Fluid Mechanics
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Lecture - 30
Incompressible Viscous Flow Part-II

Good morning, I welcome you all to this session of fluid mechanics. Now, last class we started the derivation of equation on motion for a viscous flow, that is flow of a viscous fluids, and what are the steps that we recognize. First you write the equation of motion; that means left hand side is the acceleration times the mass, and the right hand side is the total force in a particular direction of coordinate axis. So, this force is composed of as you know the body force and the surface force. And the surface force was written in terms of the stress components, and in a viscous flow the stresses are not only the normal stresses as pressures, but it is the normal stress plus the shear stresses. Normal stresses has only pressure is the case for an inviscid flow, but in viscous flow it is the normal stresses and shear stresses.

Then we recognize the most important point of deducing the equation of motion is to relate the stress quantities in terms of the strain rates, because you know from kinematics of fluid the strain rates are defined in terms of the velocity components. Therefore, if you can relate the stress with the strain rates or finally with the velocity gradients, because strain rates are related to velocity gradients, then finally we can deduce the equation of motion in terms of the velocity components, and we want this in terms of the velocity components and the pressure. So, that from these we can find out the velocity and pressure filled in a flow.

Now, this is the most important and most complicated part of the deduction of equations of motions for a viscous flow. Now, the entire deduction is beyond the scope of this syllabus, but what we recognized that we should know the assumption based on which the deduction is made for a class of fluids and what are their relationships. And we recognize that assumptions were like that, we first assume that these relationships or the equations of motions rather will be developed for a class of fluids which obey a relations, linear relationship between stress and strain rates. Those fluids are known as Newtonian fluid. Number two is that these relationship is really a physical law, that means it is invariant with the transformation of coordinate axis.

Number three is the fluid obeys Stokes law that means the second coefficient of viscosity is 0 which we discussed last class, that known as Stokesian fluid and fourth one is that the ultimate relationship or the equation of motion in which way you see both the relations are equation of motions or relationship between stress and rate of strain must reduce to hydrostatic conditions when hydrostatic conditions must reduce to hydrostatic equations when hydrostatic conditions are substituted.

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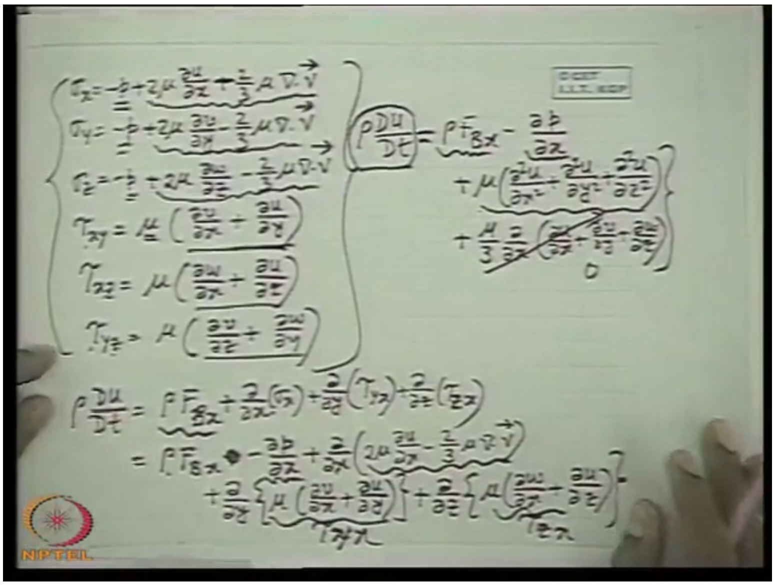

So, with these assumptions we finally, obtained these equations you just see now sigma x is minus p plus 2 mu del u del x. del u del x is linear strain rate in x direction is simple to recollect minus sorry minus two-third mu into divergence of the velocity vector that means del u del x plus del v del y plus del w del z. In a Cartesian coordinate system sigma y is minus p plus 2 mu del v del y minus two-third mu into divergence of velocity vector. This is simple to recall, you have to recall these always 2 mu this is related the normal stress in z direction with the linear strain rates in z direction minus two-third mu divergence of the velocity vector.

And out of three stress quantities we recognize there are six stress quantities, three normal stresses, three shear stresses which is equal to mu times gamma dot x y that means del v del x plus del u del y, u and v, u v w being the x y and z component of velocity x y x z is mu del w del x similarly, plus del u del z the cross differential, this is the gamma dot x z. Similarly, tau y z is mu times gamma dot y z that means del v del z plus del w del y. So, these relationships and we see that under hydrostatic conditions when u v w 0 their gradients are 0 tau x y tau x z tau y z 0 because this 0 this 0 so there is no shear stress and sigma x is sigma y is equal to sigma z is minus p.

So, under hydrostatic condition the hydrostatic equations come out from this general equation. So, this part is due to friction. So, normal stresses is not equal to minus p as it happens in case of hydrostatics or in case of inviscid flow, it is plus this quantity. This quantity is known as deviatoric stress. I defined yesterday then these are the shear rates. So, mu is the coefficient of viscosity, p is the thermodynamic pressure or static pressure as we have already recognized them. Now, if you recall the equation now.

Now, next part is to substitute this into the equation of motion in terms of the stress quantities. Let us consider the x direction equation of motion rho D u D t is rho F x B f B x that means the body force part plus del del x of sigma x plus del del y of tau x y or y x whatever is written is same thing plus del del z of tau y z sorry tau z x, alright. So, if you just substitute this F B x comes from the other physics, physics of the body force field that means del del x of sigma is it is written like this minus del p del x p is taken out plus del del x of this part twice mu del u del x, very simple minus two third mu divergence of the velocity vector plus del del y tau x y that means mu into del v del x plus del u del y.

Well plus del del z of mu into tau x z del w del x that means now we see that this is the major part which is beyond the scope of our syllabus for the for deduction. So, if you substitute then becomes a real (( )) very small as every school level algebra that you substitute and you get this. So, if you make little rearrangements these equation takes this shape rho D u D t is equal to rho F B x. So, with this we have no control then minus del p del x this can be written in another form.

This form if you write it is very clear to see that these are the stresses, this is the pressure this is the deviatoric normal stress because del del x of sigma x del del y of tau x y that means this is tau x y or tau y x whatever you call this is tau z x or tau x z this is clear. But it is usually written in this form if you make little rearrangement this becomes mu del square u del x square. Simple algebra del square u del y square calculus del square u del z square well plus mu by 3 del del x of divergence of the velocity del u velocity vector del v y plus del w del z. So, if you write this rho D u D t will be like this.

So, now here we can physically identify the term, this is the acceleration per unit volume which is equal to the net force acting per unit, this is the body force acting per unit volume, this is the pressure force acting per unit volume and this two terms together constitute the frictional force acting per unit volume, under incompressible flow situation this part will be 0 because divergence of the velocity vector will be 0.

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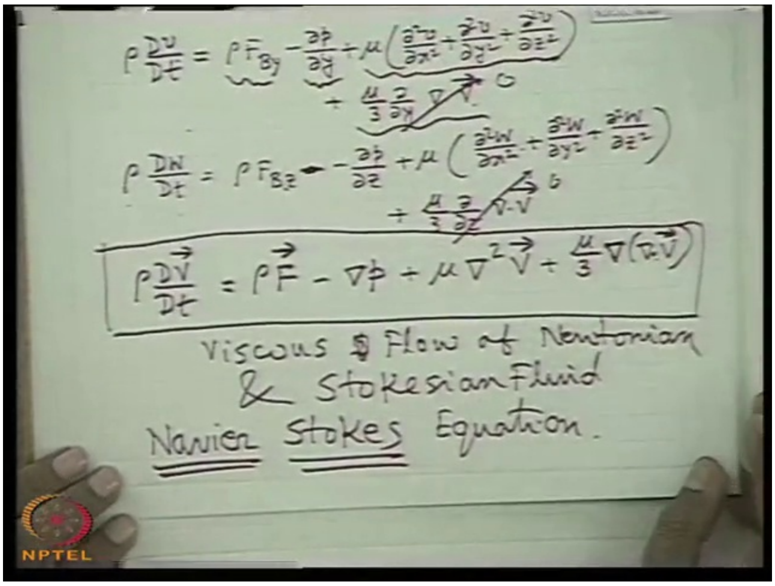

Similarly, if we write the equation for y direction we will see in the similar way that means if we recall the equation in y direction rho D v Dd t in terms of stress components and if we substitute the stresses and finally, rearrange it in the similar fashion as we have written finally, we get this equation in this form plus mu instead of u it be with v del square v del x square plus del square v del y square plus del square v. All of you must workout this just as an simple exercise so that you get this del del y of I am not writing this divergence of the velocity vector.

Similarly, for the z direction rho F B z we get the same expression sorry minus del p del z plus mu it will be with del square w del x square plus del square w del y square plus del square w del z square plus mu by 3 del del z of divergence of the velocity vector. So, if you take this along with these equations for u v for this is x direction this is y therefore, you see this is the buoyancy force in y direction per unit volume pressure force per unit volume in y direction frictional force per unit volume in y direction and similarly, the z component forces correspondingly they equate with the acceleration and mass per unit volume.

So, these are the forces now for incompressible flow obviously this part becomes 0, this part becomes 0. Now, all these three equations if we inspect it can be written in a vector form that rho D v D t whose x component is rho D u D t y component is rho D v D t and z component is rho rho D w D t so rho D u D t in one page it is very difficult to see. So, you can just write here so rho D v D t is equal to rho F whose y component z component similarly, in the x equation x component minus grat p whose x component is minus del p del x in the x direction equation y component z component plus mu laplacian del square v which is operating on v w and u depending upon the coordinate direction plus mu by 3 into divergence of the sorry gradient of the divergence of the velocity vector that means mu by 3 del del y of this in y component. So, del del z of this is z component and del del x of this comes in the x component so this can be written as grat.

So, this is the vector form of the equation of motion for a viscous in not incompressible I have not given the viscous flow of Newtonian and Stokesian fluid, of Newtonian and Stokesian fluid, Stokesian fluid. This was first deduced by two great scientists Navier and Stokes that is why they are known as Navier Stokes without any and this two persons Navier Stokes equation. Sometimes, when two persons develop something and is not there. So, Navier Stokes equations, Navier Stokes are two person, these are the Navier that is the viscous equation of motions for viscous flow.

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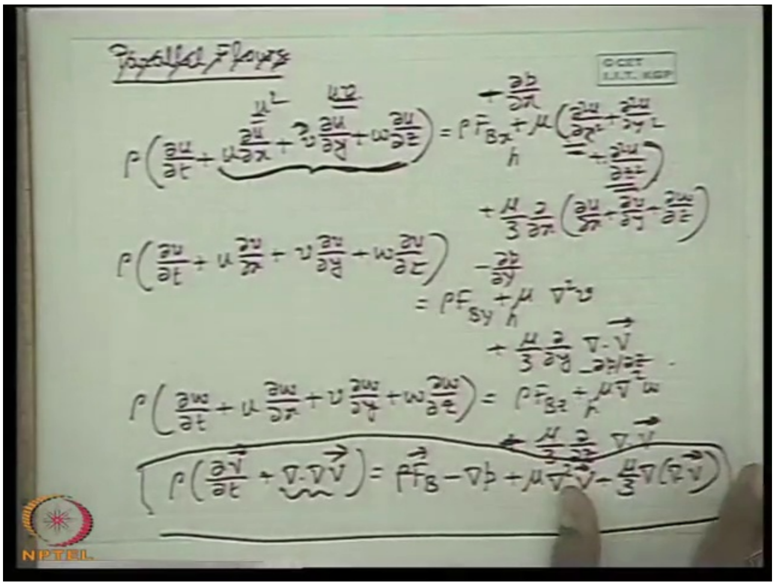

Now, we will analyze certain classes of viscous flows which is very important by the application of this equation which is very, very important. Now, first we considered certain classes of parallel flows. Now, before that we must discuss about the nature of the Navier Stokes equations and what is the nature of the Navier Stokes equation? I ask you one thing that Navier Stokes equation is a linear equation or a non-linear equation before that I think it will be better if we just see one component if I split del u del t, del u del t plus u del u del x plus sorry v del u del y plus w del u del z is equal to then the right hand side is rho F B x plus mu laplacian that means del square u del x square plus del square u del y square plus del square u del z square plus mu by 3 del del x of del u del x plus del v del y plus del w del z.

So, y direction equation will be like this rho here it will be del v del t that means D v D t is u del v del x plus v del v del y plus w del v del z is equal to similarly, rho F B y plus mu I am not writing this thing plus mu by 3 del del y of divergence of the velocity vector. This is very simple. Similarly, if you just split it del w del t plus u del w del x plus v del w del y, you know this thing already w del w del z is equal to rho F B z plus mu laplacian of w. What is laplacian this del square del x square plus del square del y square plus del square del z square minus mu by 3 I think you are very strong in mathematics, you may not be strong in physics, but what you are strong in mathematics plus mu by 3. All pressure terms are neglected I am very, very sorry plus minus del p del x you see how blunder I have done, I could have got 0 if there is an examination. So I can afford to do it as teacher you should not in examination minus del p del z sorry.

So, this can be written in vector form that rho of del v del t plus del dot grat v that means this is the convective part is equal to, I wrote this because I could have straight written this from the vector form, but you may object rho F B sir, how you get it minus grat p plus mu del square v plus mu by 3 grat of divergence of velocity. This will be more simple for you, you are strong in mathematics.

So, now you see you tell this is a partial differential equation or ordinary differential equation, Navier Stokes equation, these equation partial or ordinary? Please, quick, partial differential. What is the order of these differential equation, order of two, second order because order is determined by the viscous term, this term. So, order is determined by these viscous term whether the equation is linear or non-linear? Non-linear, yes, correct. Why it is non-linear which term is the non-linear term, constant, which term is the non-linear term? These are the non-linear terms, the inertial term. This term is linear. What is the definition of a linear or non-linear equation?

A non-linear and linear equation definition is like that if it is an algebraic equation or a differential equation even the equation, the power of the dependent variable is more than 1, if the power of the dependent variable is more than 1 in an equation that equation is non-linear. So, here the dependent variables are u v w. So, if any of the term contains a power of u v w more than 1 is a non-linear equation.

Here, you see del square u del x square is a second order term, but the order is u by x square so this is a linear in u. But it is u del u del x so these order is u square v del u del y order is u v. So, the power of the dependent variable is more than 1 therefore, this is a non-linear equation. So, the non-linearity in the Navier Stokes equation is due to the inertial term, convective acceleration term but the order of the Navier Stokes equation is determined by the viscous term, this term so which gives a, this is a second order non-linear partial differential equations. In fact till today there is no close form solution for its entirety that means a non-linear partial differential equation does not have a closed form solution.

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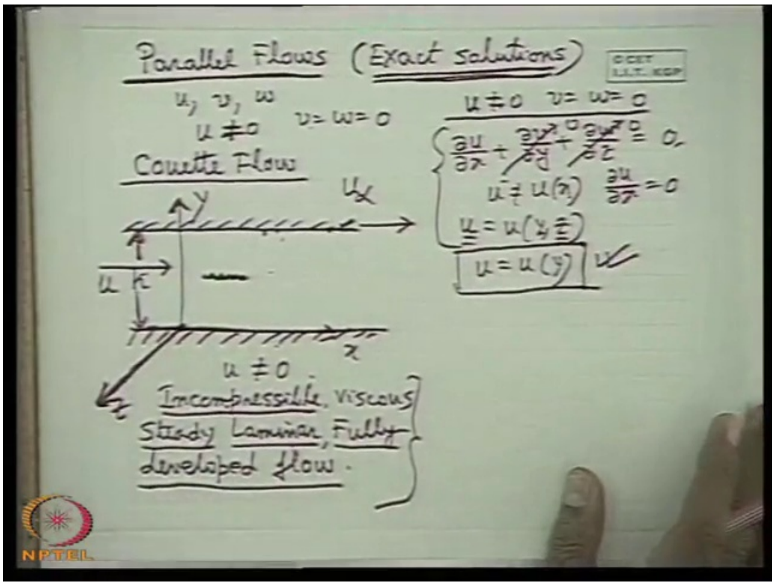

Nowadays we solve the Navier Stokes equation you must know this is the state of art with the aid of computational mathematics or computational fluid mechanics. With the advent of computational mathematics we can solve the partial differential equation of non-linear type with all form of partial differential equations, a parabolic equation, elliptic equation, hyperbolic equation fully, but that is not an full form analytical solution.

So therefore, the exact solutions of Navier Stokes equations are not possible in its entirety, but there are certain cases. but there are certain situations of simple cases of flows known for example, one of such is parallel flows where the exact solutions of Navier Stokes equation is possible.

That is why this classes of flows are termed as exact solution. These are the things you must know in the class these are not written so detail in any book. But you can tell that how exact solution is possible, a non-linear partial differential equations of order two cannot have a closed form analytical solution or exact solution, but the answer is that for those classes of flows for example, parallel flows so the equations are reduced to a very simple ordinary differential equations for which exact solutions are possible. So, physics of the problem itself reduces the equations in that form that is why those classes of problems are sometimes referred to as exact solution, a mathematical terminology is given. A few such classes of flows are known as parallel flows which we will be discussing today.

Now, parallel flows basic definition is that if you consider 3 component of velocity with respect to 3 coordinate axis let x y z w, then parallel flows mean only 1 component exist and other 2 component 0 that means flow exist only in 1 direction of the coordinate axis. That classes of flows are known as parallel flow. Such a flow we discuss a particular type of flow is Couette flow. Couette flow is such a parallel flow. First definition of parallel flow is that u is not a, u is only the non-existent velocity v w 0. Now, we consider a parallel flow situation as a Couette flow therefore, we define a Couette flow.

A Couette flow is like this, there are two plates, one of which is at rest and another one is moved with a velocity of u infinity and there is a fluid between the two plates which is flowing within this channel. Channel is formed by one plate moving with u infinity another plate is fixed that means one plate is moving relative to another and the fluid is moving and the problem is described like that, the h is the depth between the two plates. What will be the velocity distribution, pressure distribution, flow rate across any section, these are the problems to be solved.

Now, why this is a parallel flow? Now, if we consider x y z axis here better we consider not at the middle because problem is not symmetric, this is moving, this is not moving, we consider along the plate x axis. In these direction between these, within these height that means in this perpendicular direction we better take y, we take z. Now, this problem is physically defined why I am taking z in this direction mainly the problem is two dimensional in x y plane, why? If we define the depth or the width of the plane in these direction perpendicular to this plane of the paper that means in z direction is very large as compared to their spacing.

That means compared to their separation this tends h then any variation of any parameter in z direction is negligible. It is from geometry with compared to their variation with respect to y direction because this direction is very small the geometry of this problem is such that the distance between this two plate separation distance h is small compared to its dimensions in the z direction. This way a Couette flow is totally posed or Couette flow problem is posed where the velocity of the fluid is taking place only in this direction, x direction u. It does not have, no, it does not have any velocity component in either y or z direction. So, only u is the non-existence velocity.

Apart from that we considered the flow to be incompressible, you know the meaning of it, incompressible you know the meaning of it. Then viscous, steady laminar flow, this meaning you do not know at the present moment. I will discuss it afterwards when I will talk about turbulent flow, laminar flow, the flow is laminar. There is no irregular fluctuations of velocities. In turbulent flow though the flow takes place in one direction there will be an irregular fluctuation in cross direction so which is not there. Another is the fully developed flow, fully developed flow meaning will be difficult at the present moment without knowing, without having the idea of boundary layer theory.

Fully developed flow means in the entire flow region the viscous effect is prominent, there happens under certain conditions of high Reynolds number, high velocity flow the viscous effects are prominent only in the near vicinity of the solid surface and faraway from the solid surface fluid behaves like an inviscid flow, but here it is not so fully developed flow. So, these assumptions are taken and we pose a Couette flow problem as a parallel flow problem

Now, if we take x y z axis like that. Now, if we define mathematically only u is non-zero component v and w 0 which is the first line of definition of a parallel flow with respect to a Cartesian coordinate axis, the immediate consequence of continuity is what? Do you know? Just I write for an incompressible flow steady thing is automatically taken into account for continuity equation. If we write the incompressible flow continuity equation you see the immediate corollary of continuity.

What is immediate corollary of continuity? Very interesting since v w 0 means this is 0, this is 0. That means u cannot be a function of x del u del x 0. That means immediately it tells that if there is a parallel flow then the flow velocity, existent flow velocity seizes to be a function of its own space coordinates, means in this direction in which it is defined. That means it has to be a function of other two space coordinates, it is the immediate corollary of continuity because del u del x 0. That means u is a function of y and z.

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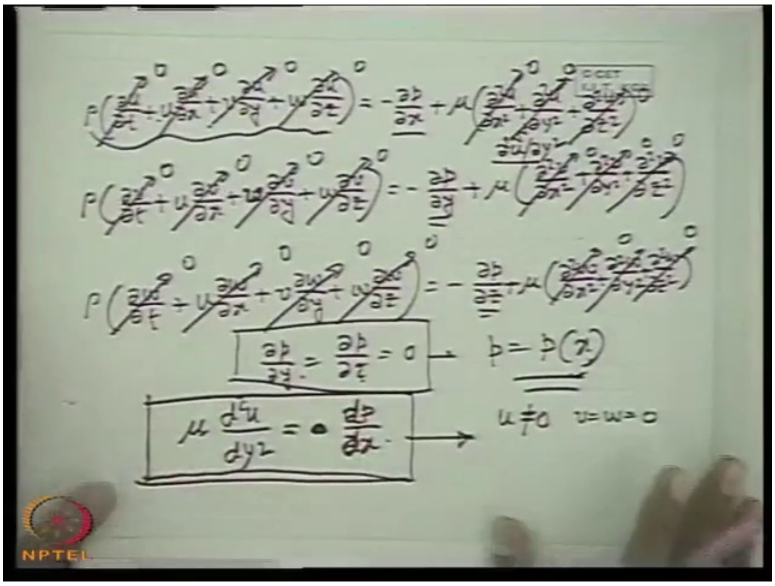

Since the problem definition is such that by geometry the width of the plate in z direction is very large that means the dimensions in z direction is large compared to the dimensions in y direction because h is small compared to the breadth in that direction. So, u cannot be a function of z variation in y direction is more than that of z with that assumption we just get a very simplified thing, conclusion that u the velocity u is a function of y only. That means u changes with y, u is a function of y, is neither a function of x nor of z. This is a very important conclusion we get from the continuity for parallel flow solution and with the help of the definition of the problem geometry u is a function of y.

Now, if we write the Navier Stokes equations what we will get we see. Now, we write one by one the Navier Stokes equation. First we write the Navier Stokes equation in x direction rho del u del t plus u del u del x plus v del u del y plus w del u del z is equal to minus del p del x plus mu. Here, one thing I am not writing the body force term because body force term is the only gravity. If I take this axis as the gravity and we take the body force, the gravity force merge into the pressure itself that it takes care of the piezometric pressure, then the gravity force are not normally written. mu del square u del x square plus del square u del y square plus del square u del z square and another term is 0 for incompressible flow.

Now, I write for del v del t y direction. Initially I consider all the three directions are equally important u sorry v sorry v del v del y plus w del v del z is equal to minus del p del y plus mu del square v del x square plus del square v del y square plus del square v del z square. Similarly, I write the z direction equation, all the term including all the term u del w del x plus v del w del y plus w del w del z is equal to minus del p del z plus mu del square laplacian part del x square plus del square w del y square plus del square w del z square, alright.

Now, you see what happens. First of all I write this see that, del u del t is 0, steady state. This term is 0, why? Please tell me this term is 0, why? del x 0 del u del x 0 though u is not 0, this term is 0 v is 0, this term is 0 that means identically the inertial term is 0. I do not know this. So, here you see since we have already proved that u is a function of x this is 0 and this is 0 come to this. Sorry I am sorry I am sorry blunder, this is called blunder, there are different definition of errors, the worst definition is blunder that means there is no correlation, there is no logic for error, there is no logic for this error del square u del y square. So, y term can I write this 0? del v del t can I write this 0, can I write this 0, can I write this 0 because there is no v component. Then can I write this 0 because there is no v component.

Similarly, when there is no v w component the inertial force and these viscous force in this direction does not come. Therefore, first I write the consequences from y and z direction equation of motion is that del p del y del p del z is 0, that means p is not a function of y and z. So, this gives that p is a function of x and u is a function of y only. Therefore, from these equation I get mu this side, I am writing first d square. Now, I will write d not del because already I have got in my hand u is a function of y. So, I will write in terms of ordinary differential equation and p is a function of x that means minus del p del x. No minus plus del, so this is the most interesting result and the conclusive results of parallel flow.

Therefore, what we get for parallel flow? So, far I have not come to Couette flow, for any parallel flow with respect to again and again I am telling with respect to a Cartesian coordinate system where u is the non-existent velocity, we get u as a function of y, p as a function of x which means p is not a function of y and z, p is only a function of x, p varies only in the x direction and u varies only in the y direction and the governing equation of motion is the unique and one equation which is mu d square u d y square is del p del x sorry why del p del x, it is d p d x.

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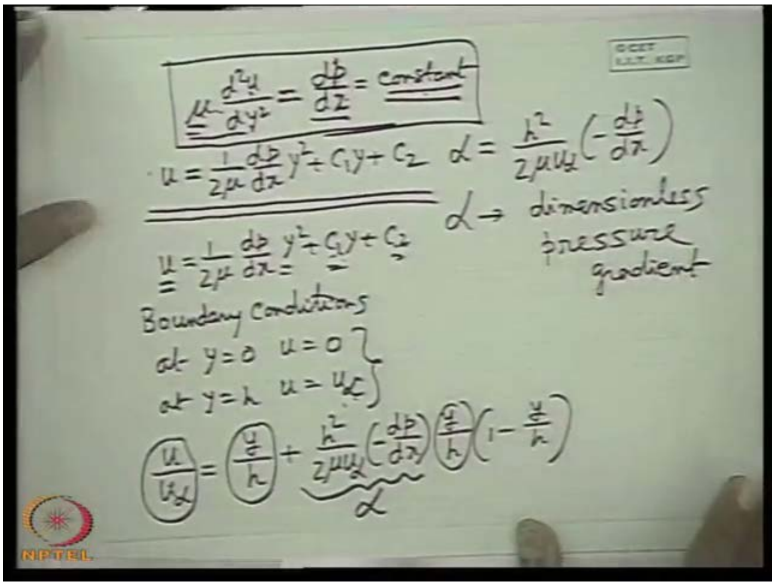

So, I get the equation mu d square u d y square is equal to d p d x. I come now to the Couette flow situation, the Couette flow situation, I come to the Couette flow situation. Now, I solve these equations for the Couette flow. Couette flow definition is this, mu d square u d y square is d p. Now, if we solve it, integrate it, it is very simple integration, school level integration. If you do it you get u is equal to mu is constant 1 by 2 mu. Two times u integrate d p d x, d p d x is constant into. Now, one thing, yes, which is very,

very important before integration which is very, very important. Now, we see mu d square u d y square is d p d x before integration one logic is required, what is that logic? This is equal to a constant.

That means mu d square u d y square is equal to d p d x is constant, that means d p d x is constant or d square u d y square is constant. Logic is like this, this u is a function of y we know. So, d square u d y square will be either a function of y or a constant depending upon the functional dependence. That means if u is a quadratic function of y then it is a constant otherwise if it is tripped y to the power 3 it will be a function of y. So, there are two alternatives either this is constant or function of y.

Now, p is a function of x. So, d p d x either will be constant or a function of x. If p is a linear function of x this is constant or if it not a linear function of x this will be a function of x. But if the two equality has to be maintained so this side cannot be a function of y and this side cannot be a function of x. So, from these equation itself an intelligent student can tell that d p d x is equal to mu d square d i square is constant means p is a linear function of x and u is a quadratic function of y such that the equality is to be maintained with only a constant C which is invariant with x and y

So, with this logic only I can integrate this keeping d p d x is constant with respect to y, that means y square plus C 1 y plus C 2, true, no problem. So, the logic is that this is constant. Now, this is the starting point of equation for again I write u is equal to 1 by 2 mu d p d x as a constant y square plus C 1 y plus C 1. So, two constant, so this is the starting point. Now I define it to find out all the hydrodynamic parameters for a Couette flow. So, what first we have to do to solve the differential equations first, then C 1 C 2 has to be found from boundary conditions.

What are the boundary conditions? Please tell me what are the boundary conditions? Boundary conditions is that at if I defined x axis like this and y axis like this, so tell me the boundary conditions means the values of u at some referred values of y we require two boundary conditions to evaluate that two constants at y is equal to 0 is 0, no slip condition. At y is equal to h there also no slip condition u is equal to u infinity because the relative velocity is 0, the velocity here.

So, if you put these two equations straight forward these equations gives C 2 0. Second equation, second condition will give the value of C 1, you get after some rearrangement u by u infinity is y by h plus h square by 2 mu u infinity minus d p d x into, well into y by h into 1 minus y by h with some rearrangements. After placing this simple algebra, this we can minus d p d x. So, this term we can, we substitute as alpha. Alpha is equal to h square by 2 mu u infinity.

Now, you see one thing that if you rearrange the equation in this form, this is dimensionless, this is dimensionless, this is dimensionless. So, that means this part is dimensionless, obviously if you equate the dimensions, this is dimensionless. So, alpha is known as dimensionless or non-dimensional dimensionless pressure gradient, dimensionless pressure gradient. Therefore, we get u by u infinity is y by h. Therefore, we get this u by u infinity.

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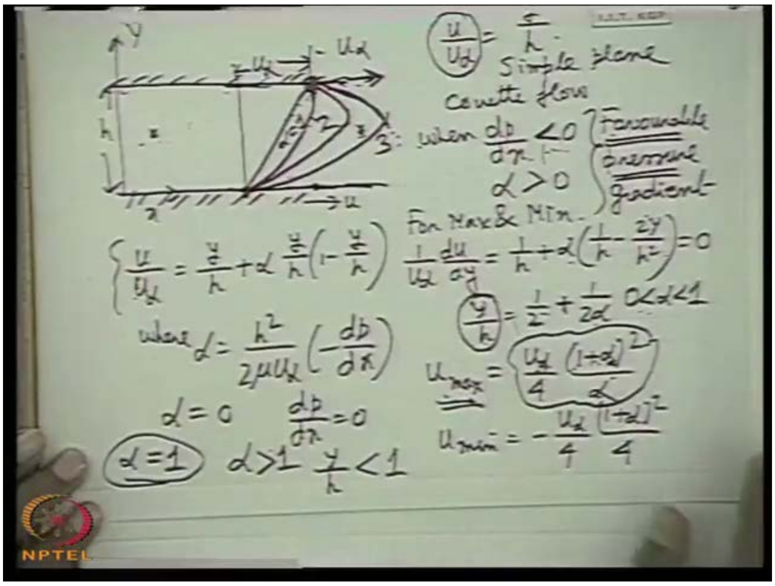

Therefore we get u, then now let us see that again if I draw the Couette flow. So, this is u infinity, this is h. Now, what we get? We get u by u infinity solution is equal to y by h, y is measured from this axis, this is the y and this is the x. That means this is x, this is y plus alpha into y by h into 1 minus y by h. This is the velocity distribution where alpha is equal to non-dimensional pressure gradient.

Now, you see one thing very careful. Now, we get a family of curves u by u infinity as y by the dimensionless u with dimensionless coordinate y. That means if you plot u by u infinity versus y by h then we get a number of curves with alpha as the parameter. Let us consider for different values of alpha what are the nature of this curve when alpha is 0, when alpha is 0, when d p d x is 0 there is no pressure gradient. Then we get a very simple relation u by u infinity is y by h. One thing is clear from these mathematics that even if there is no pressure gradient, flow takes place because we get a velocity distribution which is linear which physically means that if the pressure gradient is 0 that means throughout the pressure is same in the direction of flow.

Still flow is possible by the dragging action of the plate because the upper plate is moving with a value of u infinity, even there is no pressure gradient the movement of the upper plate causes the flow. But in that case we get a simple linear relationship. That means if we plot this here, we will get this type of, even if we plot here in this u then we get a linear relationship that means it starts from 0 and end at u infinity, let this is u infinity, let this is u infinity. So, this is, this is known as simple or plane Couette flow plane, Couette flow.

Now, there are two cases we will consider when d p d x is negative that means alpha greater than 0. When d p d x is negative alpha greater than 0, this is known as favorable pressure gradient. Why negative pressure gradient is known as favorable pressure gradient in fluid flow? Please, why it is known as favorable pressure gradient, always negative pressure gradient is told as favorable pressure gradient why? Because it, please, it it aids the flow, negative pressure gradient means mathematically it is negative that means the pressure is low in the downstream, high in the upstream because in the increasing direction of the axis the pressure is decreasing, that is the negative pressure gradient always the flux takes place with the negative potential gradient.

Of course, the fluid does not flow with pressure gradient as the potential gradient is the energy gradient, but pressure gradient negative means here, for example the pressure here is positive more pressure here is less. So, negative pressure gradient means which is a yielding force that means the negative pressure gradient yields a force on a fluid element which is in the direction of the flow. That means negative pressure gradient favors the flow, that means the pressure force is in the direction of the flow is in the direction of the flow that is why always you remember it throughout your lifetime fluid mechanics at favorable pressure gradient means negative pressure gradient, mathematically d p d x less than 0 so alpha greater than 0.

When alpha greater than 0 we will be interested in different velocity profile, but before that we try to find out one very interesting thing. Now, physically when there is a pressure gradient imposed on it apart from these u infinity so the two positive, two things are added, one is the dragging effect of the plate plus the pressure gradient. So, there is a chance that somewhere in the flow the velocity may be more than u infinity.

So, let us find out what is the maximum and minimum velocity conditions, therefore, for maximum and minimum condition we see that if you differentiate it 1 by u infinity d u d y is equal to 1 by h plus alpha into 1 by h minus 2 y by h square. And for maximum and minimum this has to be 0 and you get an expression which is like this, y by h is equal to half plus 1 by 2 alpha and you will see that when alpha greater than 0 the maximum condition will be satisfied with this condition and therefore, there will be always a maximum velocity when alpha greater than 0, that means with favorable pressure gradient.

When alpha less than 0, I will come afterwards when there is an adverse pressure gradient that means d p d x is positive, that is alpha less than 0 that means d p d x is positive means pressure gradient is positive. That means the pressure here is more than the pressure here, that means an adverse pressure gradient. So, gradient mathematically means this minus this, downstream minus upstream. So, downstream is less than the upstream means negative that means favorable, downstream is more than the upstream means adverse pressure gradient.

So, in case of adverse pressure gradient d p d x greater than 0 alpha less than 0 the minimum condition will be satisfied. That means the flow will have a minimum velocity and for a favorable pressure gradient flow will be having a maximum velocity that you see from this second derivative. And if you find out that maximum and minimum velocity by putting these values of y in the velocity distributions you will get this u infinity, this is simple job 1 plus alpha square by alpha and u minimum will be the same expression with a negative sign because always velocity is 0 at this wall. So, any minimum velocity in the flow field will be less than that, that means minus, that means in the opposite direction.

You see that mathematically it comes like that. Now, few interesting things I will derive. Now, first you see very carefully you observe this, this may not be written in many books, not even my book. Now, when alpha is equal to 0 we see that u by u infinity y by h that means velocity monotonically increasing, there is no mathematical maximum in this curve because this is a monotonically increasing curve, but forcefully the curve is truncated here because there is no flow regime after y by h is equal greater than 1 that means y more than 1 h that means practically this is the maximum velocity which is attained at the upper plate u infinity.

Now, you see when alpha greater than here. Now, you see when alpha is equal to 1 what is happen? y by h 1 that means the maximum velocity is occurring at the upper plate and its value is also u infinity alpha is equal to 1, 1 plus 1 2 whole square 4 by 4 so alpha is equal to, this is 1 4 by 4 u infinity which means for alpha is equal to 1 the difference is that the graph, the, this gives a 0 slope that means the same everywhere alpha is equal to one curve, the velocity is lower than the u infinity starts from the 0, it attains u infinity here with a slope making 0 because it attains a mathematical maximum at y by h is equal to 1.

When alpha greater than 1, that is all cases are favorable pressure gradient, alpha greater that means alpha when between 0 to 1, this gives this within this, when alpha greater than 1 you see what is y by h? y by h is very simple y by h when alpha greater than 1 y by h is less than 1. It should be half plus something less than the half which physically means that a maximum velocity will occur at a distance y by h less than 1, that means somewhere within this flow field that means below the upper plate.

Therefore, the picture will be like this. So, for all values of alpha, so the maximum will occur. So, if you go on increasing the value of alpha that means favorable pressure gradient, you will see maximum will be more and the value of maximum is that for example, if you take alpha 2 put there you get the maximum value and which is more than u infinity. You see when alpha is equal to 1 it is exactly u infinity, when alpha less than 1, but greater than 0 it is less than u infinity. But you see this alpha greater than 1 so this value is more than u infinity u max and that occur here and as you go on increasing alpha this will come away from the flat plate, so this is one part.

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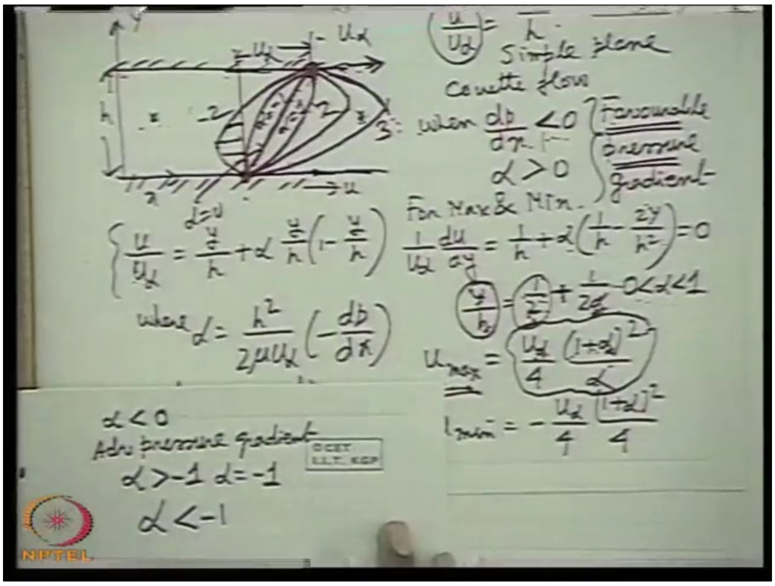

Now, you see what happens when alpha is less than 1. Please, see that what happens when alpha is less than 1. When alpha is less than 1, I will otherwise you cannot say when alpha is less than 1 that the alpha is less than 0 sorry not less than 1, when alpha is less than 0 that means adverse pressure gradient, adverse pressure gradient. Adverse pressure gradient when alpha is less than 0 is the adverse pressure gradient, in that case what is happening the minimum velocity will occur, but you see when alpha is greater than minus 1, when alpha is greater than minus 1 what will happen? y by h is less than 0 alpha is greater than minus 1, less than 0 means it is beyond the flow region.

So, that means the minimum will occur here only. But when alpha is equal to minus 1, if you put here y by h 0 that means it is just here, that means just the opposite one, that means the minimum velocity will be here only 0 velocity, but with a mathematical minimum here, that means slope is 0. But when alpha is less than minus 1 that means minus 2 minus 3 then y by h is greater than 0, alpha is greater than minus 1, that means minus 1 minus 2. So, this will be greater than 0 because this will be greater than half sorry this will be yes, this will be greater than minus 1 that means minus 1 minus 2 half plus half alpha will be always giving a positive value because this is more than this. That means the negative will that means the minimum value will occur somewhere here. That means this is the reverse here. Let for example, minus 2 alright. So, this is alpha is minus 1. So, this straight line is alpha 0, straight line part is alpha is 0. So, flow reversal will take place.

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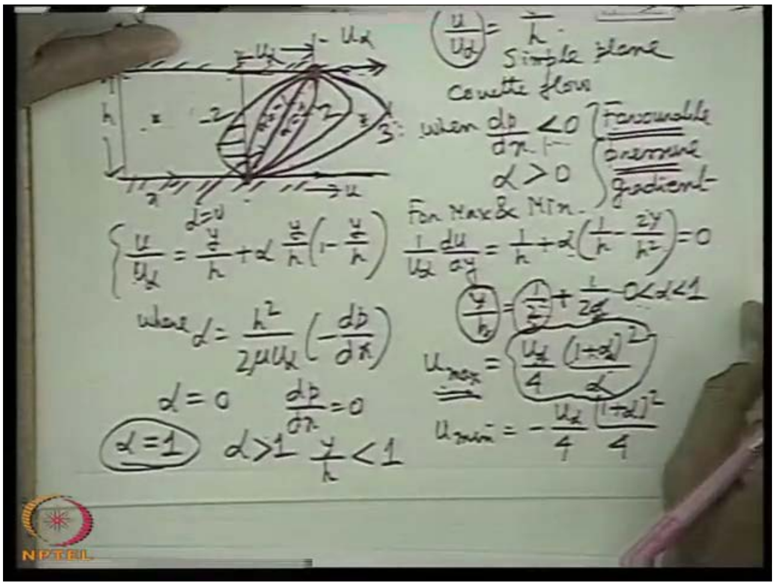

So, these are the very interesting phenomena or interesting features of the velocity profiles. Alright, all of you have understood. So, next class I will continue again the Couette flow.

Thank you.

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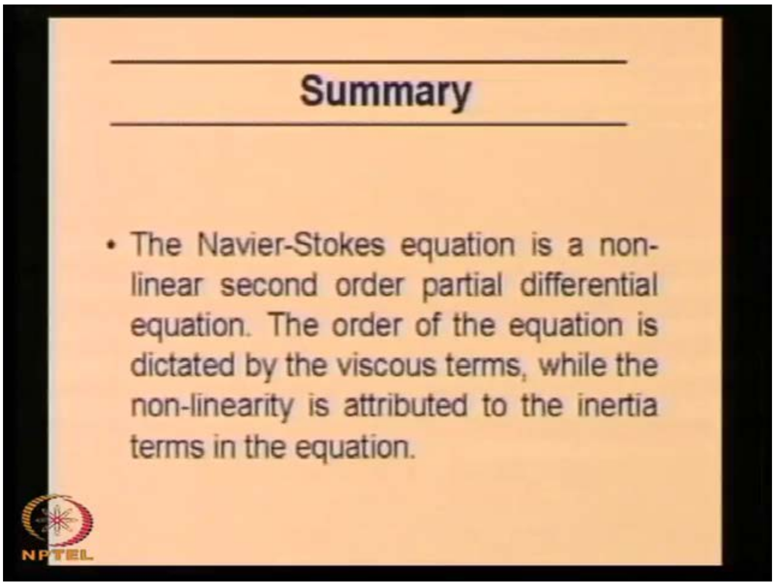

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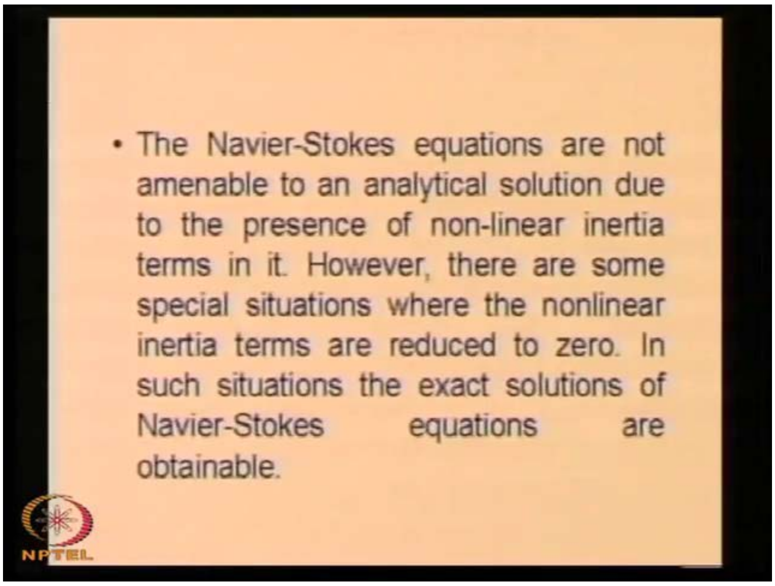

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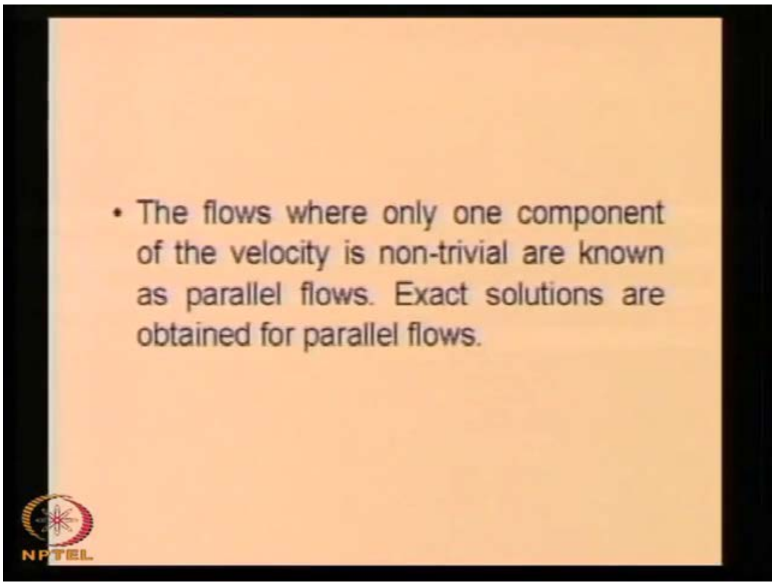

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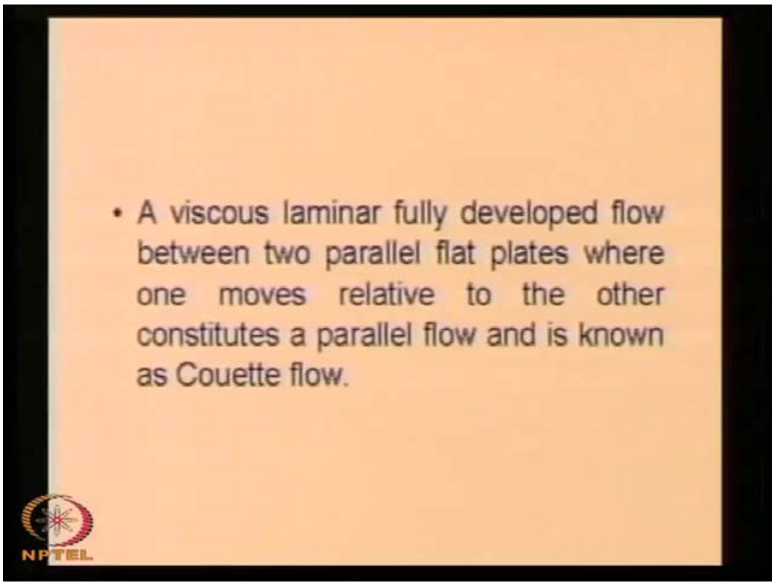

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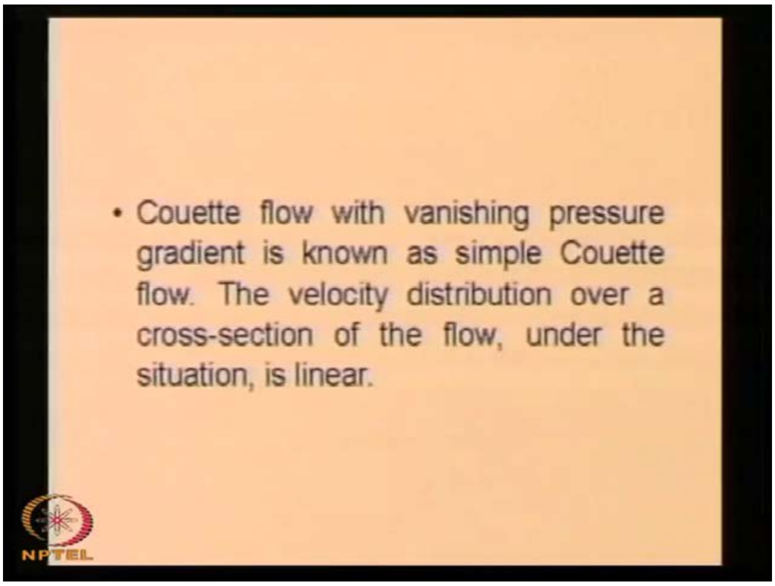

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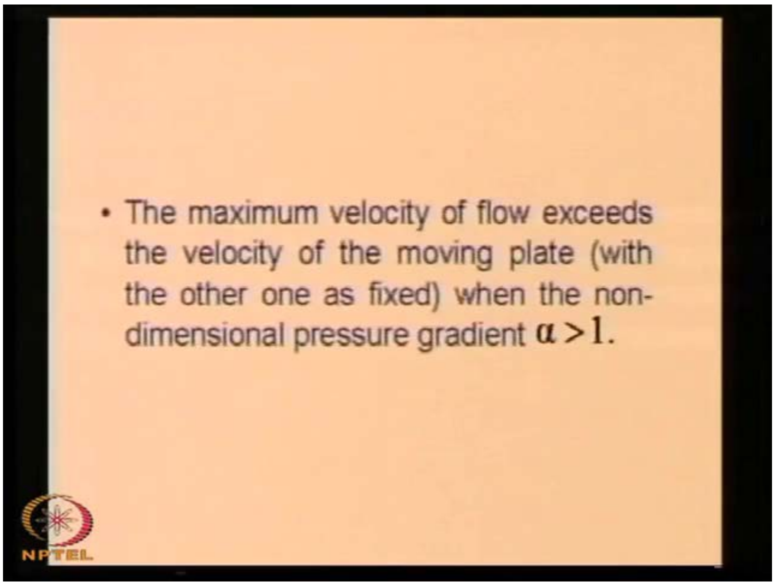

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