definition this is the available head n p to the power half h to the power five by four this is the available head this is these are in all the this is the this is the power developed this is the head available these are all out input parameter or you can tell these are operating conditions that have to be specified for a turbine this is the power develop this is the head available correct this is the by definition head available what is e by m please tell me the value of e by m which we derived earlier e by m is v f one square cot alpha one cot alpha one plus cot beta one, and v f two is v f one.

So, v f one square by two well. So, that becomes two v f one square two v f one square one plus cot alpha one into cot alpha one plus cot beta one divided by two two v f square plus v f one square. So, if takes. So, half will come here. So, two v f one square or two v f take common so…

Sir two will be inside the bracket.

Two will be.

Inside the bracket.

Well. So, two.

One plus two very good.

Divided by two.

Divided by two.

One divided by two sir.

Outside the bracket.

No sir that is.

That’s all right.

That is all right sir v f square divided by two.

V f square divided by.

That is no sir.

V f square divided by root.

Just let me write it this is equal to two v f two square let us write this one.

V f one.

Sir v f one.

If you take v f one square as common, then half two v f one square as common, then what will happen.

We cannot take v f one square as common.

If we take two v f one square, then it is all right one plus.

Half plus.

Why half plus.

One by one.

One four plus.

One four plus.

Because ah.

One four plus.

If I take v f one square common, then what will happen that is half plus cot alpha one well cot alpha one plus cot beta one.

Cot beta.

One plus half. So, that we can write this is ok.

Yes sir.

V f one square into what we can write.

Sit it is.

Two plus same all right keep it like this do not you keep it like this v f one square common one plus.

Half plus.

Half plus.

Half plus.

Half plus.

Sir plus is there any.

Half plus this term ok. So, therefore, v f one square by two into one plus two you are correct cot alpha one a simple algebraic steps nothing greater in it cot beta one well. So, this is the nothing; that means, if you take v f one square by two common one plus two yes cot alpha one cot alpha one plus cot beta sorry. So, now, if you put this value to this n s t.

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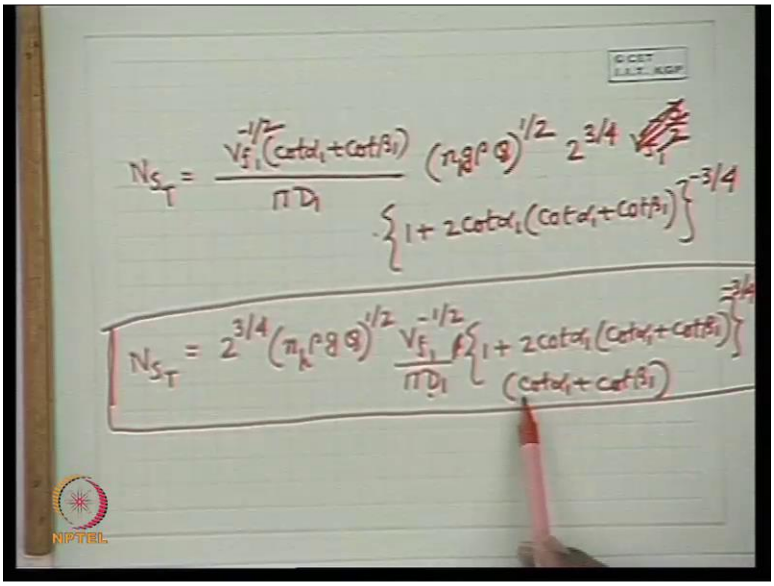

Now, we got this value, and n s t is equal to n s t carefully will have to do otherwise we will again mistake in this simple algebraic steps at this stage you have to be very careful while doing this pi d one, then we can write eta h rho q you please see rho q into g also g eta h rho q g to the power half, then h to the power minus three by four. So, two is there in the dominator. So, two to the power three by four, and v f one to the power.

Minus.

Minus three by two very good minus three by two three by four minus three by two, and there is another v f one. So, it will be ultimately v f one to the power.

Sir, its minus half.

Minus half very good into what will be there one plus two cot alpha one well into cot alpha one plus cot beta one all right this to the power minus three by four. So, this is the final expression let me write again. So, this will be two to the power three by four, and all this almost constant quantities eta h rho g q to the power half, then v f one to the power minus half divided by pi d one this remains as it is, then this quantity in the bracket cot alpha one into cot alpha one plus cot beta one to the power.

Now, in this context by like to tell you that this expression of n s t this is the conventional expression which expresses the n s t in terms of the angles of the vanes or the guide vanes guide vanes are outlet angle, and the rotor vanes angles are inlet, and the flow velocity v f one which is the major of the flow rate the rotor diameter at the inlet, but you do not have to memorize this formula, but you have to know this steps how it is being reduced. So, the expression for well the expression for hydraulic efficiency, and the expression for degree of reactions all this things the hydraulic efficiency expression.

As you are seen the efficiency of degree of reaction you do not have to memorize this equation to use in the examination. So, you have to reduced this equation, and problems will tell you that how your concepts will be applied to reduced these equation under different situations not that you will have to memorize this typical equations for specific speed degree of reaction or hydraulic efficiency, and all this any question please.

One more term is there.

Where?

Inter between term.

Yes yes yes I am sorry one extra term one term is missing here you can write cot alpha one good cot beta one this is the as it is without any index ok.

Yes sir.

Today I cannot tell you there is nothing that where I can check the final expression there is no point of memorizing the expression anymore, but you see in this case that whether it is nor not. So, it is all right cot alpha one plus cot beta one thank you any question please any question today, any question whatever I have told .

No sir.

Degree of reaction in the deduction of degree of reaction hydraulic efficiency no question.

No sir.

Thank you.