

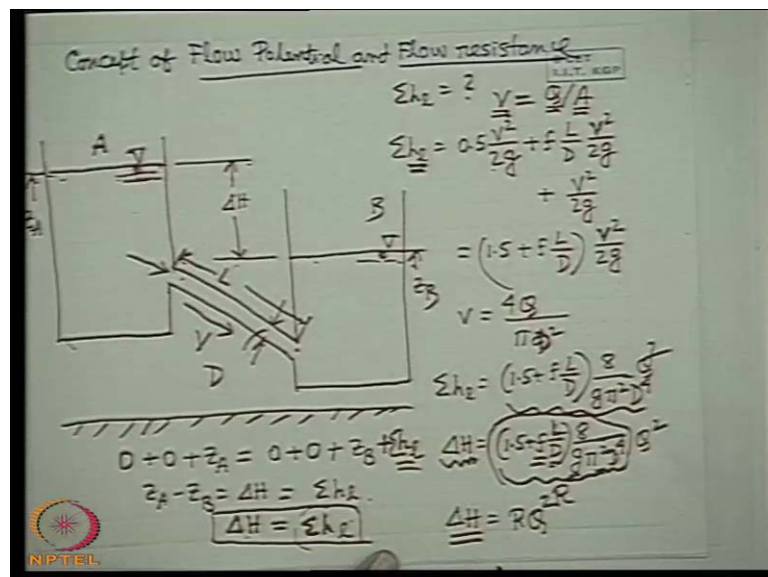
**Fluid Mechanics**  
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**Lecture - 34**  
**Application of Viscous Flow Through pipes Part – II**

I welcome you all to this session of fluid mechanics. Well in the last class, we have just started the discussion on viscous flow through pipes, and we have recognized that how the viscous flow through pipes are solved in practice the semi-empirical approach through the definition of a friction factor. First we recognized that flows are turbulent flows for which a complete analytical solution is not possible. So, these are solved semi-empirically through a definition of friction factor, and we then recognized three classes of problem with which engineers are interested. One class of problem is that a flow rate is given through a given diameter of pipe and given length of pipe; we are interested in finding out the pressure drop and corresponding power required.

Another class of problem that pressure drop is given over a given length and given diameter; we have to find out what is the flow rate. Another class of problem that pressure drop is given, flow rate is given, length is given; we will have to find out the diameter of the pipe to accommodate that flow rate over that pressure drop.

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So, these are the problems that we recognized and how they are solved with the simple

equations through the definition of a friction factor;  $f$  Darcy's friction factor that we have done. Today, we will start what is meant by flow potential and flow resistance. So therefore, you see the concept of today, we will start concept of flow potential and flow resistance; this concept of flow potential and flow resistance. Let us consider a situation like this. Let us consider two tanks which contains the free surfaces of water like this and this is connected by a pipeline. Let us consider the elevation of this free surface in this tank; let this tank is A, this tank is B. A is  $Z_A$  and that of B is  $Z_B$  which may be measured from any reference arbitrary reference data; that  $Z_A$  and  $Z_B$  are the elevation.

Now you see if we maintain a tank A and tank B like that constant head, then water will flow from this tank to this tank; that means water will flow in this direction and it is a common sense now today you can just say that, this difference of head that is  $Z_A$  minus  $Z_B$  which I represents as  $\Delta H$  is the head causing the flow, because this is the head difference which causes the flow in through the tube. So, this is the head causing the flow and if you consider the flow path for any fluid particle starts from this surface to this surface, it has rest here. A flow is induced then it flows through the tube and comes again in a static condition, stagnant condition here. So therefore, this head difference that is the head causing flow is ultimately consumed in the friction; that is the resistance to the flow of fluid. This just comes from the very common sense of physical picture.

Now how can we get this from the application of Bernoulli's equation? Let us write the Bernoulli's equation between point A and B considering the fluid to be a viscous; very simple the Bernoulli's equation the pressure head is 0. If we consider the pressure atmospheric pressure as 0 that any pressure is the gauge pressure, so pressure rate is 0. So, there is no velocity; velocity head is 0, only the potential head  $Z_A$ . Similarly, at this point along this path the pressure head is 0, the velocity head is 0. At this point there is no atmospheric pressure, no velocity plus the elevation head from the same datum is  $Z_B$  plus you know the modified Bernoulli's equation; Bernoulli's equation for ideal flow.

If we write the same Bernoulli's equation that is conservation of energy for a viscous flow, we just write the loss of head; that means, the energy loss per unit way through this fluid path that  $\sigma h_l$ ; that means, there may be various losses. So, all the losses that are encountered while it flows; fluid flows from this point to this point. So, from which we can tell that is  $Z_A$  minus  $Z_B$  which is equal to  $\Delta H$ ; that means the difference in this liquid surface levels is equal to  $\sigma h_l$ . So therefore, we see that  $\Delta H$  is  $\sigma h_l$

$h_l$ , which means this difference of head is maintaining the flow from this point to this point; that means liquid is continuously flowing, which means it just equals to the frictional loss or other losses due to friction itself; that means total loss in the flow of fluid; that means sum of the total losses is equal to the head causing the flow; this the basic equation.

Now head causing the flow is given. So, main part is to find out what are the total losses in the flow circuit or the flow path. So let us find out this, what is this. Now our next job is this. So, Bernoulli's equation tells that the head causing flow must be balanced by or equal to the total resistance in the flow path. What is the total resistance? First of all you see here, if we consider  $V$  is the flow velocity;  $V$  is the average flow velocity which is defined by as  $Q$  by  $A$  flow rate by cross-sectional area;  $A$  is the area of this pipe.

So, now first when it enters here what is the first loss that is entry loss; that means  $\sigma h_l$  first loss that is encountered by the fluid is entry loss which is  $0.5 V^2$ . We discussed it earlier that when fluid approaches from a much larger area to a narrower area, the loss is known as entry loss; this is nothing but the loss due to contraction. When the upstream area is much bigger than the downstream area, then this is entry loss and its value is  $0.5 V^2$  plus the loss in the friction; frictional loss in this pipe.

As we have discussed in the last class, how to express this; if we take the length  $L$ , pipe length  $L$  and the diameter of the pipe as  $D$ , the total frictional loss over the length  $L$  of a pipe of diameter  $D$  can be expressed as  $f L$  by  $D$  into  $V^2$ , loss of head. This is frictional loss; that means loss of energy due to friction per unit head; that means this is in terms of head meter unit; that means  $V^2$ . This we discussed last class.

When it comes here; that means which come with an exit velocity, but it ultimately enters into another large area. So, this is again an expansion loss, but a special case of expansion loss where the downstream area is much larger than the upstream area, which is known as exit loss; and whose value, a special case of expansion loss we discussed earlier is  $V^2$ . So therefore, from this path to this path; so these are the losses. So therefore, we can write the losses this way; this  $1.5$  plus  $f L$  by  $D$  into  $V^2$ .

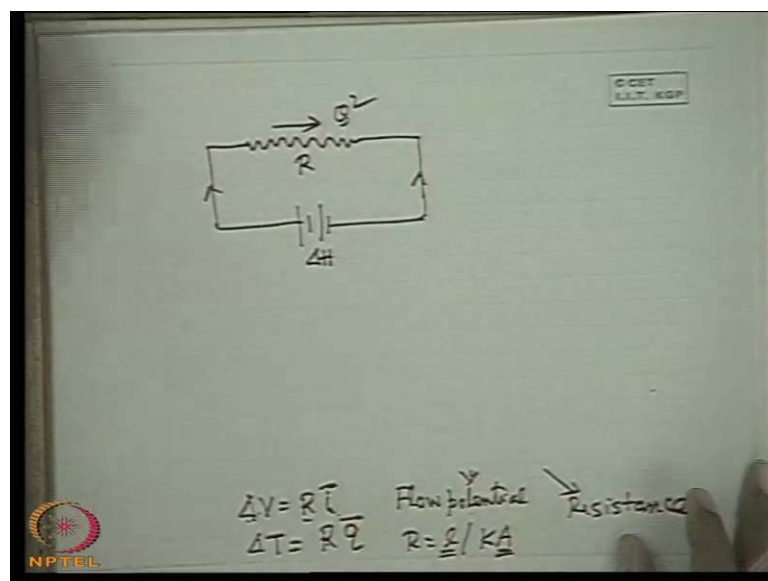
Now usually in fluid flow through pipe network system in this type of flow circuit, we replace  $V$  in terms of  $Q$ ; this is because it is not the flow velocity which always remains constant. The constancy of flow velocity requires that the area should be constant, but

under all steady condition it is the  $Q$  which is constant; that means if the pipe suddenly expands, contracts or there are two pipes joined in series. One pipe diameter is less, another pipe diameter is bigger or pipe gradually converges or gradually diverges, the velocity constants are not there; velocity changes because of the change in cross-sectional area.

But what remains constant; that the flow rate  $Q$  in incompressible flow, under steady condition the rate of volumetric flow remains constant. So therefore, it is usual to substitute  $V$  in terms of  $Q$  and  $A$  for a pipe of circular cross section is  $\pi D^2$ . So, if you just substitute this you get  $h_l$  is  $1.5$  plus  $f L$  by  $D$ , so  $16$ . So,  $8$  by  $g \pi^2$ . So,  $D$  to the power, sorry big  $D$ , I am always using; so  $D$  to the power  $4$  into  $Q$  square. Now if this expression you now substitute here, we get  $\Delta H$  is equal to  $1.5$  plus  $f L$  by  $D$  into  $8$  by  $g \pi^2$  square  $D^4$  into  $Q$  square.

Now you see that if we consider the friction factor to be constant irrespective of the flow rate  $Q$ , geometries are fixed  $L$  and  $D$  for a given pipe of given length and diameter,  $g$  is a constant,  $\pi$  is a constant. We can tell this is the head causing flow and this part is creating a resistance  $R$ , if we define this as  $R$ ; that means this entire part  $1.5$  plus  $f L$  by  $D$  into  $8$  by  $g \pi^2$  square by  $D^4$ . Then we can write  $\Delta H$  is equal to  $RQ^2$ , where you see that  $\Delta H$  is the head causing flow,  $Q$  square is the flux; square of the flux that is the flow rate and  $R$  is the resistant.

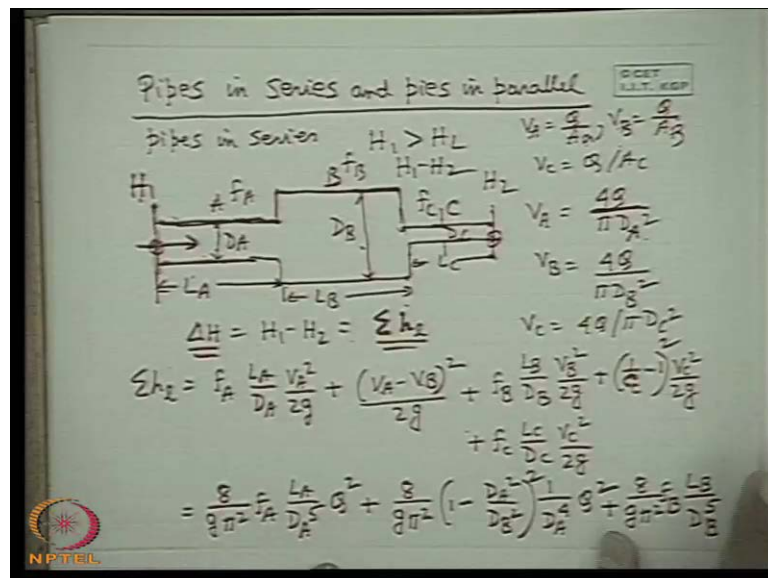
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From this equation, we can define delta H as flow potential; that means this is the potential due to which the flux takes place and this is the flow resistance that a flux suffers in course of its flow. So, only difference of this equation with general phenomenological equations of potential and flux is that this equation is non-linear, that potential difference is resistance into the square of the flux. If you compare this equation with the equation of electricity, we see that delta V is equal to R into flux I, not square. If you see the heat transfer equation delta T is equal to R into heat transfer rate q, where R is equal to l by K A; K is the conductivity, l is the length of the conductor, A is the cross-sectional area.

So, in both the cases we see the potential is equal to resistance times the flux; the linear relationship between potential and the potential difference and the resistance. Heat is only here where that relationship is non-linear Q square. So, Q square has the flux; this can be expressed by a network system like this. So, this can be expressed. So, this fluid flow problem can be expressed as a fluid network system like this; that means this is delta H and this is the Q square. So, this is the delta H and this is the Q square and this is the flux and this is; the R the value of R is this, value of R is this one.

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Now we consider next pipes in series. If you have any difficulty please tell me, pipes in parallel; what is meant by this. Earlier one is alright; all of you have taken it. Flow potential pipes in series and pipes in parallel. Now it is found that in many practical

applications or situations, the fluid flows through a series of pipes or through pipe joints rather through pipe joints; many pipes join together either in series or in parallel or a combination of this; that means fluid does not flow only through a single pipe of very long pipe, single pipe of very high length and a single damage. It is not like that; it flows through a number of pipes joined together either in series or in parallel or a combination of both, how those problems or solved.

Now, lets us see the pipes in series. First we consider the pipes in series; pipes in series means pipes are geometrically connected in series; that means let us consider like this that one pipe, then it is connected to another big pipe, then it is again shortened to a narrower one. So, diameters meet in. So, these 3 pipes let A, B, C they are connected in series. Very simple problem; the problem is actually this type of problems are tackled just like industry people, there is not much high academic or high theoretical background. Just its empirically or semi-empirically solved; for example, let the pipe A has got length  $L_A$ , diameter  $D_A$ ; pipe B has got length  $L_B$  and diameter  $D_B$ , different pipe,  $D_B$  is greater than  $D_A$  as it is shown in the figure. This is  $L_C$  and  $D_C$ .

Let fluid flows in this direction; that means let this is at entry point where head is  $H_1$  and this at the outlet point where head is  $H_2$  and  $H_1$  is greater than  $H_2$ ; that means total energy here is more than this that may be connected to a higher reservoir. This may be currently opened to atmosphere at the ground level, so that fluid flows. But we will propose the problem like that; there is head  $H_1$  which is higher than  $H_2$ , so that fluid flows in this direction. This head is maintained constant and this head.

So, fluid is flowing under a difference of head  $H_1$  minus  $H_2$  through a series of pipes A, B, C three pipes connected in series. Here I know that the head causing the flow or flow potential is  $H_1$  minus  $H_2$  and according to the earlier equation, this must be equal to the losses; that means if I write the Bernoulli's equation considering one point here, another point here, we will definitely be having this equation that  $\Delta H$  is  $\sum h_l$ ; that means always we can write this equation that head causing the flow or the flow potential is equal to sum of the losses.

So therefore, we will have to find out the losses through this pipe. This is the only job for any pipe network problem. What are the losses? Now what is  $\sum h_l$ , first here  $H_1$ . So, first is we are considering a point  $H_1$  if there is a larger reservoir comes entry loss;

this we are not taking. We are considering now, as if this is the point where the head is  $H_1$ . So therefore, starting from this point if we consider, this first comes the frictional loss in the pipe A; so which is  $f$ . Let  $f$  friction factor is also different for different pipe  $f_B$ ,  $f_C$ ; so  $f_A L_A$  by  $D_A V_A^2$  square by  $2g$ .

Now one thing, the velocity here for different pipes will be different because  $V_A$  is equal to  $Q$  by  $A_A$ , similarly  $V_B$  is equal to  $Q$  by  $A_B$  and  $V_C$  is equal to  $Q$  by  $A_C$ , where  $A_A$ ,  $A_B$ ,  $A_C$  are the cross-sectional areas of the pipe;  $V_A^2$  square by  $2g$  plus what is the next? Next when it comes here, there is an expansion losses what is that;  $V_A^2$  minus  $V_B^2$  whole square by  $2g$ . After expansion losses what is other loss, what is next loss that the fluid flow encounters; that is the frictional loss for a length  $L_B$  for the pipe B; that means,  $f_B L_B$  by  $D_B$  and  $V_B^2$  square by  $2g$ .

Next when it comes here, just at the entrance there is a contraction loss because fluid comes from a larger diameter, larger cross-sectional area to a lower cross-sectional area. The value of which become  $1 - C_c^2$ , you know what is  $C_c$ ; the coefficient of contraction, minus 1 whole square into  $V_C^2$  square by  $2g$ . This we have derived term stream velocity. After coming from that, it comes to lower diameter pipe; that means the frictional loss  $f_c L_c$  by  $D_c V_c^2$  square by  $2g$ .

Now if we substitute this value; that means  $V_A$  is equal to  $4Q$ . Now I told that  $V_A$ ,  $V_B$  we can eliminate in terms of  $V_A$ ; either  $V_A$  or  $V_B$  we take  $V_A^2$  square by  $2g$  and  $V_A$  by  $V_B$  is  $D_A$  by  $D_B V_B$  by  $V_A$  from continuity that we can do. But we should eliminate all  $V_A$ ,  $V_A$ ,  $V_B$ ,  $V_C$  in terms of  $Q$ , because  $Q$  is the flow rate which is constant. We want to establish a relationship of frictional losses that the total losses in terms of the flow rate  $Q$ ; so  $4Q$  by  $\pi D_A^2$  square; similarly,  $V_B$  is equal to  $4Q$  by  $\pi D_B^2$  square; similarly  $V_C$  is equal to  $4Q$  by  $\pi D_C^2$  square.

That means simply area of the pipes are replaced by their diameters. So, if you put this we get this one as  $8g\pi^2 F_A$  this is you can do it afterwards, but can you see immediately  $F_A L_A$  by  $D_A$  to the power 5; that means simply I am replacing  $V_A$  by this  $Q$  square plus if I take  $V_A^2$  square as common and put  $V_A$  is equal to  $4Q$  by  $\pi D_A^2$  square. So, I get this  $8g\pi^2$ . So, if we take  $V_A$  common  $1 - V_B$  by  $V_A$  which is  $V_B$  by  $V_A$  is  $D_A$  by  $D_B$ .

This is because  $V_A$  into  $A$  is  $V_B$  into  $D_B$ ; that means  $V_B$  by  $V_A$  is  $D_A$  square by  $D_B$ .

B square, we get this thing 8 by g pi square 1 by D A to the power 4 into Q square. So what I have done, I have taken V A common; so V A square 1 minus V B by V A whole square. Now V A square I have substituted V A this and V B by V A is D A square by D B square, because V B by V A is A by A B because V A A is V B A B. A by A B means D A square by D B square whole square; now it is alright, plus this term will be 8 by g pi square f B L B by D B to the power 5. This is this term. Q square is missing, the Q square.

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$$\Delta H = H_1 - H_2 = \sum h_2$$

$$V_c = \frac{4g}{\pi D_c^2}$$

$$\sum h_2 = f_A \frac{L_A}{D_A} \frac{V_A^2}{2g} + \frac{(V_A - V_B)^2}{2g} + f_B \frac{L_B}{D_B} \frac{V_B^2}{2g} + \left(\frac{1}{C}\right)^2 \frac{V_C^2}{2g} + f_C \frac{L_C}{D_C} \frac{V_C^2}{2g}$$

$$= \frac{8}{g\pi^2} f_A \frac{L_A}{D_A^5} Q^2 + \frac{8}{g\pi^2} \left(1 - \frac{D_A^2}{D_B^2}\right)^2 \frac{1}{D_A^4} Q^2 + \frac{8}{g\pi^2} f_B \frac{L_B}{D_B^5} Q^2 + \left(\frac{1}{C}\right)^2 \frac{8}{g\pi^2} \frac{1}{D_C^4} Q^2 + \frac{8}{g\pi^2} f_C \frac{L_C}{D_C^5} Q^2$$

$$\sum h_2 = \left\{ f_A \frac{L_A}{D_A^5} + \left(1 - \frac{D_A^2}{D_B^2}\right)^2 \frac{1}{D_A^4} + f_B \frac{L_B}{D_B^5} + \left(\frac{1}{C}\right)^2 \frac{1}{D_C^4} + f_C \frac{L_C}{D_C^5} \right\} \frac{8Q^2}{\pi^2}$$

$$\sum h_2 = RQ^2$$

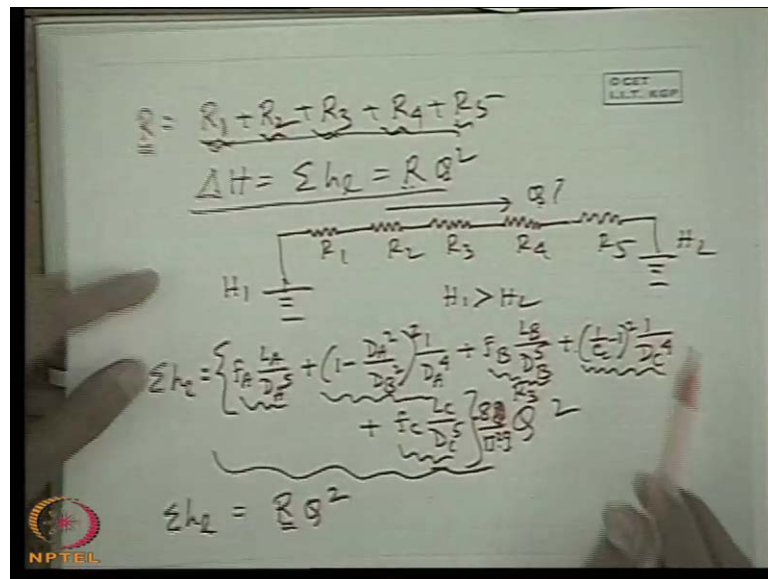
Then this can be last term. What is the last term; last term is last but one term 1 by C C minus 1 whole square. So, V C square by 2g will be giving the 8 by g pi square. V C square by 2g is written as 1 by D C to the power 4. So if I replace V C by this equation plus last term is similarly if I replace V C by the same expression 8, Q square why I am missing; it is a very important term, g pi square f C L C by D C to the power 5 Q square.

So, always Q square is there; that means we can write delta h l is equal to this thing 8 by g pi square I can take common, so that I can write f A L A by D A to the power 5 plus 1 minus D A square, simple school level algebra, 1 by D A to the power 4 plus, whole square again missing something is missing for me no, f B L B by D B to the power 5 plus 1 by C C minus 1 whole square into 1 by D C to the power 4 plus f C L C by D C to the power 5; so this into Q square. So this term in the bracket, first, two, three, four, five; that means this is equal to RQ, 8 by pi square g Q square. So, if I take this as total



resistance R. So, here it is  $\sum h_l$  is equal to  $R Q^2$ .

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So same equation we get, but here what R is composed of several resistances; this is R 1. So, this is the resistance offered by the friction in the surface friction in the tube one, another resistance R 2. So, several resistances are joined in series; that is the resistances offered due to expansion, expansion in the flow. Another resistance R 3 if we separately take, of course while defining R 1, R 2, R 3 for my convenience I am taking 8 by pi square g common, you can multiple it there 8 by pi square g, plus R 3.

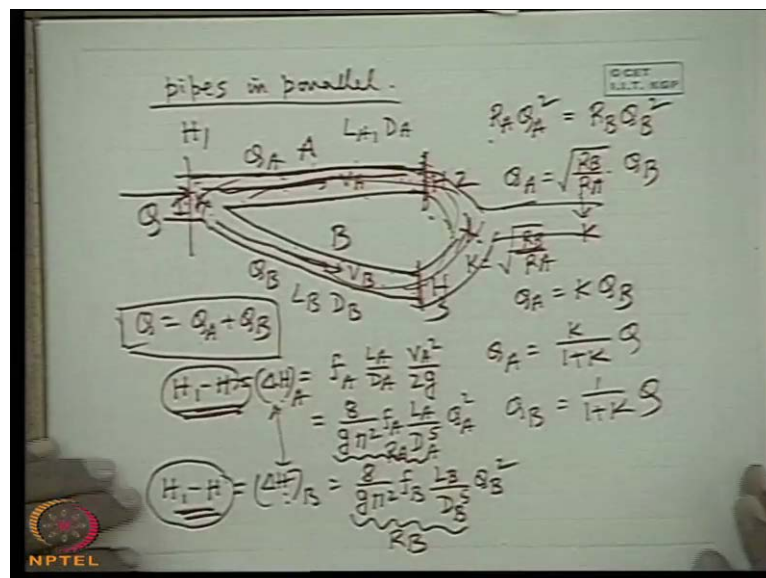
So, there is another resistance that is due to contraction R 4 and another resistance due to friction R 5. So therefore, we see sum of the resistance; that means, when pipes are in series, just like electrical conductors are in series, the resistance is different; resistances are added to give the total resistances R and we write that delta H is equal to sigma h l is equal to RQ square; that means this can be made equivalent to an electrical circuit like this. We can show the elemental resistance, the component resistance 1, 2, 3, 4, 5. So, component resistance we can show R 1, R 2, R 3, R 4, R 5, and the flow rate is Q square. One end the potential is more, another end the potential is less; H 1 is more than H 2. So, H 1 is greater; this is the simple network system.

So, from the figure itself without going for mathematics one can first recognize, what are the resistance; number 1 resistance is the frictional resistance, number 2 resistance is the expansion loss, number 3 resistance is the frictional resistance to pipe D, number 4

resistance is the resistance due to contraction, and number 5 resistance is the resistance due to friction through it and when it comes here the head is  $H_2$  considering this velocity.

But if there is a tank and we just give the head there and there is a tank where I define the head there, then we have to take entry loss and exit loss that you will have to make it out from your intelligent. Here this is the head  $H_1$ ; this is the head  $H_2$ . So therefore, I am not considering any exit loss, I am not considering any entrance loss. So, these are the five losses they are arranged in or they are arranged or they are connected in series. So, that this defines the problem. So, therefore, potential difference is equal to resistance into flux and in this case resistances are the sum of the resistance.

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Now pipes in parallel; pipes in parallel the problem is simple that geometrically the pipes are parallel means that they are branched up and they are meeting at the same point; that is a parallel pipe like this, that is one pipe and let there is another pipe, let there is another pipe that may be of different diameter they are joined in parallel; that means when the flow approaches for example, here head is  $H_1$ . The flow approaches let the pipe A and B, so they are connected at one end at the same joint and they are connected at another end at the same joint or simply they may go out. So, this is the pipe in parallel; that means this two are again connected, this may be connected at the same point rather I just express it that there is a head same  $H$ ; let this head is  $H$ .

That means at the outlet end, head is also same; that means either they are opened to atmosphere, that is also a parallel pipe or they are connected again at the same joint and then they are connected to another pipe in series. So that this two pipes in parallel means, at the two ends they are having the same head. You please do not talk. If you have to ask anything, you please ask me. You can leave the class if you want to talk, I am just giving you the opportunity to leave my class because I am sorry; those who want to talk in the class please I tell them to leave the class and IIT level I do not accept this thing talking in the class. Just continuously talking in the class is very shameful affair at this position at IIT please.

$H_1$  and the other ends, this is the basic definition of a parallel pipe; please listen to it carefully, that other end the head is  $H$ . So, in this case what we do if there are two pipes A and B and they are being specified by their length  $L_A$  diameter  $D_A$ , length  $L_B$  diameter  $D_B$  and if the velocities of flow is  $V_A$  through this pipe and  $V_B$  through this pipe, then what is the basic loss. Loss is that if  $Q$  is the flow rate, so in the series case; pipe in series the same flow was flowing through the flow rate was flowing the pipes but the velocity is changed because of the change in the cross-sectional area.

But here the flow rate is not same, flow rate is divided. Here it is  $Q_A$ , here it is  $Q_B$ . So, flow rate is divided. So, that  $Q$  is equal to  $Q_A$  plus  $Q_B$  from continuity; total flow is divided into 2 paths. But what is the constant that the head loss over the length of the two parallel pipes is same. Now if I write for the first pipe that is  $H_1$  minus  $H$  is equal to  $\Delta H$  is equal to loss; some of the losses in the first pipe. Consider only the friction loss; in this configuration there is no other loss. If this pipe parallel path could contain another pipe in series with different diameter, then expansion loss other things could come. But if I consider only the frictional loss, then in a simple case for understanding basic understanding of the pipes in parallel; that is  $f_A L_A$  by  $D_A$  into  $V_A$  square by  $2g$ .

This can be written in terms of  $Q$   $Q_A$  it can be written by  $8$  by  $g$   $\pi$  square  $f_A L_A D_A$  to the power 5, if I do any mistake you please remind me that is acceptable, but do not talk unnecessarily. Similarly, if I write the basic equation of fluid flow through pipe network system that the total head loss is equal to the sum of the flow resistance and that thing similarly for this path of the pipe; that means let this is 1, this is 2, this is 3; that means through 1 to 3, where the pipe B then  $H_1$  minus  $H$  will be  $\Delta H$  for pipe B, this

is for pipe A, and we get the similar relationship  $8 \text{ by } g \pi \text{ square } f \text{ B L B D B}$  to the power 5 into  $Q \text{ B square}$ .

We know  $L \text{ B D B}$  and  $L \text{ A D A}$ , but we do not know the divisions of  $Q \text{ A Q B}$ ; this can be made by making this two equal. This is the basic definition of the parallel flow; that is head loss over the length of the pipes joined in parallel are same. So, if we define this quantity as  $R \text{ A}$  and if we define this quantity as  $R \text{ B}$ , so that we can write  $R \text{ A } Q \text{ A square}$  which is equal to  $\Delta H \text{ A}$  is  $R \text{ B } Q \text{ B square}$ . Why; because  $\Delta H \text{ A}$  must be equal to  $H \text{ 1 minus } H$  and  $\Delta H \text{ B}$  is equal to  $H \text{ 1}$  and  $\Delta H \text{ A}$  and  $\Delta H \text{ B}$  is same.

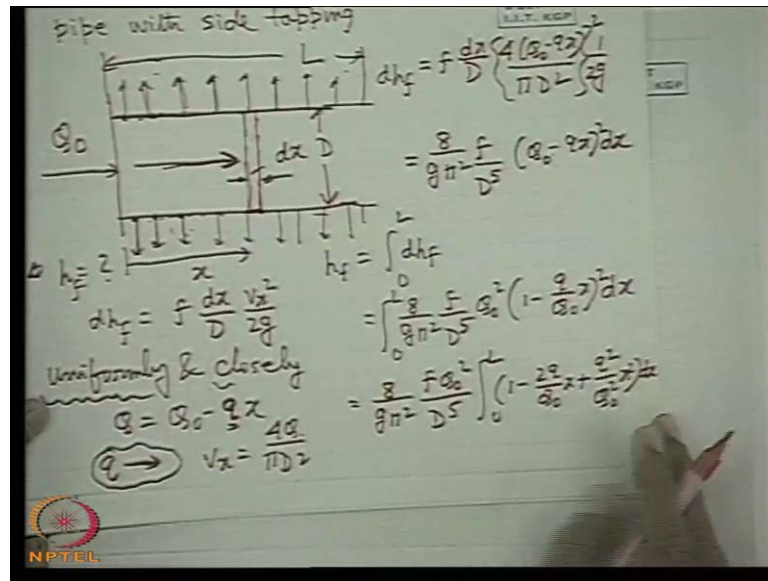
So therefore,  $R \text{ B}$ ; so that we can write  $Q \text{ A}$  is root over  $R \text{ B}$  by  $R \text{ A}$  into  $Q \text{ B}$ . Let this quantity be defined as  $K$  root over  $R \text{ B}$  by  $R \text{ A}$ ;  $K$  is root over  $R \text{ B}$  by  $R \text{ A}$ . So therefore,  $Q \text{ A}$  is equal to  $K Q \text{ B}$  and we have another relation  $Q$  is  $Q \text{ A plus } Q \text{ B}$ , very simple. So, that we get  $Q \text{ A}$  is  $K$  by  $1 \text{ plus } K$  into total  $Q$  and  $Q \text{ B}$  is  $1$  by  $1 \text{ plus } K Q$ ; that means if I know the geometry of the pipe, if I know the total flow rate, I can find out the division and how I can find out; if I equate the losses. There may be other losses also, so other losses are there, accordingly I will equate the losses.

So, in the entire this line if there are other pipes in series, the combination of series and parallel the flow rate will be  $Q \text{ A}$ . Similarly through these entire line, if there are another pipes in series flow rate will be  $Q \text{ B}$ , but  $Q \text{ A Q B}$  will be at the ratio relationship that root over  $R \text{ B}$  by  $R \text{ A}$ , where  $R \text{ B}$  and  $R \text{ A}$  are the flow resistance in path B and path A, two parallel path and total flow rate must equal  $Q$  is equal to this  $Q \text{ A plus } Q \text{ B}$ ; that are the branching.

So, it is the similar relationship as in an electrical circuit just the Kirchhoff's law that at a junction, the sum of the algebraic sum of the flow rates are 0 and the resistance times the potential difference across any close loop is 0; that is there by which formula we are equating this and this. This is a close loop. Actually we are presenting this two heads are  $H$  same; that means as if they are mixing like, they are joining.

They are joining like this; that means these two tubes joined and then they go; that means between these two points, the head difference is same. So, the same way as we do for electrical circuits we can solve the hydraulic circuits for pipes in series and pipes in parallel; only difference is that here the law is  $\Delta H$  is resistance times  $Q$  square and we will have to recognize the flow resistances from the fluid mechanical point of view.

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Now we will consider a very interesting case that pipes with side tapping. In certain cases, it has been found that while fluid flowing through a pipe it is being drained up; it is very simple it is being drained up through its side holes. The fluid is being drained up through its side holes. Let this is the flow direction; because of which one thing is appreciated that flow rate changes from section to section in the direction of the flow as it is decreasing, because the fluid is going out how to find out. The problem is that how to find out the delta h or simply h the head loss over this pipe, when there is a draining of fluid from the side tapping.

To do this type of problem what we do, we take an elemental strip of length  $dx$ . We consider a section where the fluid enters with a value of  $Q_0$  flow rates, after which the fluid is drained up. This is the inlet section, so that we represent this section at a distance  $x$  from the inlet section of  $dx$  length. So, a frictional loss over this  $dx$  length of the pipe will be  $dh_f$ ; let  $dh_f$  frictional loss will be what  $f L$  by  $D dx$  by  $D$ , if we recognize  $D$  as the diameter of the pipe,  $dx$  by  $D$  into  $V_x^2$  by  $2g$  where  $V_x$  is the velocity at this cross-section at  $x$ . Why this  $V_x$  has the function of  $x$ , though the diameter remain same, that is uniform cross-section; the flow rate is varying, because the flow is being drained up through the side tapping. So that  $V_x$  is changing.

Now we can integrate this over a length of pipe  $L$  if the problem is posed like that; that for a length of pipe  $L$ , find out the total frictional head loss. So I can integrate it,

provided I know the  $V_x$  as a function of  $x$ ; to know this, we will have to make the problem simple. How? If we assume this draining up; that means the holes through which it is draining up is uniformly spaced and closely spaced, both uniformly and very closely spaced this is very important; uniformly and closely spaced then we can assume that the fluid is drained at a uniform rate over the entire length  $L$ .

So that we can write the flow rate at any cross-section  $x$  is  $Q_0 - qx$ , where defining a parameter  $Q$  as the rate of flow coming out per unit length or rate of flow drained up per unit length. So, for a length  $x$  it is  $qx$ ; this we can define, provided this is drained up uniformly and this spacing is very, very close, as continue on approach type of things has very close spacing and uniformly; so that the fluid is uniformly drained out per unit length. So, that  $Q$  is that parameter defining rate of fluid drained up per unit length. So,  $Q$  is this.

So, if you define this then  $dh_f$  is equal to  $f dx$  by  $D$ , then what will be  $V_x$ . So,  $V_x$  will be as usual  $4Q$  by  $\pi D^2$  and this  $Q$  is  $Q_0 - Qx$ ; that means  $4(Q_0 - qx)$ , this is by  $\pi D^2$ . So, this has to be whole square,  $fL$  by  $DV^2$  into  $1/2g$ ; that means if we write this we get this expression. Again the same expression  $8$  by  $g\pi^2$  square into  $fL$  is this  $fL$  by the  $q$ ; that means  $f$  by  $D$  to the power  $5$  and then we get  $Q^2$  square in place of  $Q$  square, we get the  $x$ ; so we get this. If we just simply it  $fD$  to the power  $5$   $Q_0 - Qx$  whole square  $dx$  we get it.

So therefore, to get the  $h_f$  total frictional loss; that means we integrate  $dh_f$  over the entire length  $0$  to  $L$ , which means we integrate  $0$  to  $L$   $8$  by  $g\pi^2$  square  $f$  by  $D$  to the power  $5$  we take  $Q_0$  as common that is  $Q_0^2$   $1 - q/Q_0 x$ , which is very simple now. It is now very simple school level integration; that means  $8g\pi^2$  square  $fQ_0^2$  square  $D^5$  common and  $0$  to  $L$  so this will be  $1 - 2q/Q_0 x + q^2/Q_0^2 x^2$ .

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$$h_f = \int_0^L dh_f$$

$$dh_f = f \frac{dx}{D} \frac{v^2}{2g}$$
 Uniformly & closely  

$$Q = Q_0 - qx$$

$$v = \frac{4Q}{\pi D^2}$$

$$h_f = \int_0^L \frac{8f}{g\pi^2 D^5} Q_0^2 \left(1 - \frac{q}{Q_0}\right)^2 dx$$

$$= \frac{8f}{g\pi^2 D^5} \int_0^L \left(1 - \frac{2q}{Q_0}x + \frac{q^2}{Q_0^2}x^2\right) dx$$

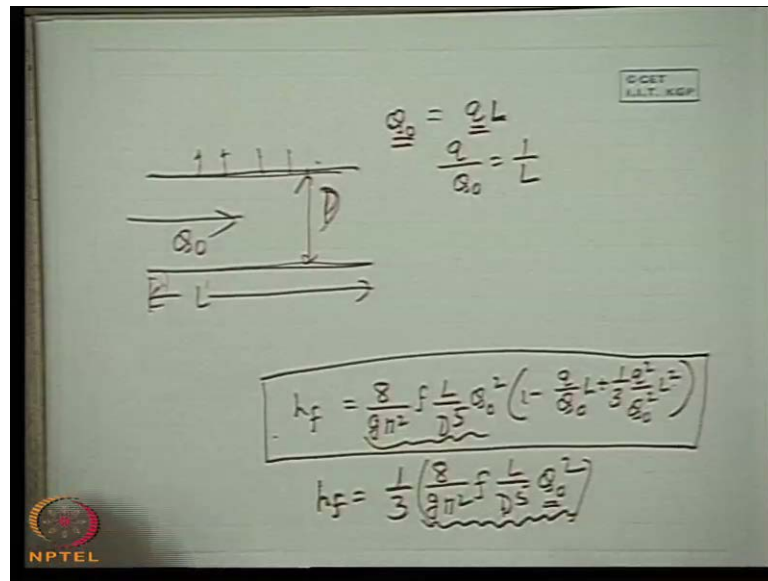
$$= \frac{8f}{g\pi^2 D^5} \left(L - \frac{q}{Q_0}L^2 + \frac{1}{3}\frac{q^2}{Q_0^2}L^3\right)$$

$$h_f = \frac{8f}{g\pi^2} \frac{L}{D^5} Q_0^2 \left(1 - \frac{q}{Q_0}L + \frac{1}{3}\frac{q^2}{Q_0^2}L^2\right)$$

If we do this, then what we will get;  $8$  by  $g$  pi square  $f$   $Q_0$  square by  $D^5$  now  $D \times L$ ; that means it is  $L$  minus twice  $x$  means  $x$  square that is  $L$  square; that means  $Q$  by  $Q_0$   $L$  square and this is plus  $x$  cube by three; that means one-third  $q$  square by  $Q_0$  square, what is that,  $L$  cube; that means this can be written as  $8$ , so  $h_f$  over the length  $8$  by  $g$  pi square  $f$   $L$  by  $D$  to the power  $5$ , I take one  $L$  common,  $Q_0$  square then  $1$  minus  $q$  by  $Q_0$   $L$  plus one-third  $q$  square by  $Q_0$  square  $L$  square. So, this is the required expression for the head loss over a length  $L$ .

If the fluid is being uniformly drained out; that is that holes are very closely spacing and uniformly drained out from side tapping, then this is the head loss over a length of fluid, length of pipe  $L$ . Now if it so happens, the problem is such that the fluid which enters here is completely drained out over a length  $L$ ; that means when you reach a length  $L$ , there is no fluid coming to here in this direction; that means entire fluid is being drained off, then what will be the value of  $h_f$ .

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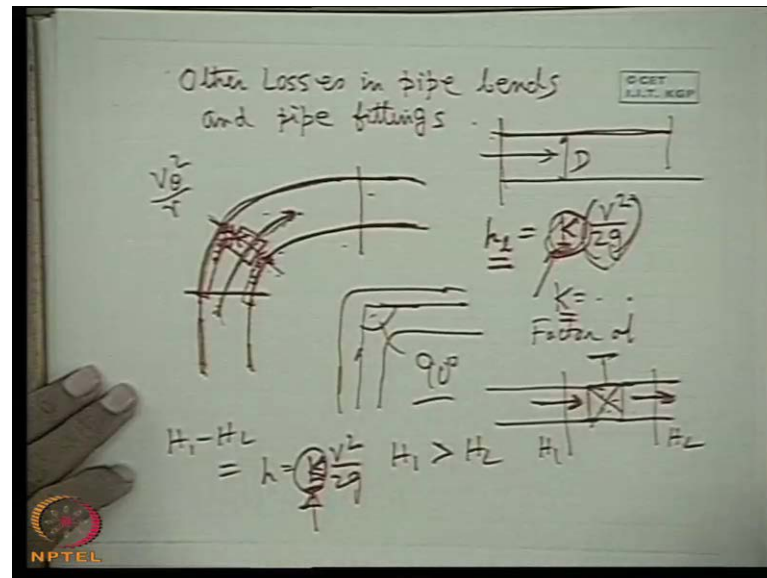


If this is the problem physically given, so what is the mathematical translation of this physical statement; that means  $Q_0$  must be equal to  $q$  into  $L$ , this is the mathematical statement; that means  $q$  by  $Q_0$ , because the entire fluid which is entering the section is being drained off over a length  $L$ ,  $q$  is the drainage rate per unit length. So,  $q$  by  $Q_0$  is  $1$  by  $L$ . If you put it here, you get  $h_f$  is equal to one-third of this quantity. Because  $q$  by  $Q_0$  is  $1$  by  $L$   $1$  minus  $1/3$  and this becomes is equal to  $L$  square,  $L$  square cancel, so one-third. So, one-third comes and it becomes equal to this one.

What is this value if you recollect, just immediately you can recognize that this is the frictional loss over a pipe of length  $L$  where there was no side tapping; that means the fluid flows uniformly with a flow rate of  $Q_0$  over a length  $L$  same length  $L$  and the diameter  $D$ , this is the friction length. So therefore, this is the head loss. So therefore, we see that loss of head in this case, when the entire fluid is drained up over a length  $L$  for a given diameter  $D$  is one-third of that which could have taken place if the fluid flows without any draining up from the side tapings. So, this is a very useful conclusion.



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So next I will mention you that losses; other losses in pipe bends and pipe fittings. You have often heard that if there is a pipe bend in practical situations, always you will see there is a pipe bend. Obviously you will have to bend the pipe, because some instrument is here, you will have to bend the pipe, you have to measure something here. So, always you will have to change the direction of the flow in the practical situation. So we provide a pipe bends, with the flinch, we just fix the pipe bends. So, pipe bend means when a pipe line, a short pipe line which changes its direction; this is known as pipe bend.

What happen in this case that if you try to find out what is the total energy here, and if we try to find total energy here, we will see there is a loss in energy, which is not exactly same as that of the frictional loss over this length; that means if I just plot this straight pipe of the same length of the same diameter, then the losses which takes place when the flow takes place through this pipe. Here the losses are more; that means because of the bend, the change in the direction of the flow losses are taking place. What is the mechanism of this loss I tell you; usually these losses are expressed, the loss due to pipe bend as some factor times  $V$  square by  $2g$ ; that means a some fraction of the kinetically or dynamically.

In practice, it is always given that for a pipe bend of this degree; 90 degree, 45 degree, the pipe bends are specified by the degree of deviation; that means this is a 90 degree pipe bend; that means the pipe flow directions is totally changed by 90 degree; that

means the deviation is 90 degree. Depending upon that for 90 degree, 60 degree pipe bend. The value of K is that, which means that the loss of energy, that head loss frictional head loss due to the pipe bending is equal to K times the viscosity, but it is always expressed as a fraction of the dynamic head. Fraction now factor; this is not fraction I am sorry as a factor because this is more than one in many cases.

So, the basic mechanism of this is that when the fluid flow takes places in that direction. So, a centrifugal force is acting on it. So, which causes both a centrifugal and centripetal acceleration and that centripetal acceleration causes a radial pressure gradient in the fluid; that means if you take a fluid element, you will see because of this tangential velocity when it goes through a bend, there is a velocity in the azimuthal direction for which there is a centripetal acceleration, as you know the value of which is  $V \theta$  square by r.

So therefore, this acceleration can only be balanced provided there is a pressure force like this; that means the radial pressure for radial pressure gradient is generated. In a way that it is maximum here at any plane like this and it is minimum here; that means maximum pressure; that means pressure is higher at the outer wall and lower at the inner wall. So, this makes it possible to have a tangential velocity. So, whenever fluid flows along a curve line, so you will see that there is a radial pressure gradient acting on them and this pressure is maximum at the outer wall and minimum at the inner wall at any cross-section.

If you see what is the history of pressure variations along the wall, you will see the pressure tries to increase from some point here along the wall, so that the pressure gradient can be set up to cause for this motion in the curve path. Similarly here also from some point here, the pressure goes on increasing and both the cases the fluid flows either along adjacent to this wall or this wall we will see the fluid flows against an adverse pressure gradient, for which there is a loss somewhere here, somewhere here which is the separation loss as I have discussed earlier, for which there is a loss of energy.

And along with that, we can express this one afterwards at the higher level or advanced level, we will see that when you will solve this; that whenever there is a radial pressure gradient when the fluid flows to a curve line there is a secondary flow; that means in this plane, there is a circulation, that is a secondary flow in a circular path takes places, which

gives a spiral motion of the fluid along with the motion in the axial direction for which what happens; this velocity gradient at the wall becomes stronger. Through the boundary layer concept, we tell the boundary layer becomes thin. The velocity gradient becomes steeper at the wall, which causes the shear stress to be more.

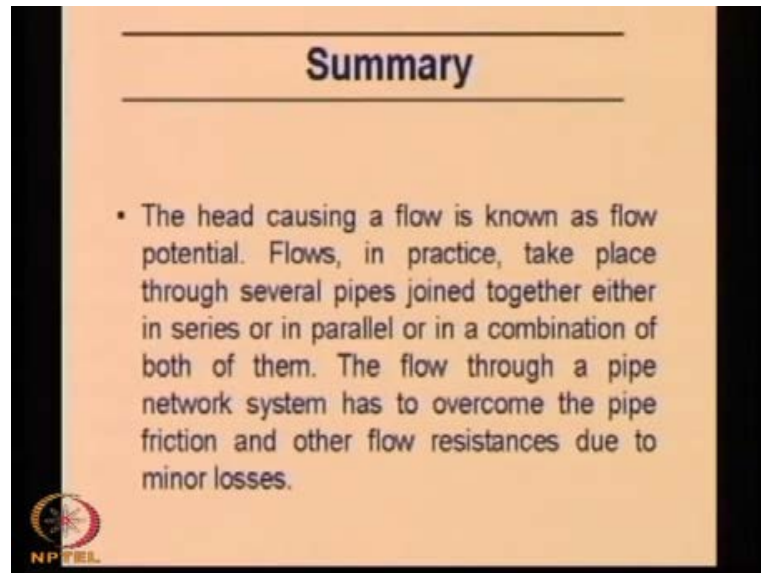
So along with this, the separation losses along with this increase in the skin friction, because of the change in the flow configuration along a curve path makes a loss, which is additionally more or additional to that of the usual frictional loss over the same level, which is known as the bend losses and expressed as a factor  $K V^2$  by  $2g$ . Other losses taking place due to pipe fittings, when there is a wall which controls the flow; if you control the wall, the flow is reduced. If you close the wall, flow is stopped. If you practically open the wall, close the wall, again the flow takes places at a reduced rate.

So, if you change a wall position giving more restriction to the flow of fluid, it creates a loss of energy; that means if  $H_1$  is there,  $H_2$  is there;  $H_1$  is more than  $H_2$  or  $H_2$  is less than it. So, this loss of head it is  $h$ . Due to pipe fittings wall, this is also expressed as  $K V^2$  by  $2g$ . So for different types of wall, putting different form of restriction in the flow passage to control the flow rate gives different values of  $Q$ . So, in solving problems we will use the value of  $K$  for wall fittings and pipe bends.

Thank you.


Summary:

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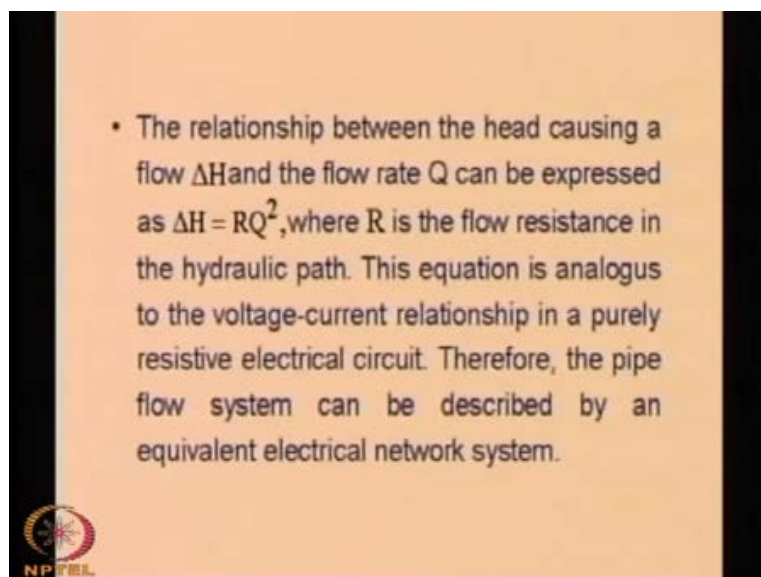


**Summary**


- The head causing a flow is known as flow potential. Flows, in practice, take place through several pipes joined together either in series or in parallel or in a combination of both of them. The flow through a pipe network system has to overcome the pipe friction and other flow resistances due to minor losses.

 NPTEL


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- The relationship between the head causing a flow  $\Delta H$  and the flow rate  $Q$  can be expressed as  $\Delta H = RQ^2$ , where  $R$  is the flow resistance in the hydraulic path. This equation is analogous to the voltage-current relationship in a purely resistive electrical circuit. Therefore, the pipe flow system can be described by an equivalent electrical network system.


 NPTEL

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- The loss of head due to friction over a length  $L$  of a pipe, where the entire flow is drained off uniformly from the side tapings, becomes one third of that in a pipe of same length and diameter but without side tapings.

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- An additional head loss over that due to pipe friction takes place in a flow through pipe bends and pipe fittings like valves, couplings and so on.

Problems:

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Problems

(Objective types with multiple choice)

1. For pipe arranged in series

- (a) the flow may be different in different pipes.
- (b) the loss of head per unit length must be more in a bigger pipe.
- (c) the velocity must be the same in all pipes.
- (d) the head loss must be the same in all pipes.
- (e) the flow rate must be the same in all pipes.

[Ans: (e)]




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2. In parallel pipe system

- (a) the pipes must be placed geometrically parallel to each other.
- (b) the flow must the same in all pipes.
- (c) the head loss per unit length must be the same in all pipes.
- (d) the head loss across each of the parallel pipes must be the same.
- (e) none of the above is true.

[Ans: (d)]




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3. Two identical pipes of length  $L$ , diameter  $D$  and friction factor  $f$ , are connected in parallel between two reservoirs. The size of a pipe of length  $L$  and of the same friction factor  $f$ , equivalent to the above pipe is

- (a)  $0.5 D$ .
- (b)  $2.0 D$ .
- (c)  $0.87 D$ .
- (d)  $1.32 D$ .

[Ans: (d)]




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4. Two pipelines of equal length and diameters of  $10\text{ cm}$  and  $40\text{ cm}$  are connected in parallel between two reservoirs. If the friction factor  $f$  is the same for both the pipes, the ratio of the discharges in the larger to the smaller pipe is

- (a)  $4$
- (b)  $16$
- (c)  $32$
- (d)  $64$

[Ans: (c)]



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5. The diameter of a pipe is reduced by 10 % due to deposition of rust. If the friction factor remains unaltered, for a given difference of head in the reservoirs, this would result in a reduction in discharge of

- (a) 10 %
- (b) 14.6 %
- (c) 23.2 %
- (d) 31.6 %

[Ans: (c)]

