Fluid Mechanics
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Lecture - 39
Flow of Ideal Fluids Part - I

Good morning. I welcome you all to this session of fluid mechanics. Today we will start a new chapter that is flow of ideal fluids. We will describe here a brief discussion on the flow of ideal fluids. Now what is an ideal fluid? We have already recognized the definition an ideal fluid is a fluid, which has know viscosity. And therefore, flow of such fluids known as inviscid fluids which has know viscosity is known as ideal fluids. In addition to this non viscous nature of the fluid, that means 0 viscosity, we will impose here two other properties of the fluid that incompressible, that density remains constant and at the same time irrotational, the rotation in the flow of fluid is 0.

So, therefore we will mainly discuss the flow of an ideal fluid with incompressibility, ideal incompressible; that means inviscid, incompressible and irrotational flow. So, let us recall few of the corollaries of the incompressibility, and irrotationality in a flow field.

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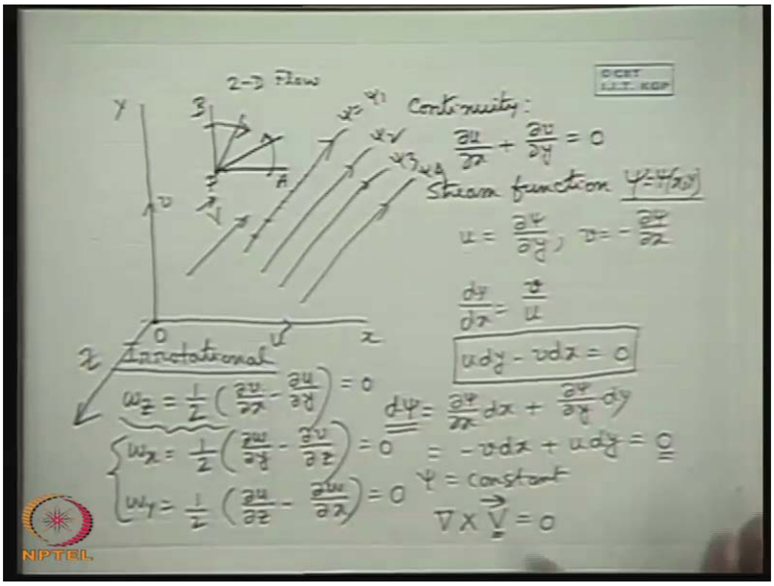

Let us concentrate our analysis on a two-dimensional frame of reference. Let us take a two-dimensional, x and y in a Cartesian coordinate system. We consider a two-dimensional flow in a Cartesian coordinate system described by x and y axis.

Now, in this case we know that a incompressible flow, for an incompressible flow the equation of continuity, the equation of continuity conservation of mass, for an incompressible flow is del u del x plus del v del y is 0, where u and v is the velocity in the x direction and y direction; that means, velocity vector v has the component u and v in along x and y axis respectively.

Now, as a consequence of this continuity equation we define a function, stream function, we have discussed in details about this earlier stream function psi which is a function of space coordinates x y. In such a way that the velocity components are defined like this u is del psi del y and v is minus del psi del x; one is positive another is negative. So that, the psi function that is the stream function automatically satisfies the continuity. You see that if we substitute u and v. From this definition we will get that this automatically becomes 0; that means, stream function automatically satisfies the continuity, this we have already recognized earlier.

Now, you see that we know that for a stream line this stream function psi remains constant along the line. Now, if were call the definition of a stream line, stream line definition we know that the tangent to it at any point represents the velocity vector. And the equation of a stream line is that d y d x in a two-dimensional frame is equal to v by u because the slope of this stream line gives the direction of the velocity vector. So, for a two-dimensional case, it is like this which gives the u d y minus v d x is equal to 0. This is nothing but the equation of a stream line.

Now, let us see that if we expand psi in a differential form, we can write d psi, psi being a function of both x and y; we can write del psi del x d x plus del psi del y d y. Now, del psi del x by definition is v minus v and del psi del y by definition is u d y. Since the equation of a stream line; that means, along a stream line u d y minus v d x is 0 so d psi is 0. So, along a stream line d psi is 0; that means, psi is equal to constant.

So, therefore, we know that this stream lines for example, if this be the stream lines that any instant in a steady state, this stream line remains same at all instant of time. So, psi is equal to constant; that is psi 1, let this is psi 2, let this is psi 3; that means, we can ascribe a single value of psi for each and every point along a stream line. So, this we have discussed earlier.

Now, if the flow is irrotational, if the flow is irrotational, now in a two irrotational flow means we have discussed earlier that is a flow where rotation is 0; so rotation has got three components along the three axis, coordinate axis; now in a two-dimensional flow there is only one component of rotation; that is the rotation about the z axis. How do you define rotation? We defined it earlier the rotation is the arithmetic average of the angular velocity of two linear segments which were initially perpendicular to each other; that means, if we take a point p and a p and b p are the two linear segments mutually perpendicular to each other.

Then because of the flow field this tendency of rotation of these linear elements in the opposite direction; for example, if we take the arithmetic average of the angular velocities of these two linear elements, we get the rotational component that we have already derived earlier. This with respect to z axis; that means, an axis perpendicular to the plane of this figure; that means, at p the axis is perpendicular to the parallel to a z axis here which can be represented like this. It will be half del v del x minus del u del y.

Now, we can recognize the other component of the rotation; that means, x; that means, in y z plane, rotation in y z plane, this is rotation in x y plane. That is rotation about z axis; rotation in y z plane that is about x axis, rotation like this. It will be y and z; that means, del w del y minus del v del z. Rotation in about x axis that is omega x del w del y minus del v del z. Similarly, with respect to y it will be del u del z minus del, y it is; that means, del w del x; del u del z minus del w; so three rotational component. And for a irrotational flow all this three components become independently 0. For a two-dimensional case we do not come across this, defined in an x y plane; so rotation is only this; so this becomes 0. So, this three together is written in the form if you recollect is 0 where V were is the velocity vector which has got three distinct component u v and w.

Now, for an irrotational flow del v del x minus del u del y is 0 or we can tell that curl of velocity vector is 0. Now, a consequence of this mathematical relationship is that this vector V can be expressed as a gradient of a scalar function phi. So, this is the mathematical theorem that the curl of any vector is 0; that vector can be obtained or can be expressed as a gradient of a scalar function. So, this scalar function phi is known as the potential function. If this vector is the velocity vector in case of irrotational flow curl of this velocity vector is 0. Then velocity vector can be obtained as a gradient of the scalar function phi which is known as velocity potential function, velocity potential function or simply velocity potential.

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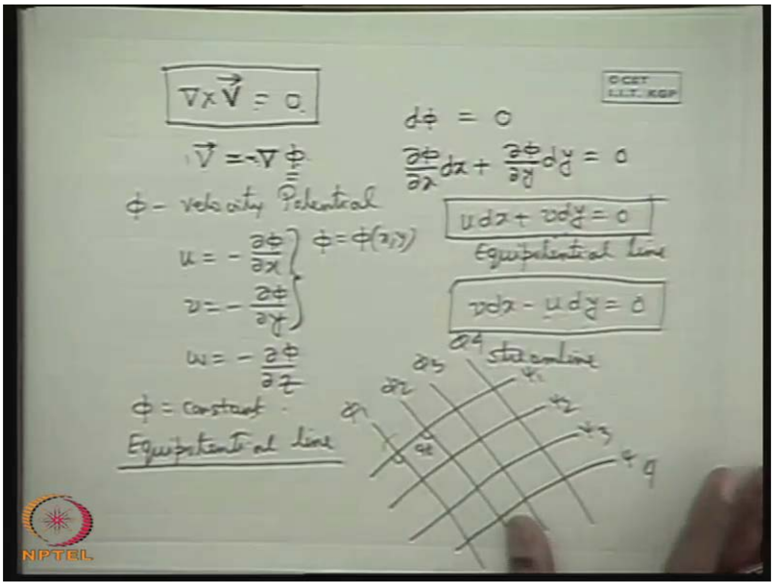

Now, a negative sign is deliberately used to make the things similar to all phenomenological equations, where any flux is proportional to the negative gradient of the potential function. So, therefore, a negative sign is implied. So for that, if you just expand this; then we can write for three-dimensional cases with respect to a Cartesian coordinate system; simply we can write u is equal to minus del phi del x, v is equal to minus del phi del y and w is equal to minus del phi del z. So, for a two-dimensional case we can deal with this two; u is equal to minus del phi del x, v is equal to minus del phi del y. And the curve or the line along which phi remains constant, is known as equipotential line, is known as equipotential line, equipotential line.

Now, let us find out the equation of equipotential line. Equation of equipotential line means phi is equal to constant; this is the equation, but we try to derive it in terms of u and v; what to do? You write it in a differential form that d phi is 0. Now what is d phi? d phi is, because phi is a function of x and y. This is the scalar function point coordinate x and y in case of two-dimensional system. In three-dimensional flow it is a function of x y and z. So, it is del phi del x d x plus del phi del y d y and that must be equal to 0 along a equipotential line.

So, del phi del x is minus u, del phi del y is minus v. So, therefore, we see u d x plus v d y is 0. So, this is the equation of equipotential line, equation of equipotential line, equation of equipotential line. Similarly, equation of stream line is v d x minus u d y is 0. This is the equation of stream line, this is the equation of stream line. So, immediate inspection of this two equation said that, they are mutually orthogonal; they are perpendicular to each other, u d x plus v d y is 0 and it is v d x minus u d y is 0. The slope of this line at any point then slope of this line at any point are perpendicular to each other.

So, therefore, we can say that if this be the stream lines, this be the constant stream lines psi 1, psi 2, psi 3, psi 4. Then the orthogonal families of curves will be the constant; five lines. So, they will be, this corners will be 90 degree. So, they mutually form a orthogonal sets of trajectories. So, this is constant psi line; these are the constant five lines.

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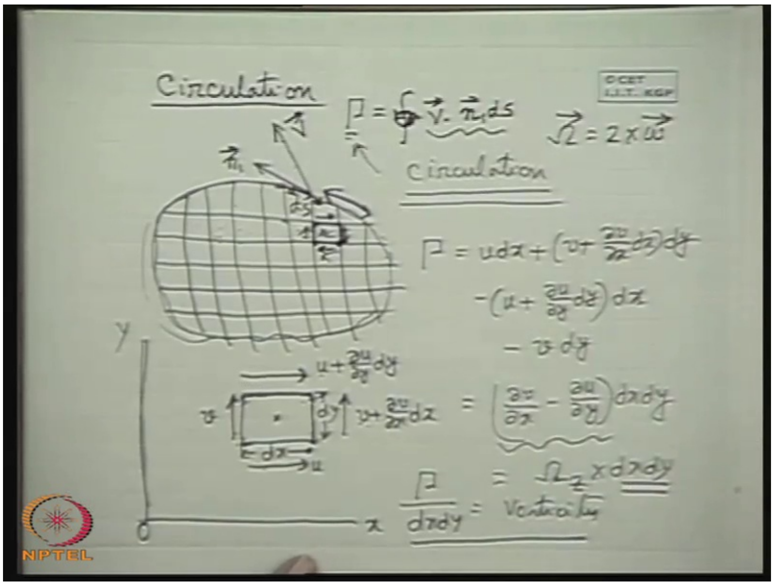

Now, we come to the concept of circulation; what is meant by, circulation, what is meant by circulation? Let us consider in a flow field, a closed-loop, a closed-loop and bounded by a closed contour. Now, if we designate some small elemental length as d s where, the velocity vector is v bar; then the contour integral of this v dot let us define a vector, normal unit vector along the tangent to this; that means, along the tangent to the curve at this point; that means, along the direction of this linear element d s and unit vector n 1 we define, then n 1 d s; which means that, the elemental d s is given a vector concept by multiplying it with a unit vector along the elemental line; that means, along the tangent to that line.

Then, this quantity if we make a contour integral over this closed contour; then we define this as the circulation gamma, which is the symbol. So, gamma is the circulation, circulation. It is simply means that, the length a linear elemental length multiplied by the component of the velocity along that length. So, velocity vector may have in any direction component of the velocity vector along that length and if we sum up all such quantities product of the velocity component, along a linear, along a infinite small elemental length and product of the length times the velocity component along that direction and sum it up over a close contour. Then this is known as circulation around that close contour or close circuit. This is known as circulation.

Now, we can show that the circulation for in a any close contour can be found out by the circulation of small grids or small contours, generated within that big contour, close contour; that means, for example, if this close contour is now divided into a number of small circuits, this big circuit is divided in number of small circuits and you see that, if we have the velocity component like this, now what happens? Before that I must say that when we make this integral, we take certain direction for this integral along the close contour; if we make in this direction usually taken as positive conventionally. So, this is given a direction like that a contour integral or a close contour integral along a close contour is given with a sign. So, this is positive along this sign; that means, conventionally taken the anti-clockwise direction or rather this way you take an anti-clock wise direction. This is not anti-clockwise; so this is an anti-clockwise direction. Let us consider the anti-clockwise direction; that means, this direction anti-clockwise direction as a positive.

Now, you see therefore, for a small circle the, therefore the circulation will be defined the velocity vector multiplied by this length; velocity vector multiplied by this length; this velocity vector multiplied by this length; this velocity vector multiplied by this length. Now you consider the common edges; that means this edge the contribution of the velocity in v at this edge in the circulation for this grid, is in one direction and for this grid is in other direction. So, for all this common surface it is like that it contributes in the calculation of circulation for one grid, one close circuit, in one direction and another adjacent close circuit in another direction. So, that they ultimately nullify each other or cancel each other and we are left only with these contributions along the outer most boundary. So, that ultimately if we sum up the circulations of these infinites small circuits we get the circulation around the close circle.

Now let us find out the circulation in case of x y coordinate. Let us see a simple case consider a circuit, we can choose a circuit a rectangular circuit like this, of length d x and length d y. What will be the circulation? Let us consider at this plane the velocity is u and at this plane the velocity will be changed for a distance d y will be u plus del u del y d y. Similarly, let us consider the velocity at this plane will be v; positive direction. So, velocity at this plane will be changed for a distance d x as v plus del v del x d x.

Now, let us find out the circulation with the anti-clockwise direction as the positive direction. Now, what is this? This will be u for this length, this elemental length u into d x and this is, this has got an anti-clockwise rotation about any point within the circuit. This is the way the direction for the rotation has to be taken care of.

Similarly, for this element it is the velocity along this element is v plus del v del x d x. u does not have any component for this which is perpendicular to this linear element. So, therefore, it is the velocity component along the linear element times d y; this gives also in the same directional notation. So, plus v plus del v del x d x times d y. Then for this, it is an opposite direction, but the magnitude is this velocity component times the length d x; that means, minus u plus del u del y d y, this is the velocity component times d x, minus this one in the same direction; that means, minus v times this length d y.

So, if you make it simple. So, it will become ultimately del v del x minus del u del y into d x d y. This is because see v d y v d y cancels u d x minus u d x cancels, but this is our twice the rotation and this is you know is the vorticity about the z axis. We earlier defined vorticity is the two times the rotation vector; if you represent vorticity as a vector then it is two times the rotation vector. So, this is the vorticity. So, vorticity times d x d y. What is d x d y? d x d y is the area of this contour. So, we can tell that circulation divided by this area is the vorticity, this is vorticity.

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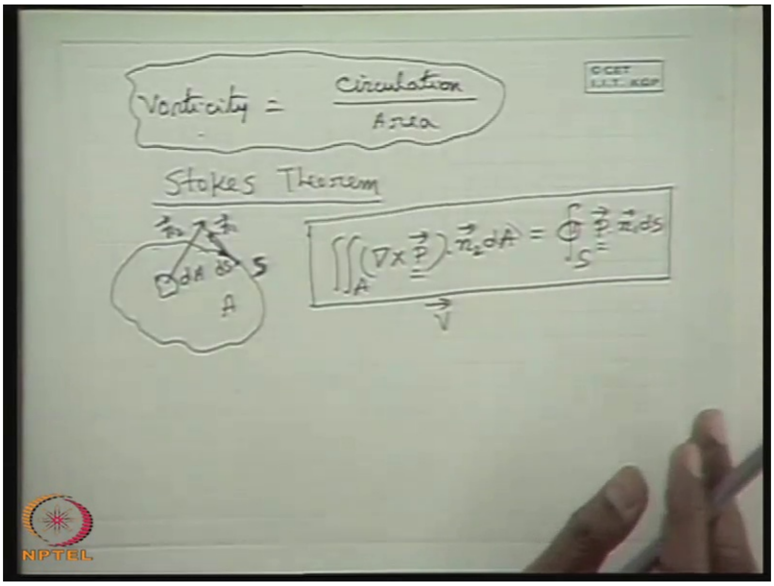

So, this is a very important theorem from which we can write that vorticity is equal to, very important theorem vorticity is equal to over a close circuit or loop in a flow field, flow field of an irrotational flow ideal that is inviscid incompressible and irrotational flow circulation divided by area.

So, this can be expressed in a general way by the application of Stokes' theory. This is known as Stokes' theorem for ideal fluid flow, but Stokes' theorem in general for mathematics tells in this fashion that if there is a closed path; closed area. It is bounded by a close surface S and if the area is A and if a vector P is defined throughout this domain continuous vector space, then Stokes' theorem tells that, vorticity times the area is the circulation, Stokes' theorem takes that curl of this, if you consider a small area d A and if you consider some for example, in two vector which is perpendicular unit, unit vector in a direction perpendicular to the area. Then dot n 2 d A is the closed contour with any direction as the positives anti-clockwise is usually positive into this vector P dot n dot n one d S. Where d s is a small elemental length and n 1 is the vector unit vector as I have told earlier along the tangent to this line, along the tangent to this line.

So, this is the strokes theorem which relates the, this is over the surface A, this is over the contour S which relates the line integral with the surface integral we have read in mathematics probably. So, this Stokes' theorem when vector P in general takes the velocity vector in case of an ideal flow gives, the Stokes' theorem as an ideal flow then this left hand side becomes a vorticity and this is circulation. So, vorticity is circulation per unit area.

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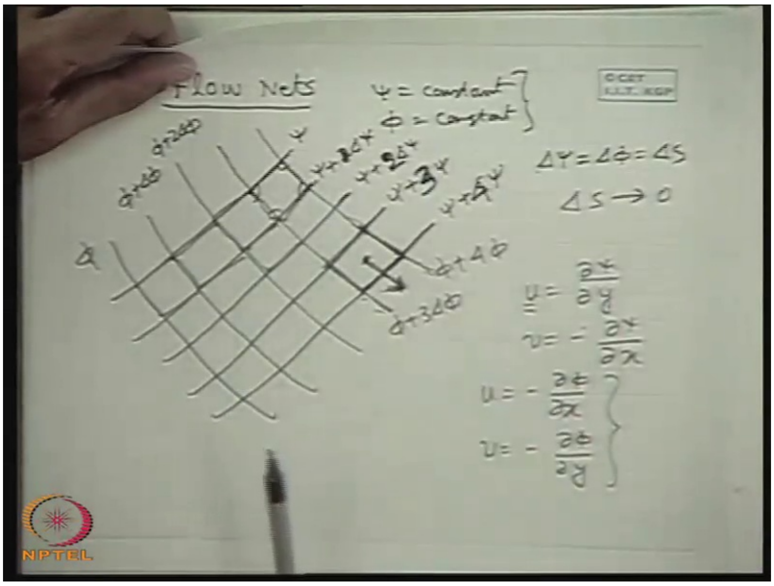

Now, after this I will give you the concept of flow nets, concept of flow nets. What is flow net, concept of flow nets. Now, flow nets are that in a flow field we have seen that psi is equal to constant line that is stream lines and phi is equal to constant line that are equipotential lines are orthogonal.

So, a series of such lines in a flow field comprises a flow net, a flow net; that means, let this is psi, this is psi plus 2 del psi with an interval of delta psi. If I draw the equipot, sorry, if I draw the stream line the constant, sorry, constant psi lines that is five lines I draw for example, four lines psi, psi plus 2 delta psi, sorry, it will be psi plus delta psi, it will be 2 delta psi, it will be 3 delta psi, it will be 4 delta psi. So, with an interval of delta psi I draw it.

Similarly, if I draw the equipotential line which are orthogonal to this set five. This lines is five plus delta phi, then this line phi plus 2 delta phi, then similarly phi plus 3 delta phi, five plus 4 delta phi. Then this two sets of lines form a mesh close a network of such closed meshes. So, these are known as flow nets. This entire network is known as flow net; this gives a quadrilateral whose angels are 90 degree.

Now, in a special case when del psi is equal to del phi is equal to some del S and del S tends to 0; that means, psi and phi lines are very close each other this goes to a perfect square. Now, certain things are obvious from the qualitative picture of this; now for example, you know that u is equal to del psi del y; v is equal to minus del psi del x similarly, u is equal to minus del phi del x, v is equal to minus del phi del y.

So, you see the closeness of the psi lines and also phi lines represents the fluid at higher velocity flow, higher velocity flow in a flow field where velocities are high. So, for example, here this psi lines will be very close; that means, if u is high for a given delta psi, delta y will be less or for a given delta y, delta psi will be high; that means, for a high velocity this psi lines will be close together similarly phi will be. For example, here if we define the velocity component, for example, we have to find out the difference for example, here the velocity component here in this direction is the difference between this psi function and this psi function divided by this length, divided by this length; then delta psi divided by this length. Similarly, to define the potential from the potential phi cost and phi lines this is the difference between this phi lines and this phi lines divided by this distance.

So, therefore, you see we can very well find out that, the closeness of this psi and closeness of the phi will indicate that a velocity is very high. Again I am telling the velocity component here will be found out as del psi del y for example, that this psi function minus this psi function divided by this length, this length because its normal to the velocity whereas, in terms of phi this value of phi lines, the value of phi along this lines, value of phi along this lines divided by, the divided by this length in this case this line. So, therefore, we can is, we can construct flow nets in such situation.

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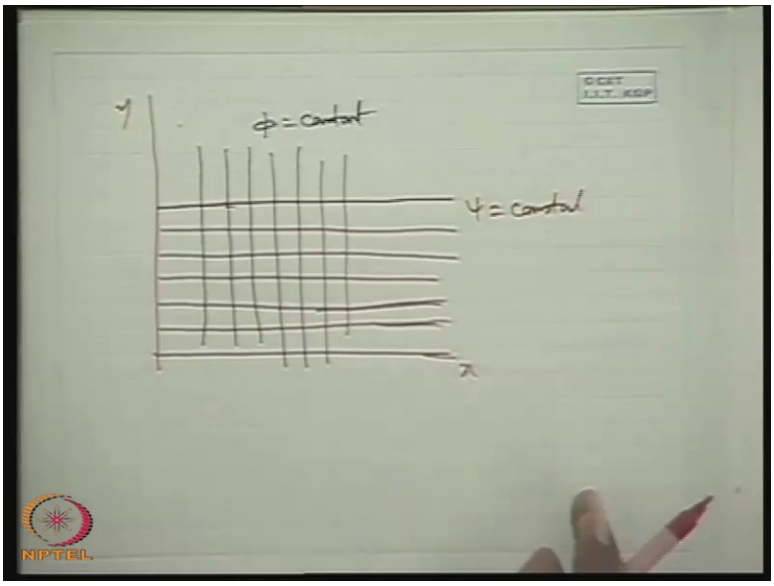

So, another type of flow net for example, if the x y, if the stream lines are along the x axis. So, then this will be the flow for example, x y; a very this will be the flow net, structure of the flow net with the, with the similar interval and if this be the, so obviously, in that case the phi lines will be the perpendicular lines along the y axis. So, this will corresponds to. These are phi is equal to constant lines the vertical lines are phi is equal to constant line, and similarly the horizontal lines are psi is equal to constant line. So, typical square with del psi and del phi remaining same that this and this.

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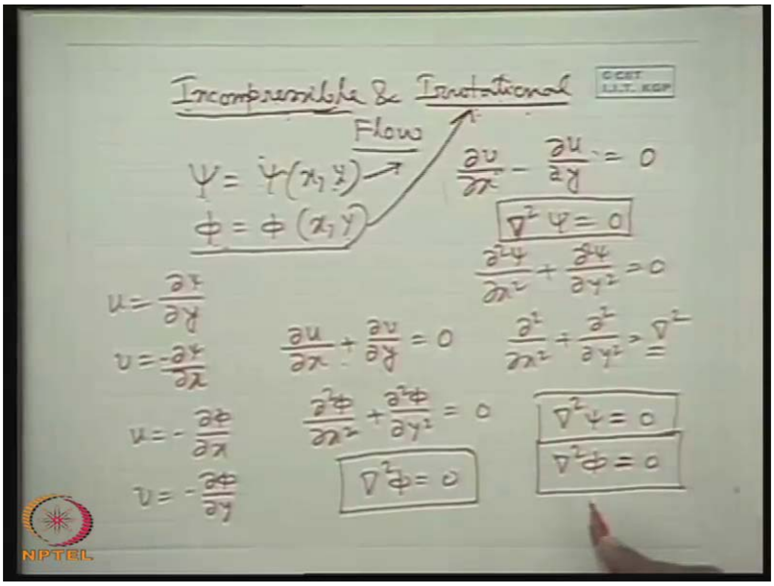

So, that this gives a flow net for a flow field, where the fluid velocities are along the direction of the x axis. So, stream lines are parallel to the x axis. So, this is the concept of flow net. Now I will describe, I will tell you one thing that with the definition of psi and phi function, you must know one very important thing that, we have defined that for an incompressible flow, incompressible and irrotational flow that for an incompressible. Now the question comes like this, for an incompressible and irrotational flow, now for an incompressible flow the psi function exist; incompressible to dimensional flow. So, existence of psi function comes from an incompressible to dimensional flow; that means, the function automatically satisfies the continuity equation. Similarly if the flow is irrotational, then the existence of potential function comes. So, if flow is not irrotational this function will not come; if the flow is not incompressible the psi function will not come.

Now, the flow is both incompressible and irrotational, then what happens? Due to incompressibility the psi function exist. It is the consequence of incompressibility. So, if I substitute the psi function in the condition of irritation, irrotationality what we get, del v del x minus del u del y. It is not satisfying automatically because it is a consequence of incompressibility not of the irrotationality. So, if we substitute this, psi you recall that u is equal to del psi del y and v is equal to minus del psi del x. So, if you substitute this you get del square psi is equal to 0; that means, del square psi del x square plus del square psi del y square is 0; that means, the laplacian, this is known as laplacian you know, that this operator in a two-dimensional case del square, this is known as laplacian vector of a laplacian, laplacian psi is 0. So, therefore, psi satisfies the laplace equation in case of irrotational.

Similarly, if we see that due to the irrotationality phi function exist; that means, phi function is the consequence of irrotationality. So, if simultaneously fluid is incompressible what equation does this phi function satisfy; that means, irrotational incompressible flow equation means if you use the equation of continuity for which psi satisfies automatically, but not phi, phi originates from this phi satisfied automatically this equation if you substitute this phi in terms of phi with the definition is that u is minus del phi del x and v is equal to del phi minus del phi del y. We get the same equation, same type of equation not the same equation; del square phi del x square plus del square phi del y square is 0; that means, laplacian phi is 0.

So, therefore, for an incompressible and irrotational flow this psi function satisfies the laplace equation and phi function also satisfied. So, to evaluate the psi function and phi function, we have to solve only the laplacian psi is equal to 0 and laplacian phi is equal to 0 with requisite boundary conditions.

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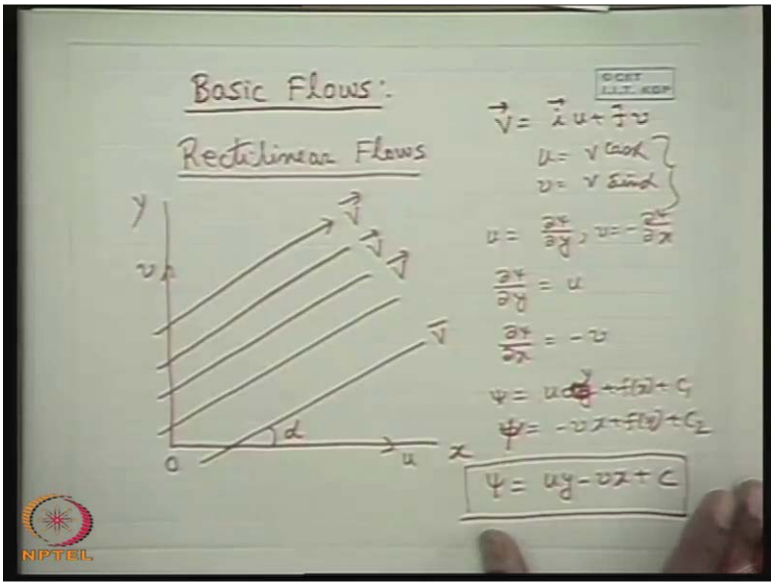

Now, after this I will discuss certain basic flows, certain basic flows, basic flows. First we discuss rectilinear, rectilinear flow. Rectilinear flows are flows in straight line and the velocity vectors are parallel to each other; that means, if we define it in terms of a two-dimensional Cartesian coordinate system. Most generalize type of rectilinear flow is that along a straight line parallel to each other the velocity vector is like this. Sorry, that is the velocity vector. This makes an angle alpha with the x axis. So, the velocity vector is can be written as u plus v. So, u v is the component along x and y axis; where, u is equal to v sign and cross alpha and v is equal to v sign alpha.

Now, how to draw the stream lines or stream function, when velocity potential evaluate this stream function and velocity potential function; very simple. So, u is equal to del psi del y; that means, del psi del y and v is equal to minus del psi del x. So, del psi del y is u what is u v cos alpha or I can write simply u, u which is constant. In case of rectilinear flow v is constant along a straight line. So, similarly del psi del x is minus v.

So, if you integrate this two partial differential equation. So, you get from here psi is equal to u y plus function of x plus sum constant c 1 and phi is equal to minus v, I am sorry, it is u into y after a minus v x plus function of y plus c 2; comparing this two this psi, sorry comparing this two psi is equal to u y minus v x. So, this will not of a function of x will be this, function of y will be this plus sum constant. So, this is the equation of the stream line.

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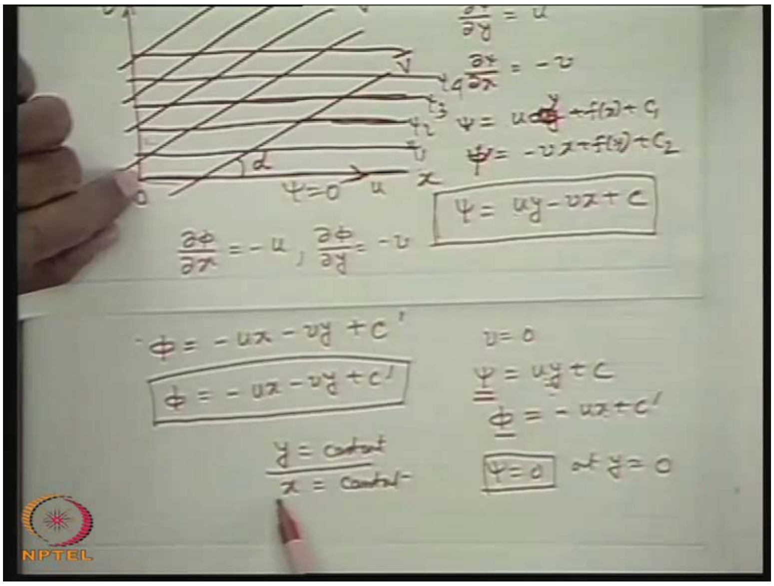

So, if you want to find out the equation of the velocity potential in the similar way, we write that del phi del x is minus u and del phi del y is minus v and if we integrate it, which phi is minus u x minus v y plus another c, let this is c this is c dash. So, therefore, we see that two equation u y minus v x plus c and we get phi is equal to minus u x minus v y plus c dash. So, these are the two equations for stream line.

Now, in a special case for example, if the v component is 0, so velocity is only in the x axis along the x axis psi is simply u y plus c and this phi is simply in that case if we is 0 minus u x plus c dash; that means, a constant psi line in that case will be u y plus c is constant; that means, in one word y is equal to constant and a constant phi line will be minus u x plus c dash is constant or in other way x is equal to constant; that means, a constant phi stream lines are the lines parallel to x axis because y is constant.

So, c may be made 0 if I define psi is equal to 0 at y is equal to 0. I can define the value of psi 0 at any arbitrary location, because it is the difference in psi which is most important to find the velocity, because we are finding out the velocity in terms of the differentiation; that means, I can define this psi on along the x axis; that means, coinciding the x axis x axis itself is the line psi is equal to 0 for this flow. So, therefore, this is psi 1, psi 2, psi 3 psi 4. Similarly, x is equal to constant and phi is equal to 0; I can define the y axis as phi 0 and all lines parallel to y axis are different phi 1; are different phi 1, phi 2, phi 3, phi 4. So, this you can find out the psi and phi function.

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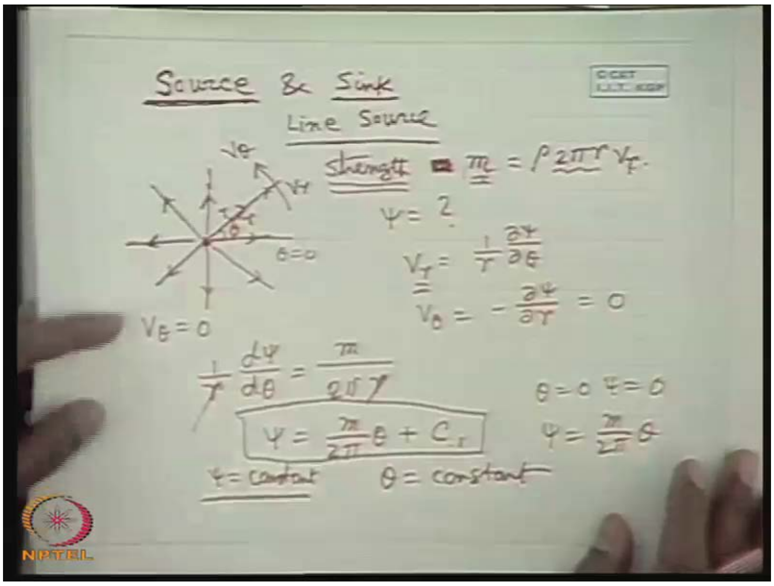

Now, we come to the another type of flow that is source and sink. What is a concept of source, which is very important. We have already define the vortex flows. So, I am not going to repeat the vortex flows, the free vortex and force vortex flows which constitute the flow of ideal fluid.

Now source and sink, now source and sink. First off all source, source and sink. Source and sink. Now source is define like that in a two-dimensional flow, source is like this from a point where source flow is like this, where the stream lines are coming out radially; that means, the flow are coming out radially and uniformly in all direction. Then streamlines are radial lines with uniform spacing; this is the concept of source, where from the fluid is emanating in all radial directions uniformly.

So, that is the concept of source and in a two-dimensional concept I can write or we can write a line source; that means, in a two-dimensional plane it is a line source; that means, it is a line perpendicular to the plane of the figure, which is seen as a point in a two-dimensional plane end on; that if a line is their perpendicular to this plane, then this lines seen end on will give a point p which is the point source, point source from where the fluid is emanating radially outwards uniformly. So, this is the basic definition of a source. And sink is just reverse of the source; that means, where fluid is approaching or coming towards that point uniformly from all radial directions, that is the sink. That is the negative version of the source.

Now for a line source, the strength of the source is defined as. So, a terminology is known as strength of the source, strength m usually. So, strength sorry, sometimes m sometimes we can define as gamma. Let us define as m the strength. It is nothing but the mass fluoride coming out from the source. Now, what is the mass fluoride coming out per unit length? In case of two-dimensional flow if you consider a linear element perpendicular to this plane, per unit length will be the density of the fluid times twice pi r at any radial location r twice pi r. So, this is simply the definition of the mass fluoride type in terms of mass fluoride m is equal to row twice pi r.

So, therefore, we can define the strength of the source as the mass fluoride which is twice pi r times in unit length times the radius. So, this is the flow area and this is row into sorry V r. So, into V r; that means, twice pi r into unit length is the surface area times the V r times the row this is the mass fluoride. So, this is the m is the known as strength of this.

Now, you find out the expression for stream function psi. Now here, as the fluid is coming in all radial directions uniformly, it is better to adopt a polar coordinate system that a point here by giving r has the radial coordinate and theta has the azimuthal angle and we consider any arbitrary reference line has theta is equal to 0 from where the azimuthal angle theta is measured.

So, if you recall the definition and if we consider the V r is the radial direction velocity and V theta is the azimuthal direction velocity. Then if you recall the stream function in terms of the V r V theta is equal to V r is 1 by r; similar way we have done for a x y coordinates or Cartesian coordinate V theta is minus del psi del r. So, these are the definitions of V r V theta.

So, from here we can see that for a source by definition V theta is 0. So, therefore, this is 0 which means psi is not a function of r del psi del r is 0. So, therefore, psi is a function of theta only. So, we can change this partial derivative to a ordinary differential form; that means, a total derivative d psi d theta is V r. Now, V r in terms of this strength of the source, that the mass fluoride which remains constant in the flow field m by twice pi rho 2 pi rho r. So, that is the V r. So, if you multiply it, so r r cancels; we get psi is equal to m by 2 pi theta plus constant c, c 1 for example.

So, therefore, a constant psi line, psi is equal to constant if I put we will give m by 2 pi theta plus c 1 is constant; that means, theta is equal to constant ; that means, along a constant psi line theta is constant; that means, these are the radial line. So, therefore, the psi lines constant psi lines are the radial lines theta is equal to constant. And this is the expression for the psi, stream function psi. If I consider at theta is equal to 0, psi is equal to 0; then c 1 is equal to 0 psi is equal to m by 2 pi theta and accordingly we can ascribe the values of psi for different thetas.

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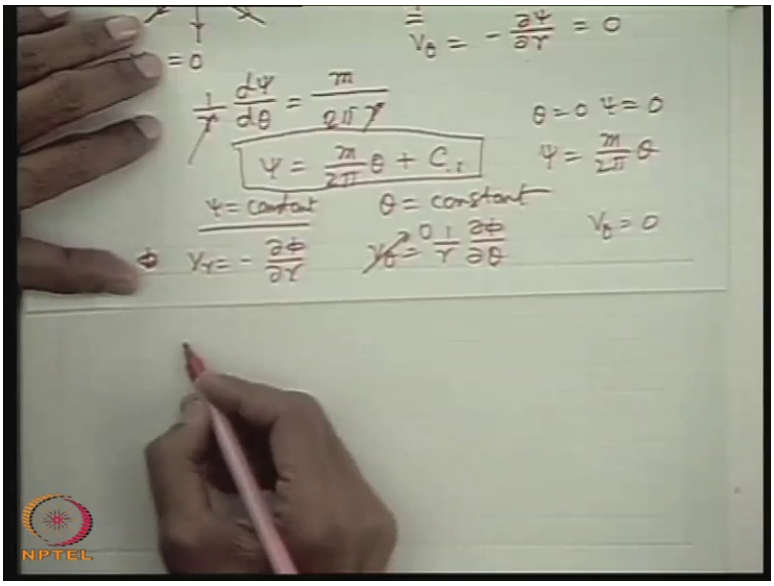

So, well now if I find out the values of phi; in this case what you will see? We will see the values of phi, in this case will be psi is equal to, sorry I can write V r is minus del phi del r and V theta is 1 by r del phi del theta. And if you make that definition that V theta is 0 for a source flow. So, this is 0. So, phi is not a function of theta phi is a function of r.

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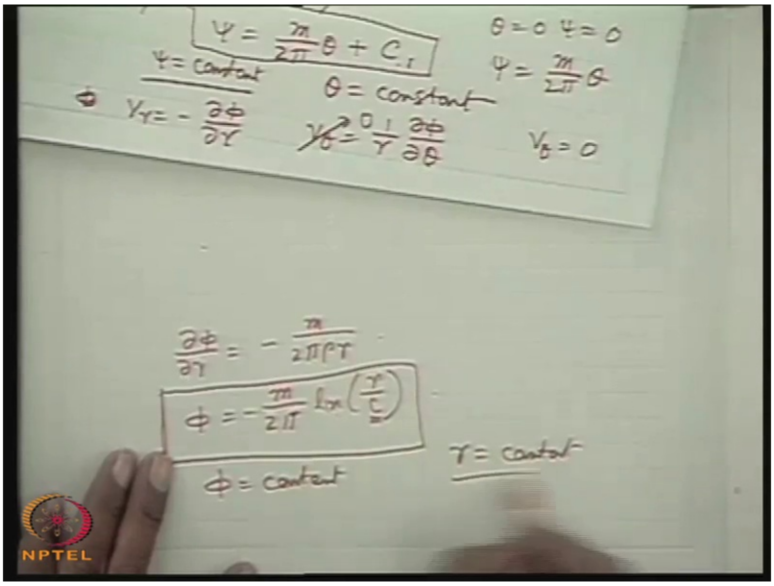

So, therefore, I can write del phi del r is minus V r. Again V r is minus, you remember that V r is m by rho twice pi r, m by twice pi rho r. If you multi, if you just integrate it. It will be like this m by twice pi minus l n r by c, we just induce some constant. So, m by 2 pi l n r by c. So, this is the expression for velocity potential function. So, phi is equal to constant lines, constant line will be the line along which this is equal to constant, which means l n r by c constants which means r is equal to constant.

So, r is equal to constant line will be the phi lines; that means, phi lines will be the concentric circle is an; obviously, we have already recognized that psi lines and phi lines will form orthogonal families of curls. So, if the radial lines or the psi lines are the concentric circle will be the constant phi line. So, this is the expression of potential function, velocity potential function. This is the expression for stream function in case of a source. For a sink, it will be just the reverse. So, with a negative sign this expressions will show the expression for stream functions and the velocity potential functions for a sink, qualitative picture will remain the same.

Thank you.