

Fluid Mechanics
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Lecture - 17
Conservation Equations in Fluid Flow Part V

Good morning I welcome you all to this session of fluid mechanics. Now last class we were discussing about the different forms of energy that a fluid particle or a fluid mass possesses in codes of a flow; that means, when a fluid particle is flowing or a fluid mass is flowing, what are the different forms of energy if we recognize a mass of fluid flowing can it have; number 1 is internal energy which is because of its intermolecular energy which is because of the kinetic energy and potential energy of the molecule that any substance has an intermolecular energy, at any temperature more than the absolute 0. So therefore intermolecular energy is the first number 1, number 2 is the kinetic energy, because of the velocity any particle or any lump of fluid does have a kinetic energy which is given by the mass times the a velocity square by 2 m b square by 2.

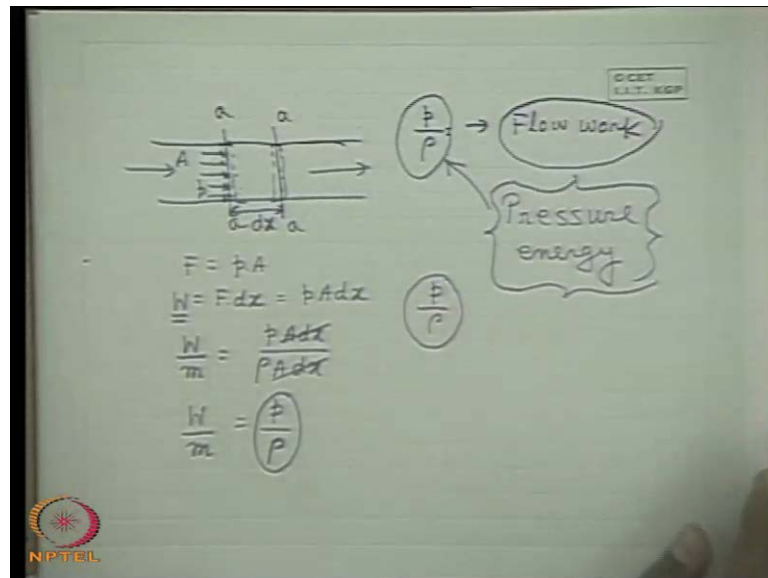
Then we know the potential energy; the definition of potential energy is the energy which a particle or a mass possesses by virtue of its position in a conservative force speed. If we consider gravity as the only body force speed, so this potential energy is the potential energy due to gravity which can be expressed for a mass m as m g into h or z whatever you call, where j dot h is measured from an arbitrary datum, where we take the potential energy to be 0.

So apart from this 3 usual terms of energy or 3 usual forms of energy, there is another form by which a fluid particle or fluid mass possesses energy in codes of flow is known as the flow work. So first better to recognize it in terms of the work, then I will come to the terminology of energy. Now one has to know that a fluid particle to flow through a medium it has to continuously push its neighboring downstream particle, otherwise you cannot go; just consider in a queue, if we have to move you have to continuously push your downstream particle or downstream neighbor, so that you can make your way through.

Similarly the thing happens in a fluid flow, for example if we recognize a layer of fluid in a flow, then we will always consider if this layer has to go or move in the flow it has to push its neighboring downstream layer to make its way through. So, therefore this way

each and every layer does work on its downstream neighboring layer; and this work is capable of doing because of some energies stored in it that is the way the energy is stored in a fluid particle.

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Now let us recognize it mathematically that let us consider a pipe or duct through which the fluid is; let us consider a layer here at any instant where the pressure is p . Let us consider a small layer like this, now after a time t exaggerated view that is small time this layer has moved through a distance; that means this layer a has moved here through a distance dx , because of the flow it has to move. So, this movement implies that at this position this layer pushed the neighboring layer, which was adjacent to this layer downstream to this has moved through a distance dx . So, therefore, the layer a has done some work on its neighboring downstream layer which has come here, now what is the force that is being exerted by these layer a to its downstream layer? This force is pressure times the cross sectional area, if you take the cross sectional area A . So this is the force, so work done is force time the displacement. So, displacement is dx in the same direction of force; that means $p A dx$. So this is the work that is being done by any layer at a pressure p to its neighboring downstream layer to maintain its flow. So if we express this work per unit mass of the fluid element then we can think of $p A dx$ divided by a fluid element which is being displaced, basically this $dx A$ into ρ so that this term cancels.

So this becomes p by ρ , which is very important term; that means the work done per unit mass of a fluid element for its flow at any section is given by the term p by ρ . So this p by ρ is known as, so therefore this p by ρ is known as the flow work; that means one layer has to do this work. This we conceive in terms of a energy in this way that by virtue of some energy this layer is capable of doing work on its neighboring layer. So therefore this amount can be thought of as energy inherently stored in this layer because of the existing pressure p , this is the only way one can think of p by ρ as a form of energy and a little loose term we tell it as pressure energy. That means the energy which a layer possesses by virtue of its pressure at an existing section because of which it has the capability of doing work on its neighboring layer to push its way through is known as the pressure energy, this is the only definition probably you can have for pressure energy.

You can have this definition rarely in any book, but in my book it is expressly written this is the only way you can define the pressure energy. So, therefore, we see in a flowing fluid because of the existing pressure at any section the energy per unit mass of a flowing fluid is p by ρ .

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$$e = u_i + \frac{v^2}{2} + \frac{p}{\rho} + gz$$

$$\left(\frac{dE}{dt}\right)_{\text{system}} = \frac{\delta Q}{\delta t} - \frac{\delta W}{\delta t} = \frac{\partial}{\partial t} \iiint_{CV} \left(u_i + \frac{v^2}{2} + \frac{p}{\rho} + gz\right) \rho dV + \iint_{CS} \left(u_i + \frac{v^2}{2} + \frac{p}{\rho} + gz\right) \rho \vec{v} \cdot d\vec{A}$$

General Energy Equation

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So therefore now we can recognize what are the different forms of energy that a fluid particle mass possesses, these are like this; if I denoted the energy per unit mass we can write this is u_i where u_i is the intermolecular energy per unit mass plus the kinetic

energy per unit mass plus this pressure energy per unit mass plus the potential energy per unit mass. So these are the usual 4 forms of energy that a fluid particle or mass possesses which have been expressed in terms of per unit mass; this is intermolecular energy per unit mass, this is kinetic energy per unit mass, this is pressure energy per unit mass and this is potential energy with only body force field as the gravitational force field potential energy per unit mass.

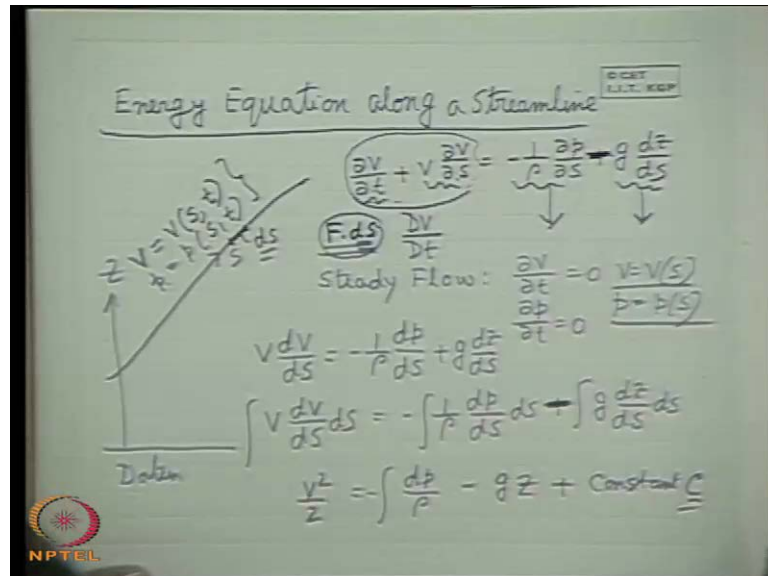
Now if we recall back the energy equation applied to a control volume through the Reynolds transport theorem then we can see that $\frac{dE}{dt}$ sometimes small d capital D does not matter, the substantial derivative for a system which you know is equal to the total energy interaction by the system with respect to surroundings and thermodynamics tells that there are 2 forms by which the energy is interacted between a system and its surrounding that is the external to the system; one in the form of heat another in the form of work and if we consider the positive senses of heat interaction or exchange as the heat supplied to a system and work for work done by a system then this difference takes care of the net heat net energy interaction then energy coming in minus energy going out, so net energy coming in which is responsible for the change of energy for a system.

So this is precisely the statement of the conservation of energy for a system, this becomes equal to; in case applied to control volume with the help of Reynolds transport theorem is the rate of change of that you have only energy for the control volume C_V as you know, what is that η ? Now η is e here, so we write now $e + u^2/2 + p/\rho + gz$. If you recall this thing we wrote earlier only e was written now instead of e now we replaced, because now we have recognized the different forms of energy per unit mass possessed by the fluid particles. So this times ρdV plus the net rate of a flux over the control surface. Similarly this thing will come, this is $e + p/\rho + gz$; I think that is ok, times $\rho V \cdot dA$. So therefore this is precisely the general energy equation, general energy equation; this is known as general well energy equation, general energy equation good.

In case of steady flow this first term will be 0; that means $\frac{dQ}{dt} - \frac{dW}{dt}$ will be this 1. So this is the, so after not for any problem we have to start from this general energy equation and you will see what are the special forms under special situation. Now though this is a general energy equation, sometimes we want a little

simple convenient form of the energy equation which is the energy equation as on a streamline.

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What is energy equation along a streamline? So this question comes what is energy equation along a streamline along a streamline, energy equation along a streamline what is this? We want to derive the energy equation along a streamline sometimes that is more simple and useful also. Let us consider along a streamline we derived the Pillars equation, let the direction is S; now along a streamline means the velocity vector is only along this direction.

So we can write that velocity is a function of S, the displacement along the streamline and the time. Similarly the pressure is a function of a S and t, these are the only 2 variables in a fluid flow; fluid flow means flow of fluid for an in visit fluid, ideal fluid. There is no shear stress, only pressure is the surface process and velocity. So, velocity is a function of a centy, p as a function of a centy and we derived the equation of what is that equation of motion that is the oilers equation this way $\text{del } V \text{ del } t$ plus $V \text{ del } V \text{ del } s$ is recalled to minus 1 upon rho del t del S plus next term $g \text{ d } z \text{ d } S$; that means if we have a velocity field described by a centy as a function of a centy, pressure as a function of a centy they must satisfy these equation which is the equation of motion along this streamline, and this equation is known as oilers equation according to the scientist Oiler who first deduced this.

Now we know from physical sense that the work done by the application of force is equal to the force times the displacement in this same direction; that means $F \cdot ds$; that means, if we multiply the force with the displacement in the same direction of force then we can find out the work done by that force and that is gives the concept of energy. So therefore, with this physical concept we can tell mathematically that if we integrate this equation by multiplying with dS , a small displacement dS then we arrive to the equation of energy. Again we see that this term is the term representing the force; obviously, because this term this 2 terms, this is the acceleration term, temporal and convective. In total this is dV/dT , what is the acceleration? It is force per unit mass; it is energy is also like that. Similarly this convective gives the physical concept of pressure force per unit mass and this is the body force that is a gravity force in this situation per unit mass and there dimensions are like that. So if you multiply with dS automatically that dimensions will be the work dimension and energy dimension, and that will physically also imply the work done by the fluid element in flowing along a streamline and that will simply give you the energy equation.

Let us do that and before doing we take an assumption for the sake of simplicity, a steady flow, steady flow assumptions we take; whenever we take the steady flow assumptions $\partial V / \partial t = 0$. So V is a function of S and p is a function of S , so all parameters ceases to be functions of time so that we can write this equation as $V \frac{dV}{dS}$, in terms of the total differential. So it becomes a ordinary differential, becomes an ordinary differential equation $\frac{dp}{dS} + g$. So this is the form of the Oilers equation in steady flow, very simple $\partial V / \partial t = 0$ and we are removing the partial derivate notation to total derivative; because V is a function of S only, p is function of S only. Now what we do we just multiply, please any problem you tell me with dS and integrate it; minus $1/\rho \int dp/dS$.

So it is a mathematical operation; that means we are integrating a differential equation with respect to a coordinate S ; that means with respect to dS that means we are multiplying dS and integrating it, simple integration. What? Yes please.

Student: In oilers equation there was $A \cdot t$ of S minus g conservative.

No it is plus or minus; minus good very good, minus very very good minus. It is minus; it has to be minus good alright thank you. So now we can write this as V^2 by 2,

simple integration $V dV$; this is integration of minus dp by ρ , very good and this will be g is constant so gZ simply z where z is the coordinate measure from any arbitrary data any arbitrary data that is a vertical coordinate Z so gZ plus a constant, a constant C ; a constant C .

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$$v \frac{dv}{ds} = -\frac{1}{\rho} \frac{dp}{ds} - g \frac{dz}{ds}$$

$$\int v \frac{dv}{ds} ds = -\int \frac{1}{\rho} \frac{dp}{ds} ds - \int g \frac{dz}{ds} ds$$

$$\frac{v^2}{2} = -\int \frac{dp}{\rho} - gZ + \text{Constant } C$$

$$\boxed{\frac{v^2}{2} + \int \frac{dp}{\rho} + gZ = C}$$

Incompressible:

$$\boxed{\frac{v^2}{2} + \frac{p}{\rho} + gZ = C}$$

along a streamline

- Ideal Fluid
- Steady
- Incompressible
- Gravity as the only Body force

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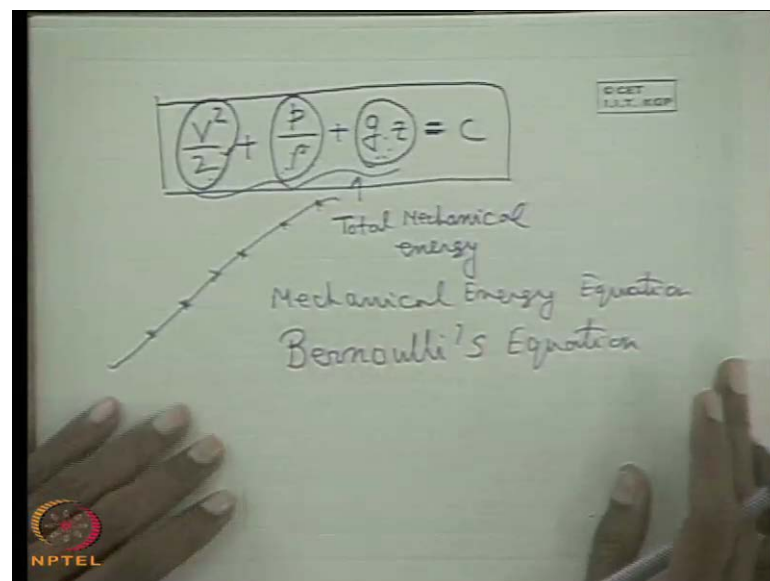
Now therefore, we can write this equation in this way that we arrived to this equation that we arrive to this equation that v square by 2 plus integration of dp by ρ , ρ plus gZ is equal to constant. So this is precisely the energy equation along a streamline; v square by 2 plus dp by ρ plus gZ is equal to constant if we take the fluid during compressible. So, far we did not take any condition for incompressibility now if we take fluid to be incompressible ρ will come out. So, v square by 2 plus p by ρ plus gZ is equal to constant. So, what are the assumptions for these equation? Now first of all this equation implies the energy equation, but first of all let us find out what are the assumptions that we did it mathematically routine mathematical steps from the Oiler's equations.

So the assumptions of oiler's equations are there, that means ideal fluid that is the fluid is in visit. Next assumption is steady, next assumption is incompressible for this form not for this form. So if we refer to this form this incompressibility will not come, incompressible and fourth assumption is the gravity as the only as the only body force as the only body force. So these are the assumption; that means, it is and also this equation

is valid along a streamline ; because we have integrated d along a streamline which means this constant is same only along a streamline; that means, now with this we can tell more briefly that V square by 2 where V is the flow velocity that means square of flow velocity by 2 plus if I tell it full in words the square of flow velocity by 2 plus the pressure by density term, plus the acceleration due to gravity into some Z vertical coordinate from an admitory datum.

The summation of these 3 terms will be constant along a streamline for the flow of an steady ideal incompressible fluid with gravity as the only body force, we can compose in a different way. So a person who can compose nicely in English he can compose in a much better way, but one should not forget that these steps; that means, steady flow of an ideal fluid, ideal an incompressible fluid with gravity as the only body force along a streamlines sum of these 3 terms must remain constant.

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The image shows a whiteboard with the following content:

- A boxed equation:
$$\frac{V^2}{2} + \frac{p}{\rho} + \frac{\rho \cdot z}{\rho} = C$$
- An arrow pointing from the text "Total Mechanical Energy" to the boxed equation.
- The text "Mechanical Energy Equation" and "Bernoulli's Equation" written below the arrow.
- A small logo in the bottom left corner with the text "NPTEL".
- A small box in the top right corner with the text "© IIT KGP".

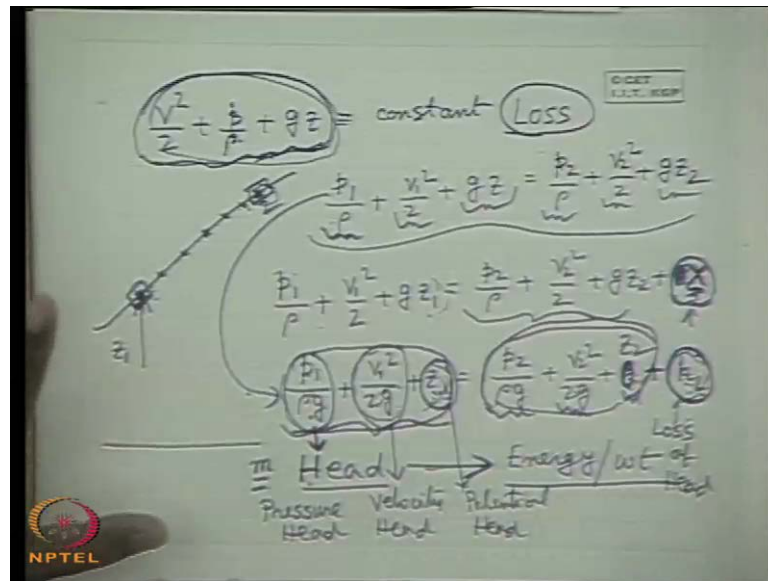
So what these 3 terms indicate, let us come to the physical aspect of this equation that V square by 2 plus p by ρ plus $g Z$ is equal to constant. Along a streamline this sum of these 3 terms at all points remain constant, this represents the kinetic energy per unit mass you see the, dimension is also like this, this represents the pressure energy per unit mass and this represents the potential energy per unit; that means the sum of these 3 energies are always remain same.

These three energies are the forms of the mechanical energy, because mechanical energy appear in these three form only that is the kinetic energy, pressure energy and the potential energy. So sum of these three terms can be, can be seen as total mechanical energy total mechanical energy, can be seen as total mechanical energy. So we can tell that the total mechanical energy comprising the kinetic energy, pressure energy and the potential energies constant along a streamline. So therefore this equation is known as mechanical energy equation, mechanical energy equation; the person who first deduced this is known as Bernoulli, Bernoulli and that is why the most popular name of this equation is Bernoulli's equation Bernoulli's equation. Now in most of the cases the flow of fluid Bernoulli's equation alright, now in most of the cases the flow of fluid takes place most of the cases means all practical cases the flow of fluid is associatively viscosity, it is not an ideal flow because no fluid in practice is having a 0 viscosity

So therefore the fluid flows with viscosity, in that case the Bernoulli's equation will not be the equation of energy for a viscous fluid; because here we have considered that the flow to be ideal or flow to fluid to be ideal or fluid to be in visit.

So therefore the energy equation for a viscous fluid we will up to take care of the viscous forces; that means, interaction between the share force and the share rate; that means, the displacement and which is more complicated. So that first of all we will have to derive the equation of motion, then we will have to go for the equation of energy and that is beyond the scope of your syllabus or your curriculum at this level. But one thing, we can still use the Bernoulli's equation or the energy equation for a frictional fluid or a fluid with viscosity at the frictional effects are there with some modifications in the energy equation in solving the practical problem; that means, solving the problems in practice where the fluids are real fluid having viscosity, how? Let us see.

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Now we know that along a streamline kinetic energy, pressure energy and potential energy is constant; that means, we consider that the along this streamline this is constant provided the fluid is frictionless, there is no frictional effect, fluid is incompressible and additionally there is no extra energy added from outside or energy taken from the fluid to the outside. That means with the surroundings no interaction of energy. So the total mechanical energy is constant this is nothing, but the conservation of energy, but if the fluid friction comes into consideration we cannot tell always the total mechanical energy will remain constant because a part of the mechanical energy will be degraded via friction in terms of the intermolecular energy. So therefore we cannot always tell this we can tell from our common sense which we have readied school level, the total mechanical energy may not constant in a frictional system because friction will incur certain losses.

That means some part of the mechanical energy will be converted to other form of energy intermolecular energy, so this will not remain constant. So therefore if we consider the 2 points; 1 and 2 we can write the Bernoulli's equation for an ideal fluid that is basically the Bernoulli's equation. If I write this one as the first one that is the convention for an incompressible flow, so ρ is constant V_1^2 by 2 plus gZ_1 where Z_1 is the height from an arbitrary datum for the 0.1, simply we can write p_2 by ρ . So if the flow is steady and ideal and incompressible we can straight equate this 2 for 1 and 2. But what for viscous flow, in case of viscous flow this is simply modify in

this term $V^2/2 + gz_1$ is equal to $p/\rho + V^2/2 + gz_2$ plus let us write this as some, oh or something here. If we write well, write X where X is the term which takes care of the frictional energy that is being lost by the fluid, frictional energy that is that has taken care of in the cutting or curtailing the energy from the 0.1 reaching to the 0.2.

Now let us first see that what are these terms; so better to understand this term we should first write it in this form otherwise there will be a difficulty mathematically. We first write this form then only we can very well, now let us write. I am writing some term here afterwards let us write this equation in this form; what are these equations? First implication this is the pressure energy per unit mass, this is the kinetic energy per unit mass and this is the potential energy per unit mass. So this is the pressure energy per unit mass, this is the kinetic energy per unit mass, this is the potential energy per unit mass and summation of these 3 are same. Now we can express these in terms of energy per unit mass.

This is more conventional way of writing these in the fluid mechanics problem, that means here we write all these terms in terms of energy per unit way; this is energy per for special energy per unit way, this is kinetic energy per unit weight, this is potential energy per unit weight in both the sides. And you now that the dimension of energy per unit weight is the linear dimension; that means, in m k unit this is meter, this is simply clear from here Z_1 is meter $V^2/2g$ is also meter.

So this term is known as head; that means, head is a terminology used for fluid mechanics; this is not our head from this well head as a this human head is not like that, head is the energy per unit weight, head is the energy per unit weight. So energy per unit weight is explained or defined as head you can see energy per unit weight is the head that is the definition of head in fluid flow. So therefore we can tell this is the pressure head, this is the velocity head that is kinetic energy per unit weight, that is kinetic head or velocity head and this is defined as the potential head.

So, pressure head; pressure head, this is velocity head or sometimes kinetic head, velocity head plus the potential, potential head. This will be Z_2 , so these sums of these 3 heads are constant. So here, X here we write h_L ; what is this X ? X here represents the work done per unit mass, the work done by the fluid element against friction flowing

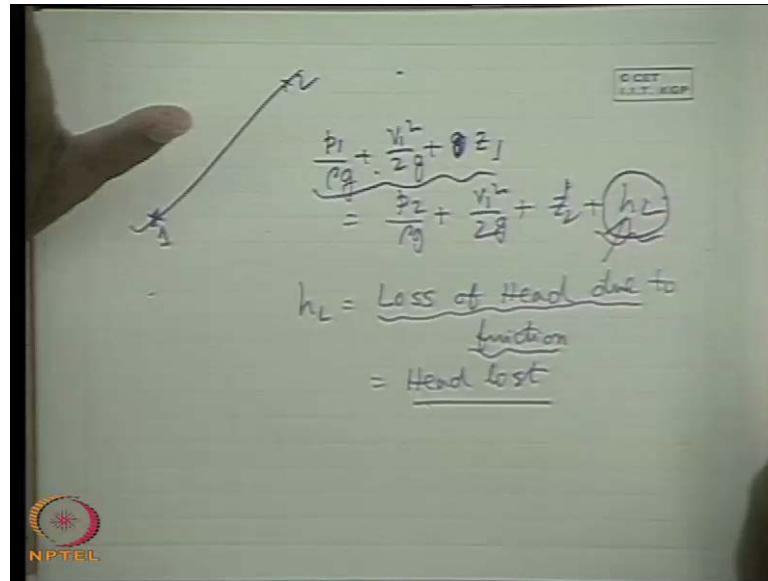
from 1 to, work done per unit mass of the fluid element in against friction from 1 to 2. Here it represents the work done per unit weight of the fluid element in against friction from flowing from of flowing from 1 to 2. Now you see, so therefore the total energy at inlet either in terms of per weight or per mass has to be greater than this energy plus the energy which has to be considered or accountable for by moving from this point to this point a particle or a mass has to suffer, because a mass has two work against a friction.

So this is the work done against the friction per unit mass, this is the work done per unit weight against the friction. So therefore a energy term as a loss it is known as loss of head or loss of X energy per unit mass. So this is loss of head, why it is loss? This is because we are considering only on mechanical energies; that means this is the pressure energy, this is the velocity energy, this is the potential energy. So we see that a sum of the pressure energy and velocity energy and potential energy with what the fluid has at section 2, sum of these is less than the sum of these 3, so where it gone? A part of the mechanical energy has being lost in doing work against friction.

So, is a very important question that why we call it loss energy can never be lost; that means, a part of this energy via friction as an agent has been converted into other form of energy which is the intermolecular energy, if we could have taken care of the intermolecular energy along with that we could have seen the balances there.

Since our only motto is on mechanical energy and conservation of the mechanical energy only, we will see the mechanical energy is not conserved; that means, the inlet mechanical energy that any point at the upstream is more than that of the downstream by another term which from the view point of mechanical energy we call it as loss. These are very important thing when I teach thermodynamics I asked that why you will call a loss of energy or loss of head? Head is nothing but energy per unit, this is because some part of the mechanical energy is being lost from the accountability of the mechanical energy and that is h_L . In fact this not a loss, this is another energy quantity which is appearing in other form rather than the mechanical energy form. So simply writing the Bernoulli's equation we simply added term loss, loss of head means this is the mechanical energy which is lost because of friction. All of you have understood this concept?

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A hand-drawn diagram on a whiteboard shows a streamline flowing from point 1 to point 2. The vertical height at point 1 is Z_1 and at point 2 is Z_2 . The Bernoulli equation is written as:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + h_L$$

The term h_L is circled. Below the equation, it is defined as:

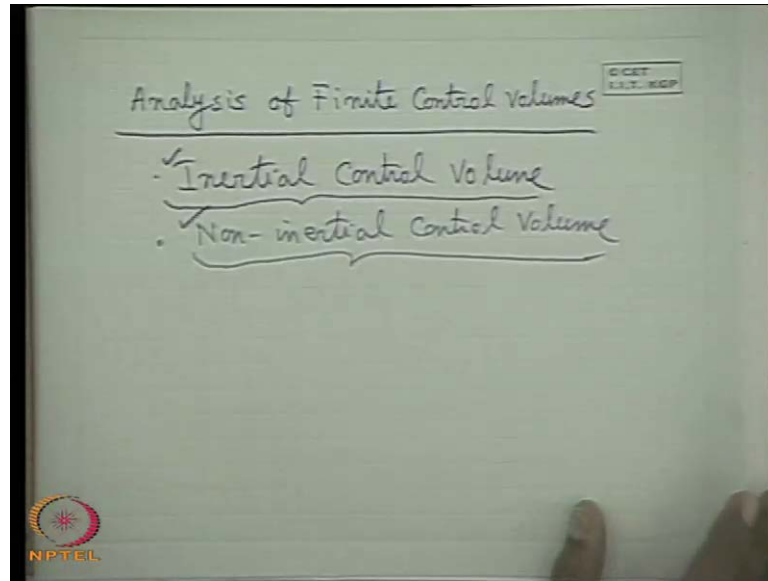
$$h_L = \text{Loss of Head due to friction} = \text{Head lost}$$

The NPTEL logo is visible in the bottom left corner of the whiteboard image.

So this way we write the Bernoulli's equation for a frictionless flow from 0.1 to 2 along a streamline that I write p_1 by ρ plus V_1 square by 2 plus $g Z_1$. Total mechanical energy at 0.1 equals to the mechanical energy, now per unit mass this is I am writing, I can write per unit weight bases also without g plus Z_2 plus h_L where h_L is known as is very important terminology loss of head due to friction, due to friction and usual terminology is h_L capital L or small l; h_L is the sometimes this is known as head loss. Head loss means in a viscous flow because of the viscosity in the fluid friction between the layers of fluids and between the fluid and the solid; so the energy, mechanical energy which is being lost or the part of the mechanical energy which is being used consumed for the fluid to flow against friction.

So this is clear, so this is known as loss; though energy cannot be lost in fluid mechanics dealing with the Bernoulli's equation or the energy equation for mechanical energy only we considering it has lost that is very important to be remembered I tell you. So after this, well the Bernoulli's equation I think the implication is over. Now I will start the analysis of finite control volumes.

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Analysis, analysis of finite control volumes; analysis of finite control volumes, now control volumes as you know we have defined what is a control volume is a fixed region in space where the fluid crosses the control volume, it has got a fixed boundary with fixed region. So control volume is specified by a fixed boundary where the fluid may come into the control volume fluid is going from the control volume, this is the definition of the control volume. But it is not necessary that the control volume will be static with respect to a static frame of reference, control volume may be an absolute rest or control volume may be moving depending upon the choice we can make a control volume moving with some velocities. Sometimes you will see afterwards that an unsteady flow to make it steady for the simplification in analysis we assume a control volume or we consider a control volume which is moving with some velocity.

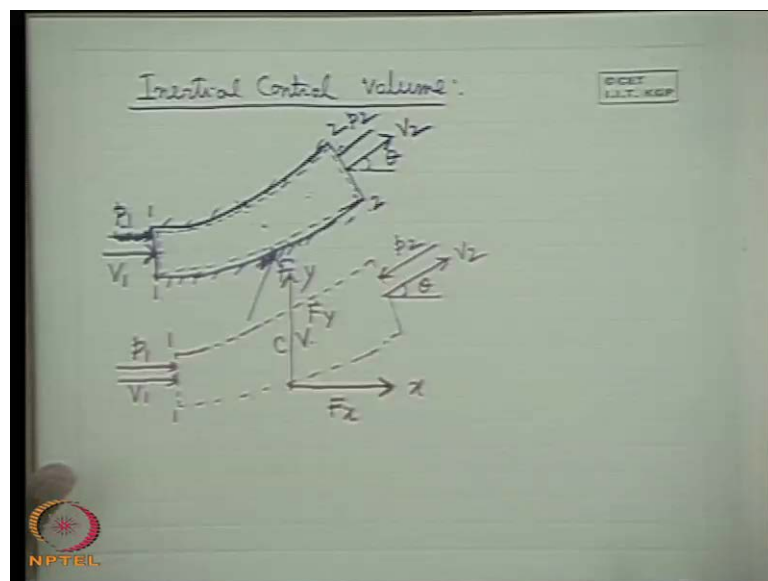
Again for a moving control volume, the control volume may move with uniform velocity or with changing velocity; that means read accelerations. Now the control volume is defined at 2 types; one is the stationary control volume, absolutely stationary or moving with uniform velocities, this control volume is known as inertial control volume; that means, control volume without any acceleration. That means either absolute stationary or moving with uniform velocity that is known as inertial control volume and another type of control volume is there where the control volume is moving with acceleration that is known as non inertial control volume.

Now therefore, we can write inertial control volume, inertial control volume that is control volume; we either absolutely stationary or moving with non-inertial control volume, non-inertial or moving with uniform velocity and non-inertial the control volume which is moving with acceleration.

Now almost all practical cases we come across inertial control volume, all though in some cases in non inertial control volume also comes into picture. Now we will consider the analysis of finite control volumes; inertial and non-inertial type through some practical problems. One of the practical problems is like this; let us consider a pipe bend flow through a pipe bend, now we will consider different practical problems where the analysis is made through inertial control volume and non-inertial. We have talked so much about control volumes.

So we have defined a Reynolds transport theorem which relates the physical loss for a system to that of a control volume and we are so much about the definition of all physical loss through a control volume, and we have defined the equation of motion for a control volume; the equation of conservation of energy for a control volume equation of conservation of mass for a control volume. Now how these theorems for control volumes are analyzed, let us consider inertial control volume.

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So through practical problems, inertial without going from inertial control volume; a general description we should go to a practical problem. A very important practical

problem is like this, let us consider the flow through a pipe bend like this pipe, bend like this, any general bend diverging or a converging a pipe bend; that means, there is a pipe bend like this and the fluid is flowing, well the fluid is flowing in this direction. So fluid comes; let this section is 1 1, let this outlet section is 2 2; let fluid comes flows through V_1 across this section we consider things to be uniform, it is a one dimensional case. Let this goes to V_2 , let this angle is theta. Let us consider the pressure here is p_1 , let us consider the pressure with which it goes out it is pressure is p_2 ; now I will not give any direction first, let us consider the pressure at the section. Now let us understand what is what happens in practice, let us consider there is a core pipe bend.

So as you know from your observation or experience that if a bend is like that, if a fluid flows through certain velocity and pressure then goes out of this pipe bend with another velocity and pressure because of the change in cross section. So if we make the bend free, bend will automatically move; that is a common sense you know when the fluid flows through a pipe, a pipe is made absolutely free then because of the flow of the fluid some force is being exerted by the fluid while flowing through the bend and bend will move. You have experienced this in your common household day that is a house pipe, if the water flows with high velocity in a house pipe when it is made absolutely free the house pipe will go on move. So we will have to hold it to keep it in a position. So, therefore it is true that if the fluid flows through this pipe bend, pipe bend will experience some force because of which the pipe bend will move. So we will have to give some force to the pipe bend to keep it, some force to the pipe bend that some arbitrary direction by our common experience for a bend if the fluid flow with some velocity and pressure goes out will see we have to keep a force in a particular direction with some magnitude so that the bend remains fixed.

Now physically the problem is practically prescribed like this, a practical person will be interested to know if the inlet and outlet conditions are prescribed what should be the force in its magnitude and direction so that the bend is met stationary.

Now physically if we think how the force comes into picture; when the fluid flows to this bend it has a change of momentum, it suffers a change of momentum, each and individual fluid mass flowing through a suffer change of momentum. So where from it gets, so bend produces the change of momentum in the fluid particles while flowing through the bend; that means according to Newton's second law the bend first he is the

culprit first who exerts force on the fluid to cause its change in momentum. So therefore, according to Newton's third law the fluid exerts what flowing through the bend the same force in the opposite direction to the bend.

This way the bend gets a force, the same way the gun recoils when the bullet is fired as you know the gun first exerts the force on the bullet to cause its change in momentum and bullet will not leave it. According to Newton's third law bullet will give exactly the same force in the opposite direction to the gun. So until and unless gun is given a force in the same direction of the bullets movement by the amount which the bullet suffers, the gun will not be fixed, so gun will be recoil back. So a person who is not sufficiently strong enough will fall down.

So in the similar manner the force comes like that. So, therefore in this case will have to first analyze what is the force acting on the fluid flowing through this bend. So for this reason we take first a control volume of fluid inscribing this bend like this. So we take a fluid element at any instant out and analyze this fluid element as the control volume.

Let this is the control volume of the fluid, where the velocity is V_1 the pressure here is p_1 acting on this control surface $1-1$ in this direction. Similarly the velocity is going out this direction is θ and the pressure is p_2 , and we consider in general the pipe bend is in a vertical plane. So now we consider 2 axis; x in this horizontal direction, y in this vertical direction and we consider the forces acting on this control volumes are F_x and F_y in the horizontal and vertical directions. Well then we can find out by applying the momentum theorem, the values of F_x and F_y which is being apply to this fluid volume because of its flow through the bend and this force consist of the force because of its change in momentum plus the surface forces acting by the mutual interaction of the surface to the fluid body.

So these are summarized through force F_x and F_y . So if we prescribe the problem like that for a control volume of fluid we can find out F_x and F_y now by applying the principle of momentum for the control volume in x and y direction, which I will take up in the next class because the time is out, so today up to this.

Thank you.