Introduction to Fluid Machines and Compressible Flow

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Lecture - 1
Introduction to Fluid Machines

Well, good morning, I welcome you all to this course on fluid machines. The number of the course is ME two six three zero zero four. Before coming to the course contents, I first like to introduce this subject before you. Well, now first we start from the basic introduction to the subject of fluid machines.

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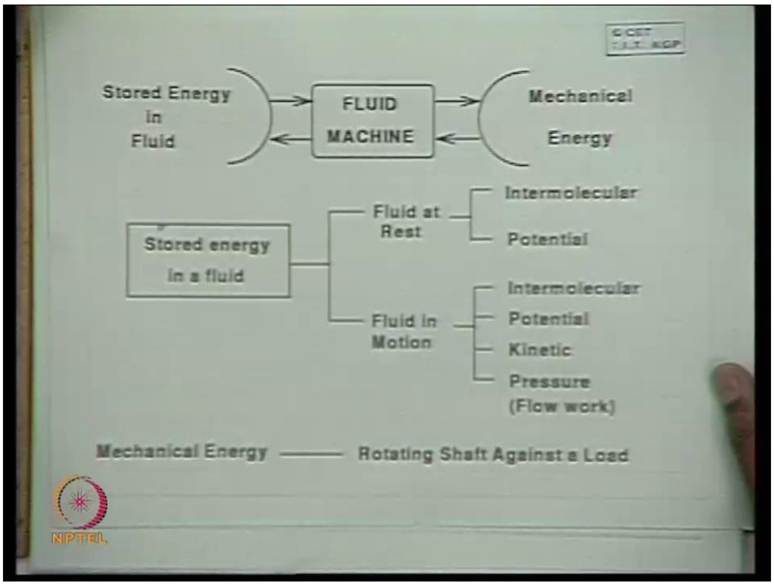

What is the fluid machine? Now, you can define the fluid machine as the system or contravenes where the stored energy in fluid is converted to mechanical energy or vice versa. That means, the mechanical energy is converted to stored energy in fluid. Now, from this very basic definition the importance of fluid machine in our engineering field is very obvious because you know, the need of mechanical energy today, which is largely used for generating electrical energy and is partly used as a mechanical energy for different engineering applications. So, the importance of fluid machine lies in the importance of mechanical energy, as we need today.

Now, next question comes about the nature of the stored energy in a fluid and what is the nature of the mechanical energy for the conversion from one to other is taking place through this system fluid machine. Now, stored energy in a fluid, now I must say, that if you recall your basic thermodynamics, in your basic thermodynamics class you have read, that the stored energy in any system, the system may be a fluid, is known as the internal energy, that is, the energy stored in a system.

So, if we look we can divide it into two parts, that when the fluid at rest, that fluid, the system is at rest, then the only way the energy is stored are intermolecular energy and potential energy. Intermolecular is the energy due to molecular motions and the potential energy of the molecules, that is, the kinetic energy and potential energy of the molecules. Usually, it is the kinetic energy of the molecules for ideal gases, you know, the kinetic energy of the molecules are due to the temperature. So, for all system or fluid at rest, the intermolecular energy is there by virtue of its temperature, all system always, that at temperature more than the absolute 0. So, therefore they have the kinetic energy of the molecules and the potential energy of the molecules, which keeps raise to intermolecular energy.

Another form of energy, which is stored in any system, even at rest, is the potential energy. Do you know the definition of potential energy? It is very simple, that it is the energy that a system possesses by virtue of its position or placement in a conservative force field. When all other conservative force fields are absent, then obviously, the gravity is the usual conservative body force field by virtue of which a system possesses potential energy.

Now, when the fluid is in motion, that means, if a system is in motion, as a specific case we consider here a fluid, what is the different form of this stored energy? In other words, I can tell, that if there is a stream of fluid, what are the different forms the energy stored in the stream of the fluid? Now, along with the intermolecular and potential energy, as we have discussed in case of fluid at rest, a fluid stream or a fluid in motion also possesses kinetic energy. It is very simple. By virtue of its velocity, know, whose magnitude is v square by 2 per unit mass.

And another type of energy a stream of fluid possesses is the pressure energy or simply we tell, flow work. Probably, you know these things, but I again tell you, that what is the pressure energy. You know, that when a fluid is in motion, any layer of fluid in a fluid stream pushes the neighbouring layer to make it over through in the flow, which means, that all the time a layer does work on the adjacent neighbouring layer downstream to it. So, this work done by the fluid layer to its adjacent neighboring downstream layer is known as the flow work and the energy by which it is capable of doing that work is known as the pressure energy. We use both the words, flow work or pressure energy and this is by virtue of the pressure of the fluid stream. And you know, the magnitude of these energies pressure times this specific volume per unit mass of the fluid.

So, these are the different forms in which the stored energy appear in a fluid mass, whether it is at rest or it is in motion, now comes the mechanical energy. So, mechanical energy is usually obtained or transmitted through a rotating shaft against a load through a rotating shaft against a load. So, these load depends upon the use.

For example, a large part of mechanical energy, which is obtained from a fluid machine in used for electrical purpose, that means, to convert heat into electrical energy. In that case, the load is an electrical load that means, the rotating shaft is coupled with an alternator drives an alternator. So, load depends upon our use. So, therefore the mechanical energy is obtained from the fluid machines or given to the fluid machines in the form of rotation of shaft against a load. So, therefore, we can now recognize the different types of energies that means, stored energy and mechanical energy during their conversion through the fluid machines.

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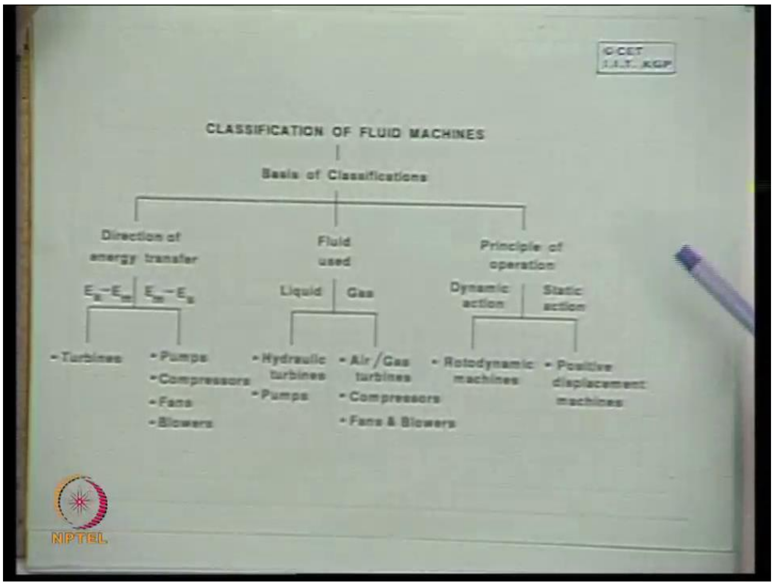

So, therefore next we will come to the classification of fluid machines, which is very important. Now, fluid machines as such is very broad, are very broad in there classification. Now, classifications therefore, are made on certain bases or we can say, that fluid machines are put into different categories on, under certain bases. So, that is why its bases of classifications are there. So, they are broadly divided into 3 bases: one is direction of energy transfer, another is the fluid use, another is the principle of operations.

Now, direction of energy transfer, as I have told earlier, that in fluid machines either the stored energy is converted into mechanical energy or the mechanical energy is converted into stored energy of fluid. So, the machines in which the stored energy in fluid is converted into mechanical energy, that means, the output of the machine is the mechanical energy and input to the machine is the stored energy in fluid, those are termed as turbines. And the machine where the mechanical energy is the input, that means, the mechanical energy is converted to stored energy in the fluid as the output are termed as pumps, compressors, fans and blowers. There are different categories. So, this will be clear in the next category of definition where the fluid is used.

Now, based on the fluid used, the fluid machines are classified into different forms. Now, you know, the fluid comprises both liquid and gas. One is the incompressible fluid, the liquid, another is the compressible fluid, gas. Now, when the machine converts this stored energy to mechanical energy and uses liquid as the fluid, then these are termed as hydraulic turbines.

The adjective hydraulic comes, because this is in general turbines, which convert the stored energy to mechanical energy, but if they use liquid, the adjective hydraulic comes. In almost all the practical purposes, the liquid used is water for these turbines. So, hydraulic almost substitute the water in its adjective sense, that is, the hydraulic turbines, sometimes water turbines. The hydraulic is the more general terminology used as an adjective to that liquid, which is usually water, that hydraulic turbines.

Similarly, for those machines, which give mechanical energy from stored energy, use gas. This may be air or any other gas, that is, a compressible fluid are termed as air or gas turbine, that means, the adjective is air or gas. That means, the name automatically signifies when it uses air as the fluid, is air turbine or it uses gas as the fluid. For example, the combustion products, as you know, in a gas turbine, the turbine fluid is the combustion products after burning the fluid in the air. So, therefore it is a gas. So, gas, air or gas turbine.

Similarly, for machines, which convert the mechanical energy to stored energy, but uses liquid are known as pumps. So, pumps are those machines where mechanical energy is the input and stored energy in the fluid is the output, but the fluid is an incompressible fluid, water or any other liquid, that these are known as pumps. Whereas, the same machines when they use gas, air or gas, they are termed as compressors, fans and blowers. This three names are there.

Now, here in this context I like to tell you, that there are differences between the compressors, fans and blowers, which we will see afterwards, that these machines uses. These machines use gas and give this stored energy in the fluid and the stored energy in the fluid are obtained either in the form of pressure, high pressure in the fluid or in the form of high velocity in the fluid. That means, either the pressure energy of the fluid is raised or the velocity energy of the fluid is raised.

Here, you must know one thing, that well, we have already read, that intermolecular energy is there, but usually, by spending mechanical energy it is not advisable from the thermodynamic point of view to increase the intermolecular energy of the fluid. Why, can you tell why? It is not advisable, that you spend mechanical energy as the input through any device to increase the stored energy in the form of intermolecular energy, why? Can you guess?

Student: That is not recoverable; cannot extract.

Professor: Well, this is not recoverable, it is partly true. But I think, it should be told in a more direct sense, that intermolecular energy is the low grade energy, whereas the mechanical energy is the high grade energy. So, we can get work from the intermolecular energy, but not 100 percent conversion is possible, according to the second law of thermodynamics. So, it is not advisable, that you spend a high grade energy to convert this to a low grade energy.

So, therefore the stored energy in the fluid, which is being generated or being converted to from the mechanical energy as either the pressure energy or the kinetic energy when the pressure energy, when the stored energy is in the form of pressure energy, that means, the pressure of the fluid is raised by virtue of the mechanical energy. In the fluid machines they are termed as compressor, so that you know, probably from your general knowledge, that a compressor always provides high pressure air, so flow or the velocity of the air is less.

Whereas, fans, in fans and blowers the mechanical energy converts the kinetic energy, the mechanical energy is converted to the kinetic energy of the fluid. So, therefore, fans and blowers provide mostly the kinetic energy of the fluid. That means, the stored energy in the fluid is in the form of kinetic energy where for compressors the stored energy in the fluid, which is being obtained from the machine, is in the form of pressure energy.

Now, the third one is the most important one, is the principle of operation, is the most important one. Depending upon the principle of operation the fluid machines are classified into two categories. So, one category is known as positive displacement machines where the principle of operation is based on the static action of the fluid. The other one is the rotodynamic machine where the principle of operation is based on the dynamic action of the fluid.

Now, in the positive displacement machines, the static action of the fluid means, what is done here, certain amount of fluid is interrupted within a given volume or in an enclosed chamber of the machine and here, the fluid mass behaves as a closed system, thermodynamically a closed system. Then, what happens is, that one of the boundaries of the system, then physically displaced to change the volume of the fluid entrapped. Either the volume is reduced or the volume is increased and by virtue of this change in volume, the energy transfer takes place between the fluid and the machine, and the pressure of fluid is increased or the pressure of the fluid is decreased.

A very common example is a reciprocating motion of piston in the cylinder, that in the piston cylinder machines you see certain amount of fluid. For example, air or it may be water, if it is entrapped and then, it is isolated from the inlet and exit of the machines during certain interval of time where it behaves as a thermodynamically closed system. During that interval of time there occurs the displacement of the system boundary. For example, in case of reciprocating motion of a piston within the cylinder, piston moves. Either it moves so that the volume of the fluid is decreased or volume of the fluid is increased by virtue of which either the work is developed by the machine or work is being imposed, done on the fluid system to increase the stored energy.

So, these kinds of machines are known as positive displacement machines because of the positive displacement of the system boundary in a closed system that fluid mass itself behaves as a closed system changes or makes it possible for the conversion of the energy. These categories of machines are known as positive displacement machines.

On the other hand, there are machines which are, in fact, in large use for our engineering applications are known as rotodynamic machines. They are based on the dynamic action of the fluid. What happens in those machines? There occurs a continuous motion of the fluid and also, of a part of the machine. That means, there occurs continuously a relative motion between the fluid and the machine. Both, the machine part moves and the fluid moves and based on the hydrodynamic principle, because of the change in momentum due to this continuous motion, the conversion of the energy takes place from mechanical to stored energy or stored energy to mechanical energy in the fluid.

So, these are the basic classifications of fluid machine.

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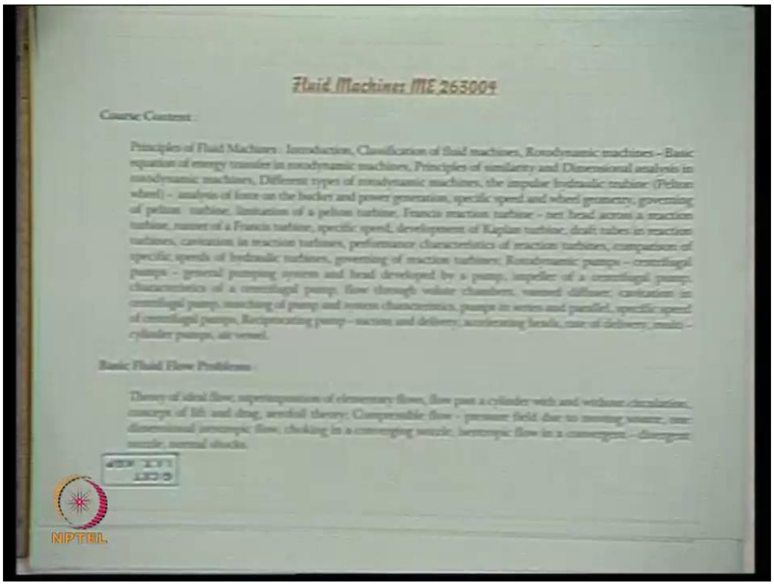

Now, after these classifications, I think, we will be in a better position to identify our course content, which probably you cannot see here clearly. I will give you this thing, the course contents individually, copy of this. Now, as such as we have seen in the definition of fluid machine, it is very broad and the single semester course cannot cover the entire fluid machines. So, these are termed as turbo machines, fluid machines, gas machines, air machines, a part of which is covered in different semester. For you also, turbo machine course will be there afterwards, which deal with gas machines.

Usually, the machines handling gas and air are known as turbo machines, that compressors, air or gas turbine. In this course, we will be dealing only with the hydraulic machines that means, the machines, which deal with the liquid as the working system, that means, hydraulic turbines and pumps on the other hand, well.

So, accordingly the course contents is like this. Probably, I tell you, you can just, first this principle of fluid machines introduction, then classification of fluid machines, which we have already covered. Then, we start first with the rotodynamic machines. Now, the basic equation of energy transfer in rotodynamic machines, basic equation of energy transfer in rotodynamic machine. This is, in general, for any rotodynamic machines where that use this liquid or gas. Then principle of similarity and dimensional analysis on rotodynamic machine. This is also, in general, for any rotodynamic machines using liquid and gas.

Then, we come to different types of rotodynamic machine. Here, here only we concentrate only on hydraulic machines for this class, for this course rather. The impulse hydraulic turbine known as Pelton wheel. Now, in this Pelton wheel, analysis of force on the bucket and power generation specific speed and wheel geometry, governing of Pelton turbine, limitation of a Pelton turbine, then we will switch over to Francis reaction turbine.

Turbines are of two types, that we will come across during our course, the impulse turbine and reaction turbine, the Francis reaction turbine. Here, we will cover net head across a reaction turbine, runner of a Francis turbine, specific speed, development of Kaplan turbine, draft tubes in reaction turbines, cavitation in reaction turbines, performance characteristics of reaction turbines, comparison of specific speeds of hydraulic turbines, governing of reaction turbines, means, up to turbines.

Then, we will go to rotodynamic pumps, centrifugal pumps, general pumping system and heat developed by a pump, impeller of a centrifugal pump, characteristics of a centrifugal pump, flow through volute chambers, vane diffuser, cavitation in centrifugal pump, matching of pump and system characteristics, pumps in series and parallel, specific speed of centrifugal pump. Then, come reciprocating pump. The reciprocating pump is the positive displacement pump, suction delivery, accelerating heads, rate of delivery, multi cylinder pump, air vessel.

Well, in this particular course at your IIT, on this course subject named as ME 263004, is named as fluid machines, a part of basic fluid flow problems are covered. This is a particular or you can say, typical design of our course curriculum at your IIT, that though the course title is fluid machines, we cover a part of the basic fluid flow problem. These are theory of ideal flow, which I believe you have read to some extent in your basic fluid mechanics course. So, I will make a hurried recapitulation of all these things.

Then, super imposition elementary flows, flow past is cylinder with and without circulation, concept of lift and drag, aero foil theory. These are extremely important topics in fluid mechanic. So, a part of it probably have been covered in your basic fluid mechanic course. There may be little recapitulation in this class.

Then, you come to a new topic, which you have not read earlier on the basis field for problem. These are compressible flow, these are pressure field due to moving source, one dimensional isentropic flow choking in a conversing nozzle, isentropic flow in a convergent, divergent nozzle, normal shocks. This is our course content for this course.

Now, before coming to this basic principle of operation of a general rotodynamic machines, I like to recapitulate the flow of fluid through a moving curved vane, the basis for which is like this. Now, I will start the rotodynamic machines, its general principle.

Now, as I have told earlier, the rotodynamic machines, there is a continuous motion of fluid as well as the part of the machine. Now, this part of the machine is known as rotor, which is a rotating element, which usually consists of a disc, rotating disc. And number of ((Refer Time: 21:43)) are mounted on the disc at the periphery of the disc and the disc itself is mounted on a sharp where the actual rotation is important. So, therefore it is the principle by which a moving fluid stream along a curved vane transfers the energy between the fluid and the vane, which is a part of the machine, part of the machine, part of the rotor of the machine, is the basic underlying or basic principle of fluid machines.

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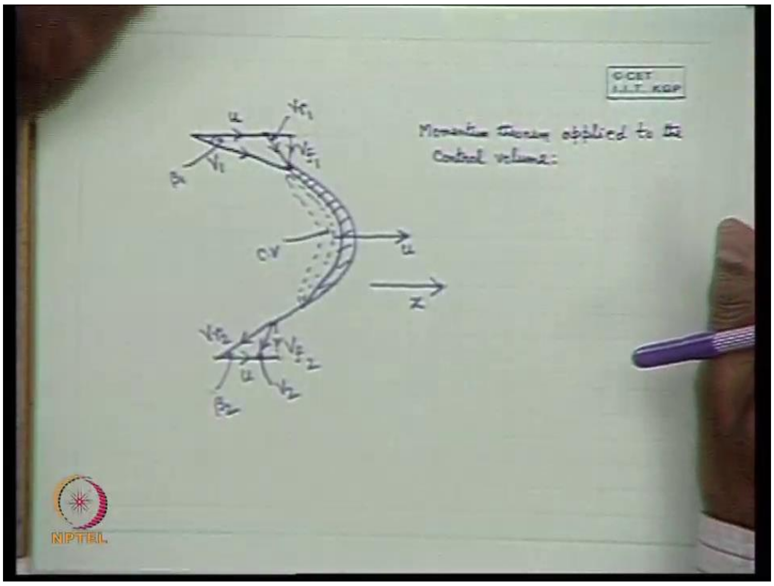


So, therefore, we try to recapitulate the principle by which the energy is transferred or the force is imparted by a moving fluid through a moving curved vane. So, let us see that. Let us, let me draw first.

Have you drawn this? I will not start before you complete your figure. I think, you can very well see this. Now, let me explain this, which will help you in drawing the figure, that there is a vane, which is moving with a velocity in this direction. Let us make this direction as a coordinate direction x, positive coordinate direction in which it is moving with the velocity u. A fluid stream approaches these moving blade with velocity v 1. This is the fluid stream approaches and it is discharged after flowing through the vane with a velocity v 2. This is the velocity v 2; this is the velocity v 2.

Now, you see, since the vane is moving, so what will happen with respect to vane? The fluid will approach with a different velocity, which is nothing but the relative velocity of the fluid with respect to the vane, which we can get from a vector diagram by subtracting the velocity vectorially from the fluid velocity, the vane velocity for which we get a typical velocity triangle.

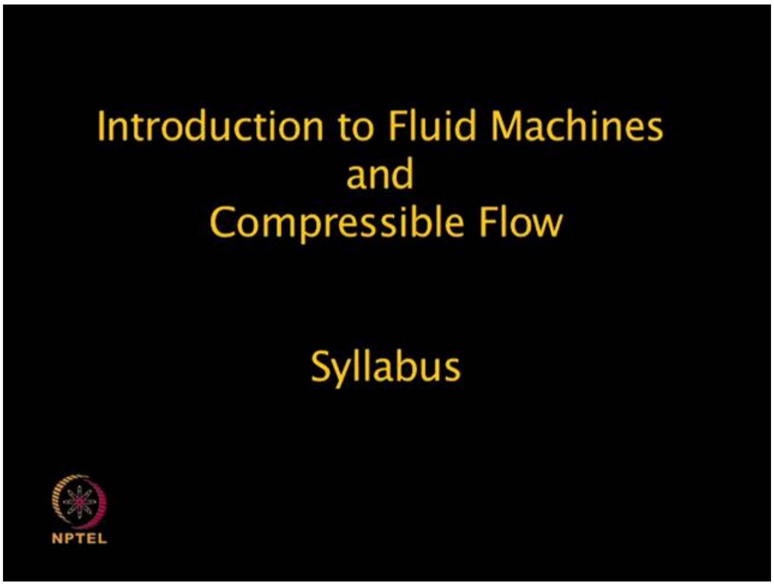
We know that that is the inlet velocity triangle where this is the vane velocity and this is the relative velocity of the fluid at the inlet. That means, this is the velocity with which the fluid strikes the vane with respect to the vane. Similarly, we make the vector subtraction of this vane velocity from the absolute velocity to get the relative velocity. This is the relative velocity of the fluid with which the fluid is discharged from the vane. Well, now we see that here as the fluid passes through the vane, there occurs a change in velocity and also the change in the momentum of the fluid, which gives rise to force exerted on the fluid or on the vane. Now, to analyze this type of problem we take the help of momentum theorem.

Now, we take a control volume. If you recall, in your basic fluid mechanics class we take a fluid control volume like this. This is the fluid control volume. This control volume is moving with the velocity u, which is the movement or the velocity of the curved vane. So, now, we make use of the movement theorem applied to this control volume momentum theorem, theorem applied to the control volume, momentum theorem applied to the control volume.

I feel the time is up for you today, so I like to end it today up to this. We will do this in next class to analyze the force that is being exerted on the fluid because of its motion through the curved vane and the energy transfer between the fluid and the moving vane. So, I think, I will stop today to this because time is up for you for the next class.

Thank you.

Student: Sir…



I have told earlier, the rotodynamic machines are those machines where there is a continuous motion of fluid and a part of the machine known as the rotor. And because of this continuous relative motions between the fluid and the rotor of the machine, it is possible for energy transfer to take place between the fluid and the rotor.

So, therefore the basic principle of this machine is based on the fluid dynamic principle, fluid dynamic principles, which is basically the utilization of useful work due to the force exerted by a fluid striking on a series of curved vane, which is mounted on the periphery of a disc that is rotating, the periphery of a disc that is attached to a rotating shaft.

So, therefore to understand the basic principle of a rotodynamic machines we should understand clearly the force interaction and the energy transfer that takes place while a stream of fluid passes through a curved vane.

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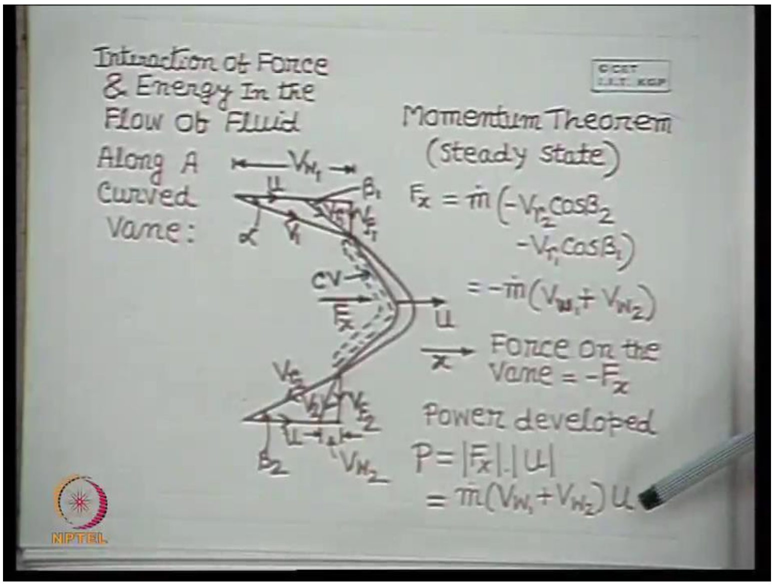

So, this is a little recapitulation of what you have already studied at your basic fluid mechanic course, that we study here the interaction of force and energy in the flow of fluid along a curved vane.

Now, see here, this is a curved vane, which is moving with a velocity u and a jet of fluid is striking the vane with a velocity, v 1 is the velocity, absolute velocity with which the fluid strikes the vane and the fluid after flowing through this vane comes out with a velocity v 2. This is the velocity v 2 since the vane is moving with a velocity u. So, the jet appears to strike the vane that means, with respect to the vane, the jet strikes it with a velocity v r 1, which is the velocity of the jet, related to the vane. Similarly, it is going out with a velocity v r 2 that is the relative velocity of the fluid with respect to the vane.

Now, these relative velocities at inlet and outlet are determined just by vectorial subtraction from v 1, the velocity u of the vane and from v 2, the velocity u of the vane. So, these vectorial subtraction is shown in terms of the velocity triangles, as we have already read, at the inlet and outlet.

Now, let the suffix one refers to inlet condition and suffix 2 refers to outlet condition. Now, you see, in this triangle this is v 1, that inlet velocity of the fluid. This is the u, the vane velocity and this is the v r 1, that is, the relative velocity of fluid with respect to vane at inlet. This component is perpendicular component to the motion of the vane, is denoted as v F 1 and is usually known as flow velocity.

Similarly, the component of the fluid velocity, absolute fluid, based in the direction of the vane motion is conventionally symbolized as v w. The suffix 1 is at the inlet. So, these v w, I tell you, this is a conventional symbol, w stands for whirl, whirling component. This is because, in actual case, this velocity of the vane is in the tangential direction because the motion of the vane mounted on the periphery is in a rotating motion. So, therefore the linear velocity of the vane is in the tangential direction and that is why, this component is known as tangential component or whirling component for which a conventional symbol W is given as the suffix.

Similar is the case in case of an outlet velocity triangle. This is the vane velocity, this is the relative velocity of the fluid with respect to vane and this component is the whirling component or the component of the flow velocity, that direction of the vane velocity and this is the flow velocity, that is, the direction of the, that is the, sorry, the component of the fluid velocity in the direction perpendicular to the vane velocity.

Now, our basic purpose in this case is to analyze what is the force exerted by the fluid on the vane or vice versa, vane on the fluid and by virtue of the vane motion, which is the what is the amount of energy that is being transferred or developed due to this force, due to this action of the fluid on the vane.

So, to analyze this, as you know, we apply the momentum theorem. Now, to apply the momentum theorem we have to take control volume of the fluid like this, which is just adjacent to the vane. Now, you see, that this type of analysis can be done on the basis of both, system approach and the control volume approach.

Now, in a system approach what is done? The Newton's law is applied in a sense, that you consider a particular mass of fluid and consider its change of momentum as it flows along the vane, find out the change of momentum in a specific direction. And in control volume same thing is done, but the version is different. We find the momentum, aflux net momentum, aflux in a particular direction and equate this with the force in that particular direction.

So, if you, if we apply this theorem for a steady state situation, the situation is steady, then you find, that F x is the force acting on the control volume in the direction x, then it will be the net rate momentum aflux from the control volume in that direction x because we are interested in the direction x, that is, the direction of the vane velocity, the force in that direction.

So, the expression on the right hand side is, either the net rate of momentum flux x, momentum flux from the control volume or from a system approach. It is the change of momentum, change of momentum in the x direction of a fluid mass taken as the system. In either way you can see it and that becomes equal to the force, is equal to the change of momentum times the mass flow rate. Now, you see, the velocity at the outlet is v r 2.

Now, here we have to consider the relative velocities because in this case, the control volume is moving with velocity u. Since the vane is moving with the velocity, this is an inertial control volume, so the coordinate direction will be fixed to this control volume. So, therefore the velocities, which we have to take are the relative velocity.

So, you see, the component of the velocity in the direction of vane velocity, here beta 2 is the angle made by v r 2 with the direction of vane velocity, it will be minus v r 2 cos beta 2 because this direction is opposite to that of the vane velocity or to that of the positive direction of this specified axis x. This is the momentum aflux minus the momentum influx. That means, v r 1 cos beta 1, beta 1 is the angle made by the relative velocity with the vane direction. So, this component is in the direction of the vane velocity or in positive direction of x. So, minus sign is that because it is the aflux minus influx. So, both the terms are with a minus signs. So, that comes out, so minus m dot.

Now, this v r 1 cos beta 1 or v r 2 cos beta 2, if you see from this triangle, so this comes out to be v w 1 and v w 2. So, therefore we say, that force on the vane is equal to minus F x. That means, this is the force that is being acted on what? That is being acted on the control volume. So, the force acting on the vane is in the opposite direction. That means, if this is the F x, it is the in the opposite direction of F x minus F x.

Now, power developed due to the motion of the vane is then, force into the velocity, that is, m dot v w 1 plus v w 2 into u; u is the vane velocity. So, this way we can develop an expression for the power developed due to the action of the fluid passing over a curved vane. I think you have understood it, alright.

So, from this two triangles you can get from here triangular relationships geometry, that it is v w 1 plus v w 2 and this, this minus sign is because the force, which is acting on the fluid element or the control volume is in the opposite direction to this specified axis, that means, in the direction opposite to the vane motion. So, therefore the force on the vane is in the direction of the vane motion. However, the expression for power developed is written as the multiplication of F x and u, they are in the same direction. So, there absolute values are taken. Well, now…

Student: Sir.

Professor: Yes, please.

Student: Sir, how v r 2, v r 2 cos beta 2 is not v w 2?

Professor: v r 2 cos beta 2 is not v w 2 plus u and here, it is v w 1 minus u, that cancels out actually. So, ultimately you get v w 1 plus v w 2. Yes, correct, v r 2 cos beta 2 is not v w 2, it is v w 2 plus u. On other hand, v r 1 cos beta 1 is also not v w 1. It is v w 1 minus u. If you substitute that, automatically it cancels out and becomes, because I felt, that you had already done it at your basic fluid mechanics course, so this thing you know well, ok. Very good, I am happy that you are asking questions.

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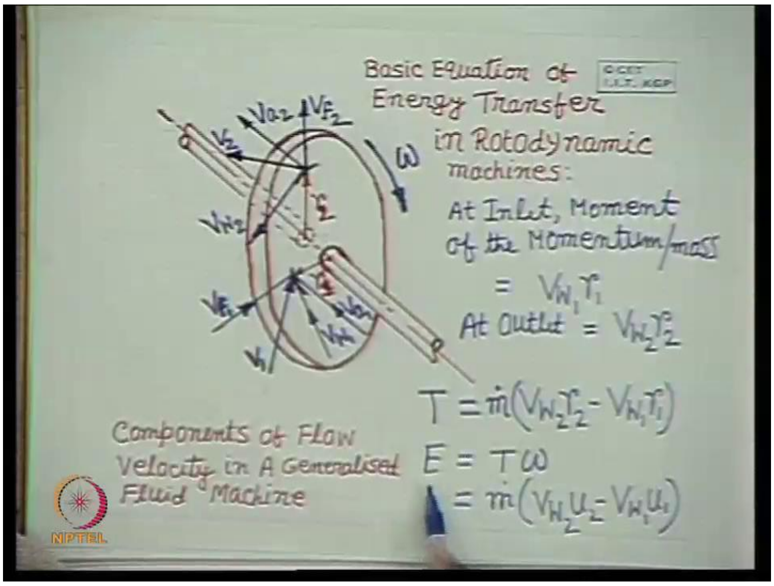

Now, we come to the basic equation of energy transfer in rotodynamic machines. Now, in rotodynamic machines what happens is, that the rotor of the machine is a rotating wheel on which the vanes are mounted and the wheel is mounted on a shaft where the rotation is important. So, in this case, the same principle is applied here and we analyze this in the similar fashion with the help of a diagram here, which is the general representation of a rotor or the representation of a rotor of a generalized fluid machines.

Now, components of flow velocity in a generalized fluid machines here, in a most general sense, we consider the rotor where the fluid enters at a velocity v 1 at any point whose radius of rotation from the axis is r 1.

Now, before that I like to mention you, there are few assumptions for this analysis. One assumption is, that the flow is steady, so there is no mass accumulation, no mass deflation anywhere in the system. And number two assumption is, that flow is uniform over any cross-section normal to the flow velocity, which is very important and that means, that the velocity vector at a point is representative of the flow over a finite area. That means, we analyze with respect to a velocity vector at a point and we assume, that this is uniform over the entire flow area, that is, an area normal to the flow velocity, so that this is the representative of the entire flow through the fluid machines, well.

So, with these assumptions now we consider, that at any point the velocity weight, that is, v 1, that is, the inlet point, a very general case whose radius of rotation from the axis of the rotation is r 1. Similarly, the fluid goes out or discharges at a point from the rotor whose radius of rotation from the, whose radius, sorry, whose radius from the axis of rotation is r 2.

Now, the velocities v 1 and v 2 can be resolved into three components. There may be in arbitrary angle at which the velocity, flow of velocities strikes the rotor, which can be reserved into three reference directions. One is in the direction of tangential, one in the direction of the tangential direction, which is the tangent to the rotor at that point. Another is the direction, which is the axial direction that means, it is parallel to the axis of the shaft. And another is the radial direction, which is perpendicular to the axial directions. So, these three mutually perpendicular directions, the velocities, are resolved. One is the tangential direction, another is the axial direction, another is the radial direction. So, these three perpendicular directions.

And accordingly symbolized as v w 1 is the suffix at inlet, that is, the tangential component, whirling component that is why, the suffix w is used. The suffix is v a, is the axial component, that is, component parallel to the axis of the rotation. And as I have told earlier, the symbol F is used. The F 1 for the inlet, that is, the flow velocity that is in the radial direction. Similar way, the velocities are resolved in tangential direction as v w 2 at the outlet. The axial direction is v a 2 and the flow direction v F 2. Now, the rotor is moving with an angular velocity omega, which is a constant angular velocity. This is the steady state problem, well.

Now, let us apply the momentum theorem or the Newton's laws of motions, either with respect to a system or control volume here. Now, here the momentum, which will be considered, is the angular momentum. This is because here the work transfer takes place due to the rotation of the shaft. So, we will be considering the angular momentum or movement of the momentum. It is very simple if you consider is system approach. Our version will be, that considering a fluid mass as it passes from the inlet to outlet, what is its change in angular momentum or if we consider a control volume of a fluid, then what will be net rate of aflux of the angular momentum from the control volume.

Now, here one thing is very important, we are not bothered about the path in the rotor, it is only the inlet and outlet that decides the change. Because if the inlet and outlet conditions are fixed, kinematic conditions are fixed and the mass flow rate is steady, so the change in momentum or the moment of the momentum, whatever you say, depends upon the inlet and outlet conditions, well.

Now, if we write the momentum, moment of the momentum at the inlet for a unit mass at inlet, what will be its value? At inlet, at inlet, at inlet moment of the momentum, moment of the momentum is equal to, that is, the moment of the tangential momentum, that means, v w 1 times the r 1 radius from the axis of rotation. It is per unit mass, per unit mass. Similarly, the same thing at outlet, at outlet, the same thing at outlet is equal to v w 2 into r 2.

So, therefore by unit mass, the change in the moment, the momentum of a fluid mass or the net rate of aflux of the moment of the momentum per unit mass from a control volume will be v w 2 r 2 minus v w 1 r 1 and that multiplied by the mass flow rates. That means, this will be the net rate of angular momentum aflux or the rate of change of angular momentum, rate of angular momentum, net rate of angular momentum aflux when we refer it to a control volume, that is, a control volume approach, control volume of the fluid or it is the net rate of change of angular momentum for a system as it passes from inlet to outlet.

So, in both the cases, that equals to the torque, that is, the angular momentum theorem, that is, the angular momentum theorem applied to a system or to a control volume, the torque is equal to the rate of change of angular momentum of a system or torque is equal to the net rate of angular momentum aflux from a control volume at steady state. So, that is equal to the torque that is being imparted on the fluid by the rotating disc.

Now, the energy, rate of energy that is being imparted to the fluid will be nothing but the torque into the angular velocity omega. And that if we multiply the angular velocity and recognize, that omega r 1 is the velocity of the, linear velocity or the tangential velocity of the rotor at inlet and omega r 2 is the linear or tangential velocity of the rotor at outlet and denoting them by the symbol u, we can write v w 2 u 2 minus v w 1 u 1.

So, therefore we see, that energy transfer per unit time the rate of energy transfer in the fluid as it passes from inlet to outlet becomes equal to the mass flow rate m dot flowing times v w 2 u 2 minus v w 1 u 1, where u 2 and u 1 are the tangential velocity, that is, the linear velocity of the rotor at the outlet point and u 1 is that at the inlet point. Because in a generalized case we have to consider, that the inlet and outlet are not in the same radius from the axis of rotation; there is not at the same radial plane.