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Lecture - 11 Kinematics of Fluid Part - II

Good afternoon, I welcome you all to this session of Fluid Mechanics. Last class we were discussing about the acceleration. So, we recall the discussion again and we continue here in this class that by acceleration in a field of fluid flow we mean the change of velocity per unit time or change of velocity, rate of change of velocity with time for a moving particle. A particle moves from one place to other place in a flowing field. What is this change of velocity? That is accounted as an acceleration.

Now, since a fluid particle takes time to move from one point to other point. So, change of velocity depends upon two factors, that the velocity distribution from point to point. That is a velocity distribution with this space coordinates and the velocity dependency time. As you know that if we consider a fluid particle, the velocity of a fluid particle at any instant is the velocity of that point at that instant with the fluid particle exists at that instant.

So, therefore by acceleration with mean or define, it is the change in velocity for a fluid particle to move from one point to other point. All the fluid particles are convicted in a flow field. So, therefore this change in velocity is due to the variation of velocity with this space coordinates along with the variation of velocity with time. And if we represent the velocity vector as a function of space coordinates and time. Then we can deduce or we deduce in the last class that acceleration composed of several is composed of several terms.

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So, if you look back, we just derive this thing with respect to a Cartesian frame of reference. Let x, let this is x; this is y and this is z. If we define u, v, w, the velocity components in a flow field, u is parallel to x, v is parallel, u is parallel to x, v is parallel to y. And w is the parallel to z component velocity u, v, w then, we started from this u as a function of x, y, z, time. Similarly, v; this is by oilary an approach we can express the flow field. This is the definition of flow field; that means, when the velocity in the fluid flow is expressed as. For example, the velocity vectors as a function of space coordinate s and time in scalar components with respect to a particular frame of difference. When the component velocities as expressed as a functional space coordinate and time. Then the flow field is described.

In general the velocity components or the velocity vector are supposed to depend on space coordinate and time as the independent variables. From this, we deduce with the help of tilastari expansions and neglecting the higher order tang the derivative of u with respect to t. That means the change of u with respect to t considering the convection comes like this del u del t. If you recall plus u del u del x plus v del u del y plus w del u del z similarly, $D \vee D$ t is equal to del ν del ν del ν del ν del ν del ν del ν . This we deduce the last class w del v del z. Similarly, D w D t we can use either small or capital does not matter usually use del w del t. These are the partial derivatives u del w del x plus v del w del y plus w del w del z.

Now, let us concentrate on one of this, what does it mean? This is that change of u with time that is x components of velocity, it composed of del u del t mathematically. If we see it is very simple that u is a function of x, y, z, t. So, it is the partial differential of u with t keeping x, y, z constant; this term is u into del u del x. That means the partial differential of u with x. So, a mathematical function is given, we can evaluate this del u del x del u del y del u del z u v w. So, we can totally evaluate this tang which represents the acceleration in the x direction similarly or x components of acceleration y component of acceleration and z component of acceleration.

Now, what does these physically mean? This is the rate of change of velocity at a particular point with time. That means this is the rate of change of velocity at a particular point with times if you multiply by delta t in both this size which was the earlier state in the deducing these equation. That means del u del t into del t; that means, this is the increment of u because of an increment of delta t increment in time. Because one particle flows from one point to other point, it has displacement d s. That means this coordinate a displacement or the position vector changes by d s that is by d x d y d z during a time D t.

So, that is the increment due to time delta t. Similarly, this is the increment due to its displacement delta x. This will be the increment for its displacement delta y if you multiply with delta t if you recall the earlier state. This will be the increment with respect to delta z. That means, due to delta z, some of all these correspond to the total changing the u. So, if you divided it by D t in a limit D t tians to 0 these represent the rate of change of x components of velocity with time. This is the rate of change of u with time is a partial differential physically it indicates it is the change of u component of velocity at a particular point to its time. When the point is fix that means the x y z is fixed. And these three components represent as whole the change of rate of change of velocity due to the convection of the particle, because of this positional change. So, this term is known as temporal acceleration or local temporal or local or local temporal or local acceleration.

So, this term del u del t del v del t these at the temporal or local acceleration which physical means that change of velocity at a point with time. That means, if a particle is not allow to flow or move from one point to other point. Then also because of this term if there is a change of velocity with time at a particular point, it will suffer a change in velocity. And this will be the contribution for the acceleration. But if the particle is allow to flow which actually happens any particle in a flow field is flowing is convicted with the flow. So, these three terms together define the convective acceleration. That means, this is the acceleration component due to convection in the flow field. That means, if that velocity is do not change with time then velocity (()) velocity changes its fizz coordinates. Then this will be the contribution to the acceleration.

So, therefore we see the total acceleration this is known as total or substantial. So, total of substantial acceleration which is basically the acceleration. Total of substantial acceleration is equal to temporal acceleration plus the convective terms, convective acceleration. This is same for the y component. So, total of substantial y component acceleration is equal to the temporal acceleration y component, temporal acceleration plus convective acceleration. Similar is the case for z component through.

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Now, let us write in a general way for the sake of mathematics. This if I take u as the variables then we can write in terms of the operator. This equation if you recall. So, I can write in this form that operator wise that D D t is del del t plus u del del x plus v del del y plus w del del z. That means, what I like to say, that if we see this now well. So, you see that if it is operated with you. So, you give this term operated with v gives this term, operative with w it gives this term. That means, mathematically you can tell this is a total derivative total or substantial derivative this is known as total or substantial derivative.

So, this is known as local derivative, local or temporal derivative and these three terms together is the convective derivative. That means for any quantity, whose change is due to the change with time and change with this space coordinates, because of convection. So, the total change in that quantity is equal to the local or temporal change local or temporal derivative and convective derivative when it is operated with the velocity v u v w any v general. Then these become total or substantial acceleration because derivative of velocity with respect to time is the acceleration. These become local or temporal acceleration. These become convective acceleration.

So, therefore we see that local therefore again come back. Total acceleration is equal to local acceleration or temporal acceleration plus convective acceleration. Now, you tell me when the flow is stead y which part is 0? Which part of the acceleration is 0 if a flow field is stead y? Flow field is stead y means this is not a function of t then which part is 0 temporal part. That means, in a stead y flow the temporal acceleration is not there that means, there is no contribution of temporal acceleration. When the flow is uniform on the other hand, but not stead y. That means, velocity components seems to be function of x y z then the convective acceleration terms become 0.

So, only acceleration is contributed by the temporal term. So, therefore the acceleration is contributed by the both temporal. That means, unsteadiness of the flow and convective that is non uninformative of the flow. When the flow is both uniform and stead y then there is no acceleration both the temporal term is 0. And also the convective term is 0 the flow does not have any acceleration. So, any combination we can have that means we can write for a stead y and uniform flow.

If I write local or temporal acceleration, local acceleration and another one is the convective acceleration. Then I can write for a stead y flow both of them for a stead y uniform flow; local 0, convective 0. If we write stead y non uniform flow, non uniform then which one is 0 local 0, but convective exist. If I write unsteady, but uniform then unsteady means these exists and for uniform; this is 0, in most general case when the flow is both unsteady and non uniform. That means, the velocity vector is a function of space coordinates and also the time. Then both the thing; that means, the temporal and convective term exist.

So, therefore, we come to the conclusion the total acceleration in a fluid flow field which is defined as the rate of change of velocity of a particle which is been convicted in the flow field as its change of velocity of a particle with respect to time. As it flows from one time to other point is composed of two distingue part. One is the change of velocity with time; another is the change of velocity with space coordinates. If velocity changes neither with time nor with space coordinates. See, if a fluid particle moves from one point to other point it will not suffering any velocity. But if the flow field is dependent on time, but not on space that is uniform unsteady flow, the fluid particle when suffer a change in velocity. This is because what if it goes to other point it will take some time to go. So, the velocity is changing because of the time which is again same in the entire flow field, if you think it will be very clear, in case uniform stead y flow field.

So, a particle will suffer a change in velocity, though the velocities are not changing with time. But velocity is changing from point to point as it moves from one point to other point it suffers the change in velocity. This is all the concept of total and substantial acceleration, now different coordinate components.

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These things can be deduced, just I give one example for polar coordinates systems, cylindrical polar coordinates system is cylindrical polar coordinates. So, here one thing is that, if we just define a polar coordinate cylindrical polar coordinate in. Now, if it is our original x y z coordinates, a cylindrical coordinate z direction remains as it is. In x y plane we conceive the coordinate system as r radial direction ends theta as the azimuthal direction as you know it, now acceleration. Similarly, in the radial direction will be the substantial derivative that is D v at D t, but along with that another component will come v theta square by r as you know that this component. This is because what the change in tangential velocity gives rise to a change in the velocity in the radial component. This is because of the curvature of the coordinate axes.

Because here x y coordinate axes are such that they are mutually perpendicular, a displacement in x directions solely does not cause any displacement in y direction. This is because of that. Let us take this is the basic thing that if this is the v theta at this point. If v theta changes to v theta plus delta v theta it will be perpendicular to this radial direction. So, radial directions are not parallel. So, therefore, by the vector diagram delta v theta will give a component in the art direction. This is a recapitulation lesson of your basic thing at school level. So, if you divided it by delta t. So, it becomes delta v theta delta t. So, therefore, you see.

So, there is a change in the delta v. That mean, we get acceleration in the radial directions. Similarly, due to change in v r you see v r and v r plus delta v r. So, you get. So, delta v r delta theta, so this is delta v theta delta theta I have forgotten to write because for a change of delta theta; this is delta theta. So, here also, here you see that if you divided it by delta t, so delta v theta into delta theta. So, it will be giving v theta square by r. So, similarly, here also; this is the v r; this is the v r plus delta v r; this is the delta v r which is the change in the velocity for the radial direction due to that change in the radial velocity? There is a component in the tangential direction. Whose magnitude is v r delta theta, in the livid of delta t tense to 0, and divided it by delta t we get the acceleration that is v r v theta by r.

So, this component is added and this component minus D v v theta square by is added because of this coordinate curvature of the coordinate or the peculiar typical coordinate system as you know. So, acceleration in the radial direction is D v r D t minus v theta. That means, it is not solved due to the substantial derivative of radial velocity. Because you know even there is a flow with only tangential velocity it gives rise to a radial acceleration which you know known as centripetal acceleration or a centrifugal acceleration? Similarly for radial velocity and the tangential velocity combined gives a acceleration in the tangential direction along with the rate of change of tangential component of velocity with time. So, these things probably you know.

It is a recapitulation from your earlier mechanics. Now, D v r D t is similarly splitted as temporal term plus the convective term. So, therefore you see in this case, this is a particularly a very unique thing if you leap these alone. We can tell this part is the convective part and temporal part and this part is the convective part. This is because we consider v r as function of r theta z and t. Similarly, considering v theta as a function of r theta z and t, the expansion of these in the earlier way we will give del v theta del t. That is the temporal term plus the convective term. And this is a additional peculiar term or typical term because of these and z direction.

Of course, it is as similar as the Cartesian coordinate system because due to change in the tangential or radial component of velocity will not produce any acceleration in the z direction because z direction is totally perpendicular to this r and theta direction. So, they are $($ ($)$) z direction acceleration is only the substantial derivative of z component of velocity with time. And these can be splitted as earlier the similar fashion in considering v z as a function of r theta z and t del v z del t v r del v z or this way can do it. So, in spherical polar coordinate also we can express the acceleration components in the respective directions by expanding the term that we can see from any book. Now, this is all for our discussion on acceleration.

Now, we will discuss few other things known as streamline first. What is the definition of a streamline? Well what is streamline now? Streamline is a how do you define a streamline? Streamline is an imaginary line or a series of imaginary lines in a flow field which are concept or drawn imaginary drawn. Or concept in such a way that the tangent to this line at any point at any instant represent the direction of the instantaneous velocity vector at that point. Again I repeat, just this is a definition you may not have to write you can see any book the definition will be there you try to understand the thing. That means, a series of lines, imaginary lines drawn in a flow fields such that at any instant. Because instant to instant the series of streamlines may change that will be clear if the flow is unsteady that may at any instant.

If I tell that this is the series of streamlines. That means, at any point on this line, line or curve. Line is a very generalized statement of a curve mean a two dimensional frame like this. These are the series of the lines or curve such that at any point the tangent to the curve represents the velocity vector at that point at that instant. Where this streamlines are specified to that the streamlines are specified.

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So, let us see streamlines. Streamlines are like this, let us have a streamlines like that as series of lines, at any instant I tell t these are the streamlines which means that at this point if these be the tangent. This is the velocity vector, let this at this point; this is the velocity vector the same instant. So, at this point this is at each point there will be a streamline at each point there will be a streamline through that point. So, tangent to these line or curve at that point represents the velocity vector at that point at that instant.

So, therefore, this is the streamline which defines the direction of the velocity vector at a particular instant in a flow field. These are the streamlines now, streamlines as a very simple property you can well appreciate. Now, from time to time how these lines may change? This is because the velocity vector at each and every point changes from time to time. So, for an unsteady flow these mapping of streamlines change, changes from instant to instant. Now, one very important conclusion that two streamlines can never cross each other. Two streamlines I cannot show that this is the series of streamlines at a given instant because this highlights the basic definition. That means, if it intersects at one point that means, at this point this is the tangent.

And if this curve if we follow for two curve there will be two tangents in two direction. But this is physical impossible because at a given instant and at a given point there will be only unique velocity vector. So, there will be one velocity vector. That means, the direction of the velocity has to be unique it cannot have two directions at point at a given instant. So, therefore, streamlines at a given instant cannot intersect each other. Now, how to find out the equation of streamline? Now, it is very simple with this definition we can find out the equation of a streamline.

Now, if we consider the velocity vector v over and infinite small elemental length d s. That means that a point at any point the tangent is the direction of the velocity vector. And if we consider an infinite small elemental length d s of this line surrounding this point. Then we can write v cross d s, d s is the vector elemental line it has got a dilation. What is this? This is equal to because v s I tangent to d s. That means v s and d s are in the same direction. So, cross product will be 0 which means that the magnitude of v into the magnitude of s d s rather d s times the sign theta if theta is the angle between these. That means, if two vectors are in the same direction they are cross product is 0. So, v cross d s is 0.

So, this vector equation if we equate with a frame of reference $x \, y \, z$, so what we get as you know that if I define now v is equal to i u corresponding u v I am always doing a mistake x y like that with the sense of rotation does not matter much u v. And w at the x y and z component of velocity, I u plus J v plus k w if I represent this with i J k as the unit vectors in x y and z direction and similarly, if I represents the d s vector that elemental length along the streamline. Similarly, it has got x component in terms of d x d y d z in a three dimensional we can take component in x direction d x in y direction, d y and z direction d z; so then if we make the cross product of this two vectors, what you will get? We will get three scalar equations. One is that u d y; this is very simple to remember d x you can make a determinate i J k u v w d x d y d z is 0.

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As you know this simple mathematics i j k, u v w and d x d y d z is 0, which leads as u d y minus v d x 0. That I write here, that leads as u d y minus v d x is 0, it is very simple couple u be with cross coordinate then u w. What is that u w? u d z minus w d x is 0. This comes from here and then v w. That means, v d z minus w d y is 0 and it can be written in a more convenient form as d x by u is d y by v is equal to d z y. These define a plane that is the stream surface in three dimensional plane. In a simple case of two dimensional flow this is the equation of streamline. That means either u d y minus v d x is 0 or d x by u is d y by v, in a more simple form school level form d y by d x is v by u. And it is very simple because v is the y coordinate u is the x coordinate component.

So, their ratio is the d y d x that is the direction or the slope of the curve. So, if we integrate this equation or this equation same thing this three equations are same. That means, if I know u and v expressively at the function of x and y and t. And if you integrate these you get expressively the equation of a streamline. So, therefore, you see the equation of a streamline is this vector form. And with reference to the Cartesian coordinate system these becomes is equal to this. And for a two dimensional plus this is the equation for streamline now, I think it is.

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Now, I come to pathline, if any problem you can ask me pathline here mathematics is less. But you have to understand pathline actually. Now, if I give a definition of pathline you will see that while streamline corresponds to oilary. And concept of description of fluid flow, pathline corresponds to the lagrangian concept. Why the definition of pathline is nothing but the locus of a point of a given identity? That means let a point with identity p 1 as a traverses in the flow. This is the pathline. So, pathline like this, this is the pathline, pathline may intersect I will tell you. So, this is the pathline for; that means, pathlines are nothing but the locus of particles with identity. Identity particle identities fix by some this position at some reference time. So, pathlines or pathlines are the lines or the curves of different particles as it traces in the fluid flow. So, their body you see one difference is that while streamlines.

Were the lines series of lines at a particular instant which connects the direction of the velocity vector at each point in a O a that it is along the tangent to that point on the streamline here, with time is inherent. That means, if you trace a particular pathline which is the locus of p 1 that means this point and this point varies with time. That means it is drawn. So, that these there is no question of at an instant because the pathline is the locus of the particle of given identity. That means that time is inhering in joining the pathline. For these reasons what happened? Two pathlines may intersect why this is because this particle p 3 it reaches a point dr.

When the particle p 4 will reach this point at a differ in time. And a particular point, the fluid may attain differ in velocity and as well at different times. That means you can tell that this two intersect. That means when p 3 comes here this is the direction of the velocity. Because locus of the point p 3 means tangent to this is the direction of the velocity vector. But when p 4 comes here, the particle p 4 comes here; this will come at a different instant. So, the velocity vector may be different. So, the two lines may intersect. For example, a pathline itself can intersect can make a close look. That means, this is the p 1 how does it mean that a particle comes here and crosses this point at some time t 1. Then it goes and flow field the, such there may be recirculatory flow that particle may again come back to that point after some time t 2.

So, there is no harden cose rule a particle may come back again to another same point through which it passed earlier. That means, it can form a close loop as you know from you common knowledge that a locus of a convicted particle can form a close loop. So, that, this can cut itself. So, the pathline can cut itself or one pathline may intersect the other. So, the description of pathline is just the lagrangian description. That means the equation of pathline is just the lagrangian equation. That means if I know the velocity vector as a function of space coordinate and time. And if I write this ds D t as the function of s and t, then the integration of that integration of this function will give S bar as a function of some initial condition and time.

So, an initial condition will be required to find out the cost end of integration which will be given implosively by these expressions. That means, this is precisely the description of lagrangian version. And this is the pathline of is 0 particle which I am calling p 1 p 2 p 3 p 4. This is the concept of pathline clear pathline. Now, you see from this two definitions of streamlines. And pathline that pathlines refer to lagrangian concept. Whereas, streamlines refer to oilarian concept, but one very interesting thing when the flow is stead y pathline and streamline become same.

For example, in the flow is stead y fluid the locus of p 1 represents what this is the direction of velocity vector; this is the direction of velocity vector. That means, when the p particle traces different points it only shows the velocity vector existing at that point which does not change with time. So therefore, it is co inside in with this streamline. And with that concept in a stead y flow it cannot come back to the same point, at the same point like this it cannot cut after some time. Why this is because if the flow field is stead y.

So, when it comes here from this loop. This is the velocity vector, earlier time t 1. This is the velocity vector, but velocity vector cannot change with time in a stead y flow field it has got only one new velocity vector. Similarly, if the two pathline cannot cut, if you argue set at one time t 1 p 3 crosses this point. And another time t 2 p 4 crosses this point it cannot happened if this 2 line crosses cuts like that. So, then according to that at t 1 the velocity vector has got some direction at t 2 velocity vector at another direction which is violation to the stead y flow.

So, for a stead y flow the lagrangian version and oilary version becomes equal as I told earlier. So, pathlines and streamlines become identical, because when we trace the pathline of a particular fluid element. That means, if we trace the locus of the fluid element the tangent to this pathline represents the direction of the velocity vector as the fluid element passes through that point. And if the flow is stead y it will simply sold the velocity vector of that point which is invariant with time; that means. In other word these fluid particle traces the velocity vectors at different points which becomes similar to the streamlines. So, streamlines and pathlines are identical under stead y flow condition it should be understood very clearly.

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Now, I will tell you the concept of another line that is known as streakline. Streakline concept is like this. A streakline is defined as the locus of you may not have to write. The locus streakline is the at any instant is the locus of the feat or the ultimate locations of all the particles, at which I pass through a fix point. That means it is at any instant, it is the locus of the ultimate positions of the particles at that instant, which I pass through a fix point. Let us understand what is it streakline? Let us consider a fix point here through which several particle passes. One particle let as consider p 1 which passes through this point and it is moving by this line. After some time at time t 1, t is equal to 0.

For example, at time after time delta t another particle crosses the same point, this point is fixed O. For example, same point; this particle is p 2 which is coming from the flow filed, naple since the flow is unsteady. We will consider in general first that then it will not traverse the same path. It will traverse the different path because a velocity vector as change. Let it traverse a path like this it is after another time, let 2 delta t or 3 delta t some interval another fluid particle p 3.

For example, which again passes through and velocity vector is continuously changing. So, let us see that this traces the pathline that. So, these are the particles it tracing the path at any instant t I fixed. And see that a first particle p 1 which crosses this point to what t is equal to 0 it has come up to this point. And the second particle p 2 which as cross the this point O at some time interval from t is equal to 0 delta t. It has come only of to that point if I take a short line that then at during at that time t the particle p 3 which has cross this point O has come this point. So, if you join the feat of this point that the ultimate position we will get the streaklines. And this streaklines will also my diagram cross through this fix point also, because one particle at any time t at time t some particle will be they are at this point and that will be its ultimate location, which is passing through this point.

So, therefore, this point will be also joining. So, this line is the streakline. That means, which gives the locus of the ultimate position of the points at that instant which I pass through this fix point. So, including the fix point itself we draw the curve this is the streakline. Now, if the flow is stead y you see all particles which pass through O will trace the same line. Because the velocity vector at O will be in variant with time similarly, at all points velocity vector will be in variant with time. The fluid particles will be pushed in the similar direction as it deed for galior particles when it comes here it will be pushed in the similar direction as it was x variants by galior particles. That means all the particles p 1 p 2 p 3 they will lie in the same line which has been trace by therefore (()). So, therefore, this streakline, streamline and pathline become identical.

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For a Steady Flow are identical

So, therefore, for a steady flow; so with this concept we can write for a stead y flow; streamlines, pathlines and streaklines are identical. Solved difficulties are they are and disseminations are they are for unsteady flow.

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Rotation Translation and Deform ate on

Now, we come to another well aspect of fluid which is very, very important that is translation, rotation and deformation. Now, these three things are very important for a fluid flow. Now, one thing translation, rotation and deformation these three things. Now, let us discuss that the basic difference from a from the of a fluid motion from that of a solid motion is that in a solid, you see that when a solid body is moving each and every particle in a solid body. Because of the strong coefficient is moving the same velocity or the same acceleration. If it moves continuously, if a solid body is rotated each and every particle of the solid body moves with the constant angular velocities or angular acceleration these motions are known as solid body motion.

That means, if you want to describe the velocity field is the within the solid body as a function of space that is no description, because all point move with the same velocity. And these velocity may change. That means this will change, but again the velocity will be the same. That means they move with the constant velocity and constant acceleration, but for a fluid particle for fluid system it is not sure.

So, that is the basic difference which I told you at the beginning first class. And that is the reason for which in a flow field the velocities are functions of space coordinates. That means if you identify just common sense, physically you think if you identify a fluid particle or a fluid mask or a fluid body. Then you will see the different point on the fluid body is varying with this velocity. So, therefore, they are moving with different velocities as a result of which it creates a continuous deformation. What is that continuous changing its length or dimensions, linear dimensions; continuous changing in its shape. The angular dimension which is not found in a solid body, solid body may deform under static condition.

When a solid body moves its linear elements do not suffer a continuous change in its dimensions or its angles. That is the angle between the included sides, included angle between the sides do not go on continuously changing which happens in case of fluid body. This is because the velocities of the fluid particles are different from point to point which is mathematically expressed as a function that velocity vector is a function of the space coordinates. So, therefore, we see that because of these the fluid body has got deformation. So, this is the only difference solid body has got translation, solid body has got rotation.

But fluid body along with the translation and rotation, they have got deformation. But rotation of a fluid body differs from that of a solid body. In a solid body rotation the angular velocities are same in all point, but fluid bodies it may not have that is feature in general. So, next class I think I will discuss these translation rotation and deformation in general. And that will be the last topic of this fluid kinematics. So, today I close here.

Thank you.