Fluid Mechanics

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Lecture - 44
A Few Unsteady Flow Phenomena in Practice Part - I

Good morning. I welcome you all to this session of fluid mechanics. Well, today we will be going to start a new section, or a new chapter that is a few unsteady flow phenomena in practice. Although, in almost all engineering applications, the flows are steady or quasi steady in nature, but there are few occasions, where the flow becomes unsteady. Now, in few such applications therefore, the analysis of unsteady flow becomes important.

So, what is meant by unsteady flow, which we have already discussed earlier, is that the hydrodynamic parameter, or the fluid flow parameters like velocity, pressure, and the rheological properties of the fluid changes with time at any location. That is the definition of an unsteady flow; that the hydrodynamic parameters change with time, or functions of time at any particular location; that means it changes with time at all locations. So, depending upon type of change or the rate of change of this, hydro dynamic parameters with the time, these unsteady flow phenomena may be categorized in different regimes of unsteady flow.

In one, the rate of change of hydrodynamic parameter with time is very slow. And in this regime of flow, the temporal acceleration may be neglected as compared to the velocity, head of the velocity of the flow. These classes of flow, for example are filling of a tank; the emptying of a tank by allowing the water to flow through a side or bottom of defuse. These are the classes of problems, where the temporal accelerations can be neglected. That means changes of hydrodynamic parameters are very slow with time.

Now another regime of flow is that where the change of hydrodynamic parameters with time is very fast, or rapid. So, that the temporal acceleration is of considerable importance or the temporal acceleration is considerable as compared to the velocity of the flow. This is the real unsteady flow problems. So, we will have to take care of this temporal acceleration in the analysis of the fluid flow.

Third class category of flow is the flow where this change is very fast, very fast and sudden. So that, the change in density of the fluid comes into consideration, means the compressibility of the fluid comes into consideration. Even for a liquid this changes are so fast, the compressibility comes into consideration; and elastic force of the fluid becomes very important.

So, we will mainly discuss these two categories of flow; where the temporal acceleration is of considerable importance, and compared, of comparable magnitude to that of the velocity of the flow; and in another one where the flow is, the changes of hydrodynamic parameters are so fast with time. So, that the compressibility of the flow comes into consideration, or elastic force becomes important. So, let us start with the definition of inertia pressure. Certain definitions of certain basic terminology, is important. Sorry.

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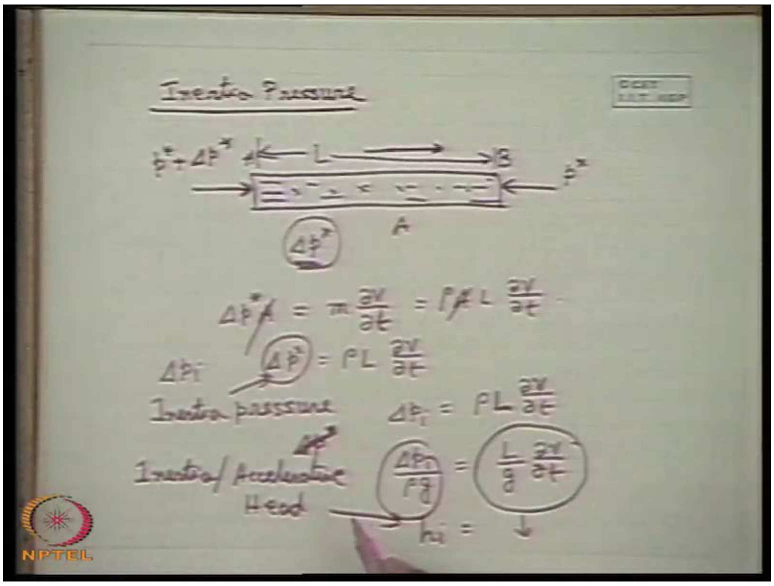

So, inertia pressure, inertia pressure, so inertia pressure is the pressure which is responsible for causing the acceleration of a fluid mass. So, let us consider for example, a stream tube that is the fluid mask of, within a stream tube which is been accelerated at any instant of time. Now, this is accelerated because of a definitely change in the trigonometric pressure acted on these two surfaces of this fluid, let us being accelerated in this direction. So therefore, the trigonometric pressure in this side will be more than the trigonometric pressure acting on this side. So delta p star is the difference in trigonometric pressure which is because of this differential static pressure plus the pressure equivalent because of the difference in elevation from A to B.

Let us find out that, what is the magnitude of this delta p star? In terms of the change in the velocity, which accelerates this fluid column at any instant. Let us consider A as the cross sectional area of this stream tube. And let us consider L as the length of the stream tube; let us consider this is L, the length of the stream tube. Then we can tell the force acting or responsible for the change in the velocity is delta p star into A; and that is equal to mass time, mass of this fluid element in this stream tube, times the change in the velocity that is with respect to time, del v del t. And we consider a uniform situation the del v, del t at all points remain the same; that means, the fluid velocity is same at all points, but it goes on changing with time. So that there appears a temporal acceleration del v, del t which is same as each and every point. So, now, we can write A as cross sectional area A; and length L; that is del v by del t. So, A is cancelled. So, therefore, we get del t, sorry, row is there; mass density row is equal to row L del v del t. So this is known as the inertia pressure; this is known as inertia pressure, inertia pressure.

And if it is expressed in terms of, and sometimes it is written as delta p i. So, therefore, we can write instead of this, delta p i is equal to row L del v del t. This is the expression of inertia pressure which accelerates a column of liquid of length L; and it is the temporal acceleration. Now, if this is expressed in terms of the head that is per unit wise delta p i by g, or this is expressed in terms of head we can write that L by g. So, this is in terms of head; this is known as, rather this can be written as h i, is this quantity; is known as, this is known as inertia head, inertia or accelerative. This is known as inertia or accelerative head, inertia or accelerative head.

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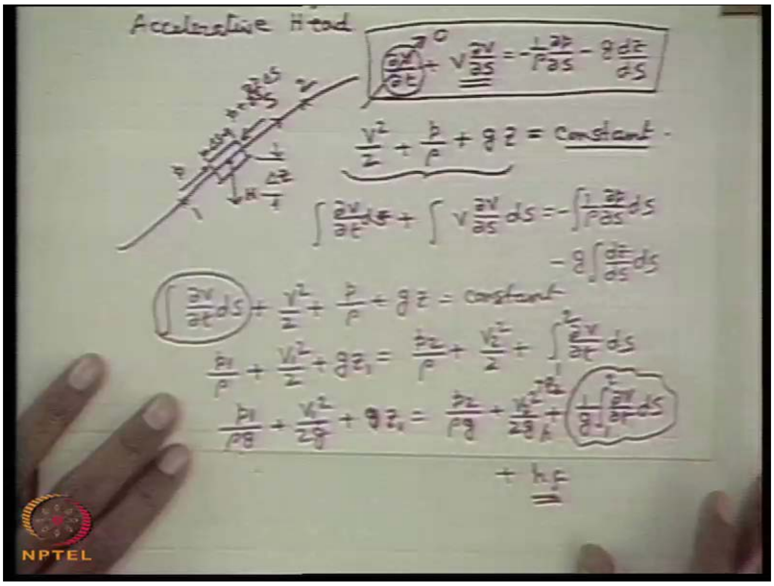

Now, we come to Bernoulli's equation, Bernoulli's equation with accelerative head, Bernoulli's equation with accelerative head, Bernoulli's equation with accelerative head. Let us, let us recapitulate the Bernoulli's equation. How did you derive the Bernoulli's equation? If you consider a stream line like this; and if we consider a fluid element in the stream line; that direction of stream line is, if you recall then we remember that if the weight is acting this way w; and the pressure forces are acting like this, that p; and this is p plus del p del s into del S, if del S is this length; and if this be the particle distance del z.

From a typical force balance on the fluid element, we derived for a general unsteady case V del v del z that is the acceleration per unit mass is minus 1 upon row del v del s for an in visit fluid minus g d z d s. This d z d s is this delta z by delta s with the limit delta z tends to 0, at a point it becomes d z d s. So, this was the equation of motion for an in visit fluid along a streamline or Euler's equation, if we recall it we have discussed it earlier. Now if we consider the flow to be steady then this becomes 0. And v is a function of s, so we write it as V d v d s. Then we integrate along this stream line, we get v square by 2, that means, integrating with respect to d s, and take this side. And considering the flow to the incompressible which we did earlier, so, it is plus p by row plus g z which is constant along a streamline. This is typically the Bernoulli's equation or equation of mechanical energy for an in visit incompressible fluid which is this v square by 2 plus p by row plus g z is constant, along the streamline.

Now, if we consider the flow to be unsteady and the situation is such that del v del t this temporal acceleration is comparable with convective acceleration; then in the integration, what we will do? We will integrate with that; that means, if we do that we will get, that integration of del v del t plus d s, sorry, d s plus integration of V del v del s d s is equal to minus integration of 1 upon row del p del s d s minus g integration of d z d s d s which gives us that integration of del v del t d s plus V square by 2 plus, considering the flow to the incompressible this terms remains same.

So, therefore, we see the extra term comes as this integration of del v del t d s. If this integration, now if this integration is made between 0.1 and 2 at two sections in a streamline, then we get p 1 by row plus, I write the this term first, V 1 square by 2 plus g z 1 is equal to p 2 by row plus V 2 square by 2, this terms pressure energy per unit mask kinetic energy, or rather we can write in terms of the g also. So, this becomes del v del t d s 1 to 2.

Now, in terms of head, I think it is better to write in terms of head, p 1 by row by row g plus V 1 square by 2 g, that is the pressure head plus velocity is, sorry, plus data main is equal to p 2 by row g plus V 2 square by 2 g plus 1 by g 1 to 2 del v del t d s. So, this is precisely the integrated form of the accelerating way. So we can write the Bernoulli's equation in consideration of the accelerating weight in this way. If we consider more over the fluid is viscous than real fluid, then along with that we take another term h f that is the loss, that is the loss, that is the head loss due to friction. So, p 1 by row g plus V 1 square by 2 g plus z 1, sum of the pressure energy per unit weight that the kinetic energy per unit weight plus data main is equal to this quantity, sorry.

I have forgotten to write data main z 2 this quantity, plus this accelerating head plus the head loss due to friction. So, therefore, we see that this way we can write the Bernoulli's equation with accelerating head.

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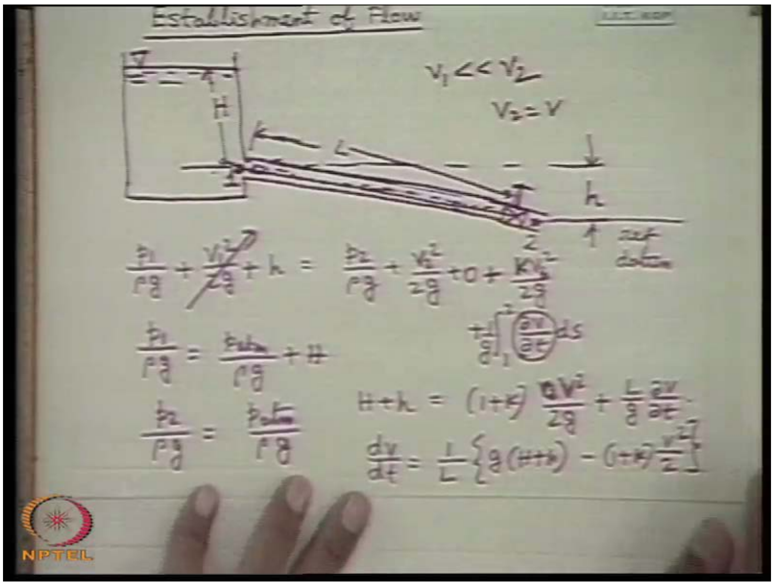

Now we come to a particular situation of unsteady flow where the temporal acceleration is practical, quiet importance is establishment of flow, establishment of flow establishment of flow, what is meant by that establishment of flow? Well, what is establishment of flow? Now, let us see this establishment of flow like this, if we consider a reservoir, if we consider a reservoir from where the liquid will be discharged by a long pipe line like this, which is inclined down in a general situation; and there is a bulb, let us see, at there is a bulb which is initially closed. Let us consider this is at a constant height, let us consider from this point this is at a constant height, H. And let us consider from this plane, this downstream point here, just after the bulb is at an elevation h.

So, now consider a case when the bulb is fully closed; then the bulb is opened then there is no flow of liquid, at this situation there is no flow of liquid when the bulb is closed. When the bulb is opened the flow will take place. Now, what happened? Initially, when the bulb is opened, this entire column of liquid in this pipe line; let us consider the length of the pipeline 2 b equal to L, the capital L. Now this liquid column in the pipeline is at rest when the bulb was closed. Now, when the bulb is opened then a pressure dropped, total pressure dropped acting on this column of liquid which is the difference between the trigonometric total pressure force, or pressure difference force acting in this column of liquid which is due to the difference in trignometric pressure at this point, and at this point. This point is at a higher trigonometric pressure; this is because of this head of liquid plus this head of, this elevation. So, therefore, the trignometric pressure, because of the weight of the liquid which gives a trigonometric pressure here, higher than the trignometric pressure there.

So, because of this pressure difference, liquid is accelerating because there is no motion force in the beginning fluid was at rest. So, initially when the bulb is just opened this liquid column gets accelerated; and this acceleration is maximum at this initial movement.

When the motion is set in the liquid, the discuss raise in force comes to appear or comes in to the picture which opposes this forces. So, that the acceleration is decreased. And ultimately the liquid attains a steady rate condition where it flows under a steady velocity or uniform velocity. The velocity does not change with time when the head is constant. So, therefore, even at a constant head when the bulb is opened, immediately the fluid flow does not attain the steady state condition. So, it requires some time to attain a steady state condition. This is known as establishment of flow.

Now let us consider that portion, or that part when the fluid in the, liquid or fluid in this column gets accelerated just after the opening of this bulb. Let us consider this situation. Now let us write the Bernoulli’s equation taking a point; now let us consider a stream line like this. This is the point two if we consider that this starts point. Here if we consider the point one. Now if we write the Bernoulli’s equation at any instant, just after opening the bulb when the flow is unsteady region, the steady state has not been reached; then we can write the Bernoulli’s equation here p 1 by row g, this is this. And if we consider this label as the data, reference data, from which we measure the potential head.

So, p 1 by row g plus potential plus V 1 square by 2 g plus h is equal to p 2 by row g, pressure here, plus V 2 square by 2 g, the h is 0. Now the frictional losses, if we consider h f, the frictional loss is proportional to the velocity of flow here. So, K V 2 square by 2 g. So, frictional head loss is proportional to the velocity here, proportionality fact is K, we have considered earlier that in case of flow through pie the frictional losses, and other minor losses has expressed as a constant with the velocity head, proportional to velocity head. So, this represents all the losses, not only the frictional losses in the pipe, but the entry losses, the losses due to bulb etcetera; plus the most important thing is the accelerative head; that means, this is from 1 to 2 del v del t d s plus 1 by g that is the accelerative head, which we discussed earlier.

So, this term 1 by g del v del t 1 to 2 d s. Now you see that, if we consider this area of the reservoir is quiet large compared to this, V 1 is very small compared to V 2. So, that we can neglect V 1; and in that case we write V 2 is equal to V that the flow velocity in the pipeline which is assumed to be uniform; that means, at any instant the flow velocity in the pipe is same, because the pipe cross sectional area is same.

From the continuity the average flow velocity in the pipe is same, which is going to be changed with time. So, therefore, we replace V 2 as V. And moreover p 1 by row g is here p atmosphere by row g plus H. Because p at this 0.1, at p atmospheric plus row g H. So, p 1 by row g is p atmosphere by row g plus H.

And similarly, p 2 by row g; that means, the pressure here is p atmospheric by row g. So, if we substitute this, we will get, the equation, that if we substitute this we will get p atmospheric by row g will cancel, we will get H plus h, in the left hand side this h will be there, is equal to 1 plus k, this two terms, V 2 no, V square we write this as velocity of flow, instantaneous flow velocity V 2 square by 2 g.

Now, if we consider this temporal acceleration is same, with the same condition of uniformity of the flow, at all locations. So, it can come out of the integration and 1 2 d s is simply the length of the pipe; that means, L by g del v del t. And v is a function of time. So, we can write del v del t as d v d t. So, we can write this as d v d t is equal to, if we take this, that side 1 by L into g H plus h minus 1 plus k V square by 2, g into H plus h minus 1 plus k V square by 2.

So, now, what we do? We write, now under steady state, if we write the Bernoulli's equation from 1 to 2, what we will get? Let us write the Bernoulli's equation under steady state from 1 to 2, with the same loss equations; that means k v square, but if we write the steady state from 1 to 2, let us consider the steady state velocity be v 0.

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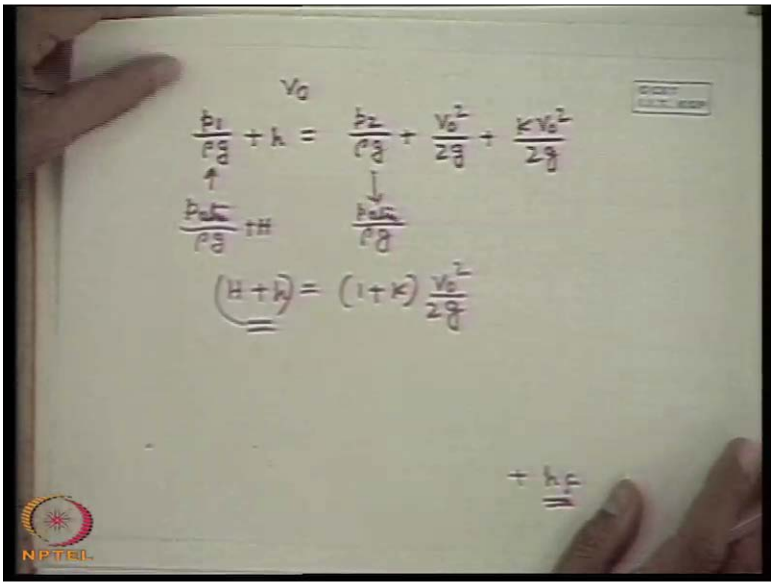

So, if we write Bernoulli's equation under steady state, what we will do? What we will get? p 1 by rho g, this velocity at this point is less compared to, much less compared to velocity in this pipe, plus h is equal to, here p 2 by row g plus the steady state velocity V 0 square by 2 g plus k V 0 square by 2 g, there is no accelerating way. And p 1 by row g is p atmosphere by row g plus H; and p 2 by row g is p atmosphere by row g.

Well. So, therefore, we get from here that H plus h is equal to this p atmosphere, 1 plus k V 0 square by 2 g. So H plus h we get 1 plus k v 0 square by 2 g by the application of Bernoulli's equation under steady state where v 0 is the steady state velocity. So, now, if we write this, from this expression, now if we write side by side what we have got by the application of steady state Bernoulli' s equation.

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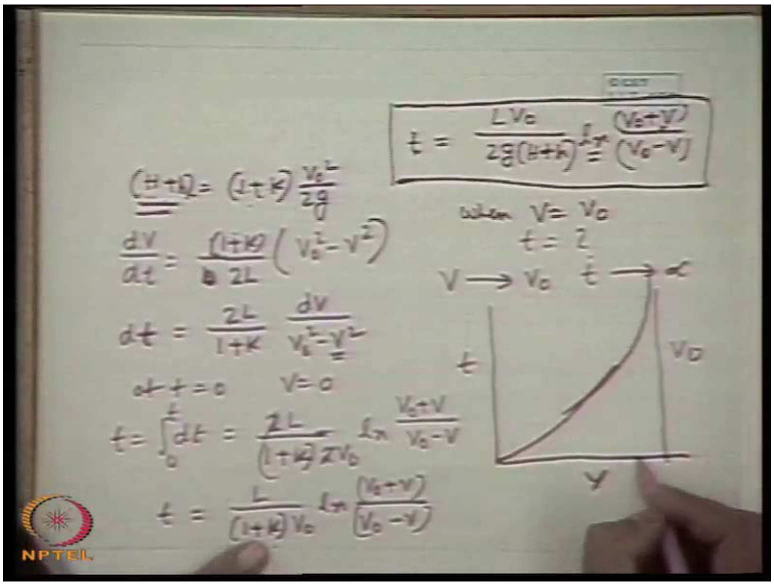

Now if we have this steady state Bernoulli's equation, that H plus h is equal to 1 plus k into v 0 square by 2 g. Then if I replace this H plus h in to g as 1 plus k v 0 square by 2 here, then I get d v d t is equal to 1 by L into 1 plus k v 0 square by 2 g in terms of H plus h, so, 1 by k, I can take common, 1 by k, even this two I can take common; that means, I can write it here 2 L into what we get v 0 square minus v square.

That means I am substituting H plus h 1 plus k v 0 square by 2 g, here I get this expression. So, now, if we write d t is equal to 2 L by 1 plus k into d v by v 0 square minus v square. So, at t is equal to 0, velocity is 0. So, if I want to find out the time t when the velocity will reach the value v, and instant any has velocity, then I have to integrate this equation. So, t is equal to integral d t from 0 to t, if you integrate this we will get, 1 plus k, here we will get 2 v 0 into l n, the integration will be v 0 plus v divided by v 0 minus v; or we can write t is equal to 2, 2 cancels; L by 1 plus k v 0, l n, v 0 plus v divided by v 0 minus v.

Now, to eliminate the k factor, again I substitute 1 plus k as, 2 g H plus h v 0 square; that means, 2 g H plus h by v 0 square; then I get, if I replace this, I get t is equal to, ultimately I get, L v 0 by 2 g H plus h, if I replace this v 0 1 plus k as H plus h 2 g by v 0 square here, I get l n v 0 plus v divided by v 0 minus v.

So, therefore, I see this is the expression for time taken to, for this liquid column to attend any velocity v from 0. Initially 0 because there is, no velocity to attend any velocity v where v 0 is the steady state velocity. So, let us find out what is the time taken to reach the steady state velocity. How we can find out? When v is equal to v 0, what is t? You see v is equal to v 0, this is 0, denominator is 0 with a length; that means, the argument is infinity for l n, this becomes infinity. So, therefore we can write v tends to v 0 when t tends to infinity, which physically signifies that after the opening of this wall the fluid will never at any steady state velocity, it will attain the steady set velocity as in two (()) .

In fact, the steady state velocity is reached as in, critically; that means, if we draw a graph for time with the velocity, if this is the steady state velocity v 0 it will reach as, it will reach as two determinants; that means, it, at any finite time it will reach a large portion of v 0, but not exactly v 0.

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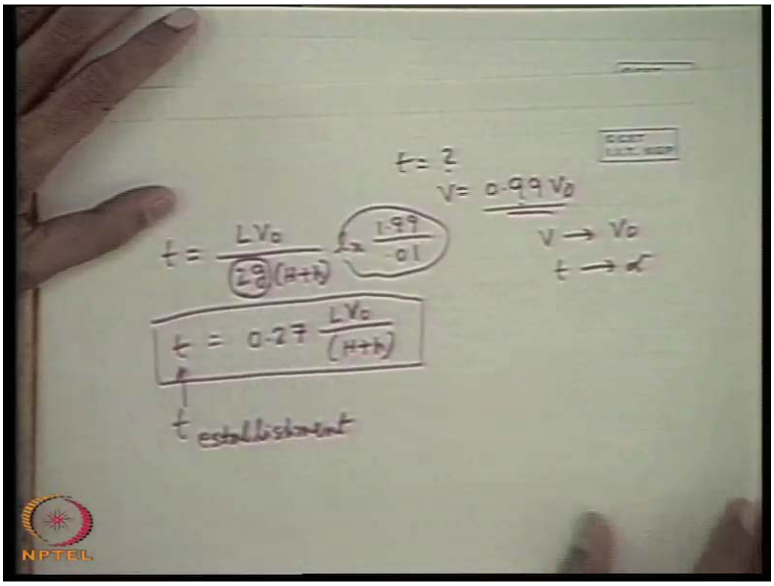

Now if we consider, let us take, a particular portion of v 0, if we consider that what is the time taken when v is equal to 0.99 of v 0 then very simple, here if we put that L v 0, v 0 is the steady state velocity that, what is the time taken for reaching 99 percent of this study state velocity? It will be 1.99 divided by 0.01.

So, it gives a finite time; and this becomes is equal to 0.27 L v 0 by H plus h; taking care of this 2 g factor and this one gives 0.27. So, finite time is there to reach 99 percent of this steady state velocity of any fraction of this steady state velocity, but v will approach v 0 only when t will approach infinity. So this is defined as a convention as the time for establishment.

So, this is the time of establishment of flow, establishment; that means there is a time required for the steady state, for the velocity of the flow to reach the 99 percent of the steady state velocity and it is given by this equation, this expression. So, equation, general equation is this; that is the time required for the fluid to reach any velocity v where v 0 is the steady state velocity from rest. And the typical response of the velocity is like that. So, if this is v 0. So, it will reach asymptotically to v 0 with time.

After this I will discuss, a very interesting phenomena, known as water hammer. Now, we have discussed the establishment of flow which is a very good example for a class of unsteady flows, where the temporal acceleration is of considerable importance and of considerable magnitude as compared to the velocity of the full; and it can be considered and should not be neglected.

Now, there are certain situations where the changes are so rapid that the fluid compressibility comes into picture and the fluid density changes. And as you know, no fluid is there which is absolutely incompressible or 100 percent incompressible, for which this bulk modulus of elasticity has to be infinite, theoretically infinite which is undefined. So, for any large value of elasticity all fluids possess some sort of compressibility. But when this change of velocity is very fast, sudden change then what happens? The density changes also fast and there is a considerable change in density which brings about the compressibility.

So, what is the consequence of compressibility, is that the elastic force comes into picture; that means, if at some location the pressure of the fluid is changed, due to a change in the flow velocity, it is not sensed by the entire fluid instantaneously; that means, the rest of the fluid will sense it after some instant of time because of this compressibility effect.

For example, if at any location in the flow of a fluid the flow is stopped, or the flow velocity is reduced, and the pressure is increased. So, this increase of pressure and reduction of flow velocity will be sensed in the entire fluid. For example, it is done at some downstream location, then the upstream fluid will come into a reduced velocity state or an increased pressure state after sometime because of the compressibility phenomena, phenomena of compressibility, nature of compressibility. So, if, this will happen, if this is very fast.

So, this time taken for the entire fluid to sense this change will be considerable. So, this is conceived by the propagation of a wave, if the pressure is increased due to a sudden deceleration of the fluid then it is conceived by the propagation of a pressure wave, in the upstream direction with a finite velocity.

So, this phenomena often comes into picture of practical cases like a hydroelectric power station. As we know in a hydroelectric power station turbines flows are sometimes altered according to the load; when the load is increased the flow has to be increased. Suddenly the flow in the pipeline leading to the turbine from the high head reservoir has to be accelerated; if the load is decreased it has to be immediately decreased, the flow has to be decelerated. So, that valve is closed, immediately it has to be decelerated.

So, this sudden change of flow, sudden acceleration and deceleration causes this wave to be transmitted in the pipeline. So, what happens in this situation, this transmission of the wave because the length of the pipe is finite, somewhere in the downstream the disturbance is created; somewhere in the upstream it comes from some reservoir. So, therefore, what happens? This wave goes to the upstream reservoir and again comes back as a reflected wave. So, therefore, a to and fro movement of this wave, or a repeated movement of this wave from one direction to other direction causes the knocking of this pipe and results into severe damages, serious damages. This problem is known as water hammer problem.

Sometimes we know that from our common practical experience that if a domestic tap which is running full with a high velocity of fluid suddenly turned off, what happens? Because of the same water hammer problem a knocking sound is heard, and the entire pipe vibrates. This is because of the same fact that when the tap is turned off, immediately the fluid velocity is sensed to 0, and the pressure is increased; but that pressure is sensed for the upstream fluid by the propagation of a pressure wave which propagates upstream and again comes back somewhere in the reservoir or high head, overhead tank, it comes back. And this repeated movement causes the knocking sound and the pipe to vibrate. This phenomena is known as water hammer phenomena.

Of course, the main water is unfortunately little less hammer because this phenomena happens to any liquid, any liquid where the flow is suddenly decelerated or accelerated. So, that the compressibility effect comes into consideration. And this repeated to and fro movement of a pressure wave takes place. But conventionally this phenomena is known as water hammer, considering in almost all the practical cases water is the working fluid.

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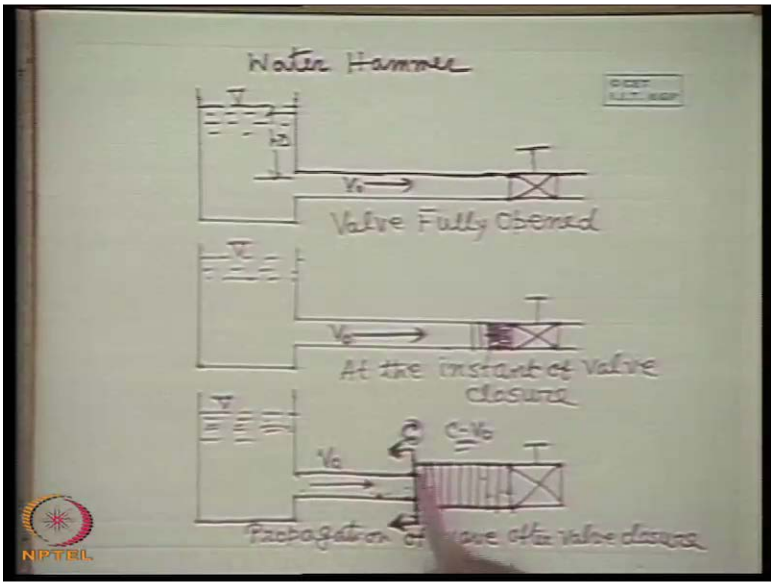

Now, let us consider the problem of water hammer in practice. Let us come to the problem of water hammer in practice. Let us see. So, this is a typical water hammer problem, water hammer. Now let us see that there is a reservoir where the liquid is flowing at a certain head, constant head h. Let this is a constant head h, let h 0 is the constant head where it is flowing, and the valve is fully opened. Now when the valve is fully opened the liquid is flowing at a some steady state velocity which is very high when the valve is fully opened through this pipe; the flow rate or the flow velocity depends upon this head and the resistance to the pipe.

Now, what happens, when the valve is closed? Immediately the fluid adjacent to the valve comes to rest and its pressure is increased. But I have told earlier also, and I am telling it again and again, so, after immediately the valve is closed, the entire fluid cannot come to rest instantaneously; and its pressure cannot rise instantaneously because of the compressibility effect.

So, therefore, at the instant the valve closure only the fluid particles adjacent to the valve is closed and this velocity is arrested. So, what happens in that case, this fluid which is flowing in upstream region with the velocity v, for example, this is the velocity v, it pushes this fluid, compresses it, and its pressure is increased. So, what happens? Up, in this wave what happens? The, for example, the fluid here compresses it; and again it comes, its pressure is increased and when it pushes this fluid in the subsequent part it is getting compressed; and its velocity is arrested; and its pressure is increased. So, this way layer after layer the fluid is getting compressed.

So, this way it is done; and this conceived by this propagation of the pressure wave from the downstream to the upstream section; as the fluid pressure is increased, so, what happens? The pipe diameter may increase depending upon the rigidity of the pipe which is modulus of elasticity.

So that, in this downstream part of this propagating pressure wave, that at any instant, if we consider the pressure wave at this location which means, downstream of this part the fluid has come to rest; this pressure is increased; and a little enlargement of the diameter has taken place. That means the kinetic energy of the fluid corresponding to this v 0, v 0 or v, whatever you call v 0, has been transferred to this pressure energy or elastic energy of the fluid; and to the, also elastic energy of the pipe. So, upstream part is moving with the same velocity v 0 and with the initial pressure. Here this is the wave which is moving with a velocity c in this direction; this velocity is the velocity usually denoted with relative to this fluid.

But absolute velocity will be c minus v 0. So, that we will discuss afterwards. So, with some absolute velocity the pressure wave is moving. So, it will take sometime, because the pressure wave is moving with the finite velocity to make the entire column to come to rest, and with increased pressure. So, this is the situation, propagation of wave after valve closure. Now before analyzing such situation physically, we must know what is this velocity of this disturbing pressure wave, moving from downstream to upstream; and what is the magnitude of this pressure rise, when the fluid comes to rest, because of a sudden closure of the valve. Let us deduce that first.

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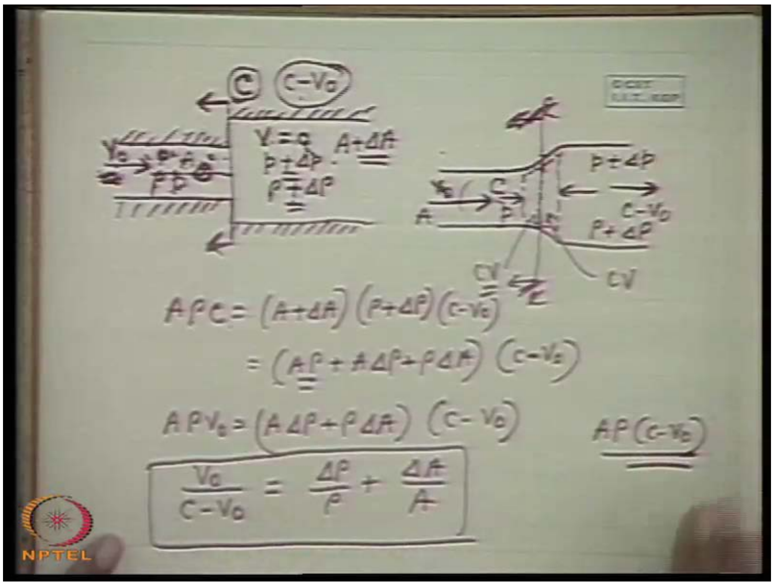

Now let us consider the situation like this. This is the pipe. And suddenly this is increased and the fluid is rest. So, let us consider. So, this is the situation. Let us consider the situation like this. Now, let us consider the fluid is flowing with a velocity v 0 in the upstream direction; let us consider this is the pressure wave which is moving. So, upstream part of the pressure wave is the undisturbed fluid with velocity v 0; let its pressure is p; let its density is rho; and the cross sectional area of the pipe is A. So, density is rho. So, pressure is p; and the cross sectional area is A. When the fluid has, when the pressure has crossed from this side, so, the downstream part has already come to rest; the cross sectional area is increased. So, here v is 0. Let us consider Y v 0, or consider simply v. So, v is 0.

So, pressure here is more, p plus delta p; similarly the density here is more, rho plus delta rho; and area is A plus delta A. These are the quantity, corresponding quantities at the upstream where, sorry at the downstream at the pressure wave while the velocity is 0. Now let us consider the pressure wave is moving with a velocity c relative to this fluid. Let us consider v 0, what we considered because otherwise it will make a application. So, here v is equal to v 0, here v is equal to v 0; that means the initial velocity. So, let us consider the pressure wave is moving with a velocity c related to this undisturbed fluid; that means, with this v 0. That means the actual velocity of the, or absolute velocity of the pressure wave will be c minus v 0. Why I have taken c, just like that? Why not c as the absolute velocity? So, accordingly we can tell with relative to that it is moving with c plus v 0. This is because in the formula deduced we will get an interesting expression relating to this velocity of this pressure wave, disturbing pressure wave with respect to the undisturbed fluid. That is why we have denoted c as the velocity of the pressure wave with respect to the undisturbed fluid. So, c minus v 0 is absolute velocity. So, let us consider this situation.

Now, if we make an analysis for the control volume. Now here let us make these things more clear. Actually there will not be such an abrupt change in this. So, change will be like this. So, this is the, like this is the, now let us consider a, this is the pressure wave which is moving with c with relative to this velocity v 0. Now, what we are doing? So, if we analyze this situation by a control volume analysis; you know earlier we have seen this type of situation is unsteady, why? Because a point here for example, having a velocity v 0; rho, density rho; pressure p; area a. So, all these parameter will change to these values when the wave has passed through. So, this wave for all the points in the upstream, or even all the points in the downstream which were earlier with this values changes to this. So, with time the hydrodynamic parameter are at any point changes, they changes.

But to make this steady, we can consider a standing pressure wave; that means, there is no movement of the pressure wave; that means, we super impose a velocity c minus v 0 to the entire system. So, that the pressure wave becomes stationary. And in that case we can tell that the fluid is approaching with the pressure wave with a velocity. If we make the pressure wave stationary; that means, we impose a velocity c minus v 0 in the opposite direction. So, that means, in that case this is the, fluid is approaching with a velocity c, and is going out with a velocity c minus v 0, because this c minus v 0 is the absolute velocity of the pressure wave which is super imposed in the entire system in the opposite direction.

So, that the pressure wave becomes standing wave. In that case all the points, the hydrodynamic parameters are in variant with time. Now let us consider a control volume like this; this is the control volume, this is the control volume. So, this side the quantities are this; and this side the quantities are like this with c minus v 0; pressure is p plus delta p. So, therefore, the pressure acting in this side is p plus delta p, pressure acting on this side is p. Now let us write the continuity equation, this is A. So, continuity equation gives A rho c, the mass flow rate through the control volume, this is the control volume I have already denoted this, is equal to mass flow rate out.

So, this side if we equate it is A plus delta A into rho plus delta rho is the density, rho plus delta rho times c minus v c. So, if we equate this we get this side A rho plus, A rho plus A delta rho plus rho delta A times c minus v 0. So, A rho c cancels from both the places. So, therefore, only thing is that A rho v 0 minus A rho v 0 this comes on this side. So, that A rho v 0, A rho v 0 becomes equal to, A rho c cancels, so, only A rho v 0 that becomes equal to A delta rho plus rho delta A, this entire times c minus v 0. From this table we can write, A rho c cancels, so A rho v 0 comes this side, so that we can write this. So, now, if we divide this quantity by, A rho c minus v 0, this quantity, both this side. So, here we get v 0 by c minus v 0 is equal to A rho; that means, we get delta rho by rho plus delta A by A. So, this is one useful relationship obtained from the continuity equation.

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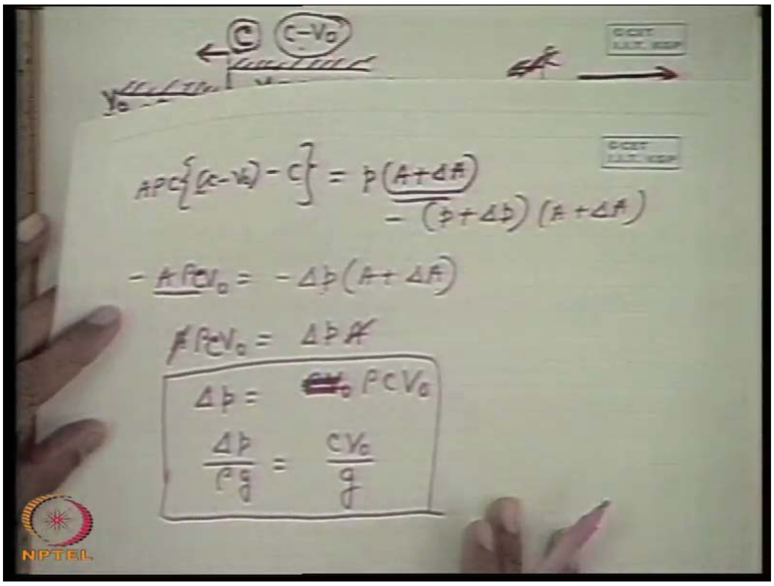

Now let us see the momentum equation. If we write the momentum equation, that means, the equation of motion for this control volume or momentum theorem for the control volume; then we can write that the mass flow A rho c times the velocity of wave flux; that means, c minus v 0 minus it is the rate of momentum a flux in this direction, if you consider this direction as the positive direction.

So, what is the net force acting in this direction? p into A plus delta A minus p plus delta p into here one thing has to be understood, that this p is acting not only this surface but on this surface also. We consider the p plus delta p to be prevailing only at this enlarge section. So, this section transition from this lower cross section to higher cross section; this part also the pressure acting on the control volume is p. So, therefore, the projected area on which the component of the pressure will come in this direction. So, which will be equal to p times the total area; that means this area plus the projected part of this area. So, that ultimately A plus delta A is the area over which p, the pressure p is acted.

And this side of the control volume p plus delta p; over this same area A plus delta A. So, if you do that we get a minus c c cancels minus v 0 is equal to, this side if you see p into A plus delta A is there. So, minus delta p A plus delta A. So, neglecting this two higher order terms which we did for continuity equation also, we can, we can write A rho v 0, A rho v 0 this is equal to delta p A, A rho v 0 is equal to delta p A.

Now we can write delta p is equal to rho v 0 because A A cancels rho v 0. So, we can write delta p is equal to rho v 0. Now delta p by c, so, a rho c c minus v 0; so I have done one mistake that here A rho c will be there, I am sorry, A rho c, A rho c v 0 because a rho c v 0. So, therefore, A rho c will be there. So, therefore, delta p is sorry rho c v 0 or delta p by rho g in terms of the rise in pressure head is c v 0 by g. So, this is very important formula; that this rise in the pressure head due to the deceleration of the fluid is rho c v 0 or delta p by rho g is c v 0 by g that is the change in the pressure rate.

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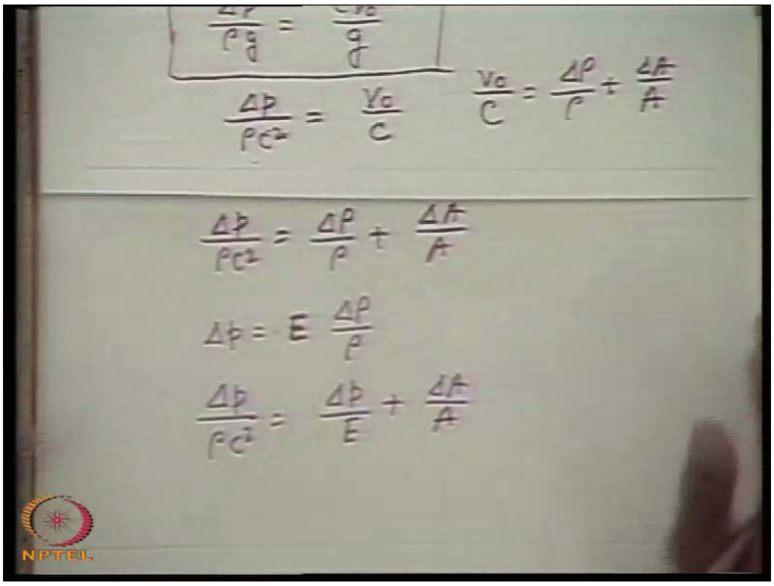

Now, if we see that this value delta p by rho 0 is c v by rho 0; and here we can see that we can write another state that delta p, from here, by rho c square is equal to v 0 by c. We can write delta p by rho c square; that means, from here from this step rho c square is v 0 by c. Now it has been found in practice; and it has been observed that the value of c is very large compared to v 0. So, from this equation, obtained from continuity we can write again that v 0 by c, considering v 0 to be small, delta rho by rho plus delta A by A.

So, if we equate this two. Then what we get? If we equate this two then what we get? That means, delta v 0 by c, from this two we get delta p by rho c square, delta p by rho c square is equal to delta rho by rho plus delta A by A; delta p by rho c square is delta rho by rho plus delta A by A. Now this delta rho by rho we can replace from the definition of bulk modulus of elasticity, it is equal to E delta rho by rho. So, this is the definition of bulk modulus of elasticity of the any medium; so for the fluid in E is the bulk modulus of elasticity. So, delta rho by rho is delta p by E. So, we can write delta p by rho c square is delta p by E plus delta A by A. Well, today I will stop here, next class.