Fluid Mechanics
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Lecture - 49
Introduction to Turbulent Flow part -II

Good morning I welcome you all to this session of fluid mechanics. So, today we will be discussing the introduction to turbulent flow, which were we have started in the last class. Now, we were discussing about the modification how Navier stokes equations with the use of the velocity component in a turbulent flow which can be prescribed as a mean component plus super imposed with the fluctuating component as suggested by Reynolds we discussed last class.

So, let us today continue that how the Navier stokes equation is modify the in case of turbulent flow using this Reynolds decomposition principal to specify the velocity components in a turbulent flow. So, before that we will see that equation for such purpose is written in a little different form from the conventional one, which we discussed in the previous chapter like laminar discuss flow theory.

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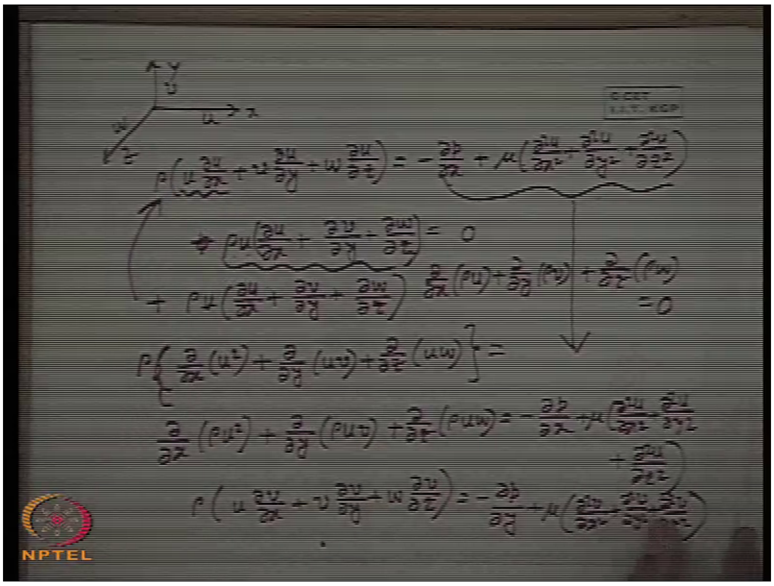

So, let us see that the Navier stokes equation, if you consider the x direction to equation for an incompressible flow it takes direction; that means, if you consider this x coordinate y and z if we take only the x coordinate. So, equation incompressible flow is like that row u del u del x; that means, this is the left hand side as you know is the share per units volume that is the acceleration times row u del u del x v del u del y. As you know previously u v w at the velocity component means x y and z direction w del u and del z is equal to the pressure force that is minus del b del x per unit volume plus the viscuss force mu we discuss del square u del x square plus del square u del y square plus del square u del z square and the other term is 0 another term is there because of compressible of the fluid for an incompressible flow, this is 0.

Now, this left hand part is written in a little different form for such a purpose a for, in modifying in this equations in turbulent flow, how it is done? Now, if we recall the , sorry the continuity equation for an incompressible flow, that is with respect to a coordinate this looks like this.

Well, now if we multiply this equation with row u throughout, then we can write the continuity equation like this multiplying the basic quantity is row u and then if we add this quantity in the left hand side which is equal to 0; that means, this quantity we add left hand side; that means, with this we add this quantity that is row u, because this quantity 0. So, we can add 0 to the left hand side without altering the equations plus 0 which is equal to this quantity.

Then we can do one, we can see one thing if we take row as common, then u del u x plus u del u del means two del u del x which can be written as del del x of u square. So, make it as second bracket plus you see this v del u del y v del u del y this term makes it del del y of u v.

Similarly, if we add up this term u del w del z and w del u del z it gives the terms del del z u w and the right hand side remains as it is minus del t del x plus del x as it is this quantity. This term can be retained by taking row inside because row is a cost and it can go within the operator. So, del del x of row u square plus del del y of row u v plus del del z of row u w. Now, I write again the right hand side to mix this equation complete; that means, the same quantity let me write mue del square u del x x square plus del square u del y square plus del square u del z square.

Now, this is the firm in which the equation is written in a slightly different firm. So, for a comprasible flow also if we use the equation of continuity for comprasible flow of which is del del x of row u plus del del y of row v plus del del z of row w and you manipulate it by multiplying u with this is equal to 0 and adding it. So, you will get the same term in the left hand side difference is that for incompressible flow row is constant and it can come out with it for a comprasible flow it can come out from this differentiates and another thing is the right hand side we will contain another term for the comprasiable flow.

So, therefore, this is one form of the equations, now similarly way if we think of the y direction equation u del v del x plus v del v del y plus del y s plus w del v del z is equal to minus del v del y plus mu times till of the velocity component in the y direction that is del v square del x square is del square v del y square del square v del z square, can we retain also in a similar fashion like that by multiplying row viewing continuity with the equation and adding it with the left hand side.

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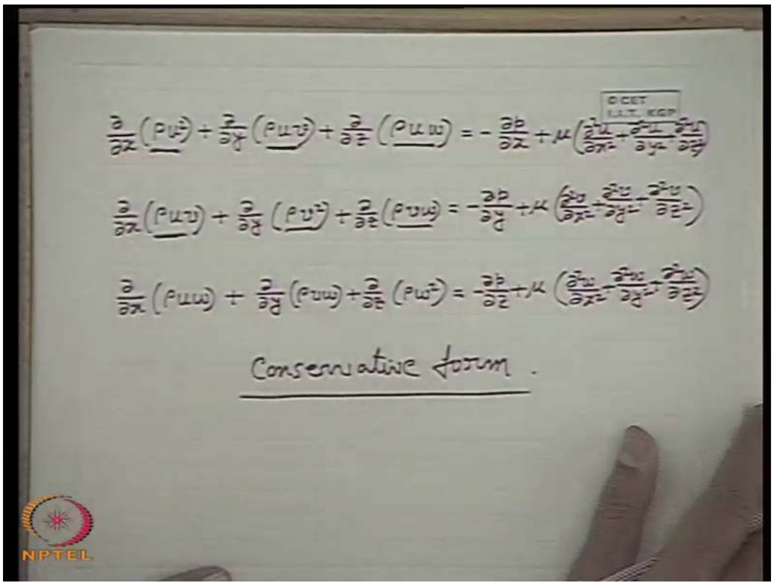

And this way if we do for all the three directions equations, ultimately we will come to this type of form of equation that is x direction this is the form we have already derived y direction the form will be like this and z direction only the left hand side will be retained in the different fashion with the manipulation of the continuity equation. So, this term clearly indicates the moment term of a flex for example, this is the x moment term a flex across the plane whose normal is index direction we call that plane as x plane.

Similarly, this is the y moment of the flex across x plane this is the z moment of a flex across x plane. Similarly, this is the y moment term of x across y plane it is the y moment term, sorry x moment term flex y moment term flex across y plane that is the plane perpendicular is in the y direction and the z is the moment term of across y plane. Therefore, these terms instead these of defining acceleration in the previous form of equation defines the moment term of flex this form of tones equations is known as conservative form.

So, writing may be us to equation in this form in any coordinate system is known as conservative form. Now, we come after that this we come to modify this equation this form of this equation.

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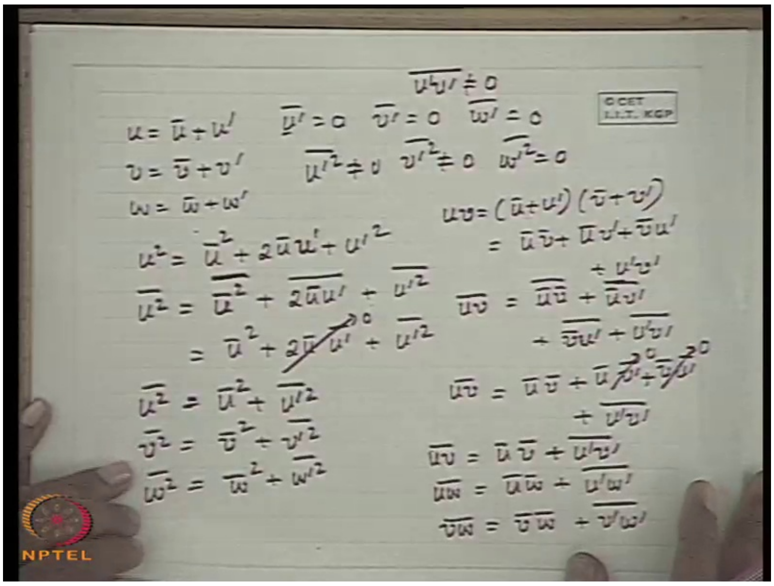

So, before modifying this equation turbulent flow, we recall turbulent flow specification, we know that u is specified by some time mean value is fluctuating component v. This time mean value this we have recognized in our earlier lecture where we know that u dash is 0; obviously, by this definition; that means, fluctuating is such in the time being value is 0, but there are certain algebraic rules for this.

Now, what is this? Let us find out u square; that means, square this quantity that this will be this square of this; that means, this square plus twice u bar u dash plus u dash square. Now, if we average this square of u; that means, time mean value of this square of u and what we get we have to average this entire term this average term and the average of all these terms is some of the averages is three term.

So, this separately plus average of this second term time mean value average means, time average value, time mean value. Now, this u bar is u bar square is u bar is time mean value which is in this independent of time, so again if you make a time mean value; that means, it is a cost end, so it is like this. Similarly, here u bar is the time mean value which is independent of time, so it can come out of this bar that is the time this becomes like this u dash square average.

Now, as per definition this is 0, so the middle term becomes 0. So, therefore, we get a very important conclusion like this plus u dash square, we know that this is 0, but u dash square average is not 0, similarly v dash square average not 0. So, average, time mean value of this square of the fluctuating components are not 0.

In the similar way we get that v square average is square of the average v plus v dash square. In the similar way we get w square average is w average square plus w average square; that means, the average value of square of any velocity component is equal to square of the average value of the velocity component plus average value of the square of the fluctuating component, this is the representation of turbulent intensity we have already discussed earlier.

Now, we will see another root of the product, now if we multiply any two velocity components let us multiply u and v, u and v what do you get? u bar v bar plus u bar v dash plus v bar u dash plus u dash minus u dash v dash bar. So, therefore, if we make a mean of u v, then we get mean of this quantity plus second quantity mean plus third quantity mean; that means, mean of the third quantity of mean plus u dash v dash fourth quantity.

Now, you see therefore, what happens? The mean of this are all time independent, so this becomes like this. So, this can come out; that means, u bar v dash mean plus this will be v bar u dash mean plus u dash v dash mean. So, v dash mean is 0, mean of the fluctuating component u dash mean is 0. So, therefore, u v mean is equal to this quantity.

And we know from the definition of turbulent flow though u dash mean and v dash mean is 0, but the product of the mean and there average is not is equal to 0. So, therefore, we get a rule that the mean of the product of two velocity component is equal to the product of their mean values plus the mean of the product of their fluctuating component and this is true for other velocity component also this is u dash w dash mean.

And similarly v w mean is v mean plus v dash w dash mean. So, with this rule now we can modify this equations, which has been returning conservative forms, now how we do it? Let us see now.

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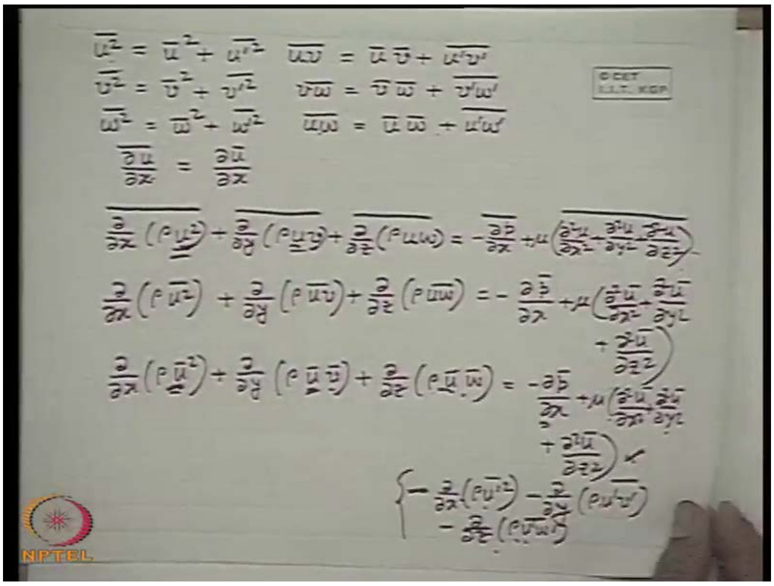

So, therefore, we have arrived as such rules for the squaring and taking the mean for the velocity components and their product and the mean. Another rule for example, if you take mean of a differential it is obvious it takes on a basic definition of this mean value it is a time average value, simple algebra it can be retained as differential of the mean.

So, with all this rules, now let us look how you do to the modifications for the , let us see the x direction to equation in the conservative for which we have just derived and the first step is that we take the time average quantities of this to equation. That means, physically we try to express the equation of motion on a time average basis; that means, we make the time average for both the left hand and right hand side.

So, term by term we take the time average well, now left hand side you see this time average value del del x of row u square by using this rule will be del del x of row u square average. Now, row is incompressible fluid, so we are not considering its fluctuations, so it is out of the turbulent quantity, so it will be only u square average. Similarly, the average of this differential of any parameter, fluctuating parameter is the differential of there average; that means, del del y of row u v average plus del del z of row u w average is equal to similarly by this rule minus del p the p is also a fluctuating quantity in a turbulent flow plus m u similarly each and every quantity the average of the sum of the three quantities is the sum of the average of the three quantities.

And again the sum of the average of this differential operator is the differential operator of the average quantity; that means, del square u average del x square plus del square u average del y square plus del square u average del z square. So, right hand side becomes simple, but in the left hand side, now we express u square average u v average by this u square average by this equation and u average by this. Then what we get, so if you replace this we get del del x of row u bar square.

So, another term we get del del x of row u dash square average, so fluctuating component now I am taking on the right hand side. So, keeping only this component the mean component in this side del del y a little algebraic manipulation we will tell you. So, here also del del y of row u v average means u average v average is u bar plus v bar u dash v dash bar dash bar v taking separately on the right hand side. This term is, here we will take it in the right hand side plus del del z of row u is equal to minus the del del x, the same quantity in the right hand side as usual u bar del x square plus del square u bar del y square plus del square u bar del z square minus the quantity which comes from here, here del del x of u dash square average bar; that means, minus if we take del del x of row u dash square average.

Similarly, from here we get del del y of this quantity row u dash v dash average well minus del del z dot; that means, from here row u dash this is very important. That means, if we see here after using this rules and then what we have done again I repeat by making time average of each and every term in the left and right hand side of the equations. We ultimately see that the time mean value of equation is the same left hand side, if we replace the velocity components by their mean values for the right hand side also plus some added terms these three terms are the added terms, these three terms, these we take for some purpose that will be explained afterwards in the right hand side.

So, apart from these three added terms the equation remains the same; that means, if we compare upto this, equations without taking the time average they are same only the velocity components are replaced by there time average values plus some added quantities or added three terms, which are these terms are fluctuating added terms average value of the fluctuating component.

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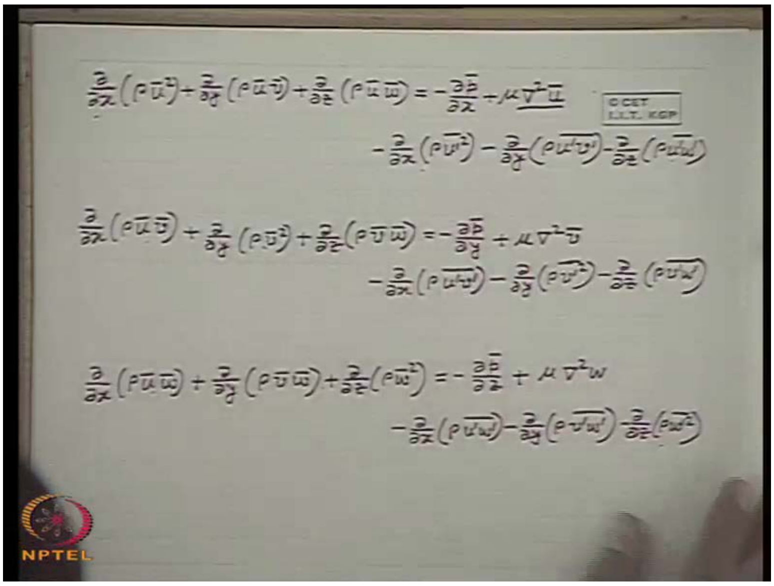

So, if this way we perform the modifications for three equations, three equations in three directions we get the same this we have derived earlier. So, del square u bar we have written in short for the want of space that is in the u, that is del u bar square del x square plus del u bar square del y square del u bar square del z square. Similarly, in y direction the equation the left hand side equation of the conservative form which was earlier done, this is the conservation form of the y direction equation, if you recall this equation. So, similarly if you replace it by the bar quantities, then the additional term, this three terms, this additional term come.

So, for z direction equation also it is the same equation of the conservative form with the velocity components which is replaced by their mean values average values, but extra three terms come of this short. So, these extra terms in the modification of the equation the time mean value of the equation. Now, you want to see the physical implications of this term, let us see the physical implication of this term.

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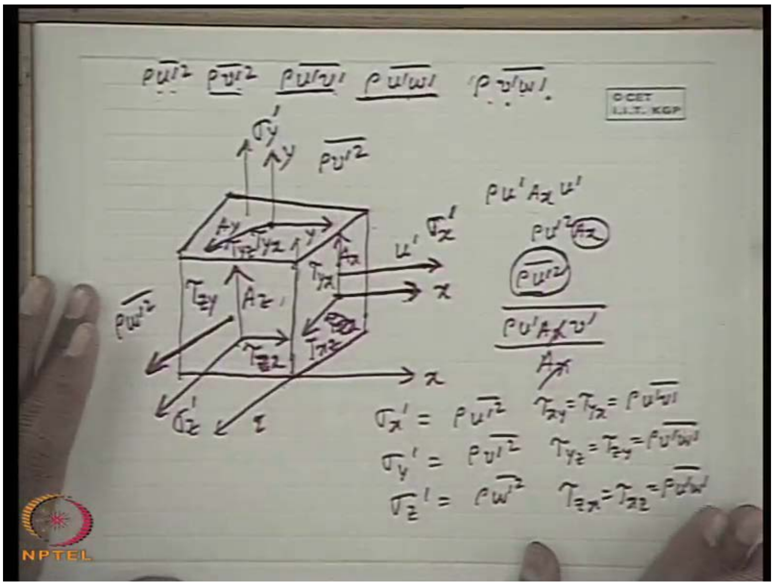

Now, if you consider what are the terms we have got, now you see the physical implication is like that what is the physical implication of row u dash square average row v dash square average row u dash v dash average row u dash w dash average these are the terms which have appeared with differential notations row v dash w dash average what they physically signify?

Let us see consider a plane x plane whose normal is in the x direction who consider some x direction. So, therefore, we see if is the fluctuating component of velocity in x direction at any extent of time, therefore row u dash is the masflex across the section here, if we consider axis the constructional area of these plane times a x these carries a x moment term which is this one. So, row u dash square a x is the x moment term at any instant moment of the component x moment in flex across a cross section whose perpendicular of direction of x. So, therefore, row u dash square average represents the average x moment per unit area a, because a x has been cancelled.

So, therefore this quantity represents the average x moment term a flex across this section; that means, across x area x plane that whose normal is in x direction. So, these is experienced as a stress normal stress as you know the moment term in flex in one direction across a section is can be conceived by a stress. So, stress will be in the same direction in which the moment of this term taking place; that means, it constitute a sigma x we give sigma x dash to is that in the sigma x dash, which arises due to more , because the stress is at any plane at if is responsible for the momentum transport across plane, because of the molecular moment term transport the laminar stress is arrived which we have already discussed earlier in the beginning of this course.

So, similarly here, because of the momentum transport associated with the fluctuating components; that means, the momentum transport by the macroscopic mass of fluid some additional stresses are coming. Now, let us see what row v dash square? Row v dash square in the similar way if we consider a y plane; that means, whose perpendicular is or normal is in the y direction this is y direction. So, row v dash square average in the similar fashion shows the y moment term of a flex y moment plane, which is conceived in terms of sigma y normal stress in the y plane sigma y dash.

Similarly, if we consider z plane; that means, this is the z direction, so here also the quantity row w dash square it represents z average the flex of the moment term across the per unit area, of course across the z plane is area as a j that is conceived in normal stress of a . What is row? u dash v dash, row u dash v dash can we conceived for this x plane as if the y moment term, average y moment term of a flex across x plane, because of the mass flow is row u dash x with the y continent of velocity per unit area cancels the average.

That means, average y moment of flex is explained which can be conceived as stress in the y direction in the x planes which is the y x. Similarly, this quantity can be conceived as an average x moment term a flex across the y plane, so that this can be conceived as x direction stress in the y plane that is y s is as stress. Similar way row u dash w dash can be consider as an average z moment term of a flex x plane and can be consider as a stress in the x direction in the x plane; that means, the stress or tow z x.

Similarly, row u dash w dash can be considered as an x moment of a flex across z plane and which can be consider as a in the z plane in the x direction tow that can be x z x, sorry tow z x and it is tow z x rather z direction x z tow z x. Similarly, row v dash w dash this can be expressed as y moment term of the flex across the z plane and can be consider as a y direction stress in the z plane tow z y and similarly this can be seen as a z moment of the flex across y plane. So, this can be expressed as a z direction force; that means, c s stress in the y plane that is tow y z. So, therefore, now we can sum up with this that sigma x dash that is an additional stress is an developed, because of this molecular moment of transport, because of the molecular moment transport associated with a fluctuating component this are like this row.

This is the average sigma x, x direction normal stress sigma y dash is row v dash is square bar and sigma z dash is row w dash square bar and we have seen the tow x y or tow y x this is also symmetrical is row u dash v dash bar. Similarly, tow y z is equal to tow z y is equal to row v dash w dash bar and tow z x is equal to tow x z is equal to row u dash w dash bar. So, you see that again 6 distinct stress components are coming additional with the laminar stress components.

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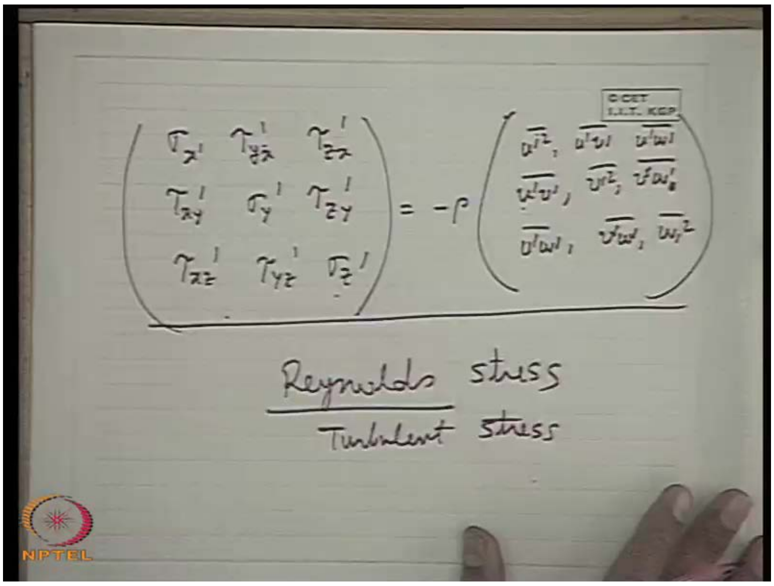

So, we can write this as a different stress matrix or stress tense like that sigma x dash tow y x dash tow this is in x direction, so therefore tow z x dash, so tow x y dash they are symmetric, sigma y dash, so tow this will be x y direction tow z y dash. Similarly, tow x z dash tow y z dash and sigma z dash. So, these 6 additional stress term which are these equal to minus of if we take row common for an incompressible fluid u dash square average u dash v dash bar and u dash w dash bar.

Similarly, this will be u dash v dash bar this are symmetric, so therefore, v dash square bar. So, v dash w dash v dash w dash, sorry w dash bar, here we get u dash w dash bar these are symmetric, then it is v dash w dash bar and then w dash square these are the normal stress diagonal element. So, this is the additional stress is developed and this is known as Reynolds stress, according to the name of the scientist derived it or turbulents stress, this is an additional stress coming along with the normal stress in the due to stress in the laminar flow.

So, therefore, now if we look we can see that this equation here this can be retain as sigma x dash tow x z dash, similarly tow it is sent x y dash sigma x y dash tow y z dash, similarly we can write tow y z dash sigma z dash. That means, del del x of sigma x dash and del del x of tow x y dash del del of x z dash, if we recall then we see that this quantity also can be replaced by del del x of sigma x plus del del y of tow x y plus del del z of tow x z which are the stresses of laminar flow.

If you recall if we start in the equation we have derived, see if we recall this term to be retained in terms of laminar stress, similarly for y direction and z direction we can write this way. For example, x direction equation; that means, minus del v del x plus mue del square u bar can be written like this; that means, because of the molecular moment term transport the laminar stress.

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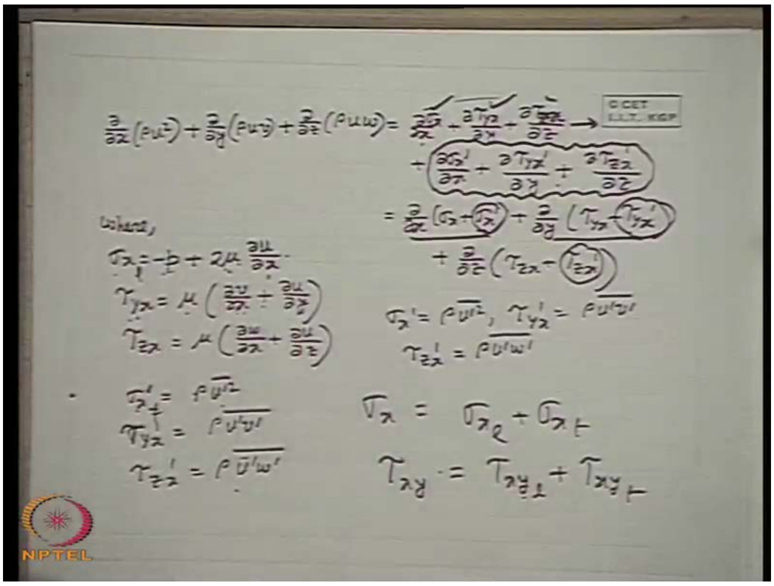

So, this is the net surface floats in this volume in x direction, similarly this is nate surface force in the unit volume in the; that means, an additional surface forces occurs in case of turbulent flow. So, this two can be added together; that means, if you see always an additional stresses are coming because of the turbulent flow.

If you record the definition of the minus b plus del u del x, similarly tow y x is mue del x plus del y in in x y plane z x plane and where as this additional stresses we have already recognized this thing the . So, the net stress sigma x can we retain as, for example laminar part, now we give this as the laminar part and this as the turbulent part, so laminar part plus the turbulent part.

Similarly, tow x y or y x whatever you call is equal to laminar part plus the turbulent part, turbulent part is the dash quantity and the laminar part is this quantity, which is the laminar part is this stressed is related to laminar viscossity mue in this fashion, mue del u del x plus del u del y twice mue del u del x, whereas the turbulent stress are related to the fluctuating component of velocity. Now, you see the most interesting task in this context comes that how do you relate, now this turbulent stress in terms of the velocity gradient of the mean flow this is because of the fact.

If you look back here, you see the most difficulty in integrating this equations to find the flow field is the occurrence of this fluctuating component, this is because the eratic behaviour of fluctuating components or its random fluctuations with time, we do not know exact mathematical formula of velocity component of time, so it is very difficult to integrate this equation.

So, if by some means we can relate this additional stresses through some parameter with the velocity gradient described by the mean flow velocities as it is done in case of laminar stresses, then we can integrate this equation and that is the full philosophy of a turbulence modeling, which relates the fluctuating components, which creates the stresses. That means, which delays the turbulent stresses with the gradient of the mean velocity through a parameter like the viscosity in case of a laminar flow and then expresses the equation of the motion to equation in the laminar form. Similar to the laminar equation and integrity and that is the sole purpose of a philosophy of the purpose turbulent modeling. Now, I will discuss different turbulent modeling, different turbulent model, one is the Eddy viscosity model.

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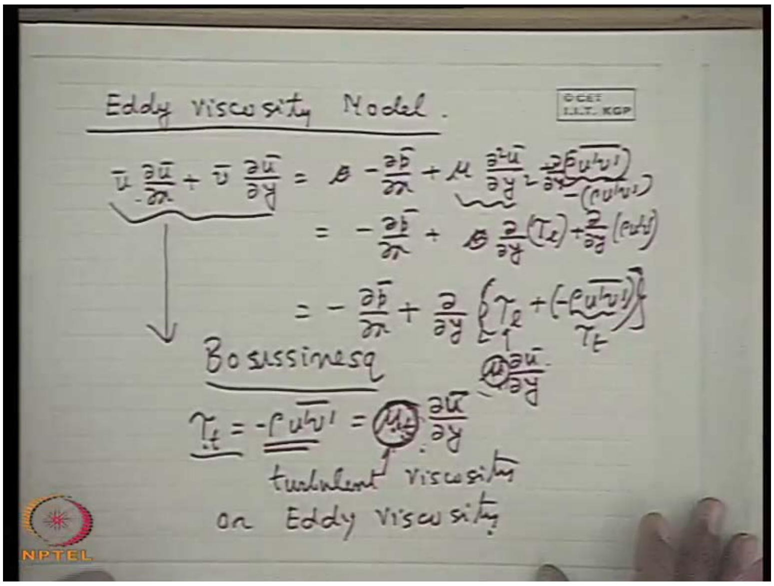

Now, if we take a very simple case of a one dimensional flow, for example you take a boundary layer equation u del u del x. So, if you make a modifications with the time as we have done for the equations, what we will get? We will get for example, minus del p del x gradient does not matter, may be they are depending on the curvature of the body, mue del square u del y square.

Which means, we can write this thing this mue del square u del y square part this part let it be like this. So, mue del square u del y square part we can write, so del del y of plus row u dash v dash bar, sorry this is the extra term for the del del y, I am sorry for the turbulent flow. So, mue del square u del y square can be written as laminar stress which is mue del u del y and ultimately this term comes plus the additional term del del y of row u dash v dash. So, therefore, you see the left hand side remains same, the right hand side pressure part is like this, it comes straight in terms of the mean pressure.

But what is the difference that along with the laminar stress we come to a turbulent stress tow t, how do you relate that? In a simple case I just explain how to delay that? So, it was Boussinesq, he was a great scientist Boussinesq, who first propose why not we tow l is mue del u bar del y.

So, similarly we define a turbulent viscossity which relates tow t is equal to, it is a minus sign, I am sorry minus row u dash v dash bar, it will be the minus sign there, so this will be del del y of minus this I am sorry minus this. So, that minus row u dash v dash bar should be related with the mean velocity gradient through a similar term like a laminar viscosity a term mue t which is known as turbulent viscosity, why not we do it turbulent viscosity or it is known as eddy viscosity eddy viscosity. But the difficulty comes that well we could have been happy just like the laminar flow equation that we know laminar flow c s t is linearly proportionally , in such a simple flow.

So, del u del y is the c r rate, so therefore mue times that is del y del mue is the constant of proportional which is a fluid property which does not depend upon the flow field. If it could happend for the turbulent flow also the turbulent stress is proportional to the share rate define by the mean velocity component we could have been the most happy man. So, that mu t become the constant of the fluid, but unfortunately even if you forcefully define the turbulent stress in terms of the velocity gradient with using the mean velocity, the mue t does not become the property of the fluid or it does not remain as constant as it does in case of laminar flow. So, mue t depends upon the flow field therefore, again the problems comes how to relate this mue t. So, therefore, we must have a relationship of mue t with the square coordinates in the flow field for integrating this equation.

So, therefore eddy viscosity gives a clue to solve this equation provided more information is known of about the turbulent viscosity or eddy viscosity; that means, as a function of the face coordinate, for this turbulent or eddy viscosity. So, this is eddy viscosity model.

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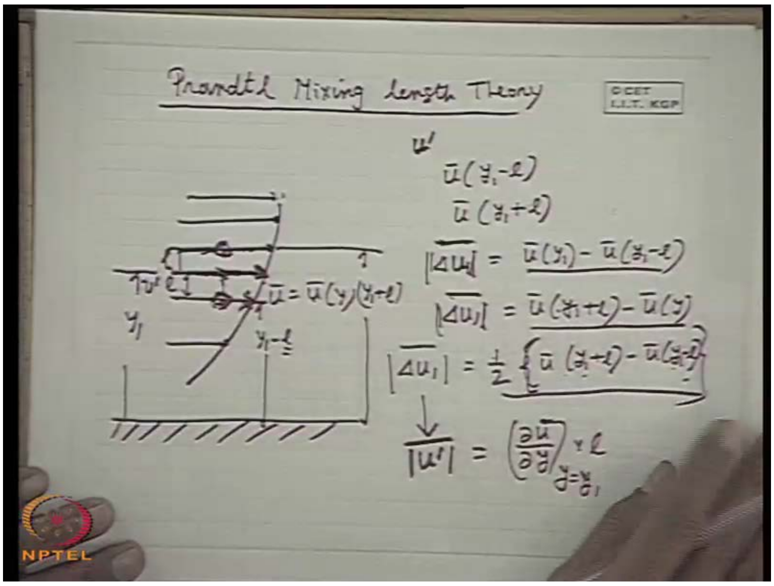

So, one step further regarding this given by Prandtl, so we will now discuss it known as Prandtl missing length theory. Now, Prandtl the great scientist who gave the revolutionary concept of boundary layer theory, so Prandtl missing length theory. What Prandtl did missing length theory, he actually proposed a short term mean free path in a turbulent flow. You know that turbulent flow the fluid particles move in all directions and there velocity is randomly fluctuating with time. So, here they to think this as a similar phenomena that happens at a molecular level, you know the laminar stress is a consequences of a molecular moment term transport.

Molecular molecules are in continuous thermo in any substance in any fluid. So, therefore, due to this random molecular motion we get the laminar stress is because of laminar momentum transport across the section. Similarly, thought of that of similar type of motions of the macroscopic fluid particles which are random in both space and time coordinates and then it thought in terms of a mean free path distance. That means, though they are number of molecules in a system which are randomly moving in here and there.

But still there is a finite still there here and small not zero and ash districally there is distance before to the , before through success collusions; that means, the molecules travels a distance before to success if collusion. Similarly, they describe that there must be a length that if fluid element travels before to successive collusion; that means, it retains this original velocity and the momentum as it is for certain distance of his trouble and that distance is known as Prandtl missing length, because after the distance it mixes another particles that's why the name missing length comes.

So, let us define the missing length like that, let us consider a velocity profile for a one dimensional flow, let we define the velocity profile like this, late we define a velocity profile like this; that means, u bar is a function of y let us define, let us define from any that is a slope past a surface of first a square from the major coordinate. Now, let us choose a particular coordinate at height where we will pay our attention and let us choose an exhausted way at one layer of here which is at distance y 1 plus l where l is such missing lane and this is at y 1 minus l, so that this distance is l this distance is l and we focus our attention at is at y 1.

Now, we see what? So, because of this, now l is the missing length where it was defined that a molecule retain the momentum, same momentum retained is original momentum while distance said. Now, we see according to Pandtl he told the section y 1 the velocity component y has a fluctuating component u dash, the value existence of u dash is due to the arrival of fluid particle from this two layer, one layer down to this layer one other layer or among this layer, below this layer above this layer. Now, let us consider this fluid particles which comes from the layer l below this layer which carries the velocity which is equal to that existing at that layer is u bar; that means, y 1 minus l.

Similarly, the fluid particles which comes from these above layer, the layer l above this layer at y 1 carries a velocity to this layer, because they retain there velocity while length l . So, therefore, a change in velocity takes place here because of the arrival of a fluid particle from a layer l below that can be retain as delta u here, delta u that absolute magnitude delta u, the average values of delta u at 1 u at 1 is what which is existing at y 1 minus u bar y 1.

Similarly, the change in the velocity component that absolute value and take his average, because absolute value and take this average it is not 0. So, d t will be u bar y 1 plus l, because the velocity of the particle which arise here from a layer above this layer here l l height above this will be this u y 1 plus minus u y.

So, therefore, we can tell the net change of this value is the mean of the this two; that means, mean of this two; that means, half of this minus this; that means, u bar we can take a second bracket y 1 plus l minus u bar y 1 minus l, so this is. So, and he then argued that this causes the fluctuating component; that means, if you express the mean value of the fluctuating component, because if you is known 0 it is equal to this and this can be retained in terms of the gradient of this velocity field at this point as del u bar del y at y is equal to y 1 times the l with a central difference scheme.

So, this can be written the difference between this two half of this two, between this two values can be written as this. Now, with this and we have some philosophy we can express the turbulents stresses, now how to express this? Now, we have to understand one thing which is given by Prandtl, that this is because of the arrival of this two particles are many particles from this two layer to this existing layer, they collide each other and then again they separate and reverse there direction. So, therefore, the v dash component; that means, the fluctuating component transverse direction is originated because of the collusions of this two particles which arrive at this layer and makes the change in velocity like this.

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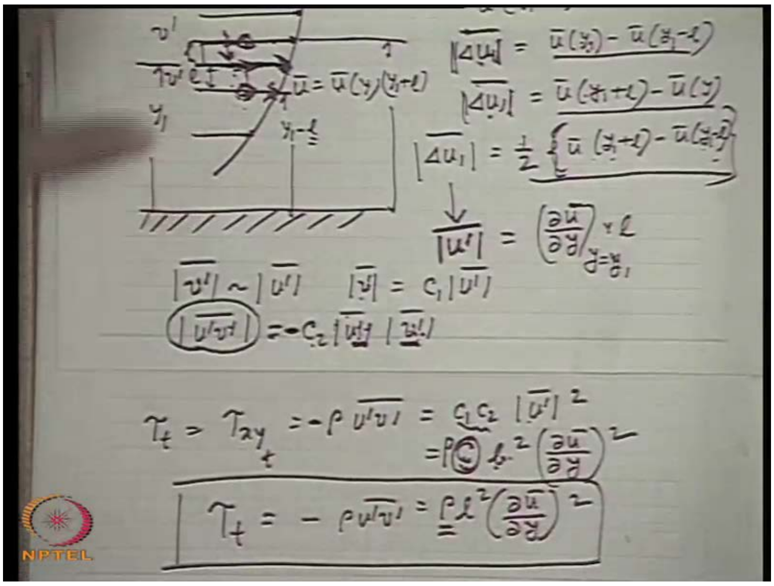

So, with this philosophy he proposed that this v dash absolute bar; that means, the mean value of the fluctuating component in y direction is proportional to this. Which means, that v dash bar that can be retain as some constant into u dash bar. Moreover, he argued that is statistical average of this product of this can be written as some constant c 2 times the product of this.

Now, if we have this two in our mind we can now write turbulent sheer stress at this is equal to tow x y at turbulent is what is that row u dash v dash average. That means, we can write c 2 and again this is proportional to again that c 1 c 2 u dash square, which can be written as 1 c and this is equal to or called two row is of course, there row c l square del u bar del u y whole square. Now, if we take this c in to l, so that missing end is define taking this c constant to it or taking care of is c into it; that means, row l square.

That means, if you define the missing length in this fashion, so he wanted only one constant will be there that is missing length is equal to minus row u dash. But here minus sign is there that is taken care of this in this session I tell you that u dash v dash is proportional to a negative quantity of the product of this, this has to be understand beyond to understand.

So, this is because this positive value of this to generate there will be a negative sign, because the u dash and v dash are in opposite sign. That means, the positive value of v dash causes a negative value of u dash and vice versa, let us verify it here. Now, when the fluid particles are arising from this layer, the v dash is positive, but when you reaches here it reduces the velocity here, delta u is 1 is negative actually.

Now, we define the absolute value that is u bar y 1 minus this, but the change in the velocity will be this minus this, because this fluid particle will written this velocity at this particle, because this is having a slow motion. Similarly, when this particle arise here; that means, this is associated with negative value of the v dash, that is the y component of velocity is negative, but when it comes here it increases the velocity influence and del u 1 that change in the x component velocity is positive, because this particle accelerates the velocity at here.

The similar way as the molecular moment at does in laminar flow, so therefore, u dash and v dash are associated with opposite signs, the positive value of v dash makes a negative contribution in u dash and vice versa. So, that when we make this statement we tell this is negative, if proportional to u dash and v dash they are mean values, absolute values mean of the absolute values with the negative sign. So, that if you make this definition minus row u dash v dash bar we get a positive value. So, this is precisely the Prandtl mixing length theory.

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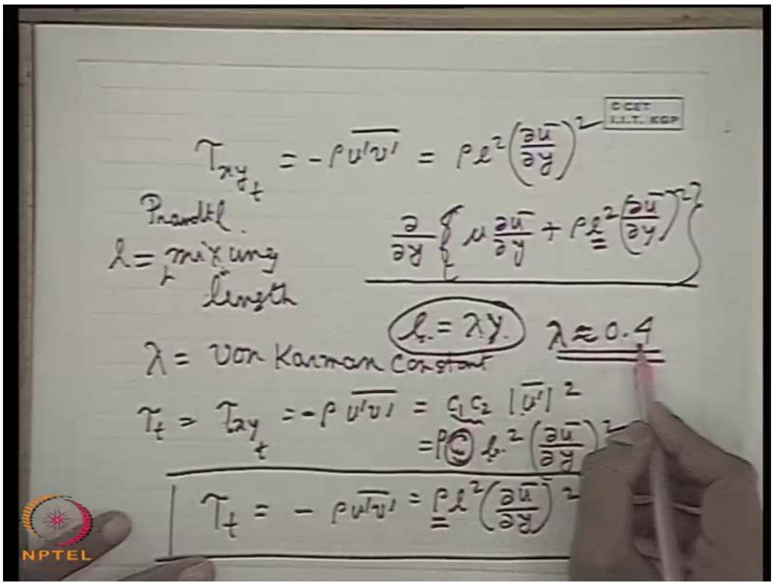

So, by the Prandtl missing length theory we see the tow x y theory in a simple case of one dimensional that is parallel flow is equal to minus row u dash v dash bar is becomes is equal to row l square del u bar. Now, here also we are not very much satisfied; that means, if we substitute it, for example well if you now substitute this here, so can u is still integrate the equation why? Because if you substitute this in our equation we get del del y of mue del u by del bar is the laminar part and for the turbulent part row l square del u bar del y whole square I understand where l is the mixing length mixing length, sometimes known as prandtl mixing length Prandtl mixing length.

But still there is a problem what is the value of l, is it constant or what is the functional relationship this is space coordinates? So, therefore a relation of that type is needed, usually it is found from experiment this l is proportional to this distance from a solid wall that measured at any distance y from the solid wall as lambda into y, where lambda is known as Von is a scientist is a German scientist in the field of fluid mechanics Vonkarbaon constant and it is its value is 0.4 approximately. So, one interesting thing is that y is equal to 0 at solid surface l is equal to 0 and there is no turbulents stress.

So, because at solid surface exactly due to the mostly boundary condition the velocity is 0. So, therefore, and near the solid surface velocity are slowly reducing and drastically coming to 0 near the wall. So, therefore, you see whatever may be the turbulent fluctuating velocities, so all this kinetic energy will dye out near the wall. So, therefore, near the wall turbulents cannot exit, so exactly at the wall the existence of turbulence is 0, so there is no turbulents stream. So, under any turbulent flow conditions at the wal there will be no turbulent stress. So, similarly at y is equal to 0 l is 0, this is satisfied by the equations, so if we take this value l is equal to lambda y and taking the constant value of the Vonkarbaon constant of lambda as 0.4, then we can integrate the equations, for a simple case this is the Prandtl mixing length analogy,

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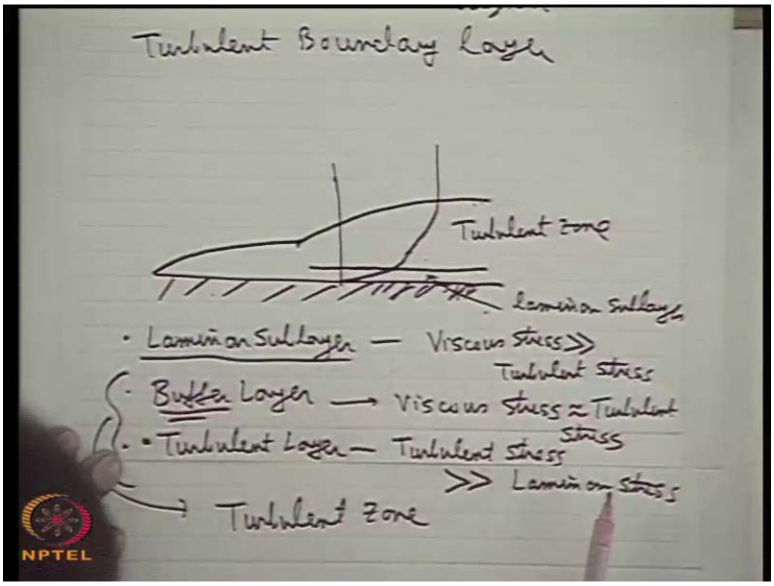

Now, I like to discuss the velocity, velocity in near wall region velocity in near wall region, velocity in near wall region, now the most important thing is that, when there is a turbulent flow, just now I have discuss near the wall, because of the viscuss interaction, so the turbulent flow is . So, therefore, if you study the turbulent boundary layer we will see that whatever may be the turbulent level, degree of turbulence, turbulence velocity is very high, the fluctuating components are very high, so that the mean values are very high, but these are aristic near the wall, so very near to the wall turbulent flow is not there is a laminar flow. So, therefore, there are the turbulence flow, turbulence velocity profile will not hold.

So, let us discuss it through boundary layer, turbulent boundary layer, if you see turbulent boundary layer then, first we see the turbulent boundary layer. Now, let us consider a flat plate, the flow takes place, now just after the leading as a flow becomes the turbulent, after some length the flow becomes turbulent, then the turbulent boundary layer grows, so here is a point where the turbulent boundary layer takes place.

So, turbulent boundary layer is also thinner though it is more than the laminar boundary layer, but it is very small compare to any characteristics length , but very near to the laminar, very near to the plate solid surface or flat plate the flow is like a laminar and it is known as laminar sublayer, it is known as laminar sublayer. Now, in fact if you see the velocity distribution, you will see in the laminar sub flow is almost linear, then it goes like this in the fully turbulent region.

So, this is the turbulent zone, this is the turbulent zone and this is the laminar sub layer and very strictly we can define the sections in to three zones, one is laminar sub layer which is very near to the solid surface adjacent to the solid surface in the near vicinity, were viscuss stress is much greater than the turbulents stress, so viscuss stress is prominent, so purely a laminar flow.

Another layer is the buffer layer, this the name itself it joins two layer buffer layer; that means, just immediate after the laminar sub layer where the viscuss stress, viscuss stress is almost in the same order of magnitude of turbulent stress. Well an another one is the fully developed turbulent layer or simply we can tell turbulent layer, where turbulent stress, turbulent stress is very, very greater then laminar stress.

So, one can neglect the laminar stress in solving the equations in that zone, usually the buffer zone the turbulent layer can be mixed up till only the turbulent zone, sometimes we tell there are two layers one is turbulent zone or another is laminar sub layer zone. Now, we have come to find out what is the velocity in the near wall region in the turbulent zone.

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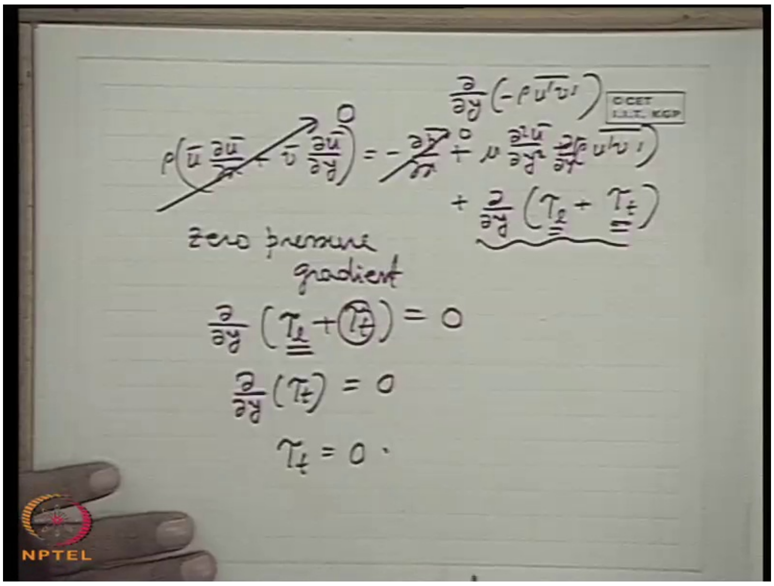

So, now to do this, if you consider a simple equation, for example the boundary layer equation which can be written as row u bar del u bar del x. For example, if you consider a boundary layer equation it will be easier for you, where we can show that the rest part, the second two part mue del square mue, for example if I write del square u del y square plus row u dash v dash bar, we can write this as del del y of this is the laminar stress, sorry again sorry minus minus of row u dash v dash bar. So, del del y this term is del del y of minus row u dash v dash bar.

So, this is you can write tow t row u of u dash v dash bar minus is the row u dash v dash bar is tow t, now tow l plus tow t. Now, if we consider very near to the wall where the inertia force influence is almost 0 and if we consider a 0 pressure gradient at the same time, 0 pressure gradient. Now, at this probably I should tell you one thing that well that probably , I have told you the in turbulent flow just a minute on negative sign, because all the time I am making this mistake negative sign, I am sorry negative sign I am correct, negative sign taken at that time, I was scared that is 0 pressure gradient, then what we can tell that this is 0; that means, del del y of tow l plus tow t near the wall is 0.

Now, if you consider the viscuss layer of the laminar sub layer is very small that it is only the tow t, then neglecting the laminar sub layer considering it is very thin we can take this 0and tow t is 0.

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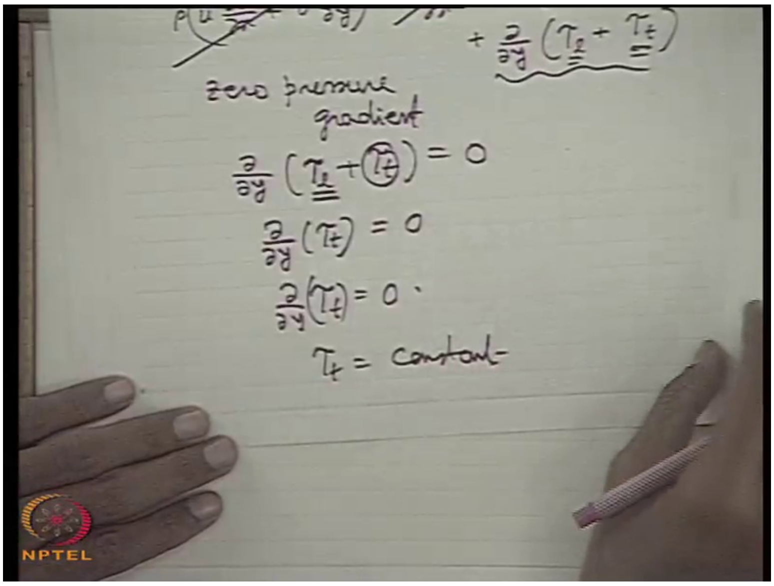

So, therefore, we arrive at tow t, sorry del del y of tow t is 0; that means, sheer stress is constant.

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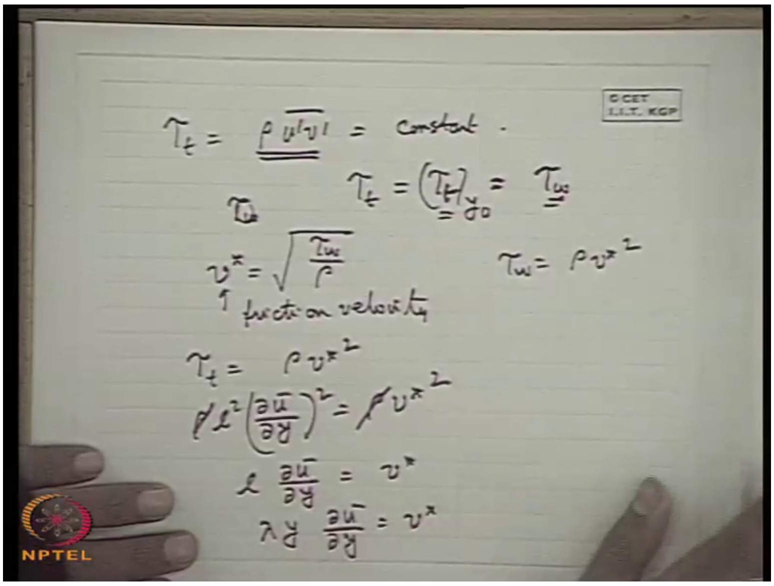

Now, with this we can now show that the turbulent sheer stress t is row u dash v dash bar now is equal to constant. Now, here we do one thing, we can find out this row u dash v dash bar in other way, now you see if we define that tow w now tow t row u dash v dash bar. So, we can write that tow t is constant; that means, through out the turbulent zone near the wall tow t is tow t at y 0 where tow t y 0 is laminar wall, which can also be written as tow w.

Considering the laminar wall is thin, so that the same there totalling the laminar sub layer or viscuss layer. Now, defining a velocity equal to root over tow w by row, which is known as the friction velocity. That means, tow w is equal to row v star square, we can write tow t is row u by using friction velocity v star. Now, tow t is what? Tow t we can write in terms of the Prandtl missing length theory, row l square, now using the prandtl missing length we can write row v square row . Now, if we take square root on both sides del u bar del y is equal to u square, now a is lambda y, so lambda y del u bar del u y is equal to v star.

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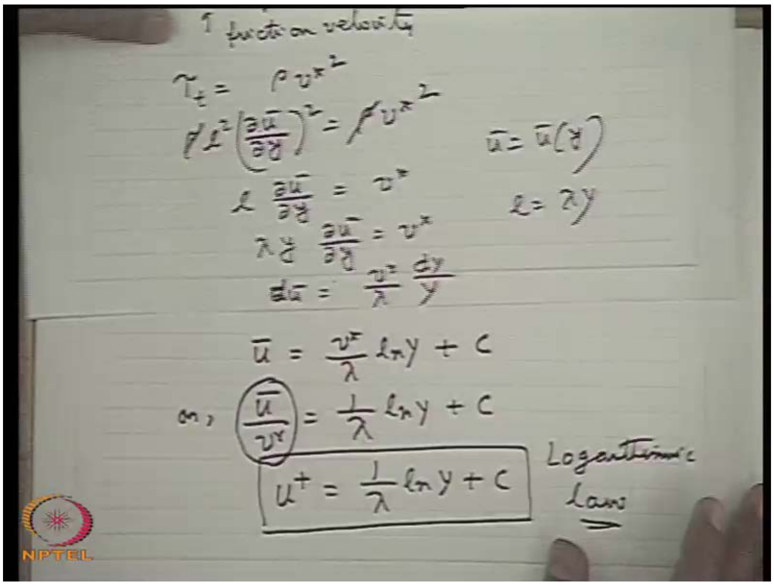

So, after this we can write del u bar is equal to v star by lambda, so this can be written as d y by y or this is t u bar, t u by d u is a function of y only in one dimensional parallel flow. So, therefore, we can write u bar is v star by lambda l n y plus or u bar by v star is 1 by lambda, lambda is co efficient l is lambda 1we have substituted here. This is a dimensional velocity and is defined as u plus 1 by lambda y plus c. So, this is the law near the wall and known as the logarthmic law, the constants are found from experiments, let us found out the law in the laminar sub layer.

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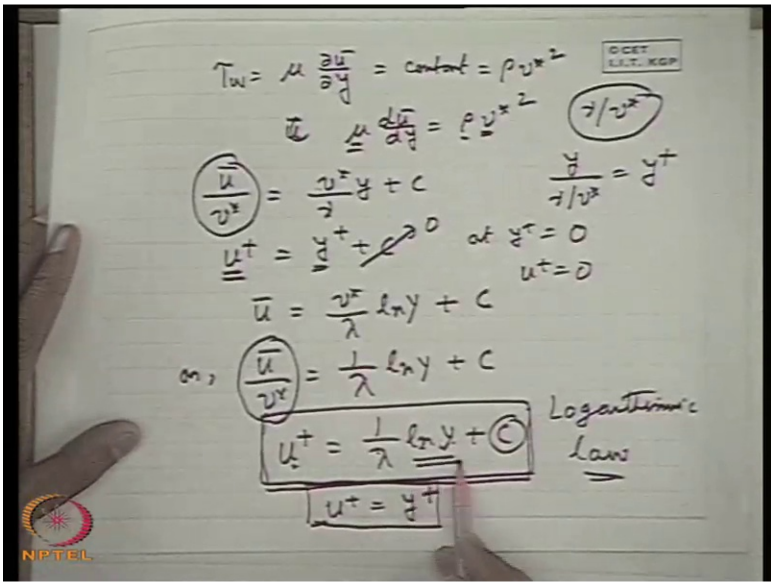

Now, laminar sub layer tow w is constant, so we can write tow w is a mue del u bar del y laminar stress is related to laminar viscosity mue del u del y and that is equal to constant. Again from the definition of the friction velocity with star, we can write tow w s v star square is equal to row v star square, from which we can try by integrating let us write mue d u bar d y is row v star square. Then by integrating this taking mue constant v star constant row constant we can write u bar by v star, if you take is equal to v star by mue by row is mue v star by into y plus v star by mue into y plus c. Now, this is a non dimensional velocity u plus, now this is a non dimensional y coordinate, because nue by v star nue by v star is a length dimension, therefore v star y by nue is a non dimensional, so y by u by star is non dimensional.

This typical non dimensional quantities are define in turbulent flow y plus plus c, for the condition at y plus is equal to 0; that means, y is equal to 0 u means u plus 0, so that c 0, so therefore you get u plus is y is equal to y plus. So, therefore, we get u plus is equal to y plus. So, this is the law for the laminar velocity profile in the laminar sub layer and this is the equation for the velocity profile of the turbulent flow in the turbulent zone near the solid o r. So, from the experiments we can find out the value of the constant to adjust to this alien term, because this is dimension less.

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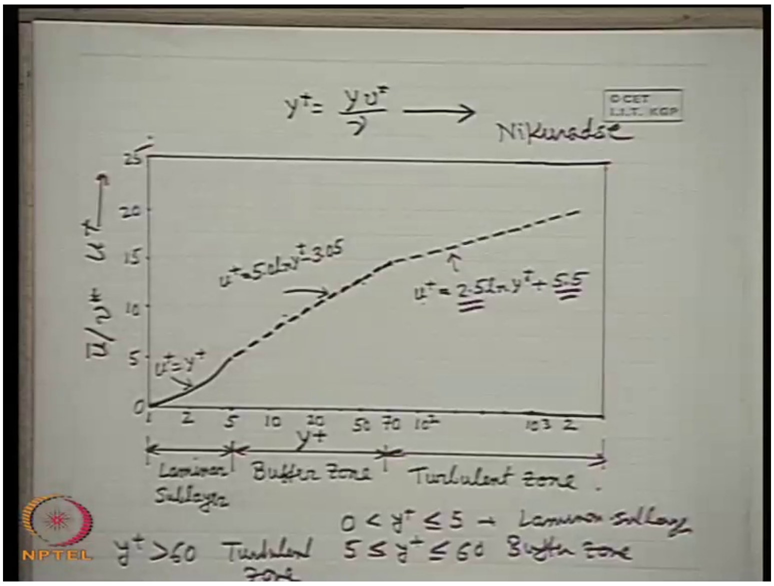

Finally the results come which was the contribution of the scientists known as Nikuradse who also made a major contribution to it. Now, if you see this is the thing, so in the laminar sub layer, now it is plotted against y plus against u plus u bar by v square that is nothing but u plus in a similar graphic, because this equation is such that in this side it is an l n, this side it is no log, so we will be showing linear profile. So, this is u plus is equal to y plus this is the laminar sub layer.

So, laminar sub layer extends 0 less than y plus less than equal to 5 is the laminar sub layer, where the equation of velocity profile is this is a curve because it is a semi log, this is the logarithm size, this is the normal graph. Then in the buffer zone which is defined to be y s which is defined to be 5 less than y plus less than 60 is the buffer zone, buffer zone the velocity profile is like this, 5.0 l n y plus minus 3.05 and in the fully turbulent zone that is this u plus, the same equation buffer zone and full determinant zone we change the constant and this is 2.5 is the cost and 5.5. So, this will be less t and this follow this rule u plus y plus in the fully turbulent zone where y plus is greater than 60 fully turbulent, fully turbulent zone y plus.

So, therefore this is the laminar sub layer velocity profile u plus is equal to y plus, this is the buffer zone this is the with changing in cost ends between the buffer zone and fully turbulent zone . This is physically, because in the fully in turbulent zone mixing is much more between the particles because of there high turbulence level, so that the velocity profile become more stream and less stream sorry and this is the total picture of the turbulent flow velocity profile near a solid war, of course with 0 pressure gradient.

Today I stop it here and this is the concluding lecture for your turbulent flow theory introduction and to turbulent flow as such turbulent flow is a very vast topic and the turbulent modellings and more details of the turbulent flow is kept beyond the scope of this basic label fluid mechanic course, this will be thought in an advance fluid mechanic course.

We have seen earlier there are topics like incompressible discuss flows, there are topics like boundary layer theory these are the advance topics. So, only introductory concept and the brief description of this topics have been given to a primary level or basic level fluid mechanic course, more detail of this advance topics we will second course of a mechanics or an advance course of fluid mechanics. So, with this I conclude this section this chapter and at the same time the entire course on fluid mechanics.

Thank you.