**Fluid Mechanics**
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**Lecture - 9**
**Fluid Statics Part – VI**

Good morning, I welcome you all to this session of Fluid Mechanics. And last class, we were discussing about the stability of floating bodies. And we ultimately came to the conclusions that if the meta centre is above the centre of gravity, then a floating body is instable equilibrium. If the meta centre coincides with the centre of gravity, it is in neutral equilibrium. And if the meta centre is below the centre of gravity, the floating body is on stable equilibrium. This, we discussed. Now we see that also we recognized or we derived an equation representing the meta central hide in terms of the dimension of the body; that is the geometrical shape of the body and its dimensions. And do if we recall the equation is like that the distance between the centre of buoyancy to the meta centre, along the old vertical line containing the centre of buoyancy, centre of gravity and the meta centre is equals to the moment of area of the plane of rotation, above an axis perpendicular to the plane of rotation divided by the immersed volume.

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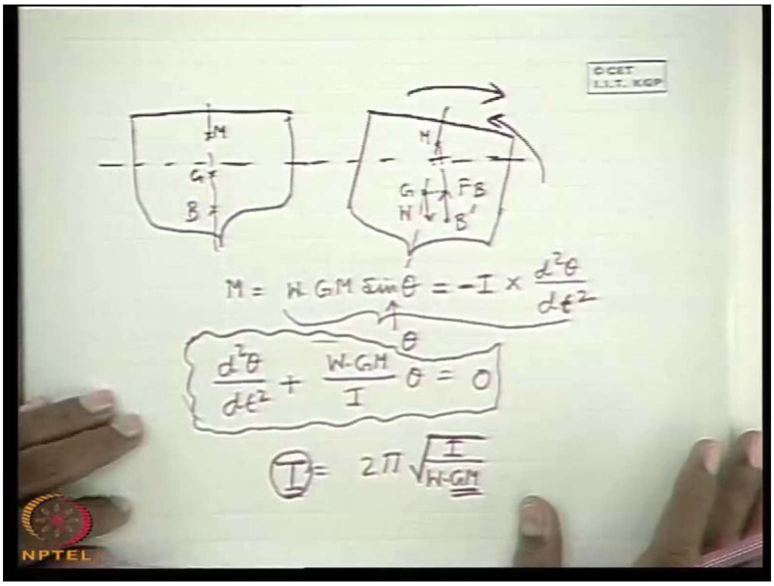

Now, we just see one interesting thing that when let us consider a floating body, let us consider a ship; let us consider a ship like this a floating. Now, we see that if this is the meta centre a stable condition, if this is the centre of gravity, and this is the centre of buoyancy, this is under stable condition. That means, under a tilted condition, under a tilted displaced condition, the ship looks like this; ship looks like this. And we have recognized one thing, that this is M, this is the mu B centre of buoyancy B dash and this is the G, through which the vertical force W acting through which M. Now, we see in this case, the restoring moment which acts in this direction opposite to the direction of tilt is equal to this distance into W or H B. Let us call W and this distance can be written for a small angle theta G M sin theta, this is the restoring moment.

Now, if I equate this with the moment of inertia and the angular acceleration from the conservation of angular momentum, the theorem of conservation of angular momentum. We can write this is equal to the momentum inertia of these body mass momentum inertia of these body about the plain of or the axis of rotation axis of rotation times into d square theta d t square. So, this is the angular acceleration; that means, theta is the angle of wheel at any time instantaneous angularly at any time with a negative sign. Because this is a retarding one because this is a couple is a restoring couple. So, therefore, the role is reducing; that means, if we consider the rolling of the ship we can write the equation for the angular position with respective to time by this considering the theorem of conservation of angular momentum.

For small angle theta, sin theta can be replaced as theta and if, if I write this these equation is ultimately transferred to d theta d square theta d d square plus W into G M G M is the meta centric height divided by I into theta is 0. So, therefore, we see that we have simple harmonic motion for the angular displacement. That means for the rolling of the ship, a simple harmonic motion whose time period t can be written has twice pi times under root of 1 by this; that means, under root of I divided by W into GM, this you know from a simple mathematic. So, this is the time period of root. So, one very interesting things comes out from this that time period of role is inversely proportional to the meta centric height; that means, if we meet the meta centric height B; that means, they give a better chance of stability then the time period will be very small. And time period very small means rolling will be fast which can cause discomfort to the passengers.

So, in ships carrying passengers sometimes the meta centric height is may deliberately low at little sacrifice of the stability to increase the period of as to increase the period time period; that means, for better comfort the time period is increased. But in one ships it is other way the meta centric height is made as large as possible to have a much better chance of stability. Whereas, the time period of role becomes very if this is very high so this is very small. So, time period of role becomes very small. So, comfort is sacrificed at the cost of stability, but for ships carrying passengers sometimes the comfort is not sacrifice that much with the respect of stability. Because stability is also first primary thing is the stability. But what elements we should give for GM, if you make GM very high with a better factor of the safety sometimes it effects the, causing the uncomfort to the passengers.

So, is this the key equation for the role? That is all regarding this meta centric height problem, but few things I must tell you at this moment. Before concluding these buoyancy thing, one thing you must know that for a floating body even if G is above the centre of buoyancy it is stable. Why? Centre of buoyancy gets a chance to sit in the direction of the tilt. So, when centre of buoyancy shift direction of tilt because of the changing in the inners volume, not total volume is distribution that stability is improved. Similarly, if the gravity also tries to change in the direction of this tilt it will give an adverse effect of stability that you can very well understand. So, therefore, the gravity the centre of gravity is not allowed to change in the direction of the tilt.

So, this may happen in a case when some cargo may move in the ship or sometimes it happens if the floating body contains inside a liquid. So, when it is giving a tilt the free surface of the liquid will be horizontal. So, some liquid volume will be moving towards the tilt. So, therefore, the centre of mass will be shifted towards the tilt. So, that will give an adverse effect to the stability. So, therefore, the, if the floating body is contain liquid, these are separated into a number of large compartments large number of compartments large sorry large number of compartment. So that the free surface almost remains the same there is a liquid change in the distribution of mass and if there are some solid particles which wants to move for example, small cargos in a big ship that has been prevented.

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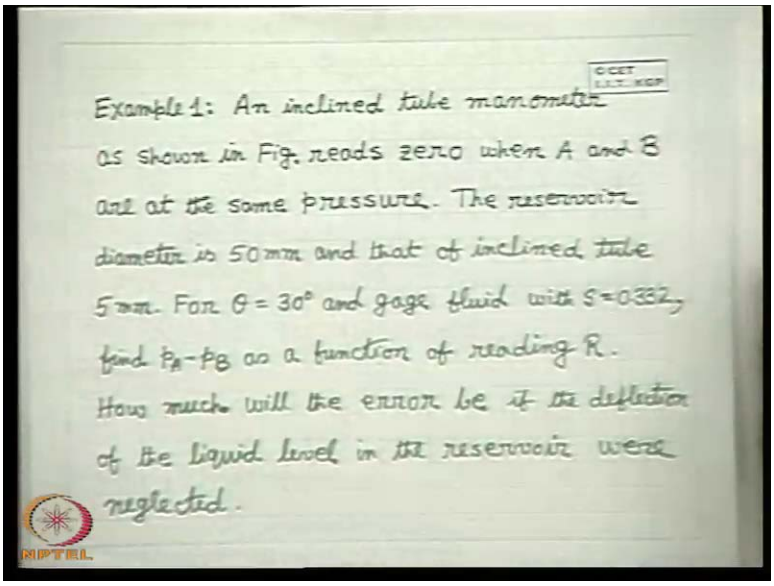


So that gravity centre of gravity it is not allowed to move in the direction of the tilt well. Now, after this I will just solve few examples. Let us see, which is very important, first is example one and an inclined tube manometer. Let us see the example one, an inclined tube manometer as shown in figure. I will show the figure afterwards reads 0 an inclined tube manometer as shown in figure reads 0. When A and B are at the same pressure this A and B that requires the figure. So that is why figure some A and B, you just recall it when at the same pressure an inclined tube manometer shown as shown in figure reads the, the reservoir diameter is 50 millimeter as you know an inflective manometer is a big reservoir and a small tube and that of inclined tube which is inclined 5 millimeter which is very small compared to the d z. For theta is equal to 30 degree and gage fluid with S is equal to 0.332, that is the specific gravity S is specific gravity of the fluid.

Find P A minus P B, that is the pressure at A and pressure at B and the difference of this as a function of reading R. R is the reading, that I will show in the figure how much will the error be if the deflection of the liquid level in the reservoir were neglected. So, this is all right, again I am reading an inclined tube manometer as shown in figure reads 0, when A and B are at the same pressure. The reservoir diameter is 50 millimeter and that of inclined tube 5 millimeter that is the inclined tube. For theta is equal to 30 degree and gage fluid with S 0.332; that means, the gage fluid is specific gravities 0.332.

Find the difference of pressure between A and B, this is P A minus P B as a function of reading R that is the manometer reading. How much will the error be, if the deflection of the liquid level in the reservoir were neglected? So, the advantage of liquid level reservoir to be neglected is that we can make the reservoir as I have told earlier not of transfer in material it may be costly. So, only we can make the inclined tube transparent. So, this problem is well understood. Now, with this problem.

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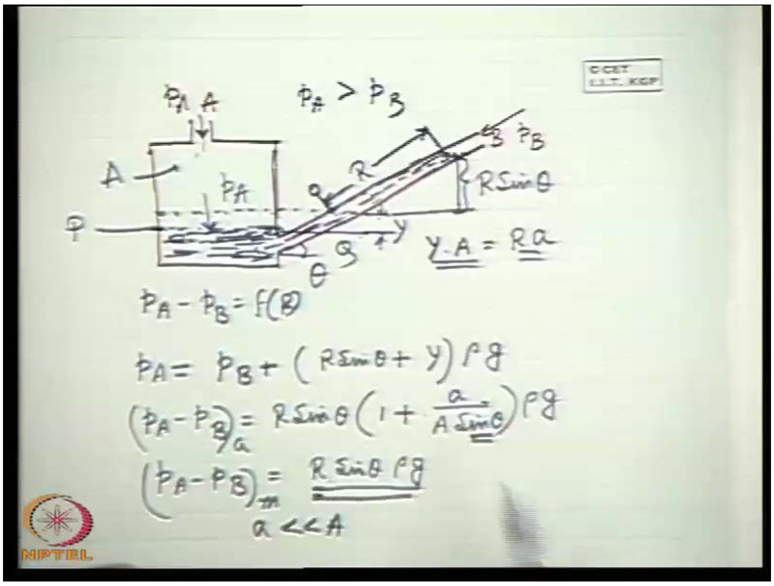


Let us see, what exactly the problem tells. Let us draw this is the sorry, this is the inclined tube and this is the system A. Let this is the point A, where the pressure is exerted be and let this is the point B, where the pressure is exacted P B; that means, A and B this 2 open ends of this 2 sides, one is the reservoir and other is the short link are connected to 2 systems is pressure P A and P B. Well let us consider, the initial level of the liquid be here, let us consider the initial level of the liquid when there were no pressure A and B, P A and P B equal that is initial; that means, initial level means, if the manomate.

This is the manometric, full of manometric liquid, then this will be at the same level; that means, the reserve it is written in the problem you see that when A and B are at the same pressure; that means, the system pressures are same they are not connected to A and B which are varying impression either way you can understand. The reading figure reads 0; that means, inclined tube manometer reads 0; that means, here the reading is 0 this, we measure like this. Now, what happens, when this pressure is applied when A and B, it is applied to that when P A is greater than P B, then what happens this limb will be decreased. And here this will be increased; that means the manometric fluid.

That means this is the manometric fluid. So, this reading we tell it as R, which is graduated along, this tilt this reading we tell it as R, this is the reading. Now, we have to find out P A minus P B as a function of R, which is very simple. Now, what we have to do? We have to find out, this is the level, now this is the. So, we are finding out the pressure at this point. So, let us consider a hydrostatic equation, we are writing at this level from both the sides. Let us consider this deflection as Y and this is definitely. So, this one is definitely, if this be with theta, this is R sin theta. So, actually truly speaking, we will have to use hydrostatic equation at this level of the liquid.

So, original level of the liquid is not very important. Now, we have to find out the difference in level both the limbs like a simple, you tube manometer from this side, what is this P A is equal to now here the height, we are not considering, consider a system comprisable fluid gas which gives the pressure state to this P A. Because you know the height of the fluid in case of a gas used density is low is neglected. So, it is not a liquid extractive system pressure which maybe here is just impressed on this surface. So, it is simply P, which is equated from this side P B plus this height; plus this height of the manometric fluid; that means, plus R sign theta plus Y. Well R sign theta plus Y into rho into G where rho is the density of the manometric fluid here. Now, what is Y? Y is from the continuity. That means the amount of liquid which is been depressed in the reservoir has gone here.

So, therefore, one can write that this Y into A is equal to where A is the area of this. Well is equal to this R, which has moved along this pi R into A; that means, this equals the same volume of the liquid which has come down here has gone up there. So, with this we can write P A minus P B is equal to what R sin theta, if we take common, 1 plus, what is this Y is equal to R A by A; that means, A by A sin theta. Any question? A simple mathematics at school level, but the concept is that at this point. We are equating the pressure from both the limbs as we do in case of a manometer U tube manometer it is a modified version of an U tube manometer.

Now, therefore, now we see that without doing that. If we have a not a transparent material with the reservoir only this part is transferring, we do not know this level we only read 0 at its initial level what is the problem statement. Then we only see the deflection; that means, the movement of this liquid in the inclined tube and only read R and try to find out P A minus P B as only R sin theta into row G, then this is the actual write A and this is the measured.

So, if we measure only the movements from 0 to this that initially, it was 0 and we moved measured the only R reading that is the what is that reading movement of the liquid in that tube. Then this will be the reading and it is common sense just you see from this 2 that the difference, this difference between this 2 will be very less provided A is very less than A. If A by A is very small than 1 sin theta is not very small in that case, because it usually varies between 10 20 30. So, if A by A is very small. So, this part is negligible this 2 will be equal. Now, the problem is to find out the error. So, if you define error as let this is delta P actual and this is delta P measured then.

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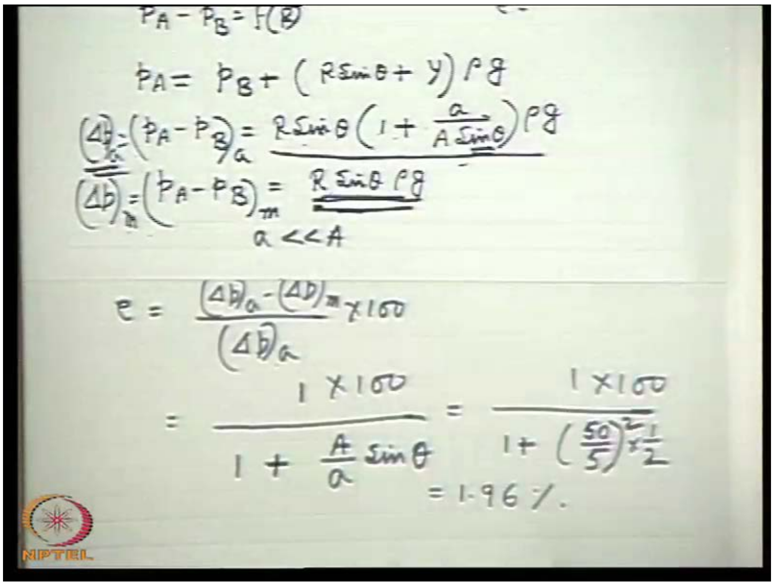

If we define the error e, well then e is equal to in terms of percentage delta P actual minus delta P measured divided by delta P actual this is the percentage error 100. If you make that; that means, this becomes this can you see the delta P actual minus delta P measure. So, if you make that this will be 1 by 1 plus A by a, a simple algebra if you make 1 by A plus; that means, delta P makes actual minus delta P measure . So, therefore, remunerated, it will be R sin theta A by a sin theta. And denominator, it will be delta P actual R sin theta 1 plus A by a sign theta R sin theta will cancel. And the, we again make the numerator 1 by dividing A by A sin theta.

So that in denominator we will get that is the simple algebra. Now, if you make the substitution here, A is this 50 millimeter and this diameter is 5 millimeters small. So, this whole square is at the diameter into sin theta half. So, this comes out to be into 100 well and this will be 1.96 percent, any difficulty? So, this is the value of delta P A, this is the value of delta P M. This is the freed mechanic after that it is a class 8 level algebra. So, you can find out the delta P A minus delta P M by delta P A into 100 becomes equal to this. So, if you put the values, you will get this 1.96 percent think that is nothing great only the concept is here. Now, come to this the second problem which is very interesting.

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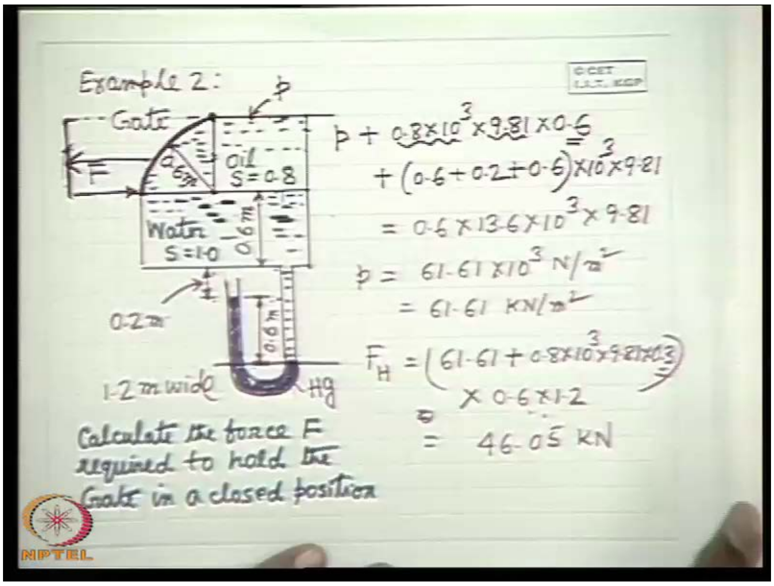


Second problem; well second problem as you see it is the problem related to the pressure on a carte surface. Pressure on a simple surface is very easy, if you have any problem you just ask me. Do not make any noise in the class. You just ask me, is there any problem, difficulty? Please. Then now you see this problem tells like this example 2; calculate the force if required to hold the gate in a closed position. There is a gate which is a sector of a circular R; that means, it is a, this is 0.6 meter; this is 0.6; this is 0.6. So, this has to be understood from the figure. So, this sector of a circular R, this gate is 1.2 meter wide, which I had forgotten to write 1.2 meter wide; that means, the gate is like this, this is plain in the perpendicular direction. This is the plain of its, plain of the oil plain of the gate.

So, in this direction perpendicular direction the gate is 1.2 meter wide; that means, it is a sector of a cylindrical surface, 1 sector quadrant, 1 quadrant of a cylindrical surface not sector. I will tell 1 quadrant of a cylindrical surface used is that is 0.6. So, this is connected to a closed box with oil and water at pressure how do you know. Because if you attach a manometer, this manometric fluid is mercury, this manometric fluid is mercury Hg. So, with the manometric fluid, we get a deflection; that means, this is open to atmosphere this part is open to atmosphere so; that means, there is a pressure in these oil and water. So, what is due to this pressure in the oil throughout? So, a pressure force is exerted on the carte gate which is a quadrant of a cylindrical surface, what is the force of this.

So, what should be our first duty to do please tell me, what should be our first duty to do in this problem? To find out, please tell pressure at this point, top point. Let this pressure at the top point is P, we should find out the pressure at this top point. So, what to do, you write the manometric equation to find out the pressure at the top point; that means this part is the water which you. So, if we write the manometric equation at this plain, what we get from this side P plus. What we will write the pressure due to this height this column of oil; that means, rho G H, what is rho density? Oil S means specific gravity; that means, 0.8 into 10 to the power 3 is the rho into G 9.81 rho G into H. What is h? 0.6 rho GH, rho G H plus the height of the water column equivalent to 0.6 here 0.2 here and also 0.6 here; that means, 0.6 all water column plus 0.2 plus.

Again 0.6, you come to this point from this straight plus again 0.6 as it is given in the problem this deflection is 0.6 times the density of the water is 10 to the power 3 H rho G rho H g. Whatever you write H rho into G 9.8 here, H is 0.6 rho is this and g is this H rho G. So, starting from this point P that is pressure P we come here. Equate from side is equal to it is atmospheric pressure 0. If we want to find out the force due to the pressure above the atmosphere, because this side atmospheric pressure force is acting. So, therefore, we take atmospheric pressure as 0. So, 0 plus only this column of liquid 0.6 and manometric fluid is mercury, whose density is rho is 13.6 to the power 3 H rho into G 9.8 imply. So, if you equate this equation, if you solve this equation P will come equal to some 60, well 61.61. You check that 61.61 into 10 to the power 3 Newton per meter square well; that means, 61.61 kilo Newton per meter square.

So, this is the P, now next part is to find out. So, this we know the P, now we should find out the pressure on this carte surface. Let us apply our idea that what is the horizontal force on this carte surface. Please tell? What is the horizontal component of force in the carte surface? Yes, if we make a projection of this area like this the horizontal force which is acting on this projected surface, projected plain surface is the horizontal component of the force acting on the carte surface. So, what is the horizontal? What is the force acting on the horizontal plain surface, please tell?

What is the horizontal force acting on the plain surface? That is the pressure at the centroid. So, centroid is at the middle; that means, 0.3 meter from the surface. So, what is the pressure force at the 0.3 meters? What is the pressure intensity; that means, this P plus rho G H rho is 0.8 10 to the power 3 rho G into 0.3. That is the pressure at its centroid; that means, centroid of the centre of area of the plain surface, which is a plain surfaces uniform plain surface, whose depth is or height is 0.6 meter. So, that is 0.3, so at the middle the centre of the area this is the pressure intensity.

So, then the force is acting not at the middle, this is the centre of the area. Into the area, what is the area now? Area is 0.6 into 1.2 very good. So, this will come out to the horizontal component 46.05 into kilo Newton, 46.05 kilo Newton. Now, the next part is the vertical force. How you find out the vertical force, please tell me? The vertical force will be equivalent to the weight of the liquid above this carte surface. Please do not talk, please tell me whether it is all right or no. If you do not understand you please ask me, Yes please. Density of finding F is at an average force that is active on the gate, not average force. So, you have not attended the class, the thing is that the horizontal component of the force is equal to force acting on a plain area which is oppositional area in to that direction.

So, the force acting on this plain area consider a vertical area is equal to the pressure at the centroid, no question of average times its area. If we integrate the force component, you will get like that which has been already told in the earlier classes. So, do not ask the silly question, that it is an average that it is the total force acting which is the integrated force of all elemental force components on the surface which finds out, which is found out to be the pressure intensity at the centroid times the area. The simple formula was derived in the class earlier. So, therefore, in any plain surface in submerge surface the total force acting on the plain area is equal to the pressure intensity at it centroid times, its area this has been derived so, no question of average and all those things.

Now, what is the vertical component of force? The vertical component of forces act on this surface, if we find out the weight of the liquid above the carte surface that also, we derived in the earlier classes, those who have attended the earlier classes they knows them. If you have not attended you please see earlier lecture. So, this was derived vertical component is equal to the weight by magnitude, weight of the surface. This is the easier way to do weight of the liquid above the surface up to the free side; that means, we will have to consider for example, a free surface. Let us make a gap, because space is not there as, if there is free surface whose height is h. This H; this H you can see this H.

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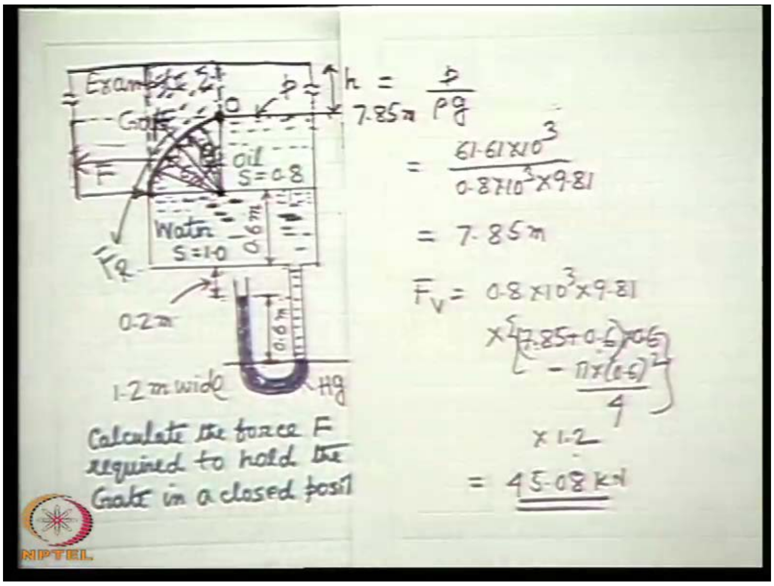


If this is the height of the free surface which is extrapolated imaginary free surface that also was discussed in the earlier class. This becomes equal to this simply pressure P here by rho G; that means, this simply equal to pressure. What is this pressure? 61.61 into 10 to the power 3 divided by the rho; that means, this we are having, we are finding out that imaginary free surface. That means, if the oil is allowed to go; that means, without this chamber there is a pressure; that means, oil free surface could have been extended to a height h from here, which is equal to P by rho g. That means, 0.8 10 to the power 3, it is very simple 9.81. And that becomes equal to that exactly becomes equal to 7.85 meter; that means, we get an h is equal to 7.85 meter. Now, it is very simple, that we can tell now that it is the weight of this liquid vertically above the surface as; that means, weight of this part of the liquid up to the free surface.

So, we will have to find out the weight of this liquid, the weight of this liquid. So, how to find out the weight of this liquid, please tell me, how to find out the weight of this liquid? That is yes, you can find out the volume weight of this liquid means the volume of this liquid, volume of this parallelepiped, whose area this area, this into this and multiplied by this minus the volume of this cylindrical part. So, therefore, we can write a F v is equal to rho, rho is 0.8 10 to the power 3 better we write rho and into the volume. What is the volume, volume? You can write that this is equal to 7.85 plus 0.8; that means, volume is 7.85 plus 0.6 very good into minus this side, 1.2 is same minus Pi into 0.6 whole square by 4. Because this is the quadrant of a circular surface quadrant of a cylindrical surface, this side 1.2; that means, this into 1.2.

So, this is the volume of this liquid, 7.85 plus 7.85 plus 0.6 into 0.6; that means, this side is 0.6 into 0.6, oh 7.5 very good; very good; very good. I am very happy; very happy, 7.85 into 0.6, then it comes meter square minus 5.6 whole square by 4 very good, into 1.2, very simple thing. So, we get the value of F d. So, what is F d? F d is 45.08 kilo Newton, this divided by 10 to the power 3. If you make, this will be kilo Newton. Now, the question is that there is a hinge here, let the point is o. Now, we have to find out, to find out F, the question was, what is the force F required to hold the gate? So, therefore, to find out a, we will have to take the movements of the forces above well, movement of all the forces. So, therefore, we must know the point of application of the forces, now one catches that do not try to find out in this type of problem.

The point of application of this horizontal force and the vertical force rather the catches like that as because this is the cylindrical path. All the forces will be perpendicular to the element and this perpendicular always passes through the radius; that means, these are the radial linewhich are always perpendicular to. So, by which, we can tell that the resultant force is a line, which is perpendicular to this and passes through the separate origin here this origin. So therefore, this is another radial line, because all the force components act along the radial line they may not be parallel, but the radial line.

So, therefore, the intelligent part of this work is to assume that one radial line is the line of action for the resultant force F. F R for example, resultant pressure forces F R, let the angle makes this resultant force with the vertical with theta with theta, then you take simply movement, then you take simply movement. But before that you have to find out what is the value of this theta? What is the value of this theta, this theta value? You can find out tan theta is equal to F H by F V.

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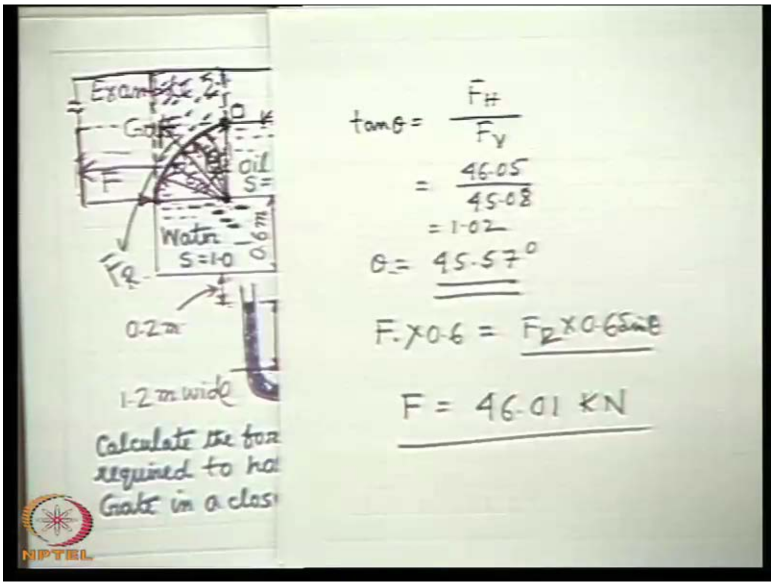


Simple with that F H that is the h component divided by F H theta and F H and F B are known. And if you write, if you use this value of F H and F B, we will see this is 46.05 divided by 45.08 and this becomes equal to 1.02 and theta is 45 point, this you check 5 7. Now, when this theta is known the resultant force is specified. Now, I can take movement about the hinge o to find it. So, F into points, what is this 0.6 is equal to the resultant force F R. They are in the opposite direction into the perpendicular distance from the hinge point to the line of action of the resultant force, which will be equal to 0.6 sin theta; that means 0.6 sin theta well, 0.6; 0.6 sin theta; 0.6 sin theta is this. So, this gives you F equal to 46.01 kilo Newton. All of you have understood. Now, we come to the third problem, well third problem is a very simple problem a uniform wooden cylinder has a specific gravity of 0.6.

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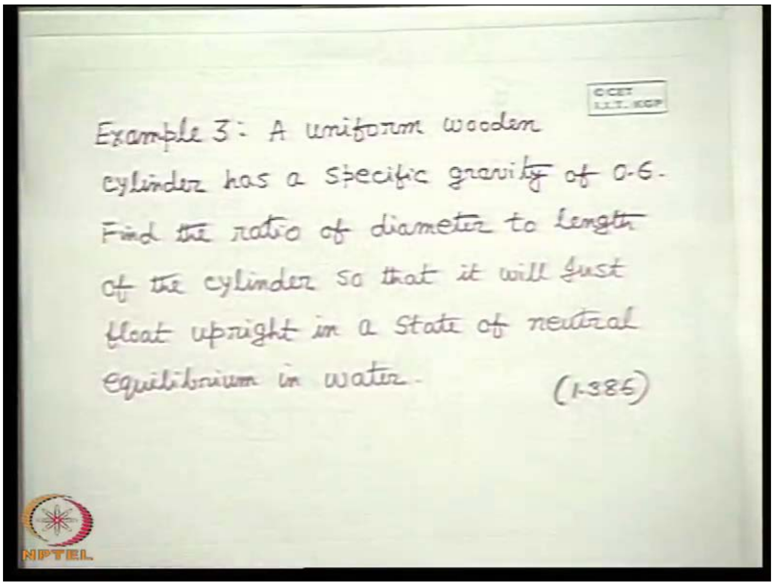


A uniform wooden cylinder has a specific gravity of 0.6. Find the ratio of diameter quick? Find the ratio of diameter to length of the cylinder? Find the ratio of diameter to the length of the cylinder? So, that it will just float upright in a state of neutral equilibrium in water probability. One of the simplest problem in meta centric height, a uniform wooden cylinder has a specific gravity of 0.6. Find the ratio of diameter to length of the cylinder. So, that it will just float upright in a state of neutral equilibrium in water well.

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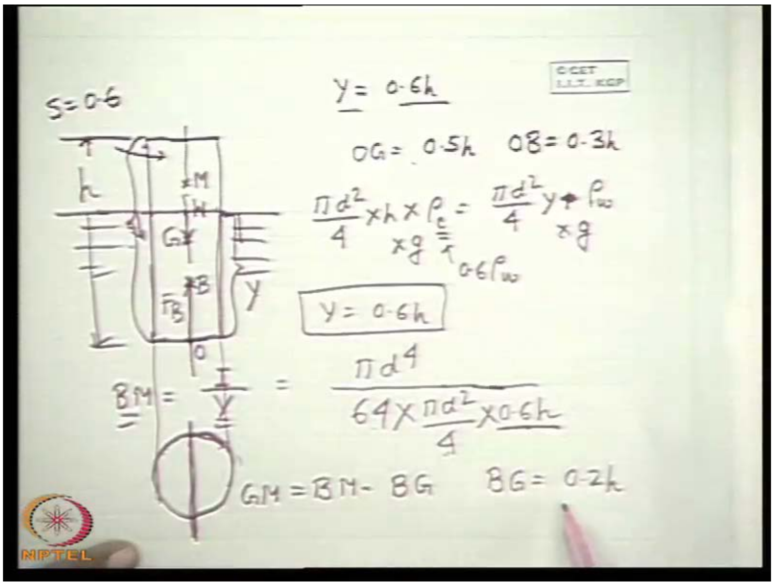

So, this is a very simple problem, oh God again, a uniform wooden cylinder consider a wooden cylinder specific gravity of 0.6. Find the ratio of diameter to length of the cylinder? So, that it will just float upright in a state of neutral equilibrium, these are the pertaining points.

Now, what is this? You consider a wooden cylinder and it is floating in the equilibrium, S is equal to specific gravities 0.6. Let us consider the height of the wooden cylinder as h; that means, so if this be Y, what will be the value of Y. The height of the floating part uniform wooden cylinder, Y will be how much 8 9 level thing 0.68. How, because weight is balanced by the weight, where is the weight X? Weight X by h by 2; that means this is the centre of gravity through which the W acts. And this led this point is o base point o G is equal to 0.5 h. Now, centre of buoyancy acts at Y by 2, this is the centre of buoyancy, this is A A B.

So, from the equilibrium consideration, W is equal to A B, what is W pi d squared. d is the diameter of the cylinder times h into density of the cylinder rho's is equal to the buoyant force. Buoyant force is the weight of the h rho into G of course, G is there G is equal to the weight of the displaced volume. What is the displaced volume pi d square by 4 into Y into rho of sorry, into rho of water into G. Now, rho C is equal to 0.6 rho of water. So, from which we get Y is equal to 0.6 h, it is very simple plus line 9 level, it is now. So, when 0.6 is the specific gravity Y is 0.6 h.

So, therefore, B is the centre of buoyancy for the uniform cylinder, it will be at the middle of this Y. That means, 0 B is equal to 0.3 h; that means, this distance is 0.3 h and 0 to G, the distance is 0.5 h. Meta centre will be above the centre of gravity for stable equilibrium and for neutral equilibrium, it is coinciding. Let us discuss in general M is above this. So, what is the value of B M, we know B M is I by Y. What is the concept of I? I is the movement of area of the plain of rotation. If I take a sectional view, I will see a circle that is the circular cross section. And we have to take the movement of area of this circle about this axis that is the axis perpendicular to the plain of rotation. So, this axis we will take plain of rotation. So, about this axis, what is the movement of area I, I is equal to pi d 4 by 6 d 4.

So, this is the movement of area of this circle, circular area of diameter D. So, this is an another diameter; that means, the movement of area of a circle about its diameter times the volume, what is the beamers volume, please tell me? pi d square by 4 into this one; that means, 0.6 h. So, this is equal to B. So, what is M G or G M, B M minus B G? Well B M minus B G so, B M minus B G understand, this B M minus B G. Now, B G is what is B G? Let us write here B G is; B G is o G minus o B. Yes, 0.6 h into pi d square by 4 y is 0.6 h. So, Y is 0.6 h, it is immerged. When you are correct, it is immerged volume that is why pi d square by 4 h, not h 0.6 h. So, B G is 0.2 h and B M is this one.

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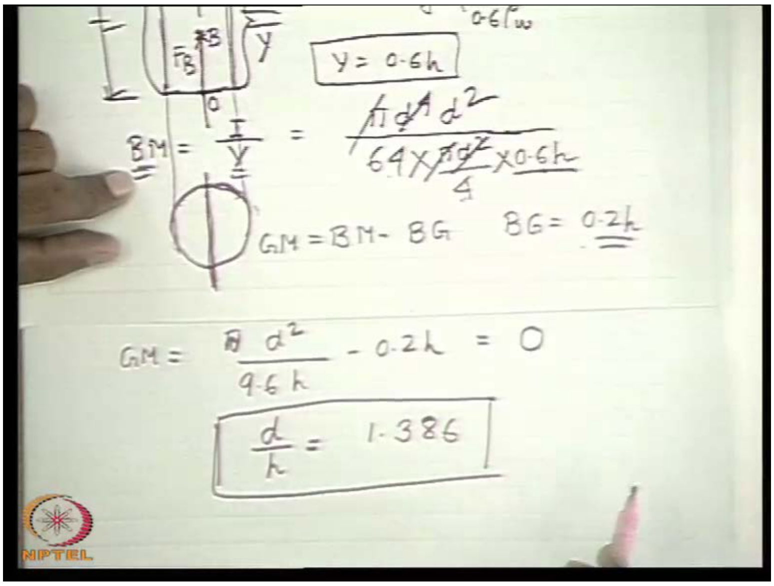


So, therefore, next part is very simple, that is g m is equal to pi. So, now, you cancel it; that means, d square by, then it is 16 into 0.6 that is 9.6 into h minus 0.2 h. This is, this has to be greater than 0 for a stable equilibrium for neutral equilibrium. According to the problem, this is 0. So, this gives d by h equals to 1.386, from this you get a value d by h is equal to 1.386. So, this is a very simple problem of meta centric height. Please tell me, whether there is any problem, today I think we have completed this second chapter. Of course, the time is short; otherwise I could have given a closure lecture today. So, next class, I will be giving a short closure lecture of what we have covered in this section and we will pass on or we will start the next section kinematics of fluids.

Well, thank you.