Fluid Mechanics
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Lecture - 43
Flows with a Free Surface Part – III

Good morning. I welcome you all to this session of fluid mechanics. In the last class, we were discussing the concept of specific energy in a channel flow. Just if we recall, the specific energy at any section in the flow through an open channel was defined, as the total mechanical energy at that section, where the potential energy was taken from the base or the base of the channel as the datum.

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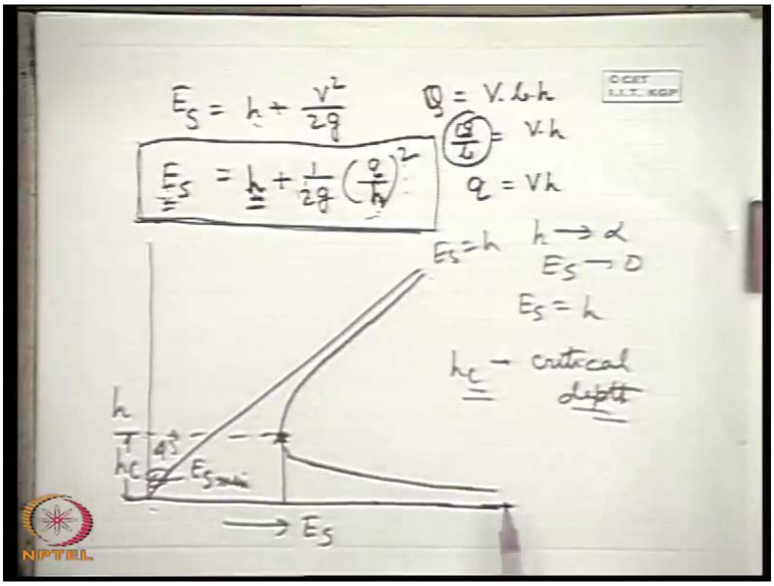

And the expression of specific energy, if you see was like that E s is equal to h plus v square by 2 g; h is the depth of flow. And then this v was substituted in terms of Q, this Q is the floor rate per unit width of the channel. So, this was the expression. And then we considered that for a given flow or discharge Q per unit, the variation of the specific energy is we deduct of channel like this. This was discussed in the earlier class, the last class. And this curve distinguished shows the minimum value of the specific energy for a given h depth of the flow, which is termed as critical depth of the flow.

Now, our next point or next task is to the find out the value of this minimum specific energy; or this depth known as critical depth at which this specific energy has to critical energy becomes minimum.

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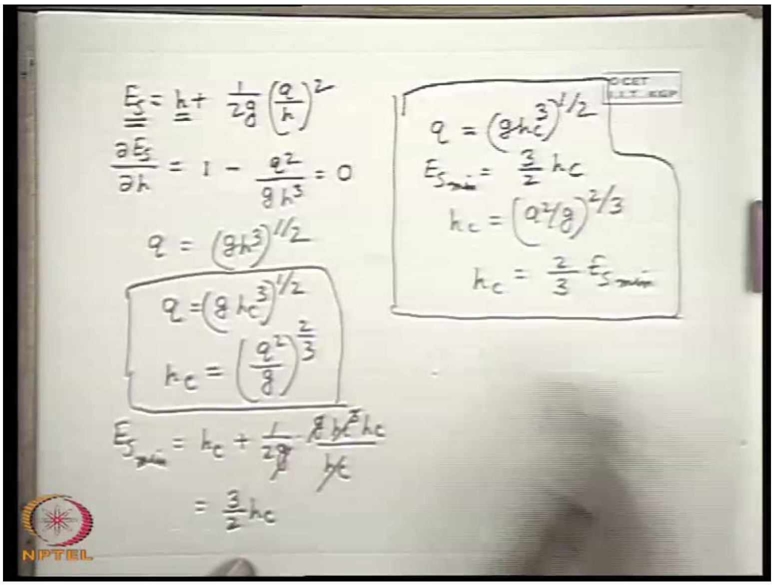

So, that can be found by differentiating this expression. Let us write the expression now again, E s is equal to h plus 1 by 2 g q by h whole square. Now for a given discharge rate, s will be minimum for a depth which can be found out if we differentiate del E s del h, and set it to 0, which will be 1 minus q square by h to the power minus 2, g h cube and that should be set to 0.

If this is made 0 then we get q is equal to g h cube to the power half. So this is the relationship between the florid with the depth h. Now this h can be expressed as h c. So, we can tell q is equal to g into h c cube whole to the power of half; or we can write h c, h c as the q square by g whole to the power two third. So, this is the relationship given. Now if we express this q substitute in terms of h c, here we will get the value of the minimum specific energy. So, minimum specific energy will occur at critical depth h c; and at that condition the flow rate is given by the expression g h c whole cube to the power half; that means, q square is equal to g h c cube; that means g h c cube. So, q square is g h c cube, divided by h c. So, g g cancels; h c h c cancels. So, this will be 1 plus half; that means, 3 by 2 h c; that means, we get E s minimum is equal to 3 by 2 h c.

And q, corresponding q is g h c cube to the power half, or h c can be written as q square by g to the power two third. And this can be written, the critical depth in terms of the minimum specific energy is 2 by 3 E s minimum. So these are the outcomes that for the minimum specific energy, for a given decision. Now, if we focus our attention in a different direction from this equation, if you consider that if specific energy remains constant, what is the variation of h with q?

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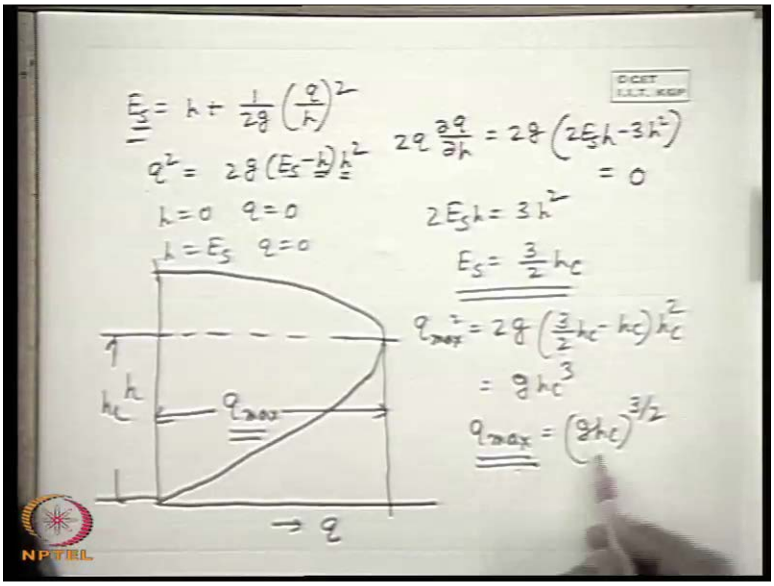

Now you see in this case, we can write q square, if I write it again E s is equal to h plus 1 by 2 g q by h whole square. So, you can write q square is equal to 2 g E s minus h into h square. Now, you see when h is equal to 0; q is 0. Now here also q varies in a fashion that it may increase with increase in h, or it may decrease with the increase in the h. Because h happens here which produce as a decrease in q which is increase, and again h produces a increase in q which is again increase. And one thing is clear that q is equal to 0 when h is equal to 0; and again when h is equal to E s by magnitude then q is also equal to 0. So, this relationship gives a curve like this; that means, if we draw a curve of q verses h, depth verses q, for a given value of specific energy, costs minimum specific energy then we get a curve like this, we get a curve like this. So this is the q verses h curve. This curve shows clearly a maximum.

The curve for constant q; that means, this curve E s verses h shows, showed a minimum that is mathematically proved that if we make a second derivative this will be positive.

That means, this has got a minimum; that means, this has got a minimum that means h verses Es given for a q. But h verses q for a given E s has a maximum. Now, the depth where it is maximum, is the critical depth; and similarly the value of this maximum florid is known as q maximum.

So, let us find out again, what is this condition? What is this critical depth in this situation? What is the value of q max? How to find out? We differentiate this expression with respect to h; that means, if we differentiate this expression with respect to h for a given E s, that is the specific energy. We get 2 q del q del h is equal to 2 g, then 2 g, 2 E s h; that means, 2 E s h the differentiation of this term minus h cube, 3 h square, and that is set to 0; which means that E s into h is equal to 2 E s h is equal to 3 h square. So, we get E s is equal to 3 by 2 h. So, you see we get E s is equal to 3 by 2 h. And earlier also we got E s minimum is equal to 3 by 2 h c; and here h is h c.

So now, if I put the expression here to get the value of q, so, q maximum square is 2 g. So, E s is 3 by 2 h c minus h c into h c square; that means, we get 2, 3 by 2 minus 1 half. So, it is g h c cube. So, q max is equal to, again g h c whole to the power 3 by 2. So, we see that at critical depth h c, the, when the q is maximum, discharged is maximum, the maximum discharged is given by g h c to the power 3 by 2. And at that condition the specific energy is 3 by 2 h c. And at the same time, if you compare these results now, at the minimum specific energy condition at a given discharge, q is given by this expression. E s minimum is given by this expression.

So, if you compare this two, with this minus two, then we can conclude that at a critical depth h c, either discharge rate is maximum when specific energy is kept constant where the, or this specific energy is minimum when the discharge rate is gets constant. So, therefore, critical condition is a condition corresponding to a critical depth h c where the specific energy is minimum for a given discharge, or discharge is maximum for a given specific energy.

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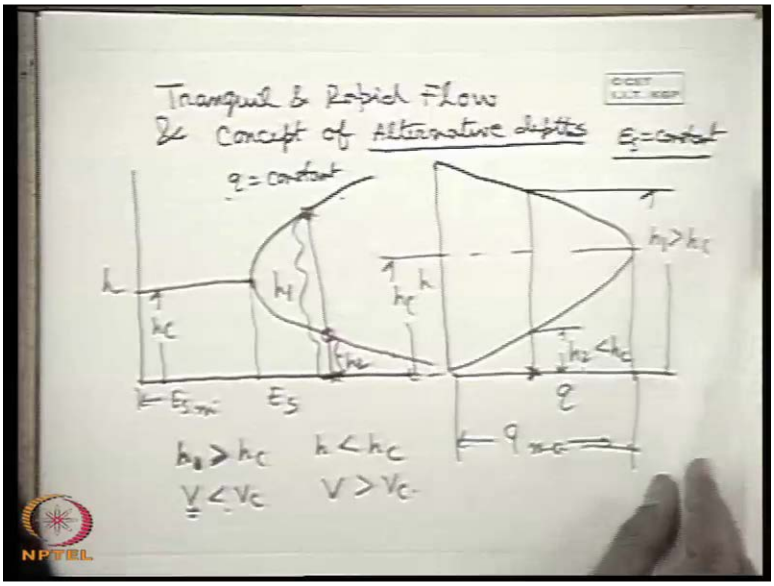

Now after this, we will try to find out, what is meant by alternative depth? And different regions of flow: tranquil and rapid flow, tranquil and rapid flow; and concept of, and concept of, and concept of alternative depth, concept of alternative depth, alternative depth. Now let us reveal again the two curves. One is E s verses h that is h verses E s, for a given q the graph is like this, if you recall, the graph is like this; that means, this is the minimum; this is the h c, this is the h c; this corresponds to the minimum E s, Es minimum.

And if you draw the graph of h verses q for a given, here q is constant, q is constant; and for a given E s, let E s is constant, E s is constant, we get a curve like this, we get a curve like this. So, this is the h c here, h c. So, this is the h c; and this is the q max. Now we see, in this curve here at any specific energy there is, for a given q there are two depths, two depths of flow. One, if you consider as h 2; another can be considered as h 1. So, there are two depths of flow. One is higher than h c that is h 1; one is lower than h c. One is this side of the maximum point; and another is this side of the maximum point.

Same is valid through here that for a given specific energy and for a given discharge rate, there are two depths. One is h 1 which is greater than h c; another is h 2 that is h 2 which is less than h c. These two depths are known as alternative depths, alternative depths. Now when h 1 is greater than h c, v will be less than v c; and when h, h 1 means h, I am writing now in terms of h, h greater than h c that is the case with h 1, then v, the corresponding flow velocity of channel will be less than the critical velocity. When h is less than h c; obviously, the flow velocity will be greater than the v c.

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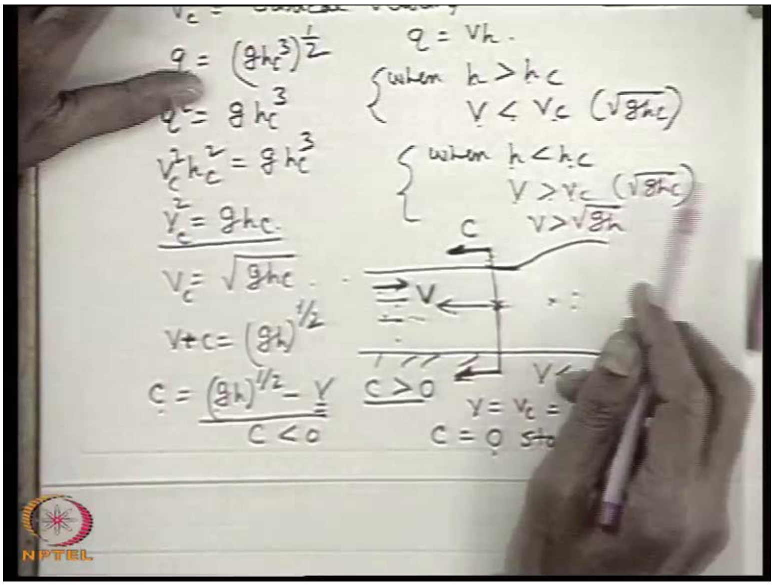

So v c is the critical velocity, critical velocity, v c is the critical velocity, v c is the critical velocity, critical velocity. How do you find out the critical velocity? You know q is equal to g h c cube to the power half. So, q square is g h c cube. What is q? q is v into h. So, q square is v square h square, and here I put critical depth h square is equal to g h c cube. So v is simply equal to, v square is simply equal to g h c. This is the v square. So, v is equal to root over g h c. So, therefore, we see this the flow velocity v is less than the critical velocity. So, this is v when h is greater than h c. So, when h is greater than h c, again I am writing v is less than v c. v c is what? v c is root over g h c. And when h is less than h c, v is greater than v c that is root over g h c.

Now, before specifying the two regimes of flow, we should know, what is the physical implication of these two? What is the physical implication of these two? Now if we recall a earlier discussion that in a channel, if there is a flow and if a disturbance is created at downstream, so, disturbance moves half stream with a velocity c such that, if this velocity is v, undisturbed velocity of the channel flow, because of the disturbance passing through this channel flow, the downstream section the height will increase and velocity will change. But we refer the condition of the hydrodynamic parameter of the channel as the undisturbed velocity of flow, v here in the upstream.

So, we know that the v plus c is equal to the g h to the power half; that means, this disturbance (( )) propagates upstream with a velocity which is g h to the power half related to the flow velocity of the channel. So, actual velocity, absolute velocity of propagation of the disturbing wave is therefore, g h to the power half minus v that is the flow velocity of v channel. Now, this v, when v less than v c; that means, when v less than root over g h, that is the case when h greater than h c, then c is positive; c is greater than 0. That means there is a possibility, all possibility that this disturbing wave reaches the upstream and changes the condition in the upstream.

But when h is less than h c, and v is greater than v c that is v is greater than root over g h, then in that case c is less than 0; c becomes negative which means that c is negative means the propagation wave cannot propagate or the disturbing wave cannot propagate or (( )) in the upstream direction. So in this regime of flow any disturbance is created in the downstream cannot propagate upstream. So this is very important physical implication.

So, the case when v is equal to v c is equal to root over g h then this propagation, absolute velocity of propagation of this disturbing wave becomes 0. This is equal to this 0. In that case, if you create a disturbance somewhere, this will create a wave which neither moves in half stream direction nor moves in downstream direction. This is the case known as standing wave; that means, in a critical flow situation the disturbance create a standing wave.

In other situation the disturbance creates a wave which either can propagate in half stream when h is greater than h c, and v is less than v c; or it will not propagate in the half stream when h is less than h c that is the critical depth, and the velocity of flow is greater than the v c, velocity of critical velocity.

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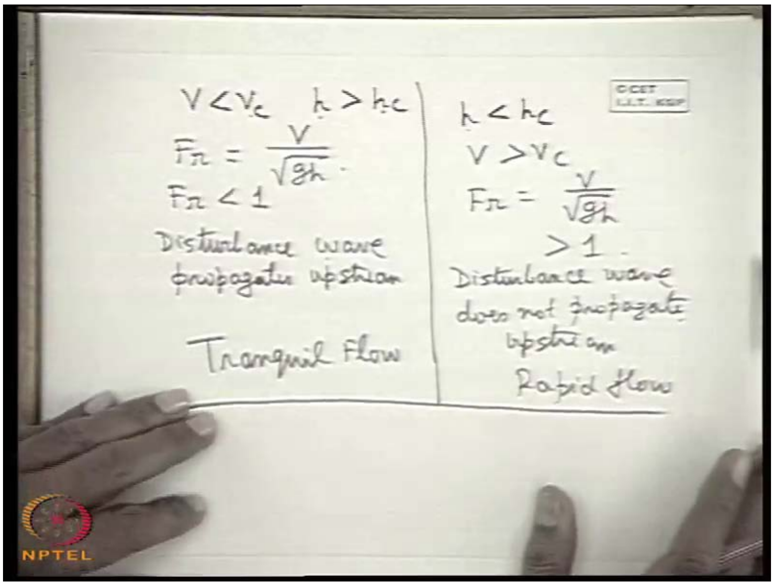

Now, we again see that when v is less than v c, that is the depth of flow is greater than h c, this is the depth, the disturbance way propagate in the stream. In that case if we define froude number. We can write v if we recall divided by root over g h. And in this case froude number is less than 1. So, I write everything for this case. The disturbance wave, disturbance wave propagates upstream, wave propagates upstream, propagates upstream.

And another situation when h less than critical condition, and v is greater than v c then froude number is equal to v by root over g h, become greater than 1. And in that case disturbance wave, disturbance wave, disturbance wave does not propagate upstream, does not propagate, does not propagate upstream.

And this flow, this regime of flow is known as tranquil flow, where the flow velocity is less than the critical velocity; and the depth of flow is greater than the critical depth of flow, and disturbance wave propagates upstream. And this flow is known as the rapid flow, where the flow velocity is greater than the critical flow velocity; and the depth of flow is less than the critical depth of flow; and froude number greater than one; and disturbance wave does not propagate the upstream. These at the two regimes of the flow.

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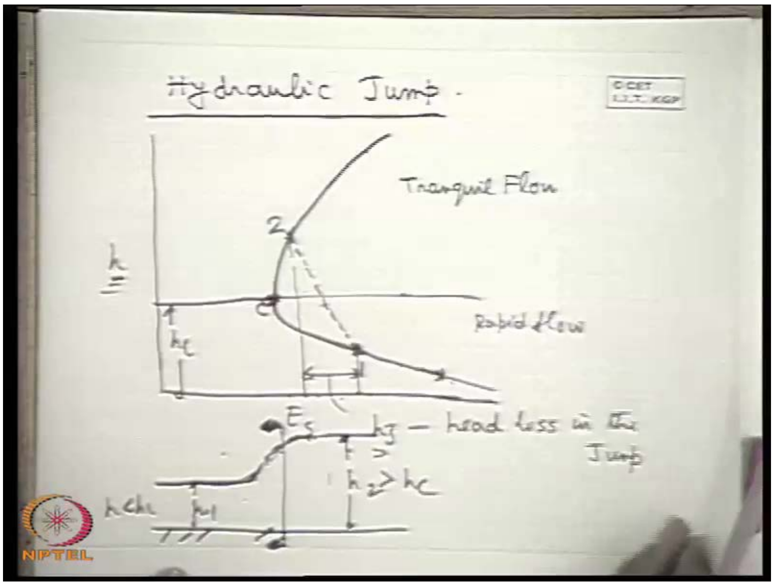

So, next, after this we will go the concept of hydraulic jump. Concept of hydraulic jump. After recognizing these two types of flow tranquil and the rapid flow, we will consider the hydraulic jump. So, by definition hydraulic jump is a transition, is a transition, hydraulic jump by definition, we can define hydraulic jump as a transition which takes place when a rapid flow is suddenly disrelated to a tranquil flow.

Now, we have to know under what situations rapid flow occurs in practice. Rapid flow occurs in practice when water is released at high velocity under the gate of a, under the (()) gate or at the foot of a speed wave. Now if this flow is disrelated by, for example by the roughness of the bed of a long channel, or by putting some obstruction on the downstream if a rapid flow; that means, the high velocity flow where the velocity flow more than the critical velocity suddenly disrelated to tranquil flow, if the type of obstruction or resistance is such that it has to disrelate to tranquil flow then the disrelation takes place very suddenly, and in an irreversible manner, and the sudden irreversible transition is known as the hydraulic jump.

Now, let us find out or investigate the hydraulic jump in detail. Now, if we see this, again this h verses, why it happens so, Es. Now the curve is like this plus. So, this is the minimum point; and this is the critical point. Let us define this as c; and this is h c; and this corresponds to minimum. Now, we are now with the definition we know this is the tranquil flow regime, tranquil flow regime where the depth of the flow is higher than the critical flow; and this is the rapid flow regime.

Now, you see, if a flow is at the rapid flow regime and if it is gradually disrelated by the action of friction, or any resistence, the same thing, resistance means it produces friction in the form of flow, so that the flow is disrelated. Now, if we have to follow the usual depth verses specific energy curve, now you see that it cannot go by the critical section, and then go to tranquil flow, why? If it goes to the critical section in the rapid flow regime we see that, specific energy decreases with the depth; that means, the depth is increasing; the flow is disrelated; velocity is decrease; then depth is increase; and specific energy also decreases.

But when it reaches critical flow, that critical flow that this point here; then it has to, if it has to follow this path to go to the tranquil flow then you see, what happens? With the disrelation its specific energy should increase which violates the law of conservation of energy, because without the addition of energy from outside, the flow cannot be disrelated to tranquil flow from the critical condition, with the addition or increase in the specific energy. So, it cannot follow the path like this; that means, the rapid flow cannot be disillerated to tranquil flow by the critical condition; that means, along this curve.

So, the only alternative is that at some location here a rapid flow for example, here suddenly goes transition to the tranquil flow which shows a decrease in energy, not where this path. So, this path is an irreversible path showing, shown by a dotted line. Let this is the point one in the rapid flow regime; let this is the point two in the tranquil flow regime. So, this from one to two it goes by irreversible path, this is a sudden transition during which there is a loss of specific energy. This loss of specific energy is termed as h j that is head loss, the loss of total energy per unit weight, head loss in the jump, in the jump that means hydraulic jump. So, therefore, this is the way by which a rapid flow is disrelated or converted into tranquil flow. Now this phenomena is known as hydraulic jump.

Now, we will first find out, what is the relationship between these two depths, that is the depths in the rapid flow and the tranquil flow in a hydraulic jump; and then we will find out, what is the head loss in the hydraulic jump?

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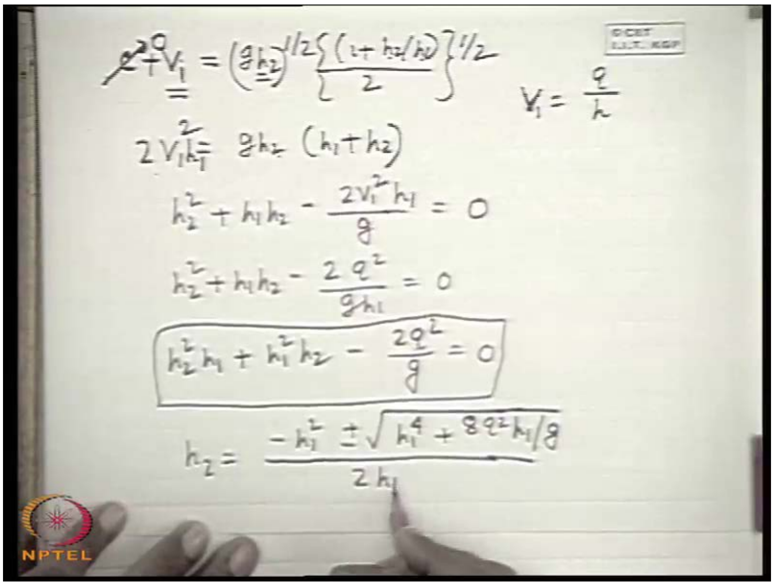

Now, if you remember that we derived some equation regarding the depth of flow, and the velocity while deriving the velocity of propagation of the disturbance wave, we will remember the disturbance wave will propagate.

If we now give the velocity v 1 then we can write g h 2, if you recollect, to the power half into 1 plus h 2 by h 1 by 2, this to the power half. So, this is the expression which we develop when there is a disturbance wave propagates in a channel, where the downstream depth is h 2 which is an increase depth from that of the upstream undisturbed h 1. This is the similar situation because the downstream depth, in case of a hydraulic jump is corresponds to a higher one as compared to the upstream depth; this is because the flow is disrelated from a rapid to a tranquil one. So, if we use this equation, if we remember this v 1 not v 1 it was there c plus v 1, I am sorry. If we put now c is equal to 0, why we put c is equal to 0?

This is because in a hydraulic jump, we can draw it here. So, if we consider a hydraulic jump, if we show a hydraulic jump in a channel like this, that it is a rapid flow with a lower depth, this h lower than h c. So, it suddenly goes through a, with a sudden discontinuity in the flow field, suddenly transfer to a rapid flow, sorry, tranquil flow where this h, let this is the h 1; and this is the h 2; this corresponds to tranquil flow which is greater than h c. So, this discontinuity takes place through a standing wave.

So, in that case we can consider this rapid transition through this point, through this path where the usual depth verses specific energy relationship is not, does not hold good. So they does not come along this curve, I just describe that. So, this transition can be conceived as a form of a standing pressure wave through which the flow passes. So, that it is disrelated; its depth is increased; and the flow velocity is decreased; so as a form of a standing wave, where this transition takes place. So, this transition is not a moving transition, it takes place at a particular location.

So, this can be made similar, or it can be made analog as, to the disturbing wave which is moving through a channel with 0 velocity. So that, in mathematical analysis we can make c 0. So, we get v 1 is equal to g h 2. Now if we square it, making c 0, making square, so it is g h 2 into h 1 plus h 2. So, if this 2, I take here, 2, and this h1 v 1 square, 2, if you square it g h 2 v 1 square h 1. So, we can write now h 2 square. So, g here; so we can write h 2 square plus h 1 h 2 minus 2 v 1 square by g, v 1 square h 1 by g is equal to, 2 v 1 square h 1 by g, h 2 square plus h 1 h 2 is equal to 0.

Now flow velocity v 1 can be written as, what is v 1? v 1 is q by h, where q is the flow rate per unit width. So, if we replace this, we get h 2 square plus h 1 h 2 minus 2 q square. So, h square, that means, g h 1 is 0. Or simply we can write, h 2 square h 1 plus h 1 square h 2, h 2 square h 1 plus h 1 square plus h 2, minus 2 q square by g is equal to 0. Now, this is an equation which is symmetric with respect to h 1 and h 2.

So, if we consider this equation as a quantity equation in h 2 then we can do one thing; we can find out the root of h 2. What is h 2 then? This will be minus h 1 square plus minus root over, this is d square minus then this square; that means, h 1 4, this is minus so, plus 8, 4 this into this, q square h 1, 4 q square by q square h 1 by g; 8 q square by g into, that is h 2 square, so h 1, h 1 4 h c divided by 4 h 1. So this is the root of this equation, to find out the value of h 2. So h 2 will be, so we can neglect this minus sign because it is minus so, negative root is physically impossible, the depth cannot be negative. So, therefore, we take only the positive root.

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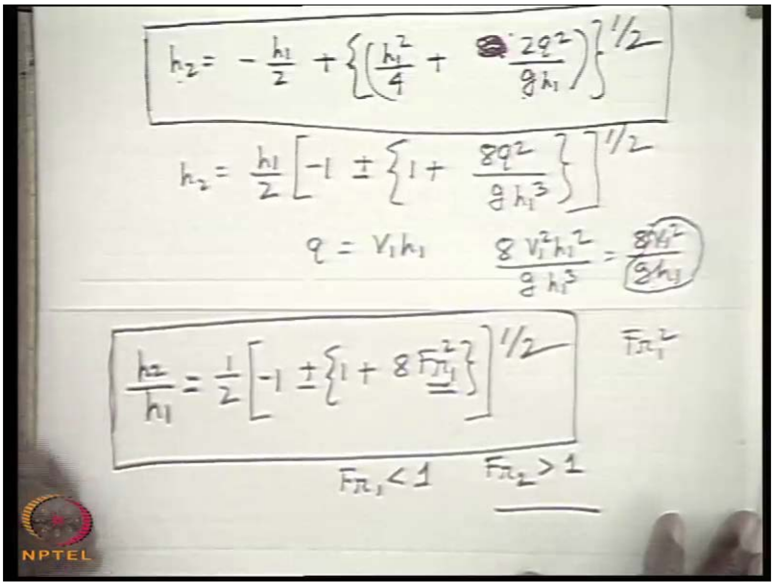

So, we get h 2 is equal to, so if we take the minus h 1 by 2 then we can write plus then if I take h 1 square common. So, h 1 so plus, or we can write within the square root rather 4 h 1 square. So, we can take h 1 square by 4 plus, what we can write? 8 q square, not 8 q square, 4 will be inside, so, 2 q square by g h 1, this to this, the same thing we can write to the power half; h 1 square by 4; that means, I am taking this 4 inside this, so, we get 2 q square by g h 1, because h 1 square will be there, so, g h1 whole to the power half. So, this is the value of h 2 in terms of h 1.

So, we can get also the value of h 1 in terms of h 2, we just simply change this topic. This is because this equation is symmetrical with this respect to h 1 and h 2. So, this can be written as h 2, as if we take common as h 1 by 2 we can take h 1 by 2 common, and then we can write this expression as minus 1 plus minus, minus 1 h 1 by h 2 we are taking common, 1 plus 8 q square, that means I am taking h 1 square by 4 as common, so, h 1 square by 4 as common, so, h 1 square comes in the denominator. So, g h 1 cube; this is whole to the power of half.

Now again, we know that this q is equal to, this q is equal to what? q is equal to v 1 into h 1. So, if we put q is equal to v 1 h 1, this term will become 8 v 1 square h 1 square by g h 1 cube. So, 8 v 1 square by g h 1. So, v 1 square by g h 1 square is froude number 1 square. So, v 1 by root over g h is the froude number. So, we can write h 2 by h 1, making this side dimensional term, half into minus 1 plus minus 1 plus 8 froude number, F r 1 froud number corresponding to the upstream condition, fraud number square I am sorry, whole to the power half.

So, h 2 by h 1 half 1 plus 8 froude number square minus 1. So, this is the final expression for the ratio of the two depths. So, h 2 is the depth corresponding to the tranquil flow when it is disrelated; h 1 is the initial depth of flow for the rapid flow, this is equal to half minus 1. So froude number 1 is always greater than, less than 1; froude number 2; that means, the corresponding to the tranquil flow, sorry rapid flow, sorry fraud number 1 that is F r 1 that is froude number corresponding to the rapid flow is always less than 1. And fraud number and corresponding to the 2 the tranquil flow is always greater than 1. So, this is the corresponding relationship between h 2 and h 1.

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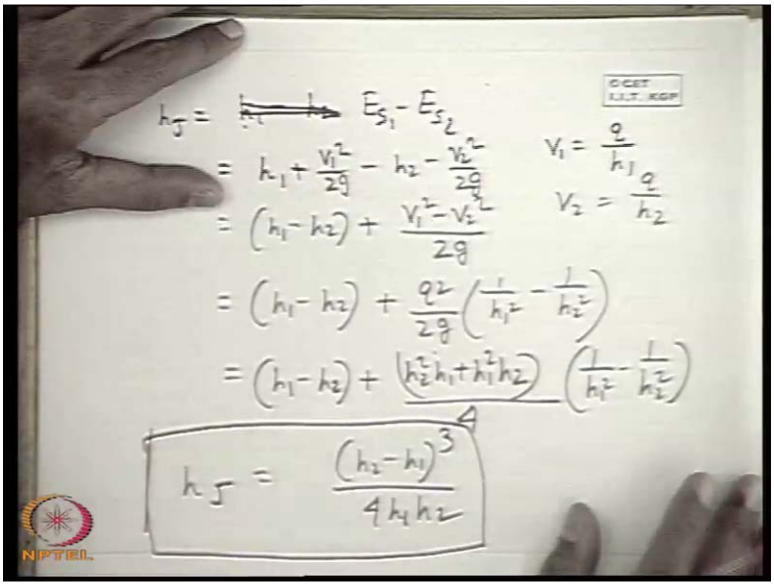

Now we will find out, what is the loss of head in the hydraulic jump? That means, now we want to find out, what is this value? This h 1 and h 2. So, this is the head loss in the jump. So, we can find out h 1 minus h 2; h j is h 1 minus h 2; that means, not h 1 minus h 2 I am very sorry, this is the E s 1 minus E s 2, or E s 2 minus E s 1. E s 1 is more than E s 2 because it is the disrelation and loss of energy. So, you see here also E s 1 minus E s 2. So, I am sorry, I am sorry.

E s 1 minus E s 2; that means, we can write, we know the expression of E s that h plus v 2 square by 2. So, let us write like this h 1 plus v 1 square by 2 minus h 2 minus v 2 square by 2. So, this can be written as h 1 minus h 2 plus v 1 square minus v 2 square by

2. Now this v 1 square minus v 2 square can be written. You know that v 1 is equal to q by h 1; and v 2 is equal to q by h 2. So, we can write h 1 minus h 2 plus, q by h 1, so q square. So, this will be 2 g per unit wave. So, q square by 2 g into 1 by h 1 square minus 1 by h 2 square.

Now this q square by 2 g, just now we have derived, can be expressed, you see here. So, q square by 2 g can be expressed from here; that means, if you see like this, that q square by 2 g, here you see the earlier one which we derived, that q square, 2 q square by g is this one. So, we can write from here q square by 2 g; that means, divided it by 4 is equal to this quantity that is h 2 square h 1 plus h 1 square h 2 by 4; that means, we can write from here q square by 2 g h, h 2 square h 1 plus h1 square h 2 by 4.

So, if we write that h 1 minus h 2 plus, this thing is this, plus h 2 square h 1 plus h 1 square h 2, it is easy to remember, by 4 times 1 by h 1 square minus 1 by h 2 square. If you make a simplification of that, you get h 2 minus h 1 whole cube by 4 h 1 h 2. And this is precisely the head loss or loss of energy per unit weight in a hydraulic jump.

So, therefore, we conclude that when a rapid flow is suddenly disrelated due to some obstruction in the flow field or by the friction in the channel head to tranquil flow, the conversion or the change in the regime of the flow takes place through a sudden transition known as hydraulic jump.

So, therefore, the rapid flow does not, does not go to tranquil regime parallely critical flow. So, the critical flow does not occur due to the, in the change of the rapid, in the change for the rapid flow to the tranquil flow. But on the other hand, if the tranquil flow is suddenly accelerated to rapid flow then the flow passes through the critical condition; that means, transition from rapid flow to tranquil flow that is the flow disrelation, gives rise to hydraulic jump and the flow does not go through the critical flow. So, the occurrence of critical condition does not take place here.

But the occurrence of critical condition takes place when the flow is accelerated from the tranquil to the rapid flow region because the specific energy and the depth of flow relationship is not invalid here; that means, it obeys all the physical laws that funds the flow. So, therefore, here we see that when a rapid, tranquil flow is accelerated to rapid flow the occurrence of critical condition takes place.

So, under what practical condition this happens, when the flow from a long prismatic channel with a mild flow approaches to a long channel of steep flow, when the flow is accelerated. And therefore, flow is acted by an additional gravity force, the gravity force is increased; in that case the critical condition occurs. Another case is that if the flow passes through a, flows through a channel with a rise in bed or over a speed wave, the tranquil flow is accelerated to rapid flow, and the occurrence of the critical condition, occurrence of the critical condition takes place at the juncture where the flow geometry is changed.

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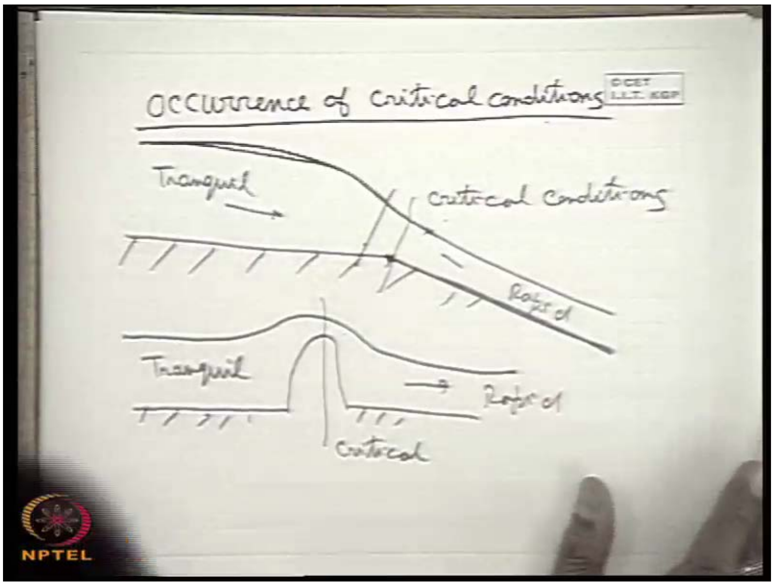

Let us consider the case like this. I tell you the occurrence, occurrence of critical conditions, occurrence of critical conditions, occurrence of critical condition when the flow is taking place through a mild flow through like this, when the liquid flows like that then, what happens? A tranquil flow suddenly changes to a rapid flow. Here the flow is tranquil; here the flow is rapid. And the critical condition takes place at this junction; that is the critical condition.

Sometimes, it happens in practice that if the slope is too steef, slope of this channel or there is an abrupt change the slope takes place, then there is an appreciable curvature of the streamline near the junction, in the near vicinity of the junction where the assumption of the hydrostatic pressure variation is not justified. And in that case, the critical condition may occur little upstream from this junction, somewhere here. Another condition is that, if there is a rise in the depth of the channel. So the flow which is tranquil initially, may goes to a rapid flow. So, this is a rapid flow; and this is a tranquil flow. So, a conversion from tranquil to rapid flow gives the occurrence of the critical flow means the flow is critical at their junction.

The flow passes through a critical condition. So occurrence of a critical flow occurs, occurrence of critical flow takes place when the tranquil flow is accelerated to a rapid flow, but not the reverse; the rapid flow is accelerated to a tranquil flow.

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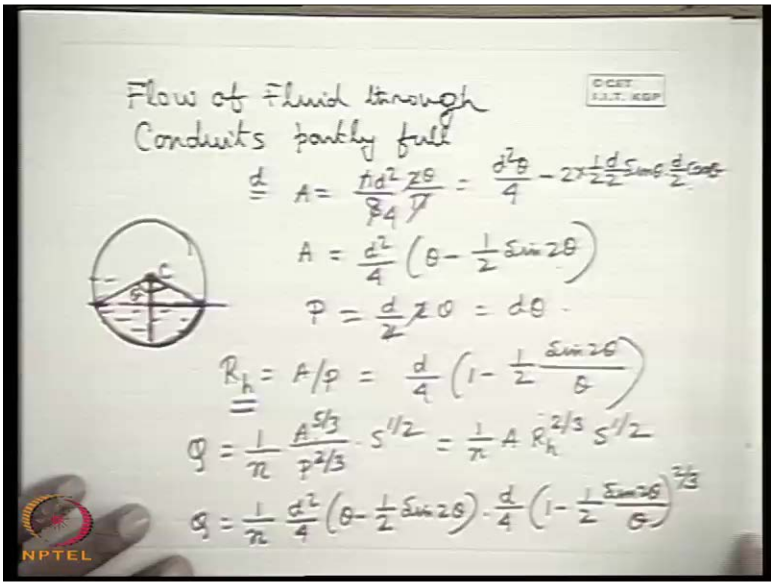

Now next, I will discuss the flow of fluids flow of fluids through conduits, sorry through conduits partly full, partly full, the flow of fluids through conduits partly full. Now, we have seen in many practical, it is found in many practical purposes or inpractice that the liquid flows a through a conduit partly full; that means, it is not full flow of liquid through the conduit. In this case, the liquid as a free surface; and the flow is governed by the principals of channel flow that is the flow through the conduit with partly full, partly filled; the conduit is not filled totally and the liquid has a free surface. So, that the principle is governed by that of the channel flow.

Let us consider a circular conduits; now, let us consider such flow through a circular conduits. Let this is the center; let this is the surface of the fluid which is flowing through this; that is the cross section of the conduits, the circular conduits. Let the free surface subtains and angle 2 theta at the centre, let this is center c. So, this angle is theta. Let diameter d, d is the diameter of this circular conduit. Then, we can write that, what is the cross sectional area of the flow? Cross sectional area of the flow is this crossection. So this cross section can be retained as this. So, pie d square by 8 is the half circle which is the cross section when this included angle; that means, the angle subtended by this arc is 2 theta at this centre, at this becomes 180 degree that is pie. So, 2 theta by pie, so correspondingly it is reduced, so that we can make it a simple school level geometric, so d square theta by 4.

So, d square theta by 4 is the, now cross sectional area of the pie, so this cross sectional area of the flow is d square theta by 4 minus; now d square theta by 4 is this area, so this is also theta, so, this area. From there we have to subtract this two triangles, so if this is, so, how to find out? This is d by 2. So this, if this is theta; this is d by 2 sin theta; so this is d by 2 cos theta; because this is d by 2; and this is d by 2 sin theta; this is d by 2. So here also this d by 2 sin theta because this is d by 2 and this is d by 2 cos theta. That means, this is the area of 2 triangles; that means, 2 into half d by 2 sin theta well, into d by 2 cos theta; d by 2 sin theta into d by 2 in cos theta, which can be written as A is equal to d square by 4, if you take common, theta minus half sin 2 theta. So, d square by 4 is taken common, then half 2 sin theta cost theta is sin 2 theta. So, this is the cross sectional area.

Now, what is the wanted perimeter, very simple wanted perimeter is 2 theta into R, yes this arc is the, we get perimeter. So R into; that means, this d by 2 into the angel subtended by the arc at this center 2 theta. So, this is equal to d theta; so hydraulic radius R h is A by P. So, if you divide this by d theta, then we get d by 4 into 1 minus half sin 2 theta by theta; this is the value of R h.

Now, if you recollect the (( )) formula where the discharge, rate of discharge was given by 1 by n A to the power 5 by 3; P to the power 2 by 3 into s to the power half, if you recollect this. And if I write this as this fashion 1 by n, one cross sectional area I take out then A to the power 5 by 3 by P to the power 2 by 3 that is A by P, whole to the power 2 by 3 which is the hydraulic radius R h to the power 2 by 3 into s to the power half.

Now, if I replace this I get, q is equal to 1 by n; this A is d square by 4 into theta minus half sin 2 theta. So, this is the area; into R h, that means, R h is this d by 4 into 1 minus half sin 2 theta by theta; that is R h, it is to the power two third, it is to the power two third times the s to the power half.

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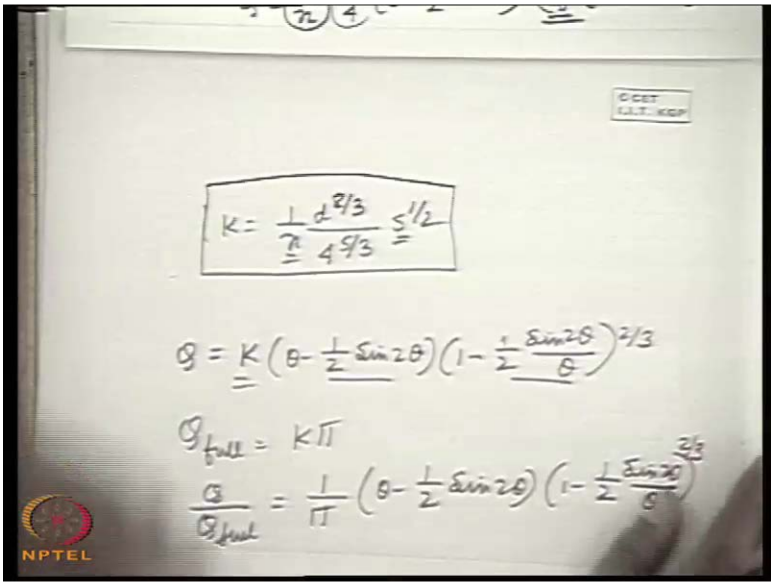

So, this can be written, taking the theta functions, trigonometry functions with a constant; that means, a k. That means 1 by n. So, this d square, this d we take care. So, this is d square, I am sorry, this d by 4 to the power two third because this d by 4 in A P. So, two third plus 2, so q is equal to k I am writing that afterwards, k into theta minus half sin 2 theta into 1 minus half sin 2 theta by theta to the power two third; from here we are writing it, where k is equal to 1 by n. So, then to the power two third plus; then d to the power 8 by 3; and 4 to the power 5 by 3 and s to the power half; that means, taking this; taking this; and taking this and 1 by any if I write a constant, because this is constant for a given roughness, for a given slope of the bed, and the given slope of the bed. Sorry, it is given slope of the conduit, given slope of the conduit and d is the diameter of the conduit. So, this is constant. So, in terms of a constant I can write Q like this.

Now Q full, when it is running full will be obtained when we will give this theta is equal to pie; then as this goes on increasing this free surface or the surface of the liquid, so, then theta goes on changing, increasing and it will be 180 degree pie. So, theta pie you see this around 0, so k pie. So, we can write Q by Q full is equal to this by pie; that means, 1 by pie into theta minus half sin 2 theta into 1 minus half sin 2 theta by theta whole to the power two-third.

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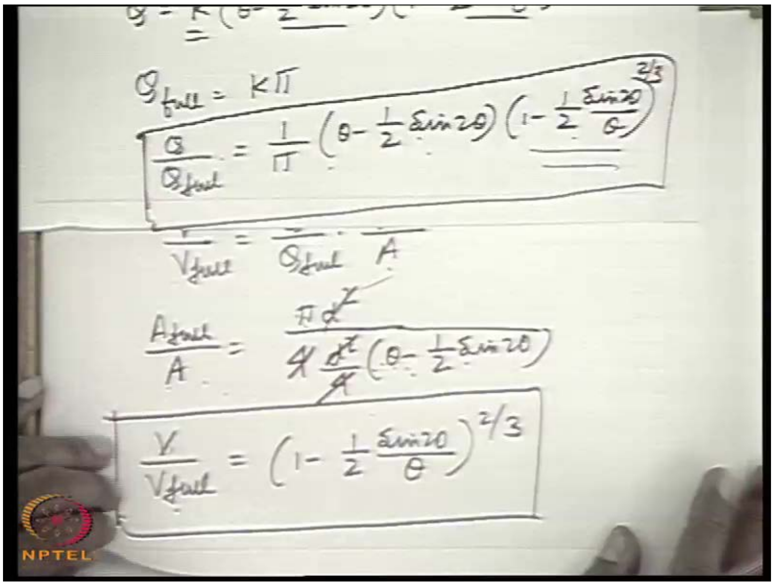

Now we can also develop the expression of, from here we can also develop the expression of v by v full. What is v by v full? It is q by q full; the ratio of discharge into v is q by A; that means, A full by A. What is A full by A? A full by A is equal to pie d square by the full area; and this area is the cross sectional area which we already developed d square by 4 theta minus half sin 2 theta. So, therefore, 4 d square by 4 theta minus half sin 2 theta. So, therefore, if you cancel this d square d square, 4 four 4, this is pie by theta minus half sin 2 theta. So, if you now write this v by v full, then v by v full, if you now express A full by A with Q by Q full then this will be cancelled, this will be cancelled; and pie pie will be cancelled.

So, 1 minus half sin 2 theta by theta whole to the power 2 by 3; that means, A full by A is pie by this. So, I write Q by Q full into A full by A by this equation. So, Q full by Q is this; and this A full by A is this one. That is the ratio of full area to the area of the constructional area of the liquid flow. So, this cancels; only this thing; pie pie cancel. So, this and this formula that is the variation of Q with the Q full that in the, when v by v full is plotted not with theta, but with the depth of flow, but with the depth of flow that is d or the diameter here not the depth of flow, I am sorry, with the diameter of the conduit.

So, what is the diameter of conduit in terms of the theta? We can write that if this is d by 2. So, we can write in terms of the depth of flow; diameter and depth.

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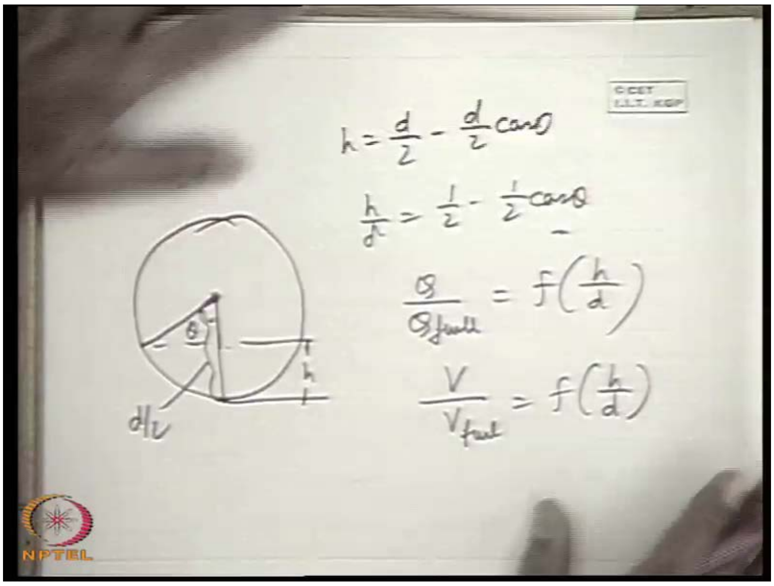

So, h, depth is d by 2 minus, this one d by 2 cos theta. So, h by d is half minus half cos theta. I can do it in a separate page. Yes I can do it in a separate page, that if this is the, sorry, this is the center, sorry this is not correct. I can do it in a separate page that. So, this is there. So, this is the depth of flow h; and this is the d by 2; and if this be the theta; then h is d by 2 minus, this one minus d by 2 cos theta; that means h by d, half minus half cos theta. So taking theta as a dummy variable we can plot v by v full, and Q by Q full with h by d; that means, generate Q by Q full; for the different values of, as a function of h by d; and v by v full as a function of h by d.

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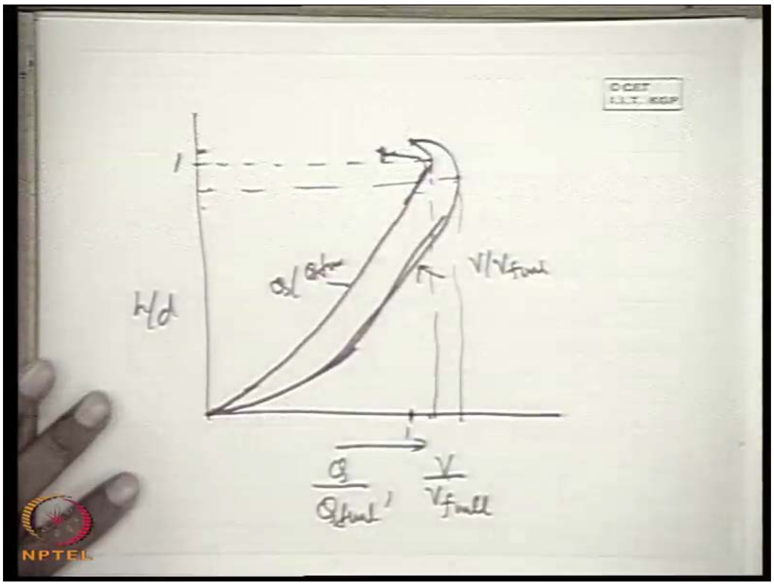

And if we plot this results we will get a curve like this, we will get a curve like this. If Q by Q full and v by v full verses h by d. So, let this is 1 and here it is 1; then Q by Q full it goes to 1 obviously, but it slightly, sorry, it reaches the value of 1 when it is 1, so I am sorry, it slightly get finished.

And v by v full if we, sorry this is like this. This is v by v full; this is Q by Q full. So, interesting feature is that the value of Q by Q full exceeds 1 at some value near the top surface. Similarly the value of v by v full exceeds 1 before the quantities fully filled; that means, we get more discharge for partly filled compared to that full flow; and we get more velocity when the fluid is partly filled, but not full flow. This is because of the fact that the friction in full flow is much more compared to that in partly flow.

Thank you all.