Fluid Mechanics

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Module - 1
Lecture - 21
Conservation Equations in Fluid Flow Part - IX

Good morning, I welcome you all to this session of fluid mechanics. If you recall in the last class, we mentioned about the class of problem, that when a fluid body is moving with uniform velocity or with uniform acceleration, what will be the force field generated in the fluid body, which we will be discussing today. So, first of all you must know that when fluid body is moving with uniform velocity, that is velocity is uniform throughout the velocity field is no longer equate to be find out, because velocity is same. Since, that case we can find out a pressure field from the velocity field by the application of Bernoulli's equation, in case of an inviscid fluid.

And in case of a viscous fluid also since the velocities are uniform, there is no shear stress developed in the fluid, because there is no velocity gradient. So, shear stress cannot come into picture. So, in either cases the fluid will behave as an inviscid fluid or you can tell it is an irrotational fluid, where pressure field is found straight from the Bernoulli's equation, that p by rho plus v square by 2 is constant in the flow field. Of course, with the elevation head, then we can find out the constant with some reference boundary condition. This is a very simple class of problem with uniform velocity, simply a rigid body translation or a rigid body rotation.

Now, if we consider a uniform translation, what is the case? That means if, a fluid body is translated uniformly, this happens for example, a fluid body contained in a tanks, solid tank in practice, that tank is accelerated in certain direction with a uniform acceleration, in some direction, it may be along the coordinate direction. So, in any direction if the tank is accelerated uniformly, what will be the force field generated or the pressure field generated in the fluid body? So, this is the precise problem for today. So, let us now analyze this problem with uniform translation.

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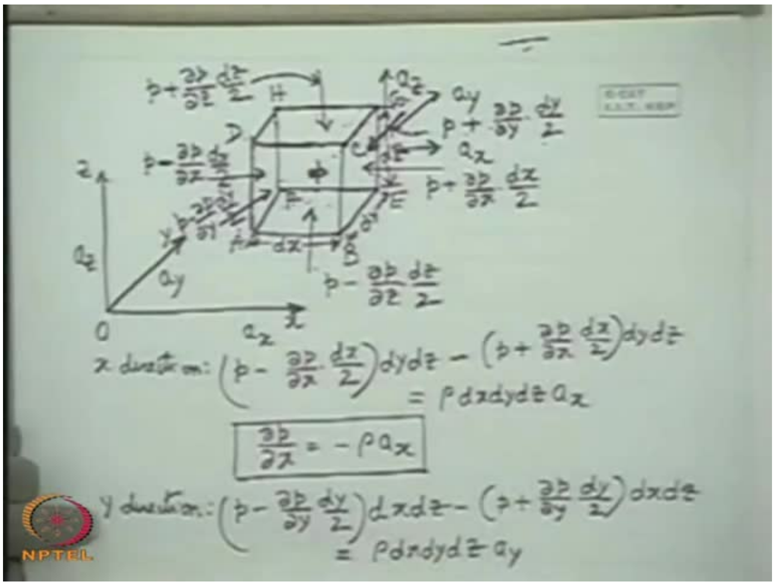

Let us consider a axis like this o x, o y, o z with this coordinate axis, let us consider a tank like this. Let us now, not tank, let us consider a fluid body. Let us consider a fluid body now, appropriate to the coordinate axis, in a Cartesian coordinate system. Now, we take a general case that the fluid mass, that means this fluid mass, which we have taken here separate fluid mass is accelerated in both x y and z direction. These are the component of acceleration, that means it is accelerated in x direction by a x, accelerated in the y direction by a y and accelerated in the z direction by a z. These are the components or acceleration in x y and z direction.

Well now, let us define as usual d x is this dimension, d y is the dimension of the parallelepiped along the y direction and let this is d z, that means d z is this one, this one is, let this one is d z. this one is d z. That means the parallelepiped of dimension d x d y and d z. Now, how to find out the pressure field? So, simple thing is that, the way we found out the force equilibrium condition, hydrostatic force equilibrium condition, it is the same one only that is, inertia force is acting because of the acceleration, this is extremely simple.

If you define the pressure at the center, it is better more generalized approach, I did earlier that p p plus del p, del rather p is defined at the center uniquely for this parallelepiped. So, then what is the force on this plane? If we now again define, to recognize the plane A, B, C, D and this one is E, this one is F, this is G, I have not drawn this with dotted line. These are the back lines, from if you see from this. However, so the plane A, F, H, D is the x plane and C, G, E, B are the x planes. Now, what is the pressure in this x plane and what is the pressure in this x plane, in this direction?

Pressure in this x plane, which is separated from the centre by a, d x by 2 distance in the positive directions, will be p plus del p del x d x by 2. Neglecting the higher order term, that means this is the increment of p due to a change in d x by 2 into positive direction. Therefore, pressure here will be p minus, that is, this is also an increment of p from that to this plane for a change in x. That is separated by a distance minus d x by 2; that means d x by 2 in the negative direction. That is why the negative sign is there mathematically. Similarly, for this y direction, there will be pressure forces for the y plane. That means for the y plane, that means the pressure force is in D, C, B, A similarly, A, G, E, F.

For A, G, E, F this pressure forces, this force, that means this force perpendicular to the face A, G, E, F, which is at a distance d y by 2 from the centre of this parallelepiped, along the y direction. Therefore, this magnitude will be p plus, I think you follow it. Del p del y into d y by 2, that means the pressure in this plane H, G, E, F that is the back plane, which is displaced from the point p along the y direction by d y by 2, because this is d y. So, this is centre point, so d y by 2 plus d y by 2, so it is increased. Similarly, the front plane that D, C, B, A or A, B, C, D y plane, which is separated from p or distance from p in the minus d y by 2, that means in the opposite direction of the positive y direction.

So, mathematically this will be p minus del p del y into d y by 2, all right? Similarly, the pressure at these planes, this pressure will be, what this will be? p plus del p del z into d z by 2. Similarly, pressure here, this pressure will be, p minus del p del z. That means with respect to a pressure p, defined at the centre of the parallelepiped, we can specify the pressure at each faces. That means 2 x faces, 2 y faces. x faces means faces perpendicular to x direction, 2 y faces faces perpendicular to y direction and two z faces. So, 6 faces, the pressures are specified like this.

Now, simply we find out the force balance equation. Now, in x direction, what is the force balance? x direction, the force balance is the net force acting in the positive x direction, must equal to the mass times the acceleration in positive x direction. Obviously by Newton’s second law of motion, that means net force acting in the x direction, will be the force. That means this pressure into this area minus pressure into this area, so that will be p minus del p del x, very simple. d x by 2 into the area, that is area of the x plane A, F, H, D is d y d z minus the force exerted in this direction, due to the pressure in C, G, E, B plane.

That is p plus del p del x into d x by 2 area is same, because it is parallelepiped. The area is same for 2 x plane and that that must be equal to rho into d x d y d z times a x. So, therefore we see from these equation, del p del x is equal to minus rho a x. So, this is one very important equation. Similarly, if we make the y direction, y direction, if we make the y direction balance, force balance, so it will be p similar way minus del p del y d y by 2 into, what is y direction area? d x d z, that means this is the force on the plane A, B, C, D pressure is p minus del p del y d y by 2 times d x d z minus the force acting on the G, H, E, F plane.

Due to this pressure, which is p plus del p del y d y by 2 in the similar fashion p plus del p del y d y by 2 and the area is same d x d z, very simple is equal to rho times d x d y d z. That is the mass of the elemental volume times the acceleration uniform acceleration in y direction.

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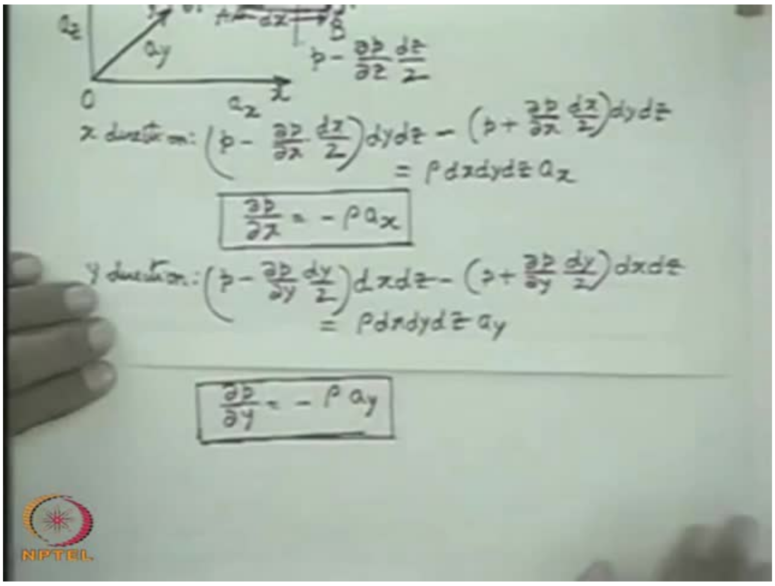

So, this gives rise to the equation del p del y is equal to minus rho a y. If you solve this, you will get. So, in x and y direction, we get del p del x minus rho x and y direction del p del y minus rho a y. now, what is in z direction? Now, in z direction if you see, z direction, here we can write, some space is there.

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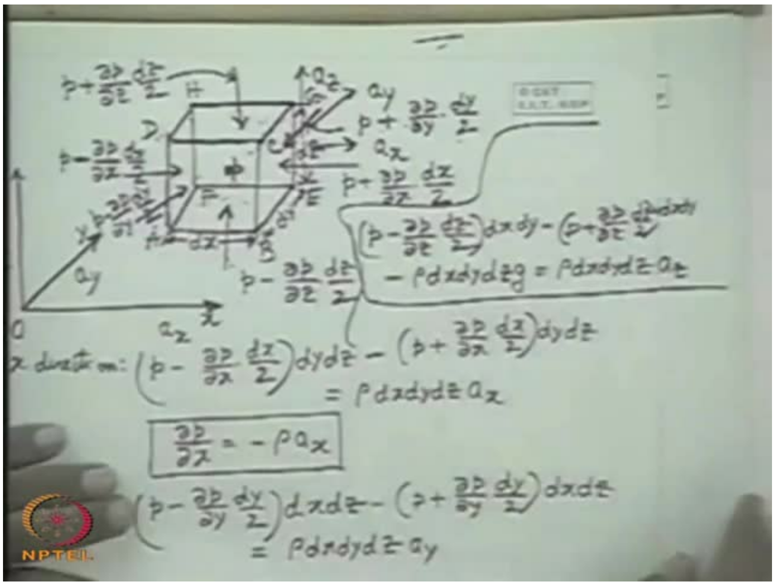

Here, we can write for z direction, z direction what is the net force acting on this volume in the positive z direction, please? p, so minus del p del z into d z by 2, d x d y, well minus force due to this pressure on this surface, this is the force due to this pressure on, A, F, E, B surface minus the force due to the pressure on D, A, G, C surface. That is p plus del p, any difficulty you please tell me. d z by 2 into d x d y, all right? d x d y, you can see that d x d y, then another force, what is that one?

Wait, that is acting downwards, so minus rho d x d y d z times the g is equal to again rho d x d y d z times a z. That means, this is the acceleration, uniform acceleration of the body in the positive direction of z axis it is given upward. So, if you solve this one, if you solve this one, you get, if you solve this one, what you get?

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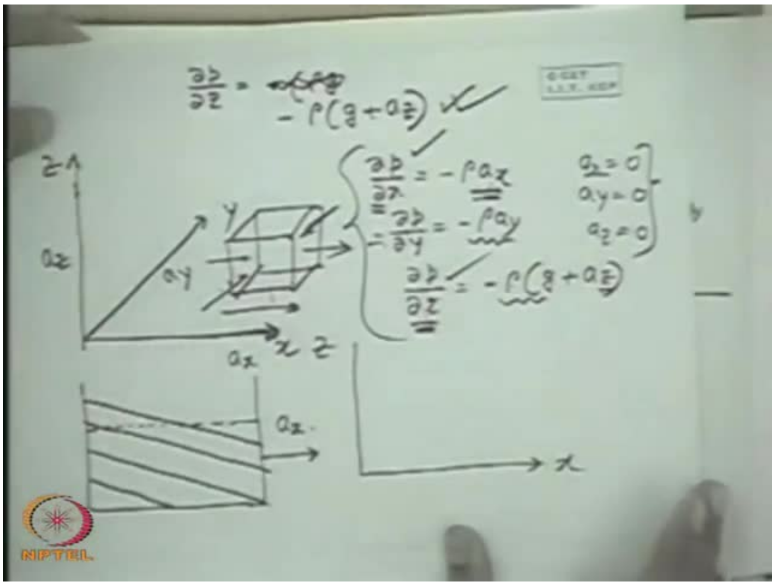

You get, well you get, if you solve this one, if you solve this one you get del p del z is equal to minus rho d x d y d z, yes. rho g is equal to minus rho g, sorry, is equal to minus rho into g plus a z minus del p del z. This is minus del p del z, if I take there, then it comes here minus rho into g plus a z from here, this is clear? Now, therefore we can write that in this case, what we get in a fluid body, which is very important, x, y and z. That means if a fluid mass is uniformly accelerating in both x, y, z, with an acceleration component a x a y and a z, then the pressure distribution in the fluid mass in x, y, z direction, now follows these equation.

If you recall or just we have derived del p del y is minus rho a y and del p del z is minus rho into g plus a z. Now, if we see these equation and compare with the hydrostatic case, now you see in hydrostatic condition del p del x was 0. That means there was no pressure distribution in x. Similarly, y direction in y, if z direction is vertical that means if a x is 0, hydrostatic condition a y is 0 and a z is 0. If there is no acceleration or in case it moves with uniform velocity, then also a x, a y, a z 0. So, whether a fluid moves with uniform velocity or does not move at all, there is no variation of pressure in any of the horizontal direction, but only it varies in the z direction, which becomes equal to minus rho g physically.

It means that it supports the weight of the fluid, the pressure difference created in the z direction supports the weight of a fluid element, but when it moves with an acceleration a x, definitely the pressure will vary. For example, if we consider a fluid element like this, so pressure should essentially vary in the x direction, which should take care for it is force acting on the body to cause an acceleration a x. So, this physically implies this thing. So, pressure should essentially vary in the y direction, so that the net pressure force in y direction should take care of the acceleration in the y direction. Similarly, for z direction the pressure should essentially vary to take care of the weight of the fluid plus the acceleration.

So, simply these are the equations of tank. Now, in most practical cases what happens is, that if we have a tank, now you see this practical case, if there is a free surface like this, initially if this tank is now accelerated. For example, let us consider the acceleration in this a x direction, now what happens? You will see the constant pressure lines will be like this, it changes like this. This is the constant pressure lines. So, to solve this type of equations, let us derive in general. In this case, what should be the equations or planes equation of the planes for constant pressure?

Let us consider this for a two dimensional. First for simplicity, let us consider a two dimensional problem, that means x and z. Let us consider a two dimensional problem, that means in x and z. If we see a two dimensional problem with a x 0, a z 0, so we see del p del z is minus rho g. So, in this case of two dimensional problem, we can solve this with, well we can solve this like this, that in a two dimensional problem of this nature.

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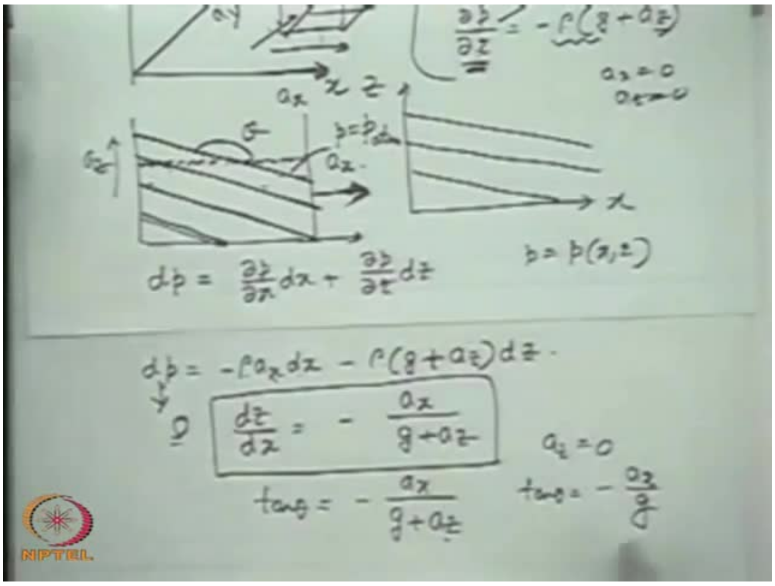

Let us write, d p is equal to del p del x. Let us for simplicity take a two dimensional case, del p del z d z that means p is a function of x z. Then along a constant pressure line, we are going to search for a constant pressure line in the x z plane. If a two dimensional case a x is 0, a z is 0, z is the vertical direction, so constant pressure lines will be horizontal, that we know. Now, here let us find out in general in case, when there is an acceleration, what is del p del x? It is minus rho a x d x. What is del p, del z? Del p del z is minus rho, as we have seen already a z d z.

So, we can integrate it to get the value of pressure at any point, in terms of x and z, but our first idea is to find out the equation of constant pressure line, along the constant pressure line d p will be 0. Therefore, we can write d z d x is equal to minus rho rho cancels, a x by g plus a z. That means therefore, we can draw lines, whose slope is like this, that means these are the constant pressure line, that means in a practical problem of tank, the constant pressure lines will be like that. That means if there is a free surface, the free surface will be like that. So, this is the slope with this, so this is theta. So, that tan theta is equal to minus a x by g plus a z.

Now, when a x is 0, that means there is no acceleration in this direction, even if there is an acceleration in z direction. So, the constant pressure lines will be horizontal because there is no pressure variation in the x direction, because del p del x is rho a x del p del x is minus rho x. So, a x is 0, there is no pressure variation, that means it is simply given the horizontal line. Try to understand, as the constant pressure line but the del p del z will be given by rho g plus a z. That means as if g is increased by an amount g plus a z when a z is the acceleration in the vertical direction.

But if there is no acceleration in the vertical direction a z is 0, only in the horizontal direction, then tan theta is equal to minus a x by g. In that case a z is 0, that means rho a x d x minus rho g d z is equal to d p. Therefore, if a tank is accelerated for example, in this direction, in one of the horizontal direction, the initial free surface, which is horizontal will now take a shape like this. So, this will be the constant pressure lines, so the free surface will be p is equal to p atmospheric. So, you have understood this? Now, with this understanding some very important conclusions will come, that when a x is 0, the surface is…

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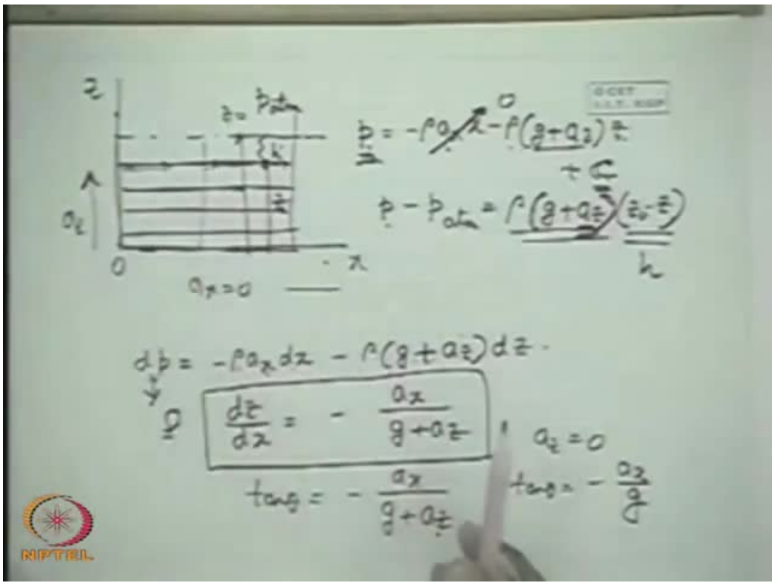

Now, again if this a x is 0, when a x is 0, 1 square, there is a z in that direction, then the liquid free surface will remain horizontal, as it is. But the pressure distribution will change because d p is minus rho g plus a z d z, all right? That means, if we integrate it p is equal to minus rho a x x minus rho, these are constant g and a z z plus C some constant arbitrary constant. So, when a x is 0, this is 0, so p is simply given by minus rho g plus a z z plus C. That means, if we consider a free surface form any datum plane, if we, or exact, if we take this as the axis o x o z.

So, if this be z 0, and if another point we take z, by applying this. Here if it is p atmospheric, a simple application of this equation, without a x we can write p at any point minus p atmospheric is simply rho into g plus a z into z 0 minus z because you will put z is equal to z 0, where p is equal to p atmospheric, defining z 0 is this coordinate taking this as axis. So, C will be found out p atmospheric plus rho into g plus a z into z 0. Then we will put that value of C and find out the value of pressure p at any other coordinate z. We will get, that means if this be the depth h, that means from the free surface we know the pressure is more than the atmospheric pressure by rho g h.

In case of a static fluid or fluid moving with uniform velocity, but in case of a fluid moving with uniform acceleration in z direction. So, contribution of z direction acceleration is to change the g, that means it is increasing g by g plus a z and take care of that, all right? If there is no acceleration in x direction, there is no variation of pressure in the x direction. So, lines will be horizontal, but if there is an a x, then the lines will not be horizontal because the pressure at this two points will be varying. As we have told, then the constant pressure line is like.

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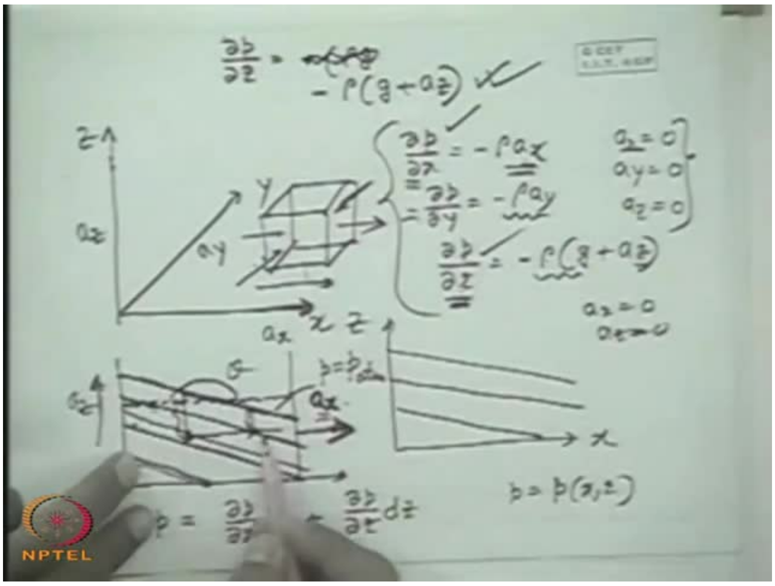

That means, in that case at two same horizontal point, the pressure will be changing, pressure will be changing because this is the free surface then. So, here pressure is equivalent to this depth and here pressure is equivalent to this depth, whether it is depth into rho g or rho g plus a z, depends upon the a z. But this will be more than this therefore, two horizontal points are not in the same pressure because there will be another constant pressure line, through this point, which is higher than this.

So, this two difference in pressure represents a net force in the x direction, that has to be, to balance the inertial force or the force required to cause the acceleration a x. Therefore, this thing should be very clear from physical sense and mathematical sense this is the constant pressure line. Now, another interesting results comes out.

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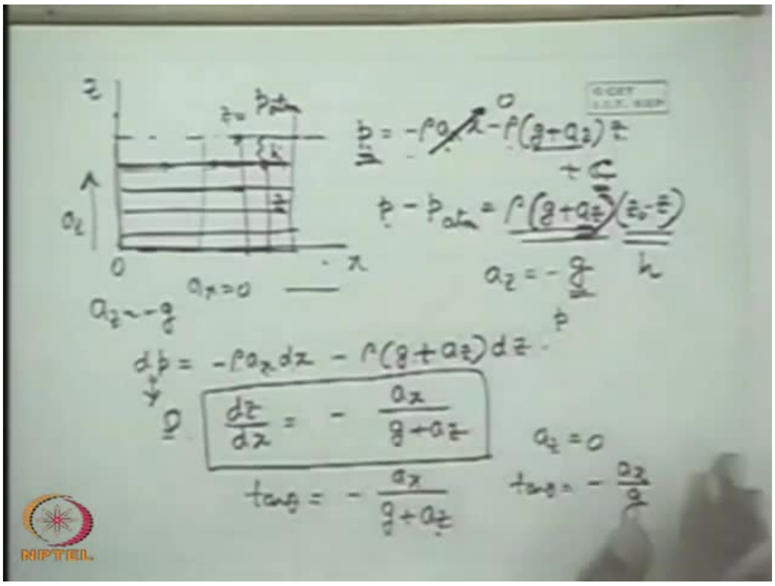

If a z is minus g, that means if the tank forget about a x a x is 0, if a tank is accelerated downwards with a acceleration equal to acceleration due to gravity, that means if the tank is allowed to fall freely, the free fall of the tank in that case, we can say a z is equal to minus g according to a mathematical nomenclature. In that case del p del z, I will not write here from the differential equation itself del p del z is equal to 0 del p del z is equal to 0 from the basic differential equation a z is minus g, that means from this equation, you can put that a z is minus g.

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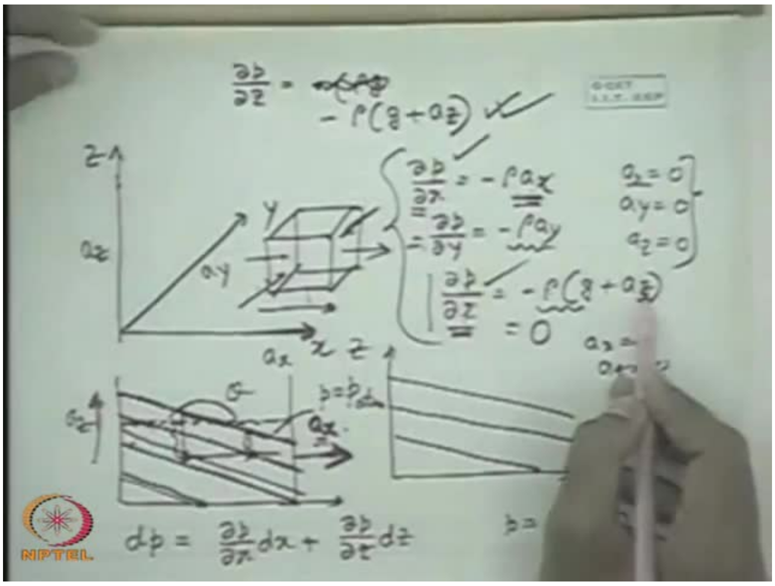

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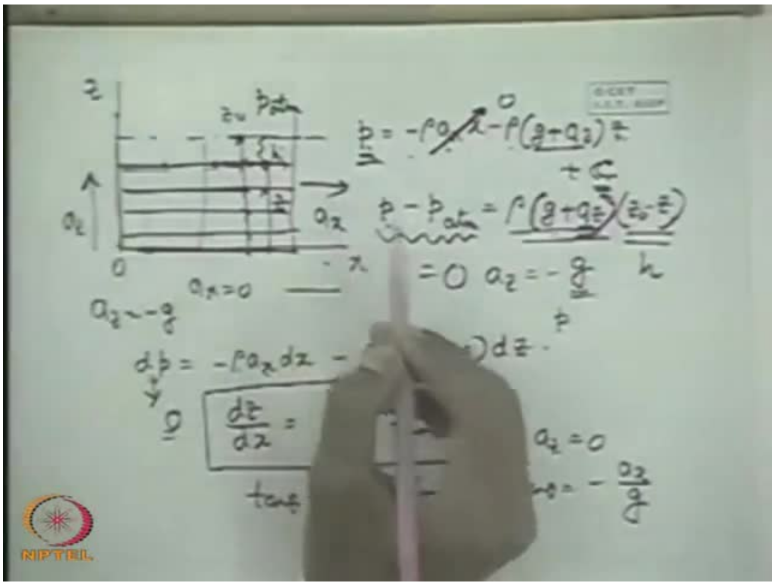

That means p minus p atmospheric is 0 for any value of z, that means in that case the pressure at each and every point remains same. Similarly, if there is no acceleration a x in these direction, then del p del x is also 0 minus rho a x, that means pressure also does not change in that direction. That means, if there is a free surface and the open tank and if you allow the tank to fall freely, the pressure throughout the liquid bodies atmospheric pressure. pressure neither changes along this pressure, neither changes along z.

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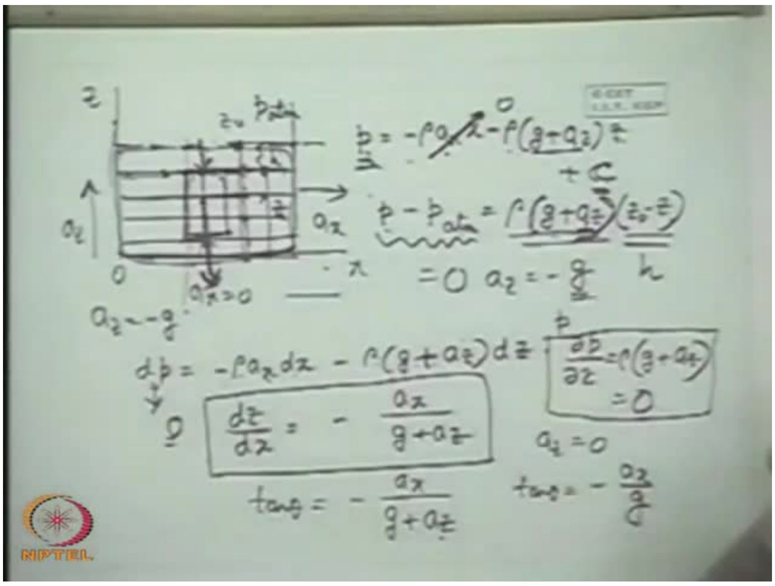

So, pressure neither changes along z comes simply from the equation, that del p del z is minus rho g plus a z. You know a z minus g automatically 0 physical explanation is very simple. That the mass or weight of the body to balance it the pressure differential is not required because the weight of the body causing, it is free fall. A body falls freely because of its weight, that means the force by which the gravitation is attracted force of gravitation or the force by which attracts the body.

That means it is because of its weight, this is falling freely. So, if the fluid body is allowed to fall freely the pressure differential in z direction seizes to exist, that means there is no pressure differential. So, pressure is uniform along the z direction, that is del p del z is 0, this is most important fact, you can experience it in your house or hall.

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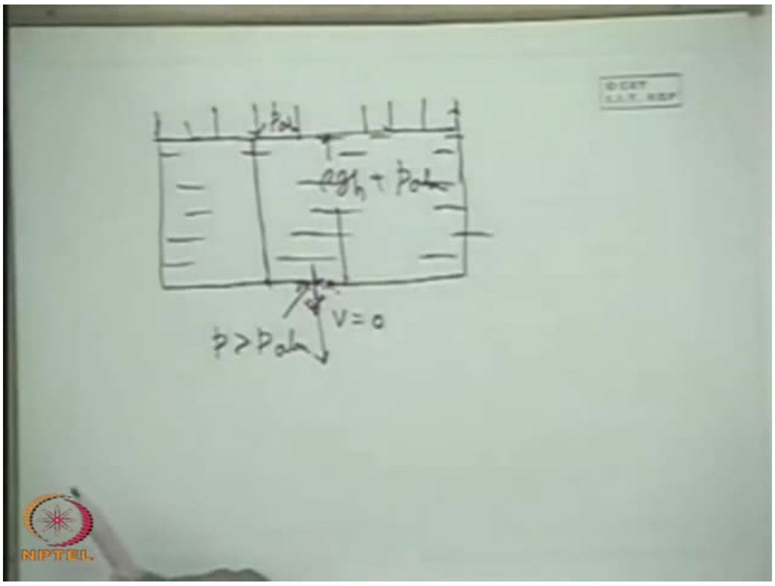

One of our colleague experience that, if you have a tank, that is written in my book and there is a hole made and you carefully fill up the tank with these hole closed by your finger, then you let the tank fall freely. If it fall freely, you just let it down go down from your from the roof of your house, you will see the water will not leak. V is 0, why water will not leak because here is the atmospheric pressure. So, water usual case leaks, when is a static condition or any other acceleration, when the pressure here will be more than, the pressure here is p, if it is more than p atmosphere, then only water will leak

So, if you fill up a container and make a hole at the bottom the water will come out like anything because the pressure here is rho g h plus the p atmosphere, if p atmosphere is pressure. But when it falls freely the pressure throughout is atmospheric pressure, because there is no pressure gradient. If there pressure is p atmosphere, so here also p atmosphere, so water will not leak from the hole. So, this is a very practical application of it, all right, Simple? Now, let us solve a problem, it is very simple. Let us solve a problem example.

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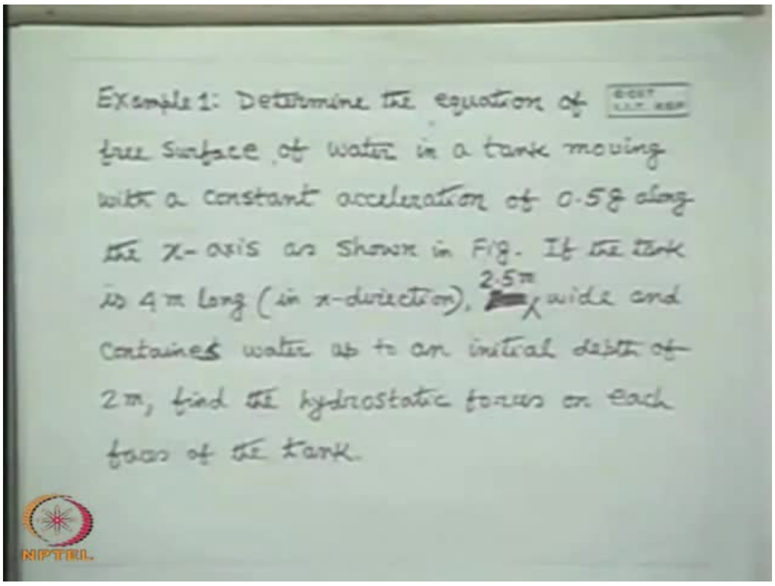

One determine the equation of free surface, this problem is of course, an worked out example in my book, but still I have given some extension to this problem. So, first part is worked out, which is a straight forward application of the equation, that we have derived just now. Determine the equation of free surface of water in a tank moving with a constant acceleration of 0.5 g along the x axis. as shown in figure. I will show the figure, you write, that means a tank is moving with a constant acceleration of 0.5 g along the x axis and water is contained in the tank. The first part is the determine the equation of free surface of water.

Next part is that, to determine the equation of free surface. You are, you do not require any other data, next part if the tank is 4 meter long in x direction, that means in the horizontal direction 2.5 meter, that I will show the x direction. Means, the direction in which it is being accelerated 2.5 meter wide, that means in a perpendicular direction and contains water up to an initial depth of 2 meter, contains water up to an initial depth of 2 meter, find the hydrostatic forces on each faces of the tank.

Please please if the tank is 4 meter long, that means I should wait 2.5 meter wide and contains water up to an initial depth of 2 meter, find the hydrostatic forces on each faces of the tank? So, this problem, if you solve the problem tells like this, okay?

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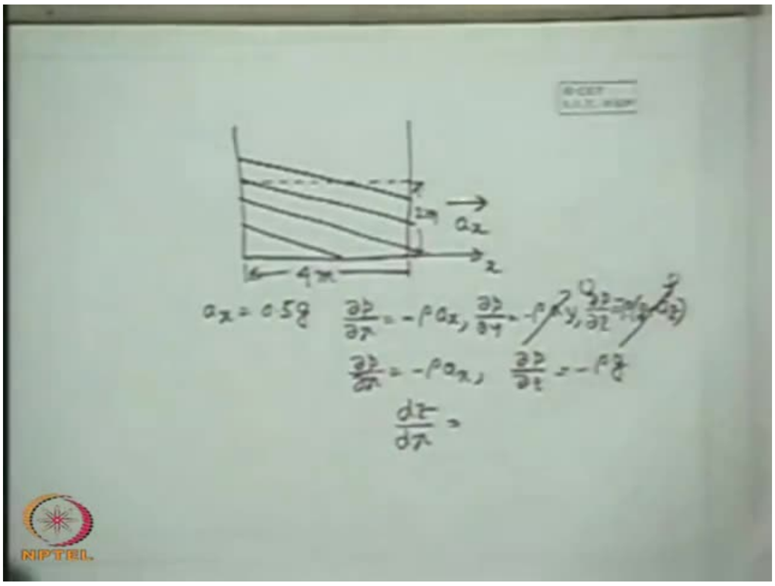

Let me draw the problem, this is the problem, that means initial depth is 2 meter, that means this is 2 meter, which is not required for the first part, but still this is x direction. Let us consider this is x direction, that means in this direction it is accelerated a x 4 meter and 2.5 meter wide, which is perpendicular to this plane. Now, initially first part is that it is given a x is equal to 0.5 g. Well, so the, we know the surface will be like this, constant pressure surface we just draw it qualitatively without knowing anything, but better, let us write the equation del p del x is minus rho a x del p del y is minus rho a y and what is del p del z? Del p del z is is equal to minus rho g plus a z, well g plus a z.

So, here there is no a y 0, there is no one dimensional, that means acceleration in this direction. So del p del x is minus, no sorry, a z is 0, so del p del z is rho g. So, del p del x is minus rho x. So, here del p del x is minus rho x and del p del z as usually, even if there is no acceleration, it is a minus sign. But we can straight forward use, that d z d x that means, d p is del p del x d x into del p del z d z, as I have done earlier, just you can see that I have done earlier. Just here you see that d p is del p del z this one. So, where I have written is d z d x is minus a x by g plus a z, that means I can use this formula straight forward d z d x. That means I can again deduce this formula from here in general a z. If it make a z 0 for the specific problem, that means for the specific problem it is minus a x by g. So, it is very simple.

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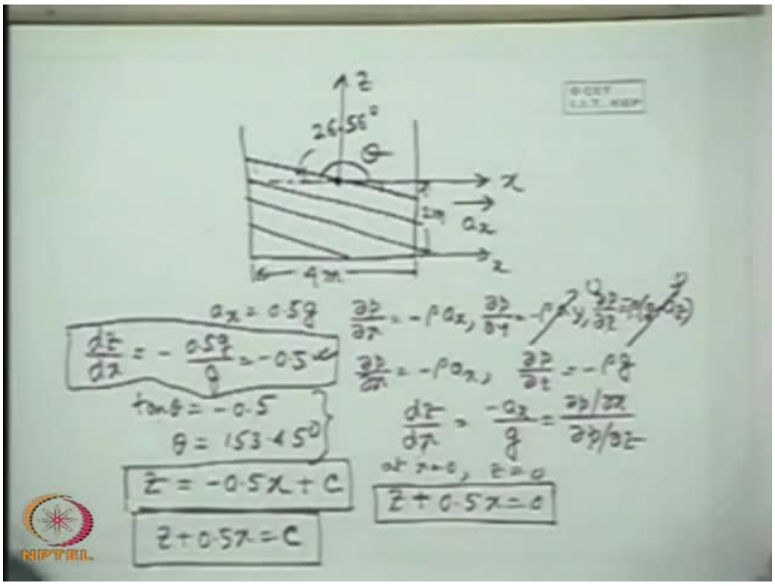

So, in this case it is minus a x by g, that means it is nothing but minus del p del x by del p del z. That is del p del x by del p del z, you can derive it. You should not write a formula directly because this will give you del z del x or d z d x, that is minus a x by g. So, therefore, d z d x is equal to minus 0.5 g. that means equal to minus 0.5. So, if I define this as theta, this then tan theta is equal to minus 0.5 and this gives a value of theta. Better I write the value 153.45 degree. I have solved it; that means this angle is 26.56 degree, which will be required. 26.56 degree, so this is the value.

So, now equation of a, this will be required afterwards, now we cannot use this. Now, better we find out the equation of free surface, you integrate this. z is equal to 0.5 x. if you integrate this. The value of tan theta will come afterwards, 0.5 x plus C. So, this is the equation, of the free surface. That means the equation of this line, which makes a slope tan theta minus 0.5, these are the parallel equations of the free surfaces with C as the parameters. Now, tactfully one can find out C and can make it 0. If we take this surface of intersection, if we take at the middle for symmetry, if we take x here and z here, then we can define that at x is equal to 0, z is equal to 0, if we take here axis.

So, that C is 0, so z plus 0.5 x is equal to 0 is the equation of the free surface. z plus 0.58 is equal to 0. so along the free surface always z plus 0.5 x is equal to 0 or you can write z plus 0.5 x is equal to C. So, the value of C depends upon a arbitrary reference for choice of the coordinate axis, this part is all right? This part is all right?

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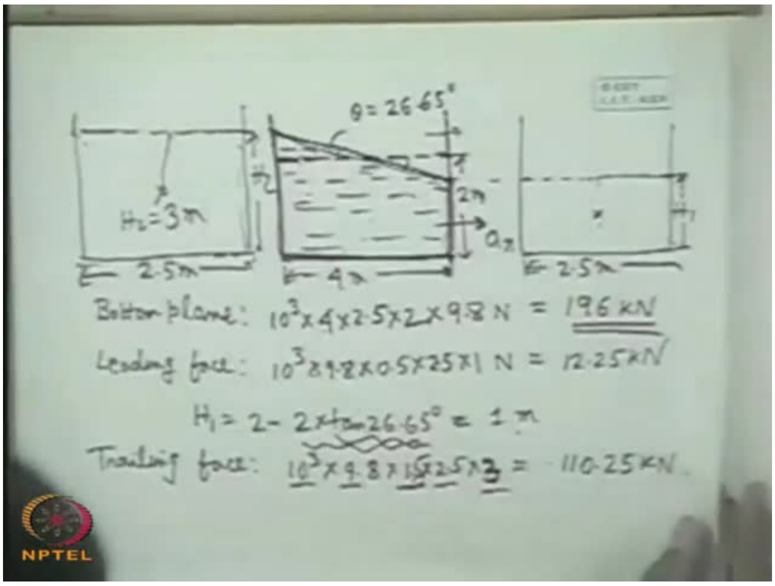

Now, next part, well I can draw the figure again for the next part. For the next part, let us consider this is, again for next part. Let us consider this again 2 meter and this is 4 meter now, the free surface we have already determined like this. So, water is carrying, carried like this. This is now the water, this is the earlier free surface, this was the free surface earlier. So, this was the earlier free surface, now the free surface has taken a shape like this, where this is the liquid, this angle. As I have already found out theta is equal to 26.65 degree.

Now, we have to find out the forces on each plane. Now, tell me first, what should be the force on the horizontal plane, bottom plane? Bottom plane is the very good, so weight of the liquid, so whichever way it is accelerated in the horizontal direction, a x, it does not matter the total weight of the liquid will be the force here. But if it could have been accelerated in the z direction, then the force will be weight plus the acceleration. That means, as if we will consider g is increased by g plus a z or minus a z downward, if there is a free fall, what is the force on the bottom? 0. You must know this thing very carefully.

So, now the bottom plane the weight is simply we will equate the weight, that means rho, that means rho is 10 to the power 3 into V. V is 4 into 2.5 into 2, it is the total volume initial depth was 2. So, V into rho into 9.8, I have taken care of all these things, V into rho into 9.84 into 2 into 2.5 9.8 Newton. So, this becomes, is equal to 196 kilo Newton. This is as simple as anything, now what are the forces on this two planes? First of all consider leading plane, leading face. What is leading face? This face that means, face leading means the acceleration is taking place, the leading face. What will be the component forces? If you see this face, in this direction, this is a face like this, where this dimension is 2.5 meter. I see from this direction this is the face, so this is the free surface.

So, free surface is this, so this height. Let this height is, what is this height? This is not 2 because water touches, so free surface is parallel here because there is no acceleration in this direction. So, free surface is horizontal here from this point, this is not 2. So, this value is what? This value, let this is H 1, this H 1 is what H 1 is? I write H 1, what is H 1 2 minus 4 2 into tan of 26.65. 2 is the half of this; that means this minus this, what is this value? This value is 1 because it is 0.5, correct? So, you see if the problem is said like that, students will not be having any difficulty.

So, therefore this is the force on a vertical surface, which is submerged up to a depth of 1 meter, H 1 is one meter and whose width is 2.5 meter. So, therefore, it is rho g into the pressure at the centroid or the centre of area into 0.5 into 2.5 into 1 Newton, so this becomes also very simple calculation, 12.25 kilo Newton. Similarly, for this trailing face. Trailing face please tell trailing face, trailing face is the same thing, only difference is the

3

Very good. Now, this is 3 meter H 2. H 2 is equal to 3 meter because 2 plus, this quantity 2 plus tan 26.25 degree. which is 0.5, that is 2 into 0.51, so 2 plus

1

Very good.

So, this remains as it is, the width perpendicular direction 2.5 meter. So, what is this value? 10 to the power 3 9.8 into 1 into 2.5 into 2

1.5

1.5 because 3 centre is 1.5 rho g 1.5 into

3

3 into 2.5 area. I am sorry, so do not, do that mistake. I may be escaped as a teacher, but you in the examination… So, 1.5 is the centre of area, so 2.5 into 3. So, after that, if you do a mistake, the deduction of mark will be less, because your concept is clear, because some silly mistake your concentration was not there, you have calculated wrong. But here, if you write 1, the way I gave written, then I will think that you have not understood, centre of area is middle of that. That should be 1.5 and area is 2.5 into 3.

So, then you can, so the mistake in calculations you can afford to do, but you should not. The probably I have got the result 110.25 kilo Newton. So, now remaining other two lateral faces are 3, where the forces will be identical. That means a vertical face like this, where the free surface of water is adhering like that. This is the water, so how to find this?

Integrating.

By integrating, very good simple by integration. Very good by simply integrating, I can use another thing, well good by simple integrating.

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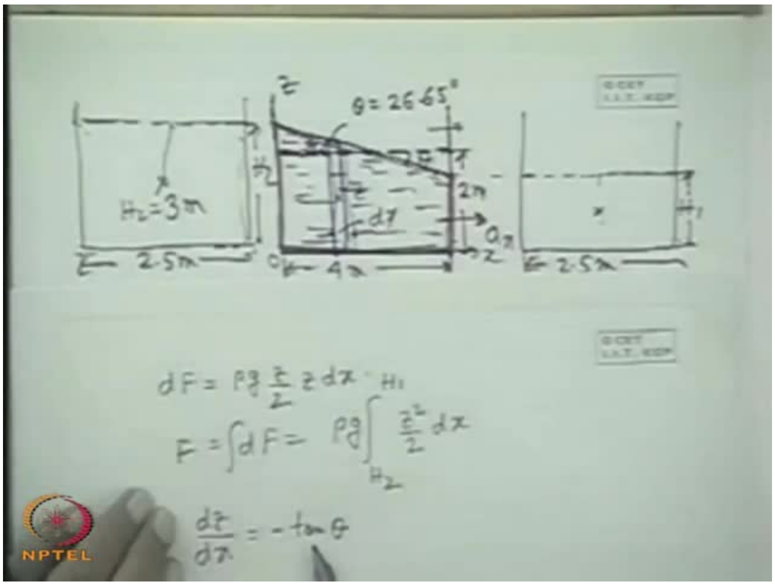

That means you can take a strip like that. For this purpose, I define this as the x axis and this axis z axis. Therefore, for this strip, the force d F is… What if this is z this coordinate of the free surface with respect to this x axis is z, that means this is z. So, centre of area is z by 2, so rho g z by 2 and its area is, if we define this thickness as d x because this is the x direction that is very simple, z into d x. So, simply F for entire surface will be d F is equal to integration.

Rho g take outside incompressible, that is a liquid and z square by 2 d x. 2 also I can, could have taken outside, though this should be H 2 minus H 1, form this I am going the positive direction. Another constant is that d z d x is equal to… What is d z d x minus tan theta? Here if I define this as the theta, now again it is as the theta, so tan theta, so simply d z d x. My no question of pressure head this is geometry because d x is increasing, d z is decreasing with increasing z z is increasing, x z is falling.

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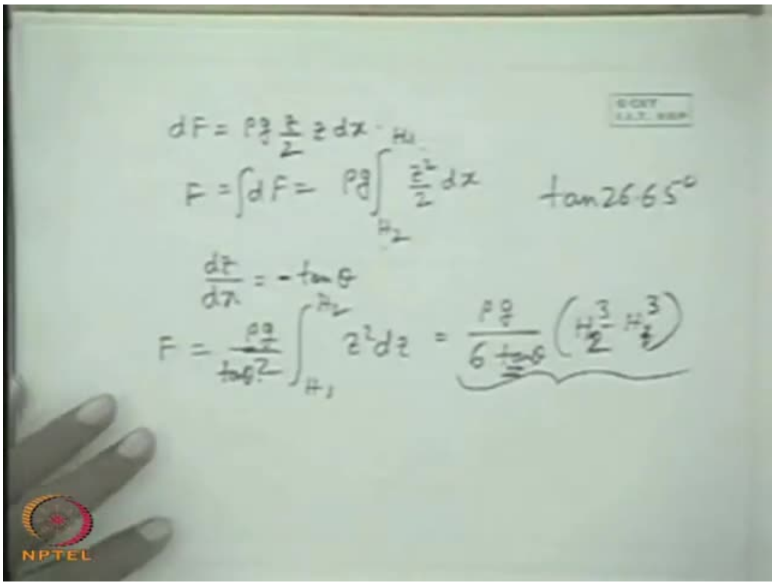

So, d x by d z is equal to minus tan theta. That means if I now substitute it, it should be, F is equal to rho g by 2. That means we can take H 1 H 2, change d x is d z by tan theta, that means you take tan theta out d z d x is d z by tan theta. That means z square d z ,that means simple rho g z cube 6 3 into 2 6 tan theta into H 1 cube H 1 H 2 we know, sorry H 2 cube and H 1 cube H 2 has gone up, because of this minus this tan theta is tan of 26.65 degree. From geometry, we have taken this theta, this thing is this, is the force acting on each of the lateral faces. That means front face and there is another back face, any problem?

This is the equation, please, no problem? So, with full confidence on you, I can leave you today for this midterm examination. So, this is up to this, In next class, I will discuss few other problems of this section and I will start the next section of this course, Applications of conservation equations in fluid flow problems in practice. So, thank you.

Fluid Mechanics
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Lecture No - 22
Principles of Similarity Part - II

Good morning, I welcome you all to this session of fluid mechanics, today we will start a new chapter, a new section, that is fluid flow applications. We have already discussed the conservation equations; that is the conservation of mass, equations for, conservation of mass, conservation of momentum and conservation of energy and we have solved some problems related to finite control volumes, both inertial and non-inertial control volumes. Now, in this section, chapter, we will discuss few practical problems of fluid flow applications, which will be solved on the basis of the equations of for conservation of mass momentum and energy.

The applications of those equations, better we can say that, it is the applications of conservation equations to fluid flow problems. At the outset, we will discuss little bit about the Bernoulli's equation or Bernoulli's theorem, which we have discussed earlier, a little elaboration on that. If we recall we earlier recognized the Bernoulli's equation, as the equation of mechanical energy or mechanical energy equation in case of a fluid flow, which tells about the conservation of energy in a fluid flow. We have recognized that, if the fluid is inviscid, obviously the mechanical energy remains constant.

If there is no energy added from outside, no mechanical energy is taken out of the fluid and if the fluid is inviscid, that means viscous action is not present, that fluid is inviscid, no viscosity, that means no friction. So, mechanical energy cannot be dissipated in terms of intermolecular energy or heat as you can tell. So, in that case total mechanical energy remains constant. We have seen that, in general these mechanical energy total mechanical energy remains constant along a streamline. We have recognized different forms of mechanical energy, as the pressure energy, kinetic energy and the potential energy.

If you recall the equation, you can tell that p by rho, which represents the pressure energy per unit mass plus V square by 2, which represents the kinetic energy per unit mass. We have to always recall in this form plus g z, where g z represents the potential energy per unit mass here. Of course, we consider only the Earth's gravitational force field as the only body force field. Therefore, the potential energy per unit mass will be g z, where g is the acceleration due to gravity and z is the elevation of the point concerned, where we are considering the energy per unit mass from an arbitrary datum.

So, p by rho plus V square by 2 plus g z thus constitutes, the total mechanical energy and which is equal to constant for an incompressible steady flow because this form of the equation, comes only for a steady incompressible flow. That is of course, important otherwise integral d p by rho, will come for a compressible flow, but p by rho this term comes for an incompressible flow. So, for an incompressible steady flow p by rho plus v square by 2 plus g z, simply the sum of the 3 3 components of the mechanical energy, is constant along a streamline.

If you recall the derivation of this equation, we did it by integrating the Waeler's equation along a streamline. Probably you can recall this, that integrating the Waeler's equation along a streamline. Therefore, the constant, which came out of these integration, was a constant. That means it was valid only along a streamline. Therefore, essentially the mechanical energy varies from streamline to streamline in general. But now we will extend this, in case of an irrotational flow; that means if we add a further constraint in the flow.

That if the flow is irrotational, already you have started, what is an irrotational flow in fluid kinematics? That a flow is irrotational, when the rotation at each and every point is 0 and you know the rotation of a fluid element at each point is defined as the Carl of the velocity vector, Carl V in different coordinate system. We can expand this Carl V physically, the rotation means the arithmetic average of the angular velocity of the two linear segments meeting at that point, which was initially perpendicular.

So, rotation is 0, means the flow is irrotational. So, if we add a separate or additional constant of irrotationality, that means if flow becomes irrotational in which its steady incompressible, then these mechanical energy is constant throughout the flow field. That means p by rho plus v square by 2 g plus g z is equal to constant, not only along a streamline, but at any point in the entire flow field. This we will derive first, which is a very simple derivation, let us do it.

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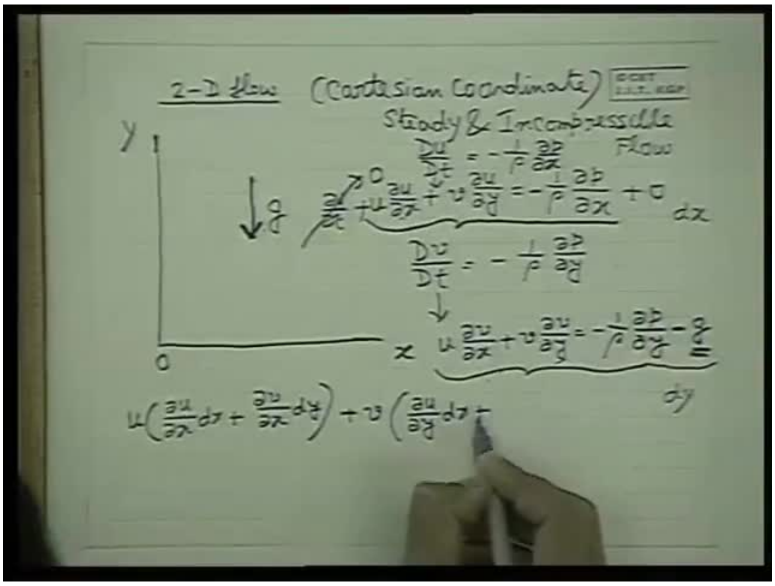

Let us see this, let us first take a two dimensional case, a two dimensional flow. A two-dimensional flow, in a Cartesian coordinate system in a Cartesian coordinate system, in a Cartesian, rectangular that is Cartesian coordinate. A Cartesian coordinate system and in this case, let us consider only x and y, no z. So, y we consider and this we consider the y axis, is in the vertical direction. y directed positive upwards, vertically upwards, so that g is acting like this. So, if this we consider as the frame of coordinate and also we consider a steady and incompressible flow, if we consider a steady and incompressible flow. steady and incompressible flow

Then if we recall the equation of motion, that is the Waeler's equation, we can write that for a steady the temporal, that you will not come u del u del x plus v del u del y, is equal to minus 1 upon rho del p del x, if you recall? So, this is D U D t, big D U D t, big D U D t that means this is equal to minus 1 upon rho del p del x this is the x direction. So, this has been expanded del u del t is 0 for steady state. So, I have not written del u del t is 0 for steady state and rho is minus 1 upon rho. Then this is the form x direction equation, y direction equation. Similarly, will be D v D t if you recall, is equal to minus 1 upon rho del p del y.

So, if you expand this, in case of steady flow, that means del v del t is 0. Then you get u del v del x plus v del v del y is equal to minus 1 upon rho del p del y. So, this two equations, are the equations for incompressible steady flow, with respect to a two dimensional Cartesian coordinate system. Of course, here I am wrong, minus rho del p del y, here g is here, acting. So, we have considered the y axis, such a way that vertically upwards directing positive. So, g is in the negative direction. So, body force term will come, which will be here minus g because these are the force per unit mass, because here these left hand term is the acceleration per unit mass.

So, first term on the right hand side is, the pressure force per unit mass. So, per unit mass the body force will be minus g why minus per unit mass it will be g m g divided by m g and it is in the negative direction of the positive y axis. Therefore, minus g this comes as the body force. If you recall, the body force here, the body force is 0 in the direction of x there is no body force. So, we have chosen the y axis, in such a way that it is directed upwards, vertically upwards, so that the gravity force come as the body force, so per unit mass basis it appears as minus g. I am sorry, so this term will come now. What we will do? We simply know that, the basic approach for finding out the energy equation is to, multiply the momentum equation with the displacement.

So, what we do? If we multiply this equation x direction equation by d x and multiply the y direction equation by d y, both this sides and add it, what we get? u del u del x d x, so we first take the u term common, then from this equation it will be del u del x d x and from this equation it will be, u del v del x d x, u del v del x d y sorry, u del u del x d x plus v del v u del v del x d y plus v del u del y d x plus…