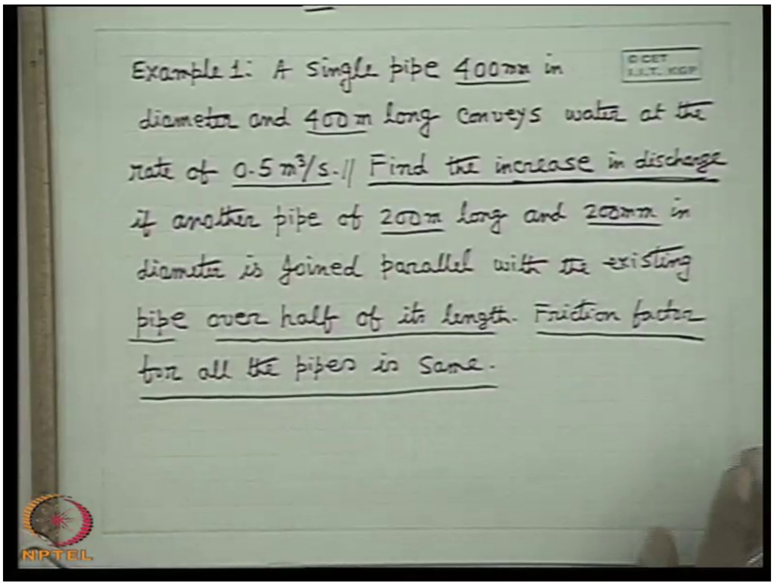
Fluid Mechanics
Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 35
Application if Viscous Flow through pipes Part – III

Good morning, I welcome you to this session of fluid mechanics. We were discussing the pipe flow problems, viscous flows through pipes. Different types of losses, while flowing through pipes fluid flowing through pipes. Today, we will be solving some problems, which will give you a better understanding of the principles we have discussed.

(Refer Slide Time: 00:48)



Example 1: A single pipe 400mm in diameter and 400m long conveys water at the rate of $0.5\ m^3/s$.// Find the increase in discharge if another pipe of 200m long and 200mm in diameter is joined parallel with the existing pipe over half of its length. Friction factor for all the pipes is same.

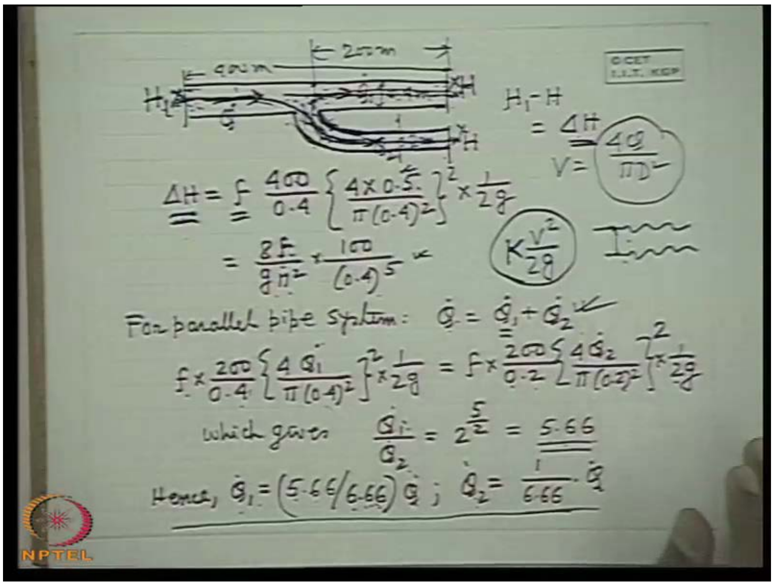
Now, let us first see the example one. You just see a single pipe 400 millimeter in diameter and 400 millimeter long diameter and length coveys water at the rate of 0.5 meter cube per second a single pipe of this diameter and this length. Find the increase in discharge if another pipe of 200 meter long and 200 millimeter in diameter is joined parallel with the existing pipe over half of its length. Friction factor for all the pipes is same.

This problem is very simple, but this problem underlines one principle that when you make a parallel joined of pipe, this increases the discharge under the same head difference. This is the principle which you have come across earlier also in electrical circuit, that if the potential difference remain same, that the parallel circuit always reduces the resistances. If I use a resistance, parallel circuit reduces the resistance. So, the equivalent resistance is lower than the lowest resistance.

So, therefore, if another pipe of 200 meter long and 200 millimeter in diameter is joined parallel with the existing pipe, of course, it depends upon the resistance of the pipe ,over half of its length. There is an increase in discharge just like an electrical circuit where there is an increase in electrical current. Friction factor for all the pipes is same this is a simple assumption given for the problem.

So, now problem is there is a single pipe 400 millimeter diameter and 400 meter long and it discharged is the first condition. And second one and another pipe 200 meters long and 200 millimeter diameter is joined parallel half of its length. Find the increasing distance.

(Refer Slide Time: 02:29)



$$H_1 - H = \Delta H$$

$$V = \frac{4Q}{\pi D^2}$$

$$\Delta H = f \frac{400}{0.4} \left\{ \frac{4 \times 0.5}{\pi (0.4)^2} \right\}^2 \times \frac{1}{2g}$$

$$= \frac{8f}{g\pi^2} \times \frac{100}{(0.4)^5}$$

$$K\frac{V^2}{2g}$$

For parallel pipe system: $\dot{Q} = \dot{Q}_1 + \dot{Q}_2$

$$f \times \frac{200}{0.4} \left\{ \frac{4\dot{Q}_1}{\pi (0.4)^2} \right\}^2 \times \frac{1}{2g} = f \times \frac{200}{0.2} \left\{ \frac{4\dot{Q}_2}{\pi (0.2)^2} \right\}^2 \times \frac{1}{2g}$$

which gives $\frac{\dot{Q}_1}{\dot{Q}_2} = 2^{\frac{5}{2}} = 5.66$

Hence, $\dot{Q}_1 = (5.66/6.66)\,\dot{Q}$; $\dot{Q}_2 = \frac{1}{6.66} \cdot \dot{Q}$

The problem is like this. H 1 is the first pipe, 400 meter length and 400 millimeter diameter; that means this is 400 meter length and this diameter is 0.4 meter; and this length is the pipe length, this length is 200 meter and this diameter is 0.2 meter. So, this is joined.

Now, in the first instant there is no other pipe. So, single pipe which conveys 0.5 meter Q per second, if I write the frictional formula. What I first assume that, H 1 is the inlet head and the outlet head which should be same for the parallel pipe is H. H 1 minus H delta H. So, delta H is the driving potential or driving head for the flow. So, when the single pipe was there I can write delta H is F L by D V square by 2 g. This is the V. Because V is always Q by A which is pi D square by 4. So, always for a pipe circular cross section this is the value of V.

So, when the Q is 0.5 meter Q per second, I just substitute this value from which we I get the friction factor. So, this I have already seen earlier, always 8 f by g pi square coming. So, from which I get the values of delta H. Friction factor is not found, because delta H is not given. So, from the friction equation for a single pipe, I make a relationship between the head causing the flow, with the friction factor in this fashion.

Now, for parallel pipe system now what are the main principles of parallel pipe system I told very simple thing commonsense, that the total flow now when the parallel pipes when another pipe is added? So, this flow rate is changed. This 0.5 meter cube per second is changed. So, a new flow is setup to the entire system. Let us consider this new flow rate is Q dot, which is now coming from the main source; that means, from here inlet plane which is getting divided in the two pipes. So, if I consider Q 1 dot is the flow rate through this pipe 400 meter diameter millimeter diameter and Q 2 is the 200 millimeter diameter pipe flow rate than, the sum of this is Q dot continuity.

Now, if I write the friction equations for both the parallel pipes, I told you earlier how we found out the flow distribution by writing the friction equation for both the parallel pipes between the two junctions. So, what we can do? The head loss between this point and that point, and between this and this, through this path is same. Heads at these points are same, through this path and through this path.

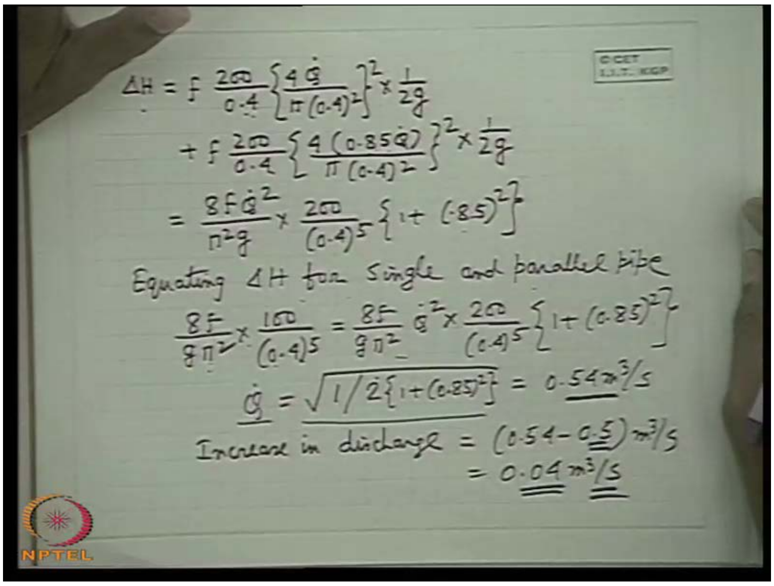
So, this head is same. So, head loss is same; that means, head loss is equated for two pipes, parallel pipes. So, for this pipe f L by D this is 200 meter length, 4 Q 1 dot by the velocity average velocity in this pipe related to the flow rate in this pipe which is Q 1 dot is equal to the head loss via this parallel pipe is L by D, the average velocity is related to the flow rate through this pipe that is Q 2 dot 2 g.

So, if you solve it we get a relationship Q 1 dot by Q 2 dot 2 to the power 5 by 2, it will everything cancels if same 1 by 2 g 1 by 2 g pi pi 4 4. So, only 0.4 and 0.2 is there. So, one is twice of the other. So, two to the power 5 by 2 this comes this. ? Therefore, Q 1 by

Q 2 ratio is there. So, Q 1 Q 2 can be expressed in terms of Q. One is 5.66 by 6.66 that is the Q 1, larger one; another is 1 by 1 plus 5.66. So, you can find out that Q 1 in terms of Q, Q 2 in terms Q.

So, therefore, the flow rates are known in terms of a single flow rate Q that is the main flow coming from this. So, now, our job is to find out this Q. So, this Q can be found out by writing the friction equation in any hydraulic path. Either you take this path the main pipe via this pipe or you can write the hydraulic equation via this path. That means, either you write the Bernoulli's equation between this point, and this one via this path main pipe or this point and this point via main pipe and then this parallel pipe, so, any of the pipes. So, that if you write the Bernoulli's equation means the friction equation. Because, delta H is f L by D this comes from the application of Bernoulli's equation only. So, if you write, then you can find out the value of Q 2 this is the trick.

(Refer Slide Time: 07:15)



$$\Delta H = f\,\frac{200}{0.4}\left\{\frac{4\dot{Q}}{\pi(0.4)^2}\right\}^2 \times \frac{1}{2g} + f\,\frac{200}{0.4}\left\{\frac{4(0.85\dot{Q})}{\pi(0.4)^2}\right\}^2 \times \frac{1}{2g}$$

$$= \frac{8f\dot{Q}^2}{\pi^2 g} \times \frac{200}{(0.4)^5}\left\{1 + (.85)^2\right\}$$

Equating ΔH for single and parallel pipe

$$\frac{8f}{g\pi^2} \times \frac{100}{(0.4)^5} = \frac{8f}{g\pi^2}\,\dot{Q}^2 \times \frac{200}{(0.4)^5}\left\{1 + (0.85)^2\right\}$$

$$\underline{\dot{Q}} = \underline{\sqrt{1/2\left\{1 + (0.85)^2\right\}}} = \underline{0.54\ m^3/s}$$

$$\text{Increase in discharge} = (0.54 - 0.\underline{\underline{5}})\ m^3/s = 0.\underline{\underline{04}}\ m^3\underline{\underline{/s}}$$

Now, if you write via this main path, the equation is like this. Delta H, I write this via this path H 1 minus h delta is f L by D up to this length, the frictional loss is this is also 200 half this is 200 then this 200 millimeter and 0.4 diameter f L by D this square, which is related to the main flow Q dot 2 g plus the frictional loss in this path.

There are no other minor losses. If minor losses are there depending upon the geometry, we will have to take care off if there is a fitting, we will have to take care of, but nothing is there, only friction in two parts L by D. It is same L same D, but the velocity is changing. So, 0.85 Q dot is the velocity. Because we found out that Q 1 dot is this Q 2 dot. So, Q 1 dot is this flow, this 5.66 by 6.66 Q dot this is 0.85. So, therefore, the flow rate through this pipe is 0.85 of Q dot. So, this is the flow rate, very simple. So, then; that means, we have an equation, connecting delta H with Q dot square. But delta H was earlier found in terms of friction factor like this, because the same head difference was there, when the single pipe was delivering 0.5 meter Q per second.

So, if we equate this, equating delta is for single and parallel 5; that means, 8 f by g pi square into 100 into 0.45 is equal to this one; that means, if we equate this one 8 f by g pi square earlier with this thing, we get simply the value of Q dot 0.54. So, you see here, now the Q dot which you were earlier 0.5 meter Q per second is now increased to 0.54. So, 0.04 meter three per second. I think this is very simple.

Now, this entry loss, where here at the junction nothing is told, due to the bend there will be some losses, but it is not told. So, how can you do it? In practice there will be loss depending upon the curvature and degree, how is a particle consideration. I am defining the theoretical aspect of this. In practice, if you make a parallel pipe joined. So, there will be some bend of the pipe and then pipe goes like that.

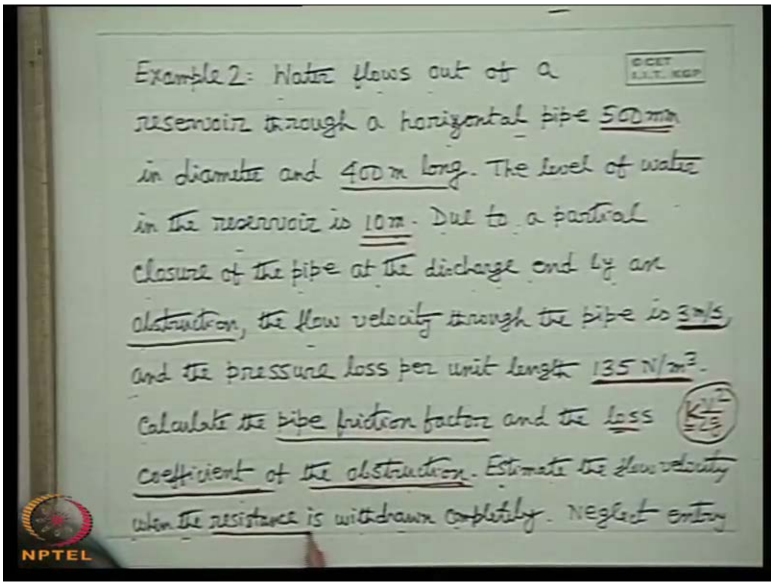
So, due to that bend there will be some losses. In a problem, if that is not told, you cannot find it. In practical application what we will do? You will know the pipe bend, what is the angle of the pipe bend? What is the diameter? Depending upon the diameter and the angle, the loss coefficients for the pipe bends are known, and then you add it.

But here nothing is told, as if two parallel lines like that. So, there are two parallel lines. These two points are same; that means, we neglect this bend losses. There will be some loss definitely. Fluid flow go like that practically a pipe cannot be made parallel like that, it cannot be super imposed on that. So, there will be a bend and this entire thing this we neglect it, because of the lack of the data, we are not told that can be added that consider a bend loss for whose coefficient is this. In that case, what we will do? You will write the equation here.

So, you will have to add K V square by 2 g, if k is the loss coefficient. Because, bend losses and many losses are expressed in terms of the factor K of the velocity here. So, this is the law, this is not given. So, we just neglect it. For example, I can explain very well, that if the problem was given like that, there is a tank and there is a head; that means, then the H 1 is given in terms of the height. So, you can consider H 1 is the height of the surface from here.

So, this is a big tank and this. In that case, I will consider the entry loss, but here H 1 is defined is the head here, just at the inlet after losses and everything. So, nothing is given here. So, if the problem is posed like that, then we will use this H 1 as the height and then we will consider the losses we will consider the entry loss.

(Refer Slide Time: 11:24)



Example 2: Water flows out of a reservoir through a horizontal pipe 500 mm in diameter and 400 m long. The level of water in the reservoir is 10 m. Due to a partial closure of the pipe at the discharge end by an obstruction, the flow velocity through the pipe is 3 m/s, and the pressure loss per unit length 135 N/m³. Calculate the pipe friction factor and the loss coefficient of the obstruction. Estimate the flow velocity when the resistance is withdrawn completely. Neglect entry

$K\frac{V^2}{2g}$

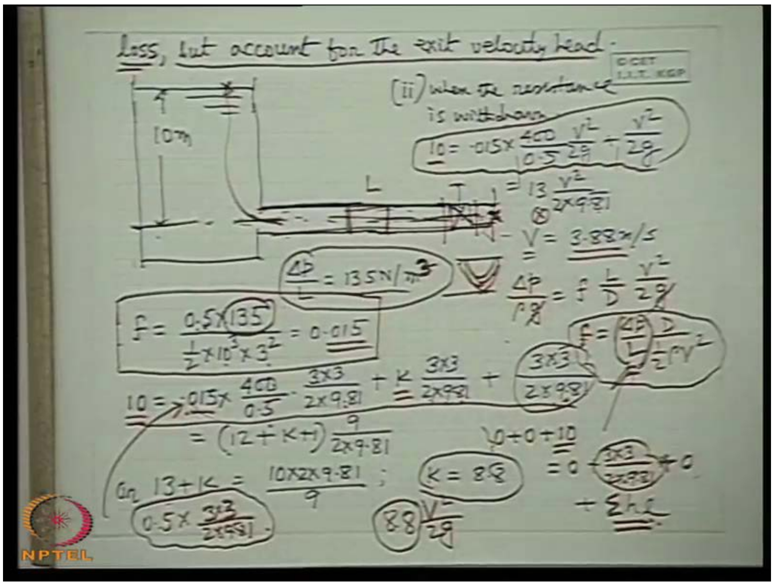
Now, example 2; water flows out of a reservoir through a horizontal pipe 500 millimeter in diameter and 400 meter long. The level of water in the reservoir is 10 meter. Due to a partial closure of the pipe at the discharge end by an obstruction, the flow velocity through the pipe is 3 meter per second, and the pressure loss per unit length is 135 Newton per meter cube. This is very important data; that means, in a pipe, this type of pipe this particular case the pressure loss per unit length is constant almost. That is why they are specifying the pressure loss per unit length in the pipe is this one.

Calculate the pipe friction factor and the loss coefficient of the obstruction. So, again we should think of the problem. This is another exercise problem of my book. Last one was also one of the exercise problems of my book. Water flows out of a reservoir; that means, there is a reservoir through which water flows out through a horizontal pipe 500 millimeter in diameter that is explicitly pipe geometry is known both diameter and length.

The level of water in the reservoir is 10 meter, means in this case the level is kept constant, steady. Problem due to a partial closure of the pipe at the discharge end it is very practical problem; that means, at the discharge end of the pipe there is a ball which is partially closed; that means, which controls the flow the flow velocity through the pipe is 3 meter per second. So, a valve is there or an obstruction you consider, valve is an obstruction. So, which makes the flow reduce and in that condition the flow velocity through the pipe is three meter per second.

And at that condition the pressure loss per unit length is 135 Newton per meter Q, what we will have to calculate for this problem pipe friction factor and the loss coefficient of the obstruction. So, obstruction gives some loss; that means, you consider the losses due to obstruction and find out what is the loss coefficient means, that K. That means, the obstruction loss, if it is expressed in terms of K V square by 2 g, this is known as loss coefficient or loss factor. In a second part estimate the flow velocity when the resistance is withdrawn complete; that means, if the valve is widely open; that means, there is no resistance given; that means, resistance is withdrawn completely then flow velocity will definitely increase.

(Refer Slide Time: 14:08)



So, this is increased. Neglect entry loss, yes the problem tells neglect entry loss. This is not correct. I could have done with the entry loss also, but I have not done with that neglect entry loss, but account for the exit velocity here. Let us see the problem, this is a reservoir. So, 10 meter head and this is a long pipe there is a valve. First case is that, now delta P by L is 135 Newton per meter cube. What is f now? If you write the equation f is L by D, now first of all you will have to find out the friction factor. From this formula, now in this problem, delta P by L is given. So, I can take any length L without any entry loss, even if entry loss has to be taken into account or loss coefficient over any length L, you know the formula is delta P or delta H delta P by rho g is f L by D v square by 2 g g g you can cancel.

So, one can find out, f is equal to delta P by L into D by half rho V square; that means, if I know delta p by L pressure drop per unit length. So, wherever for any length I can find out friction factor, simply by this formula, that I can apply anywhere in between in the pipe, where delta P by L if delta P by L value is straight forward known.

So, this is the simplest form of a problem, where f can be found out simply. By this no other loss comes into picture over any length where there is no obstruction anything. So, this formula is written as: f is equal to this delta P by L delta P by L this is D; obviously, D is what 500 millimeter; that means, 0.5 meter half rho P square. So, we get straight away friction factor.

Now, the rest part is very simple. If I write the Bernoulli's equation between this point and this point, we get the equation 10 Bernoulli's equation the pressure head is 0 velocity head is 0. So, it is 10. So, I take this center line as the data, then of course, if entry loss is there; we will have to add here 0.5 into V square by 2 g V is 3 into 33 into 3 by 2 into 9.8 actually this could have been taken into account.

But the problem categorically tells that you neglect entry loss. So, this case you should neglect, otherwise people will think that we have marked the formula and we have done it. So, neglect the entry loss 0.5 V square by 2 g should come first, but this you neglect. So, next loss is the loss due to pipe friction; that means, f L by D into V square by 2 g plus when the flow comes through this is a valve representation. So, there is a restriction. Different types of valves are there. One type of valve there is a gate type of thing which comes same; gate comes like this and closes the passage. So, it falls like this.

So, this is a restriction. So, it gives a pressure drop or the head loss whatever you call. So, this loss is expressed as K V square by 2 g K V square by 2 g plus V square by 2 g. This is the velocity head. Here, V square by 2 g. So, this writing is actually a typical writing for friction equation, but if one writes the Bernoulli's equation, he will write like that 0 plus 0 plus 10 p by rho g is equal to here P by rho g is 0, but v square by 2 g 3 into 3 into 2 into 9.8 1 plus the data made is plus 0 plus all the losses encountered in the path.

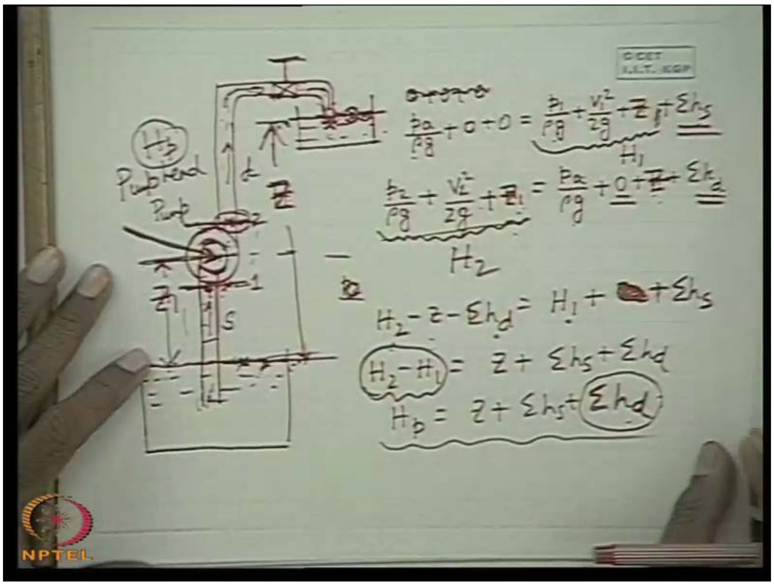
That means, this will become fast in this modified Bernoulli's equation. We write the pressure head velocity head. This is the conventional sequence data made potential head plus the losses in the downstream side point or downstream side section plus the losses if the flow is viscous; that means, a modified form of the Bernoulli's equation considering the viscosity of the fluid.

But in conventional friction equation, people go on writing as if he is following the path like that. First the fluid encounters this loss entry loss then the friction loss then the this loss plus it is having a velocity head; that means, a simple energy balance, energy accountability, which ever you look into the problem, the equation remain same; that means, this side 10 is equal to the velocity head plus h l. He would not tell that I will write the equation for a head when fluid velocity is not there.

For example, if it was in a reservoir, then this velocity head will appear also as the loss as you look into it that is the exit loss. So, if you write the Bernoulli's equation here, then it appears at the velocity head. For example, if it is connected with another reservoir and if you write the velocity head far end, where the velocity is arrested. Then it will be the exit loss there so; that means, this will be the same equation we get whatever way we tell. So, now this equation is solved, and it is only a simple equation involving only K as the unknown. So, we get the value of K as 8.8; that means, the loss coefficient is 8.8; that means, 8.8 into V square by 2 g. This is the meaning of the loss coefficient that the factor multiplying factor with the dynamic.

Now, when the resistance is completely withdrawn, it is a child’s job this is equal to without any K V square by 2 g loss; that means, it is the frictional loss plus the velocity; that means, if I write the Bernoulli's equation between this point and this point the basic frictional equation is like and then we get the velocity of flow is 3.8. So, the velocity of flow is increased from 3 meter per second when the resistance is completely withdrawn any problem in this.

(Refer Slide Time: 21:06)



$$\frac{p_a}{\rho g}+0+0=\underbrace{\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+\cancel{z_1}}_{H_1}+\Sigma h_s$$

$$\underbrace{\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+\cancel{z_1}}_{H_2}=\frac{p_a}{\rho g}+0+\cancel{z}+\Sigma h_d$$

$$H_2-z-\Sigma h_d=H_1+\Sigma h_s$$

$$\boxed{H_2-H_1}=z+\Sigma h_s+\Sigma h_d$$

$$H_p=z+\Sigma h_s+\boxed{\Sigma h_d}$$

Now, I like to discuss one very interesting thing that, sometimes that you will come across in case of a pump for example, if a pump is fitted in a pipe line system; that means, in the operation of the pump. Let us consider the fluid is taken from this sum from a lower level and there is a pump. How would you solve this type of problem? Let there is a pump which discharges the fluid into a reservoir which discharges the fluid into a reservoir, the reservoir overhead, how do you solve this type of problem? Now, let us consider the difference between this reservoirs, is H from the free surface of the lower reservoir which is known as sum and this center line of the pump from here. Let this is H 1; this also can be solved by pipe flow problem. This is a very interesting one, there is a pump in deep, intelligent students can get it.

Now, what happens? The purpose of the pump is to shake the liquid from lower level; the liquid is flowing from a lower level to a higher level. This is because energy is added here in the pump energy is added in the pump. So, with energy addition, it is possible to take liquid from a lower head to upper head; that means, the potential energy of the liquid is increased by virtue of the energy. Electrical energy in the form of mechanical energy external electrical converted to mechanical energy is imparted to change the potential energy of the fluid. That means, a fluid at a lower data made is being lifted. So, this is flowing like that.

Now, let this tube is designated at s section tube, section side of the pump this tube is delivery side of the pump. By simple friction formula, you can find out this, you can do this problem. How you can do this problem? Let us see. Let us define a section here. This section is one and this is the free surface let us define a section here that is let the section and that is the final top surface of the surface.

Now, if I write the Bernoulli's equation, between point at any point, here at this section, that the entry to the section and at this point. What can I write if this 0.0 try to understand pressure head 0 velocities at 0 and data made 0? Why data made this is the data made now pressure head I should not write 0, otherwise it will be confusing. Because pressure here is negative, pressure otherwise it will not shake from the atmosphere. So, in this case, I induce the atmospheric pressure p a by rho g plus velocity head is 0 plus data made is 0. Because, I am considering the datum from here reference datum then what is the head here. Let we define the pressure head p 1 by rho g this p 1 by rho g is less than p a by rho g.

You can understand. Because some other things will come because to create a velocity head here out of the pressure head atmospheric pressure head this pressure head has to go lower than that; obviously, V 1 square by 2 g if V 1 is the velocity in this point; that means, in the pipe if the pipe is of uniform diameter plus H 1.

Now, this is upon the center line. So, inlet to the pump and outlet from the pump does not very much in the elevation head as compared to a center line, in comparison to the difference in the head from the lower reservoir to upper reservoir. That is sum from sum to the overhead reservoir. This changes a very low. So, this is H 1. So, we define this as H 1. That means, this is the height from the sum p of the pump center line which can be taken as the height of the pump inlet and pump outlet. So, this is the equation.

Plus all the losses in the sections side, that means, sigma h s. So, this is the pipe friction loss section side there may be a bend, there may be a valve, many things may be there. So, all the losses encountered in the section side. So, therefore, I am not posing the problem as such that it will be only frictional losses depending upon the problem definitions. So, there may be number of losses.

So, this is the précised Bernoulli's equation. Now, if I write well the Bernoulli's equation from any point at 2, and a point at the free surface of the upper head or overhead reservoir. What we can write? We can write p 2 by rho g, well plus, please tell me V 2 square by 2 g, p 2 by rho g plus V 2 square by 2 g plus H 1 is equal to at this point again I have p a by rho g, a surface at the overhead reservoir only understanding, then 0, then H, then sigma h d. That means, all the losses encountered; this bend; this bend and usually there is a valve in the delivery side. You will afterwards read in fluid machines course valves are not placed in the section side.

So, there is only a step valve in the section side for some other purpose for priming purpose. But there is no such valve in controlling the flow because of capitation problem. Because, now I can tell you because, you know this fluid mechanics principle. Now, because this pressure is section pressure; obviously, from this equation and physical principle also their outer atmospheric pressure we have to create velocity and overcome frictional losses.

So, we should be careful about the losses, because velocity is fixed depending upon the capacity of the pump, flow rate of the pump. So, we should be very careful of the losses. So, that this pressure. So, this quantity should be minimum. So, this pressure should not fall below the vapor pressure of the water at the existing temperature. So, that the capitation will start water will boil off.

So, therefore, it is very essential I have told in any fluid or hydraulic, any hydraulic circuit. Where there is a chance of pressure going below the atmospheric pressure or low pressure that the pressure should not fall the vapor pressure of the working liquid at the working existing temperature. So, therefore, valves are placed in the delivery side only. So, delivery side there will be many losses.

So, all the losses including the exit velocity head, because I have not written the velocity head. You can tell me sir you are writing because I have using the same thing just I described in the last class; that means, I have not taking a point at the delivery or discharge plane of the delivery pipe. I am taking a point at the free surface of the reservoir just is discharge is in the reservoir. Where the liquid has been stagnant; that means, the velocity head which could have come if I could have written the Bernoulli's equation at the discharge plane of the delivery pipe it has come within the losses; that means, the exit loss.

So, exit loss, bend loss valve loss, frictional loss. If nothing is mentioned about any valve fitting nothing is mentioned about any bend in a problem. So, we will consider only the frictional losses, if I know the length of the pipe and diameter of the pipe. But I will not forget to include the velocity head loss that is the exit losses here.

So, now these two things show us one thing. That if we subtract one from other or make p a by rho g equal. So, what we get we get p 2 by now, if I define just p 2 by rho g plus V 2 square by 2 g plus H 1. We defined as well already, we have defined as H, no H 1 and H better we change this definition to z the elevation is z and z 1. It will be easier for you z 1 and z; that means, I just change the nomenclature that z is the difference and z 1.

Now, if I tell that p 2 by rho g plus V 2 square by 2 by g plus z 1 as H 2. What is this? Precisely these are total energy at the outlet of the pump per unit weight; that means total head; head means energy per unit weight. So, total head at the outlet of the pump. Similarly, if I tell p 1 by rho g plus V 1 square by 2 g plus z 1, which is the total energy per unit weight at the inlet of the pump. The total head at the inlet of the pump H 1; then what I can write please tell me.

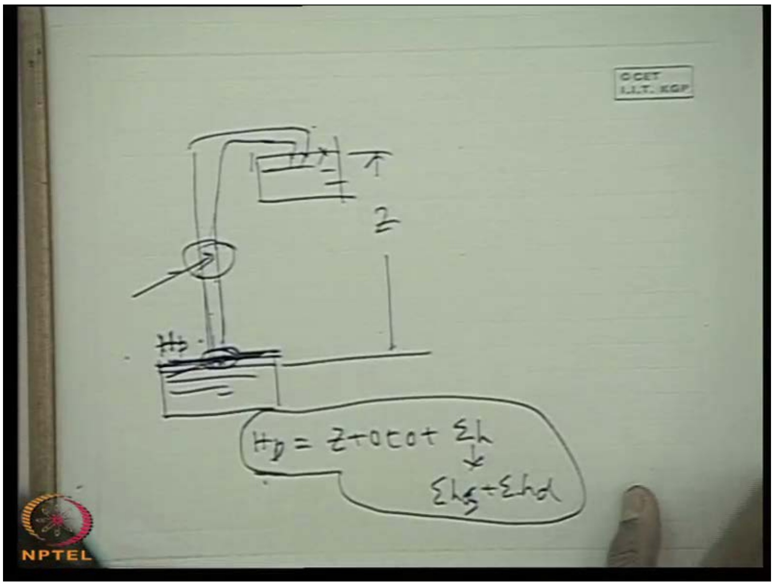
So, then I can write H 2 minus z minus sigma h d, which is equal to p a by rho g, h 2 minus z minus sigma h D is p a by rho g. That means, H 1 plus z 1, some z 1 is missing p 2 by rho g plus V 2 square by 2 g plus z 1 is equal to; this is the z 1 is equal to z 1 no, this is H 2; H 2 means a z 1 is included in the H 2 well H 2 minus z minus sigma h d this is alright is equal to H 1.

Then we can write H 2 minus H 1 is equal to what z plus. Now, this H 2 minus H 1 is the change in the total head from outlet to inlet. That means, total energy per unit weight; that means, this head is given by the pump. So, pump head if I tell that pump head is H p which means the pump develops this energy per unit weight. This is nothing, but H 2 minus H 1. So, therefore, this is the pump head which is equal to z plus sigma h s.

And from a common sense, we can tell definitely without knowing any even Bernoulli's equation nothing pipe flow problem. A intelligent student of class ten, if it is, if he is told this problem, you tell yes pump has to develop an energy which must balance or counter balance or counter wise the energy equivalent to this height. That means, the liquid has to be lifted; that means, this potential energy is created plus the losses and also to create the velocity which of course, is included in h d.

So, from common energy balance, if we will tell the pump energy should be sufficient to overcome the frictional loss to elevate fluid to this height. That means, this potential energy plus, the kinetic energy created by the velocity of the fluid which here is included in the delivery losses, a component of the delivery losses that is exit loss. So, therefore, from the physics point of view this is clear.

(Refer Slide Time: 33:08)



It means, this is equivalent of telling that this is equivalent that there is a single line H p. That means, it is equivalent of telling that there is a single pipe, which simply elevates water to a height z by virtue of which by virtue of some head given to it at this plane, this plane h; that means, it has got H p. So, if I write the Bernoulli's equation between this and this point, I will get H p is equal to z plus sigma, because if I write total head is H p which is given here is equal to here z all other things 0 plus sigma h which composed of sigma h d plus sigma h s plus sigma h d in the actual pump system.

That means, if there is a pump in between in a pipe line. We consider the entire section and delivery pipe lines as a pipe system where the input, because you will see that such a pipe system without any pump cannot make a flow from a lower level to higher level. But this is equivalent in saying that some energy is added; that means, at this free surface, we have extra amount of energy added to it. So, this pump can be taken it here, and can take the entire system as a fluid path or a fluid circuit, where we have energy here which will allow the fluid to flow up, and we can write this equation the same equation.

And this is the basis of developing the equations for a pump system, which I will be discussing afterwards which you will know afterwards. Not now in this course; in the course of fluid machines. So, it is clear to you. So, I think today I will not discuss much because, next class I have thought I will be describing the concept of dimensional analysis. But I feel that today I must first give you a closure of this pipe flow problem.

So, pipe flow problems are the practical problems, where the flow takes place through pipes of circular cross section. But the flow rate and the dimensions of the pipe are such the flows are in turbulent region. In a pipe flow, the flow becomes turbulent in a fully developed pipe flow. When the Reynolds number you know a dimensionless parameter defined as row, the average times, the average velocity.

That is the flow rate by cross sectional area times the hydraulic diameter which is the diameter in case of a pipe of circular cross section divided by mu, this dimensionless parameter known as Reynolds number which is a symbol for the dynamic similarity, which represents the criteria for dynamic similarity which I will discuss afterwards in case of flow governed by viscous force, inertia force and pressure forces.

Of course in a fully developed pipe flow problem, the viscous and pressure forces are the only dominating forces is inertia force is 0. However, we will come across with this thing afterward. So, Reynolds number, this dimensional parameter exceeds 2000 and it has been found in all practical applications the range of flow rate. And the pipe diameter is such that the Reynolds number exceeds 2000. So, in analytical solutions of Navier strokes equations in close form which is known as exact solution is not possible for a turbulent situation as such, but with the advent of computational methods or advanced numerical techniques.

We can solve the turbulent flow situations with turbulence model. But all this turbulence models require empirical information. These are based on certain empirical factors. So, therefore, we can tell even the advanced theories of turbulent based on experimental information. So, without going for such turbulent flow analysis with turbulence model; where again, we require empirical information. Engineers in practice or in floor they have developed very good semi empirical approaches in dealing with the pipe flow problems. In this view I must tell you what a hydraulic engineer needs, he needs that if there is a pipe of some diameter.

And he has to force the fluid at certain rate, what should be the pressure drop over a certain length. And depending on which what should be the power required to transmit that flow. Or you know what should be the power required; you know that this much power is available; you know this much fluid has to be fed through a pipe system, whose length you know that it has to be transported over that length. What should be the exact diameter of the pipe, what diameter of the pipe you will select?

So, these are the practical problem. So, these problems are analyzed by simple way of tackling the Bernoulli's equation modified Bernoulli's equations provided you can identify the loss different fluid losses in the fluid flow path. And these losses are defined through a friction factor or friction coefficient, Darcy’s friction factor we have defined.

And if we know the friction factor before, and then we can use the friction, and solve this type of problems, and this friction factor is a function of Reynolds number, in case of smooth pipe or in laminar flow. We have seen or pipe flow in laminar flow conditions that is Hagen Poiseuille flow friction factor is 16 by R e or Darcy’s friction coefficient or friction factor is 64 by r e 4 times the fanning friction factor which is the basic friction coefficient shear stress by dynamic head.

So, this friction factor is found for different ranges of turbulent flow from experiments, different empirical formulae. And they are nicely documented in a figure by moody known as moody’s diagram. So, we imply moody’s diagram, find out the friction factor. Sometimes the friction factor may be given. Sometimes the equations may be given friction factor relating Reynolds number from which you find out. And we solve the problem in the help of friction factor and the modified Bernoulli's equation. This is a semi empirical approach of solving the pipe flow problems we have discussed. Next class, we will be discussing the principle of similarity and dimensional analysis

Thank you.