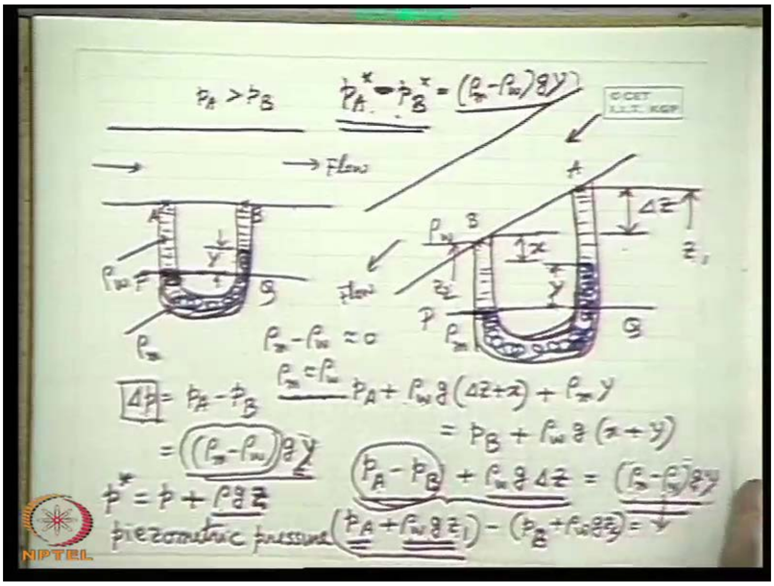
Fluid Mechanics
Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 6
Fluid Statics Part – III

Good morning, I welcome you all in this session of fluid mechanics. In the last class, if you recall, we were discussing about the manometers. A manometer is a device, which measures the pressure of a system in terms of its difference from the atmospheric pressure or the pressure difference between two points, whether the fluid is at rest or fluid is in motion. At last we discussed how the manometer registers the pressure difference between two points, when that fluid is flowing through a pipe.

(Refer Slide Time: 00:51)



Let us recall this again, that if we recall, this is like that that is there is a pipe where the fluid is flowing in this direction, it is the flow direction. Then, if the two points A and B are connected, if the pressure PA is greater than PB, then the difference of pressure is measured by connecting a manometer, which is essentially a U tube like this, sorry like this. We use a manometric fluid, which is which is having a density higher than that of the working fluid, that is the fluid flowing through this pipe. We register this is the fluid a deflection; that means, the interface. That means the separation surface between the working fluid and the manometric fluid. The interfaces stand like that. This is known as the let this is y.

If we take the manometric fluid density is rho m and the density of working fluid is rho w, we found that the delta P that is equal to PA minus PB becomes equal to rho m minus rho w g into y. That means, observing this deflection manometer deflection. That means, the difference in label of the meniscus y. We can find out the pressure difference. How to find it by writing the manometric equation? That means, we take this PQ line as the same horizontal line in this same expanse of fluid, which is the manometric fluid. Then what we do is we write the pressure here; equate the pressure here from this limb. When equate the pressure here from this limb and find out this equation.

Now, what we start at that what is the situation? We see a very interesting thing that when the fluid flows through an inclined pipe, let the flow direction is like this. The fluid flows through an inclined by where we are interested to measure the pressure difference between these 2 points A and B. Let us connect the manometer like this. Sorry, a U tube is like this. Let us connect a U tube like this U tube manometer.

Then, what we do? The manometric fluid will be having a deflection definitely like this. Let us have a deflection of the manometric fluid. So, we just this is the working fluid this is the working fluid. Let this deflection as it is we call it as y. Let from B this distance is x. A and B, there is a difference in vertical height between the section A and B. Let it be delta j just I give this nomenclature delta j. Now, from simple manometric equation; that means, if you take this line as the PQ horizontal line, if we equate the pressure from both the limbs.

Now, from this limb, if we come from this limb here first, PA plus if the working fluid density is rho w, the same nomenclature and rho m is the density of the manometric fluid. That means rho w delta z plus x; that means, rho w into g into delta z plus x. That means, this is the pressure of this column of liquid plus rho m into y, y is the deflection. This is y. This is y.

So, this becomes equal to what PB. That means, the same pressure is here at the same horizontal level within the same expanse of fluid. So, this is PB plus this pressure due to working fluid of height this x plus y. That means, rho w g x plus y. See, if we equate this, we will see PA minus PB. So, rho w g x cancels from both the sides PA minus PB. So, plus rho w g delta z is equal to rho m minus rho w into g into y.

So, therefore, we see that if we compare these 2 equations, we see that from the deflection y manometer deflection; that means the difference in level of the meniscus. If I equate this rho m minus rho w into g y, this gives an additional term from the pressure difference rho w g delta z in case of a horizontal pipe A and B at the same horizontal level. So, delta z is 0. So, we can put delta z 0. These is the special case becomes this. But, what is the physical interpretation of this term actually?

Now, if I express the vertical elevation of A and B, this we will come afterwards in our studies in our course and z 2 from a reference datum, from any reference datum, from any reference datum. If I just specify the elevation of A as z 1 and elevation of B as z 2, then I can write this as PA plus rho w g z 1, this quantity minus PA PB. PB plus rho w g z 2 is equal to this 1 rho m minus rho w. This one will come like that.

So, therefore, we see this manometer deflection multiplied by the difference of density into acceleration due to gravity gives the difference of static pressure plus some term which is rho g z one where z 1 is the elevation of that point from a reference datum. So, this term is known as P star that is piezometric pressure. That is piezometric pressure. It is defined the static pressure plus the equivalent pressure corresponding to its vertical elevation from a reference datum.

So, this is very piezo important definition, which we will come across afterwards piezometric pressure. So, always piezometric pressure is defined as static pressure plus the pressure equivalent of its elevation from a fix datum rho g. So, therefore, it registers a piezometric pressure difference PA star minus PB star. That means, the same equation we can use for an inclined pipe where if we find this quantity, this will give you the piezometric pressure difference instead of static pressure difference.

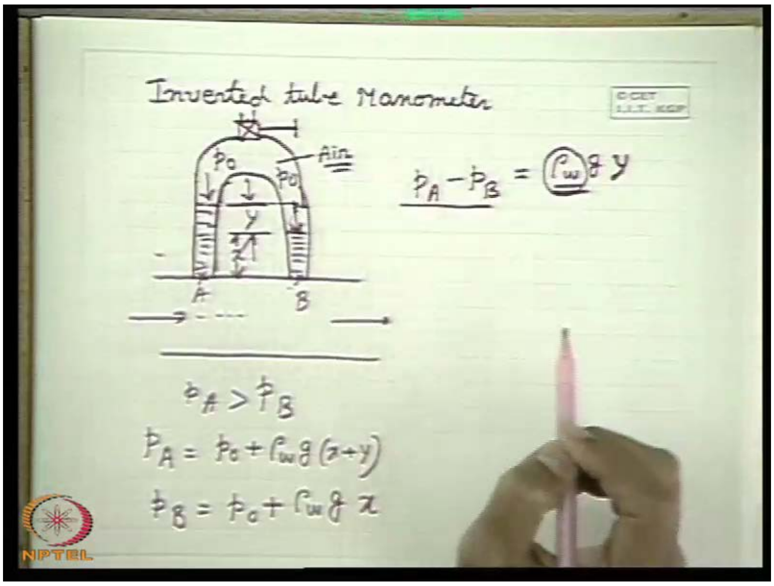
So, if I will have to find out the static pressure difference, of course, you will have to deduct these things. Sometimes, directly we want the piezometric pressure difference. In that case, the manometer reading from the manometer reading y, we can straight away find out the piezometric pressure difference in case of a horizontal pipe. The piezometric pressure difference becomes equal to this static pressure difference because delta z is 0.

Now, next we come to the next part. Here, we see that basically the pressure difference between 2 points in a fluid flow is measured by a manometer. Basically, by this equation, either it is a static pressure difference or it is a piezometric pressure difference depending upon whether the pipe is horizontal or inclined. Now, if we want to measure a very small pressure difference, then what we have, we should have we should have a substantial value of y for a small pressure difference.

In other words, you can say, if you want to increase the sensitivity of the manometer; that means, for a small change in delta P. If we want to have a substantial value of y or readable value of y, what we should do? We should make this part very small very small. So, that even what small delta P we have a large y; that means, rho m minus rho w should be very very small nearly equal to 0. Rho m that is the manometric fluid density will be very close to the density of the working fluid

In practice, it is very difficult to have a manometric fluid, whose density is close to the working fluid and at the same time defining a meniscus good meniscus. That means, a separation of surface between the 2 liquids has to be defined. So, well defined meniscus should be another reason why we cannot read y. So, a well defined meniscus between the immiscible liquids and also a close density values are not practically feasible. So, without going for changing the properties of the liquid go for some other changes in the construction of the manometer to measure small pressure difference.

(Refer Slide Time: 10:29)



This is one, which is known as inverted tube. This is done inverted tube manometer inverted tube manometer inverted tube manometer. What is inverted tube manometer? In this case, what happens if we consider a horizontal pipe and the fluid is flowing through it? What do we do if we want to measure the pressure difference between 2 points A and B? So, inverted tube manometer is placed like that in an inverted way.

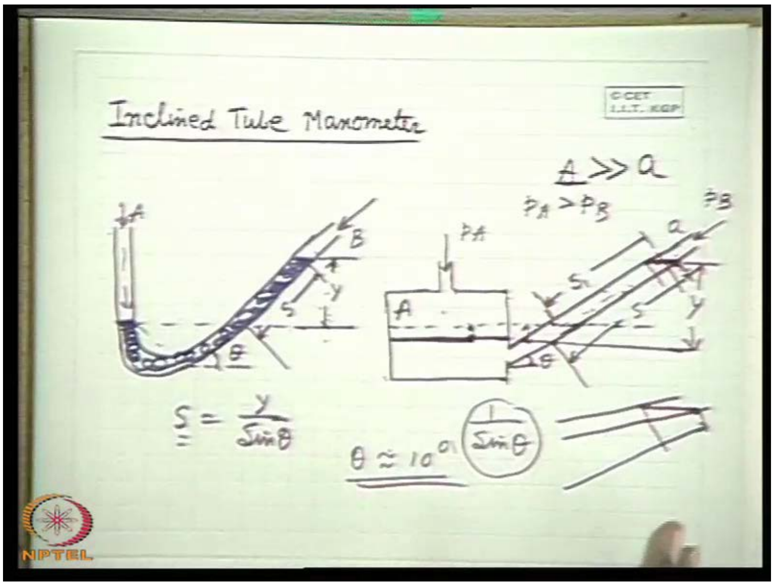
What happened is that there is a bulb inverted tube manometer contains air as the working fluid, air as the manometric fluid, air as the manometric fluid with some pressure. So, pressure of the air is adjusted by the bulb. That means, if you open the bulb, some air may come out. You know the mass of the air is reduced. The pressure in the close system of the air is reduced. So, this way, we can we can change the pressure of the air inside. So, what happened?

The working fluid depending up on the pressure will go inside. For example, the working if the pressure PA is greater than PB that will be adjusted. It is very simple. So, therefore, if we measure this height, this side, let this side is y. Then we can get simply the PA here is equal to some pressure. Let air pressure is P0, so P0 plus rho w into y. Let this is x rho w into g into x plus y. Similarly, here PB is equal to P0. The same air pressure in the closed system, the air will impress the same pressure. There is no difference in the pressure because of the height of the air.

So, you can tell say the pressure here is P0. So, here pressure will be P0 plus height of the air. So, that height of the air we neglect because the density of the air is small compared to the working fluid. If this is a liquid is already working, fluid is a liquid. So, we neglect for small height, the variation due to pressure because of the height of the air. So, the same P0 pressure is impressed here. So, that is P0 plus rho w g into x.

So, if you just subtract 1 from other, you get PA minus PB is equal to what will be there rho w g x will cancel rho w g. That means, there only appears there appears the density of the working fluid liquid. So, therefore, by this we can have that. So, this is low compared to the manometric fluid. If you use a manometric fluid in usual conventional U tube manometer, we can have a large deflection even for a small pressure difference.

(Refer Slide Time: 13:37)



Then, another modification is inclined tube manometer inclined tube manometer inclined tube manometer inclined tube manometer inclined tube manometer. What is it? It is very simple. Again, on principle, if 1 limb of the U tube is made inclined, what happened? If 1 limb of the U tube is made inclined, just imagine what will happen? That means, it is very simple to have amplification. That means, if a pressure is connected to 1 pressure point, another pressure point, let this is A, this is B with the working fluid. Then what will happen? The manometric fluid will be deflected like this. So, let us have the deflection like this. That means, for a vertical deflection y, we can read this as it is.

So, this corresponds to for example, this vertical deflection y corresponds to a deflection s along the pipe with a magnification. That is if this angle of theta, so s is equal to y by sine theta. That means, if we measure the displacement of the liquid mercury or the manometric liquid is any mercury is used as manometric fluid or manometric liquid along the pipe, then that will be amplified by a factor of 1 by sine theta with a y. So, this y will be proportional to the pressure differences PA and PB applied here.

So, the magnification is made by this inclination. So, that this s if we measure along the pipe along the inclined tube of this U tube manometer. So, we measure a greater displacement than y because of this geometrical magnification 1 by sine theta. This is an inclined tube manometer. Usually, we have to find out the difference of level. That means, we will have to find out s from this pressure is applied. That means, we will have to recognize the difference, the deflection here and the deflection here. To avoid this, what is done that one part is made with larger cross section compared to the other part?

Why? In that case, we will understand that only the movement of the liquid in 1 limb is sufficient to be read. That means, let us consider a manometer like this inclined tube manometer where this part of the limb is made with a larger cross section let A. This part cross section is A where A is very very larger than greater than A. So, pressure is being this. It is connected to a system. This is connected to another system PA, PA before the connection. Let the level of the liquid was like this. This is the original level of the liquid, manometric liquid. There is manometric liquid.

Now, after the connection with the pressure where PA is greater than PB, then the manometric liquid will be depressed here. So, manometric liquid will come here. It will go here like this. So, now, we will have to find out this deflection. If you write the manometric equation, you will see we require this deflection y to be measured. That means, in that case, we have to measure this s. Try to understand this s.

So, this s to measure we will have to know this length, but we do not do that. What we do? This part we do not make transparent. Only this part is made transparent. We see the initial level in the transparent inclined part. We see the final level. Instead of measuring this, we measure this s 1. That means, we neglect the measurement of this. Why we can neglect? How we can neglect?

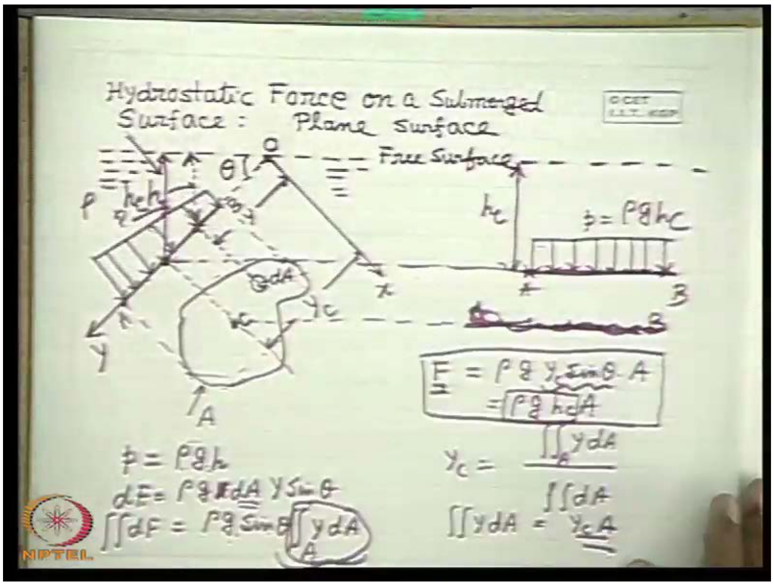
If this area is much larger than this, so this deflection means, this vertical deflection is much much smaller compared to these displacements in the tube from the continuity because the same fluid which is depressed here will be going up in this tube. As the cross sectional area is much larger, the cross sectional area is made 2 times, 2 times in the order of magnitudes 100 times larger than it. The diameter is made some 10 times more than this is well cases. So, therefore, this depression is much more negligible compared to this displacement.

So, error in taking s 1 as the measurement as instead of s; it is very, very less. So, that we need only this inclined part of the tube to be transparent and this can be made metallic if the cross sectional area is larger. So, simply what we do? We take the initial level. We take the final level. This displacement along this inclined tube is measured. So, this displacement is magnified by a factor of 1 by sine theta instead of having a vertical displacement here that we have explained I have explained. Now, here the one question comes if the magnification factor is 1 by sine theta, we can go still lower and lower value of theta.

So, we can get more magnification that for a small difference of delta y, very small difference in pressure or small difference in y. We get a larger value of s. So, there should be a limit. Limit is like that as you go on decreasing the theta; you will see this meniscus free surface of the manometric fluid will be flat. It is very difficult to find out its exact position. We will take its end point. We will take its middle point or here. So, this we will call error to the system because free surface will be horizontal.

So, if we make a higher value of theta, this surface will be such that the difference in s, that means, the distance along the limb from this two points on the free surface will be this. So, if you make and make more horizontal, the surface you see like this, then a free surface will be wide because it has to be horizontal. So, this 2 point, if you measure the displacement in this direction, this we will call substantial error. So, that theta should not be very very low. Theta is usually restricted between 10 degrees. So, this is principle of inclined tube manometer. Now, we we will discuss the pressure.

(Refer Slide Time: 20:45)



Now, next section is the pressure exerted by a surface submerged in a expanse of static liquid that is hydrostatic force exerted on surfaces submerged. So, when a surface is sub merged in a fluid or at rest, then the pressure will be exerted on the surface. What should be the pressure exerted on a surface? So, let us write that hydrostatic. This is already done. Let us see that hydrostatic force on a submerged surface hydrostatic force on a submerged surface. We should first consider plane surface. Now, let us consider a fluid at rest with a free surface. Let this is its free surface of the fluid.

Now, let consider this is a liquid and a surface is submerged in a liquid within the liquid. This is the surface at an inclination of theta. That means the plane of the surface and the plane of the free surface makes an angle theta. That means, if we see the surface from this top view, this is the surface of area A. That means that the surface is inclined surface. So, this is the free surface. This is the surface, which is inclined at an angle theta, which will be seen from the top. So, this is the view. So, this is the surface an arbitrary surface.

Now, you see in this case, what happens? You can argue that the pressure force is acting on the both the sides of the surface due to the water. At any point, they will be equal the pressure on this surface. Then pressure on the other side will be equal at this point also. They will be equal. So, the net resultant force will be 0, but this is not the case we are considering.

In actual case, a surface submerged will not experience any of force. We are interested to find out what is the force exerted on the surface if other side is exposed to atmosphere. That means, what is the gauge pressure exerted or the gauge force exerted on the surface from one side. That means we will consider as if the other side is exposed to atmosphere. What should be what could be the net force exerted on this surface? You can just think the problem in that way otherwise you can argue. In the actual case, they balance each other.

So, we are not considering the forces from the other side. We are considering forces from one side. Other side is as if open to atmosphere. Now, as you know, the pressure varies linearly with h in compressible fluid if it is a liquid. So, therefore, there will be a variation of pressure like this. That means, the pressure force will act on the surface like this. The pressure force will act on the surface. Let this is the surface. Let AB is the surface pressure force. This is the pressure force will act because the pressure at any point is irregularly proportional to the height.

Now, let us consider an elemental area. Now, first of all, before that let us consider a co ordinate axis. This point is the point. This line seen as a point is where the plane of the surface and the free surface meets the plane of the surface. The free surface meets at a line, which is seen at a point from this direction perpendicular to the plane of the paper. This is taken as the origin o and o o y y co-ordinate is taken a line along this plane along the direction of the plane. Perpendicular to this, ox axis is taken. I think there is no problem.

Now, let us consider a small elemental area d A, which is at a depth h. That means, this one this one is h. That means, this elemental area here, which is seen as the area from the surface area d A is at a depth h from the free surface. So, what is the force on the elemental area? First of all, what is the pressure exerted by the liquid on this elemental area? It is at a depth h from the free surface. Here, we are considering the force or pressure above the atmospheric pressure.

So, this will be simply rho g h. Rho is the density of the liquid rho. So, rho g h, h is the depth at which this elemental area is considered. So, therefore, the force on this elemental area will be d F will be rho g h times d A and this h from the simple geometry if I define y as the co ordinate y co ordinate of this elemental area from the x axis.

So, this h will be simply y sin theta. So, it will be y sin theta well. So, if we integrate this force over the entire area; that means, double integration over the entire area. We will get rho is constant, g is constant, sin theta is constant. Why not we take out rho g sin theta? h will not be there in this case. Rho g h, h is y sin theta rho g sin theta y d A. I think this is clear. Rho g h is the pressure here in this area. So, the force on the area elemental area d F is rho g d A that is the rho g h into d A. d A is multiplied because the pressure into area is the force and h is substituted as y sin theta. This is theta. This is y. This is h. So, y sin theta becomes rho g sin theta y d A.

If you make a double integral over the area A, this is row g sin theta y d A. Now, what is this term? This term is an implication you see that y d A over the entire area represents what is this moment of the area about x axis. So, this term is d y d A, this term is the moment of the area about the x axis. So, now, if we define a center of area, you know what is center of area very well. So, if you define c as the center of area whose y co-ordinate from the reference x axis if it is y c, then by definition y c is what? It is the moment, first moment of area over the area A divided by the total area d A. That means, the total area A; that means, y d A is equal to y c into A.

So, if we replace this here, we get a very interesting result that F becomes is equal to rho g y c into A instead of integral y d A. That means it is rho g y c sine theta into A. What is y c sine theta? y c sine theta is similarly, the vertical depth from the free surface. So, this is y c sine theta. If I denote this as h c; that means, h c represents the vertical depth of the center of area.

That means if it is y c, y c sine theta is this vertical depth. That means, if the center of the area is at a vertical depth from the free surface, h c F is equal to rho g h c. So, this is a very interesting result. That means, if there is an inclined surface, there is a inclined surface where we see the pressures at each and every point is varying because of their depth from the free surface. The total pressure due to this total force due to this hydrostatic pressure is simply the pressure at the centroid.

If one can find out what is the pressure value of the centroid times the total area, will give you the force. That means, it is it can be equivalently told that if this surface AB could have been placed horizontally, this will be at this end. So, if the surface AB, this is wrong surface AB could have been placed horizontally at a depth h c below the free surface, then what could have happened?

The uniform pressure in a horizontal surface pressure at each point is uniform. This is the pressure P. So, this P is equal to rho g h c because of this depth. That means, placing an inclined surface in a fluid submerged in a fluid is equivalent to place the same surface at the depth of its center of area. That means, if we know the center of area depth at the center of area, we can find out the pressure and times the area is the total force. That means, it is equivalent to it will experience the same force like that. We say experience the same force if it could have been placed in a horizontal position at a depth h c, which is the depth at which the center of area of this area lies.

So, this is one important conclusion. Now, next is to find out the point of application of this force. Now, it is true that the point of application of this force well I can put it here. Here itself, you can see the point and how to find out the point? Now, you see that the resultant force magnitude is known that is equal to rho g h c into A. So, if I know the value of x c, the centroid depth, depth of the centroid from the free surface, I can find out the total force. The direction of the force is always known that this is perpendicular because for all elemental surfaces, the pressure force is perpendicular to the surface.

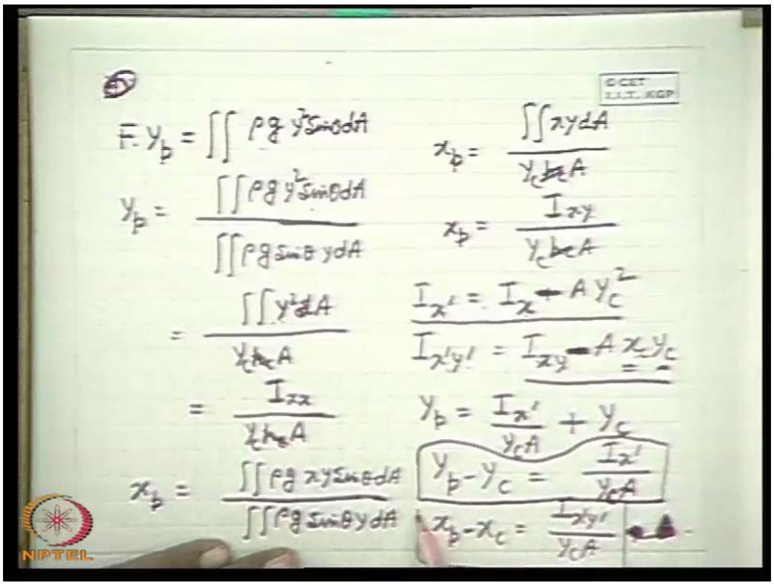
Since, it is a plane surface, the perpendicular at all points are parallel. So, a scale of summation will do. So, the direction will not change. Direction is perpendicular to the surface. Where is the point of application? It is also very simple. The point of application definitely will not coincide with the center of area y. This is because the forces are not proportional to the area. As we go along surface, we see that depth is increasing. So, even for the same area here, if we have another same area here that 2 pressure forces and the 2 areas are not same.

This is because the pressure intensity is changing, because of the depth. So, it is common sense that as we go below the surface free surface, the pressure is more and more. So, therefore, center of pressure will lie. For example, here we denote this c p as the center of pressure which will lie beyond c center of area towards the depth of the fluid.

So, let us consider this is the center of pressure. Let us define the co ordinate of the center of pressure as y p from the x axis. Let us define this as the x p from the y axis.

Now, common sense will tell us to make this c p here, not here c p would have coincided with c. The center of area of the forces could have been proportional to the area element, but it is not. So, since the pressure is increasing; that means, with the depth, that means, the lower portion of the area is having more force. So, that it will be this side from this center of area. Now, if we apply simple mathematics to find out the y p and x p what is that? That means that the resultant force multiplied with y p; it will be the sum of the moment of the component forces above the o x axis.

(Refer Slide Time: 32:49)



That means if I tell write this F into y p will be the sum of the moment; that means, integral of the component forces moment of the component forces on the x axis. What is this component force rho g d a y sine theta? Its moment will be about y axis. x axis is another y; that means, it will be simply rho g y square sine theta d A rho g y square sine theta d a. That means, y p will be integral rho g y square sine theta d A divided by F. F is rho g y sine theta d A. That means, it is integral rho g sine theta y d A.

So, this is already found as integral y d A as h c into A. So, therefore, I write is at h c into A. Here, I will write as integral y square sine theta will cancel y square d A y square d A into h c by A. What is y square d A in this case? y square d A, you see is the second moment of area of the surface above the x axis. Second moment about x axis, sometimes it is second moment of area or moment of inertia of the area about this x axis. So, x, x divided by h c into A. Is there any problem? No problem.

Now, what we get? What we do that this can be replaced by a parallel axis theorem. Before that, I let me do the x 1. So, what will be the x component? Now, x component, if x p we want to find out, we can take the moment in the similar way about the o y axis. That means, this will be simply integral of rho g x y sine theta d A divided by what is this thing? This is rho g sine theta y d A. now, if we come here, we see that this becomes x p is equal to integral x y d A divided by what is that? This we can write as h c into A.

So, therefore, we see that this is known as the product of area I x y. What is this x y d A? That is product of area. That is known as product of area. That means, if this be the area, this x co-ordinate of at any point or at any elemental area, this is the y co ordinate. So, the integration of x y d A over the entire area is known as the product of area, I just simply write it is as x y this already you know h c into. So, we know x p by a parallel axis theorem. Now, if you know, any question please you ask, you can ask at this moment.

So, before we proceed to the next one, is there any problem?

Student: This will be y c.

Yes yes yes yes. I am sorry.

This will be y c. This will be y c. I am sorry I am sorry. This is because sine theta I am cancelling, this will be y c. This is because sine theta I am cancelling from these sides, numerator and denominator. So, I cannot take y sine theta as x c. So, it will be y c because sine theta has such as cancelling from the numerator and denominator. It should be y c. I think there is no other problem.

Now, by a parallel axis theorem, you know that if I define the moment of inertia that means, I take the parallel shifting of the axis as you have read earlier. If I define x dash and y dash are the co-ordinate axis parallel to x and y through the centroid c, if I now define. You understand this x dash and y dash through the centroid c.

If I define that I x dash is the moment, second moment of area or moment of inertia about o x dash axis, then the relationship between these and the moment of inertia of the old x dash is I x plus A into y c square. That means the moment of inertia about this axis is the moment of inertia or the second moment of area above this axis plus area into y c square. That is the distance of this axis from the earlier axis y c square. Similarly, for the product of area, if I define with respect to new x dash y dash axis, this will be at the relationship I x y plus A.

The two co-ordinate will come, which will define the co-ordinate of the new axis from the old one. That means this is y c in that case. This is x c in our case. Our new axis is to center of area. In that case, y c, x c are the y and x co ordinates of the center of area. So, now, if you substitute see this interesting results come y p is equal to if I substitute this here. That means, I x, it will be minus. It will be minus. I x will be I x dash plus this will be minus. I am sorry. So, I x will be I x dash plus this I x y. That means, in the old coordinate will be in the new co ordinate plus this.

So, if I substitute here, what I will get? I will get I x dash divided by y c A plus cancels y c. So, I get y p minus y c is I x dash divided by y c A, y c into A. Similarly, if I substitute this one here, what I will get? This is I x y I x dash y dash. So, we will get x p minus x c is I x dash y dash divided by y c A plus. What we will get x c y c. That means that if I substitute here, y c will cancel A simply. x c has come here. Nothing will be there. So, I get this and this one. So, therefore, we see that y p minus y c is this one. This is a positive quantity because I x dash contain an integral with the square of the ordinate.

So, therefore, the second moment of area always a positive quantity y p minus y c is greater than 0. That means, this quantity gives you the displacement in this direction. This is y p minus y c. Sometimes, this is told as the eccentricity. That means how far the center of pressure is shifted from centre of area in the direction along the plane. So, this is y p minus y c. So, this distance is given by a simple expression I x dash divided by y c A. I x dash is the moment of second moment of area about this o x dash axis. y c is the center of area in respect to this co-ordinate. A is the area

The most interesting fact is that x p minus x c becomes 0 because the product of area about the centroid axis is 0. This means that x p is equal to x c because if I take the axis x dash y dash to centroid plane, the product of area is 0. So, therefore, x p minus x c is 0. This one can make from a common sense that if this c is the center of area. So, whatever may be the arbitrary area, the center of pressure cannot be shifted in a direction perpendicular to the plan. This is because any area in this side and another area will be always at the same tip. So, therefore, the pressure forces are proportional to the area element only.

So, therefore, the center of pressure and center of area is not displaced in a direction perpendicular to the direction of the plane. So, it cannot be disturbed. So, therefore, this is x p minus x c; that means, this is the x p and this is the x p. So, x co-ordinate of the center of pressure coincide where the y co-ordinate. That means, along the plane is shifted towards the depth of the fluid by the amount, which is given by y p minus y c is this. You have understood this plane figure, plane surface. So, next class, we will discuss the pressure forces on curved surface.

Thank you.