

Refrigeration and Airconditioning
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Lecture No. # 13
Vapour Compression Refrigeration Systems
(Contd.)

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Welcome back in the last lecture, I introduced multi stage vapour compression refrigeration systems and we discussed multi compression systems with flash gas removal intercooling etcetera. In this lecture I shall discuss multi evaporator systems and cascade system. So the specific objectives of this particular lesson are to introduce multi evaporator systems, discuss performance of typical multi evaporator systems with single and multiple compressors individual and multiple expansion valves and intercooling and finally to introduce cascade refrigeration systems.

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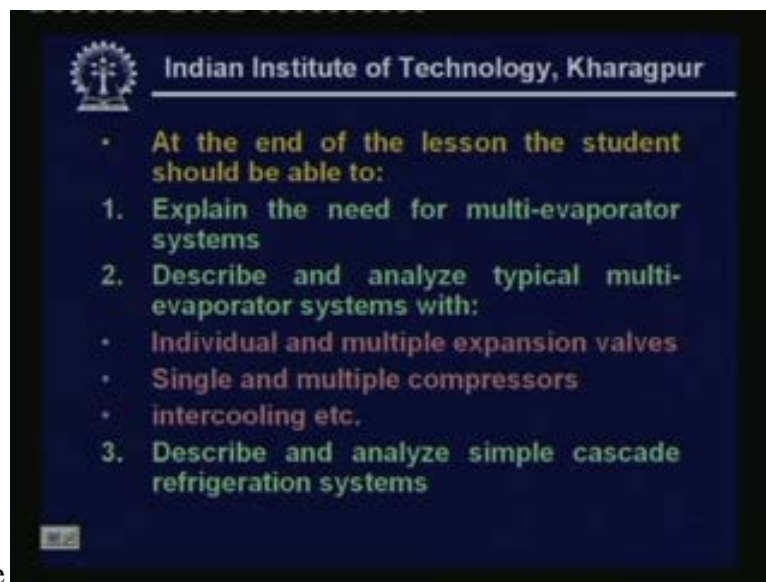


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12. Multi-evaporator and cascade refrigeration systems

- The objectives of this lesson are to:
 1. Introduce multi-evaporator systems
 2. Discuss performance of typical multi-evaporator systems with:
 - Single and multiple compressors
 - Individual and multiple expansion valves
 - Intercooling
 3. Introduce cascade refrigeration systems

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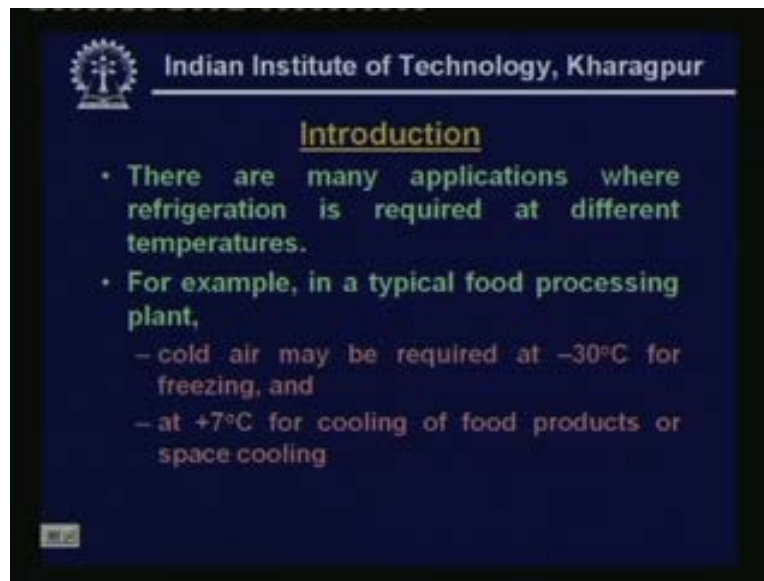


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- At the end of the lesson the student should be able to:
 1. Explain the need for multi-evaporator systems
 2. Describe and analyze typical multi-evaporator systems with:
 - Individual and multiple expansion valves
 - Single and multiple compressors
 - intercooling etc.
 3. Describe and analyze simple cascade refrigeration systems

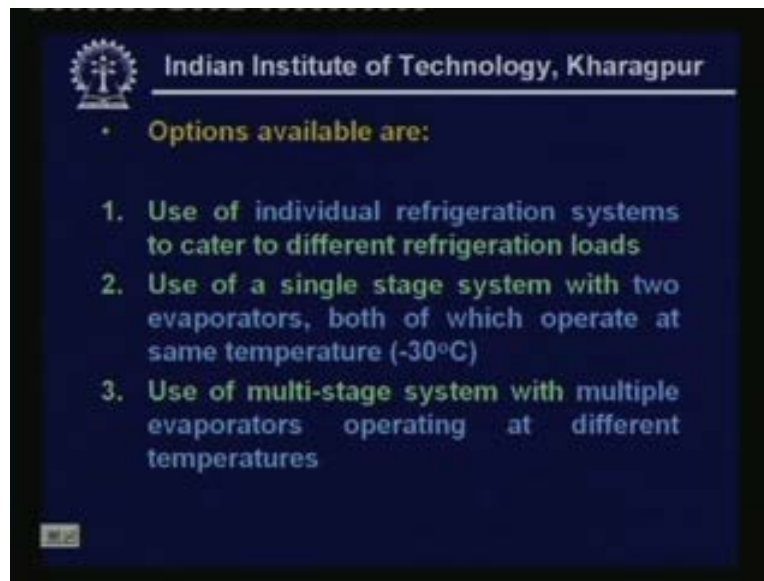
So at the end of this lesson you should be able to explain the need for multi evaporator systems, describe and analyze typical multi evaporator systems with individual and multiple expansion valves single and multiple compressors intercooling etcetera and also describe and analyze simple cascade refrigeration systems.

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Let me give a brief introduction there are many applications where refrigeration is required at different temperatures. For example in a typical food processing plant cold air may be required at minus thirty degrees centigrade for freezing and cold air is also required at plus seven degree centigrade for cooling of food products or space cooling. That means it is a typical application where you require refrigeration not at a single temperature but at different temperatures. As I mentioned in the last class another application is in the domestic refrigerator where you require about minus twenty degrees centigrade for frozen food storage and about plus five degree centigrade for fresh food storage. So these are the typical examples of applications requiring refrigeration but not at the same temperature but at different temperatures. So let us look at what are the options available for this kind of applications.

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The options available are, you number one is use of individual refrigeration systems to cater to different refrigeration loads. That means you have two individual refrigeration systems one will be catering to the very low temperature requirement and the other one will be catering to the high temperature requirement. And the second alternative is to use a single stage system with two evaporators but both of which operate at the same temperature. That means use a normal single stage system which will simply have two evaporators both will be operating at the same temperature that is at minus thirty degree centigrade. And the third option is to use a multi stage system with multiple evaporators operating at different temperatures so these are typical options available for these kind of applications.

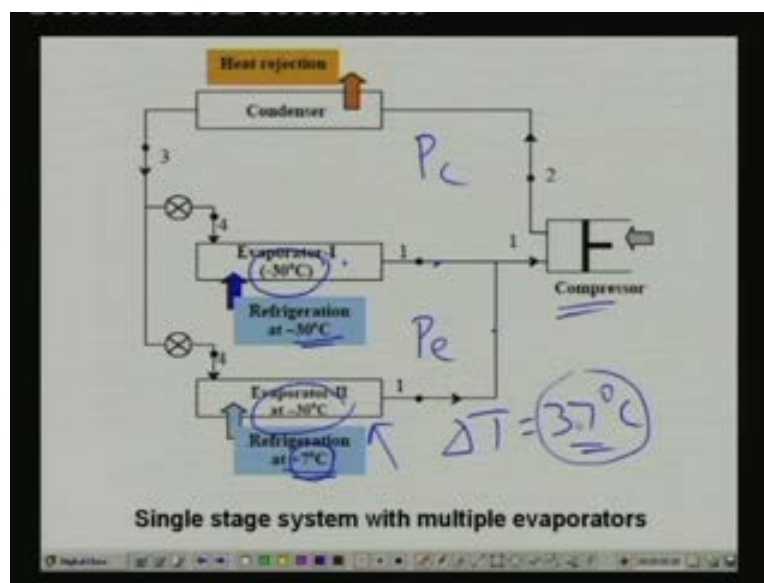
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- Use of individual refrigeration systems may not be economically viable
- Use of a single stage system with two evaporators operating at the lowest required temperature is:
 1. Thermodynamically not efficient
 2. May create practical problems such as frost formation or freezing of secondary fluids at high temperature loads
 3. Excessive water losses in refrigerated products stored at high temperature due to very dry supply air

Now use of individual refrigeration systems may not be economically viable and use of a single stage system with two evaporators operating at the lowest temperature required temperature is thermodynamically not efficient. It may create practical problems such as frost formation or freezing of secondary fluids at high temperature loads. And excessive water losses in refrigerated products stored at high temperature due to very dry supply air. Let me explain this.

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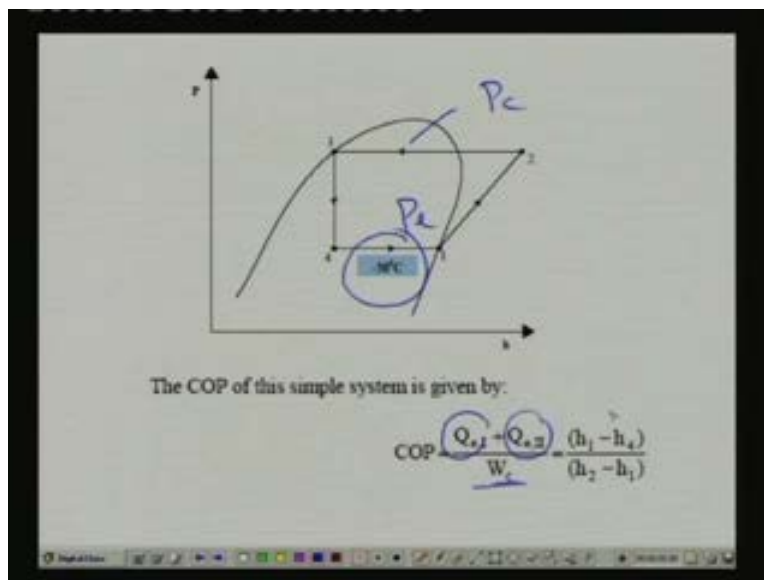


Okay. So what we have here is the simple single stage refrigeration system. So you can see that this is single stage because there is a single pressure. That means this evaporator and this

evaporator both are operating at the same temperature. That means there are only two pressures one is condenser pressure P_c and the other one is evaporator pressure P_e okay. All that we have done is we have provided two evaporators this will be catering to refrigeration load at minus thirty degree centigrade and the evaporator two will be catering to the refrigeration at plus seven degree centigrade.

So external requirement is minus thirty and plus seven but we are maintaining the evaporators both the evaporators at minus thirty degree centigrade. This is required because you are using a single compressor and you are not using any pressure reducing valve. So pressure at this point has to be same as pressure at this point that means the temperature at this point should be same as the temperature at this point okay. So this is the typical single stage system and its P h diagram.

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A simple P h diagram like this, you have a single pressure and this pressure corresponds to okay. So this pressure corresponds to minus thirty degree centigrade right the evaporator pressure corresponds to minus thirty degree centigrade and this is the condenser pressure okay. And what is COP of this system very simple COP of the system is refrigeration effect obtained at evaporator one and evaporator two. This is simply equal to the mass flow rates through individual compressors individual evaporators divided by the total power input. That means this

is simply equal to refrigeration effect $h_1 - h_4$ divided by the work of compression $h_2 - h_1$ okay.

All that we have done is we have divided the evaporator into two parts that is the only difference compare to the earlier systems. Now what is the disadvantage of this system? You can see that for example here you require minus, you require cooling at plus seven degree centigrade still you are maintaining the evaporator at minus thirty degree centigrade. That means you are provided a delta T of thirty-seven degree centigrade. Which is very large and this will lead to large irreversibilities at this point okay that is one of the main problem from thermodynamic efficiency point of view. Other problem is that air that is coming from air or water, whatever it is that is coming at this to this evaporator will be typically at high temperature okay. That will come in contact with a very cold evaporator.

So this may lead to the frosting of air that means the air the moisture in the air may simply freeze on the evaporator coil. And what is the problem with this once frost forms in the evaporator coil it will block the air flow passages. So air flow may get reduce so you require continuous defrosting of the evaporator this is one typical problem when you are using air as a secondary fluid. The other problem is if you are using a liquid as a secondary fluid then there is a danger of the liquid freezing okay. Because they all this is happening because externally you require above zero temperatures but you are operating the evaporator at sub zero temperatures okay this is a typical problem with this kind of the system.

Another problem is if you are using this system, let us say for cold storage applications in cold storage application, let us say that you are storing the vegetables at plus seven degree centigrade then you take the air from that storage space and you pass it on the evaporators. So air gets cooled and you send the cold air to the vegetables so that they can be kept cooled. But the problem here is since the evaporator is at minus thirty degree centigrade along with the sensible heat transfer there will be large latent heat transfer also. That means the air becomes not only cold but also very dry. That means you are sending very dry air into the cold storage. So when you are sending very dry air into the cold storage the food product stored there will lose water okay so ultimately there will be large water losses. So this is a practical problem if you have very cold evaporator okay so these are the typical problems with this single stage system. Okay so that is what I have mentioned here and so let us look at what are the options.

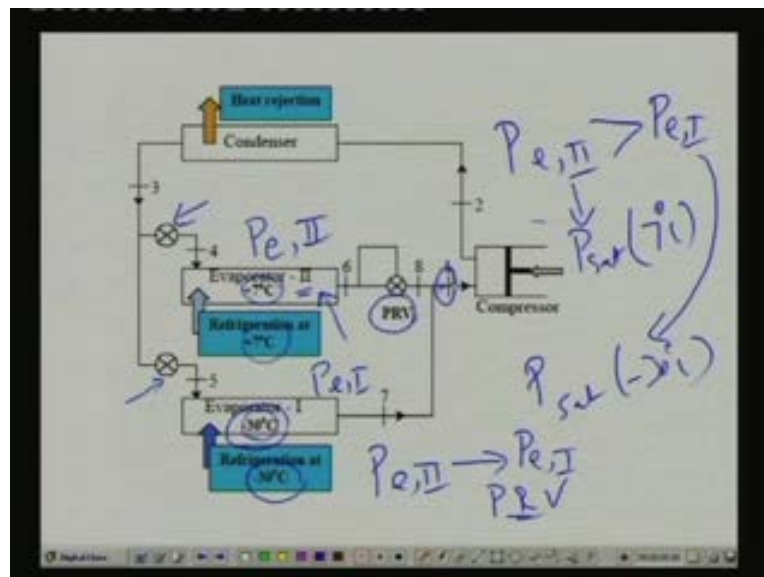
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1. Individual evaporators and one compressor with a pressure-reducing valve:
 - a) Individual expansion valves
 - Higher refrigeration effect at high temperature evaporator
 - Higher specific work input due to operation of compressor in superheated region
 - Better than single stage system due to proper operation of high temperature evaporator

So first option is we can use an individual evaporator and one compressor with a pressure reducing valve okay. I will show you the so again here there are two options you can use either individual expansion valves or multiple expansion valves. Let me explain what do you mean by individual expansion valve.

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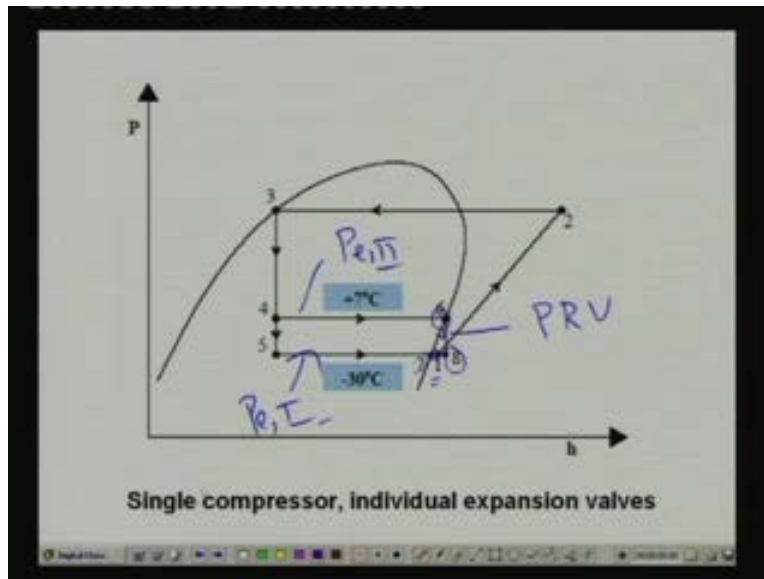
Okay, so this is the system as you can see this is a looking almost same as earlier one but there is a difference here the seven degree evaporator is being operated at seven degrees only the external cooling is required at seven degree. So you are operating the evaporator at seven

degrees. And here it is cooling is required at very low temperatures so you are operating the evaporator at very low temperature. That means the evaporator temperatures are as per the external requirements that means there is less delta T between the external medium and the evaporator refrigerants so there are less irreversibilities okay. So that is one difference between the single stage systems since you are operating these two evaporators at different temperatures obviously the pressure.

Here let me say that this pressure is P_e two and this pressure is P_e one so obviously P_e two will be greater than P_e one and what is P_e two P_e two is nothing but saturated pressure at seven degree centigrade and P_e one is the saturated pressure at minus thirty degree centigrade okay. So that is the difference between the earlier system so since your these pressures are different we have to have two different expansion valves. So we are using two individual expansion valves this expansion valve will reduce the pressure from the condenser pressure to the pressure corresponding to seven degree saturation temperature. And this expansion valve will reduce the pressure from the condenser pressure to a saturated pressure corresponding to minus thirty degree centigrade okay.

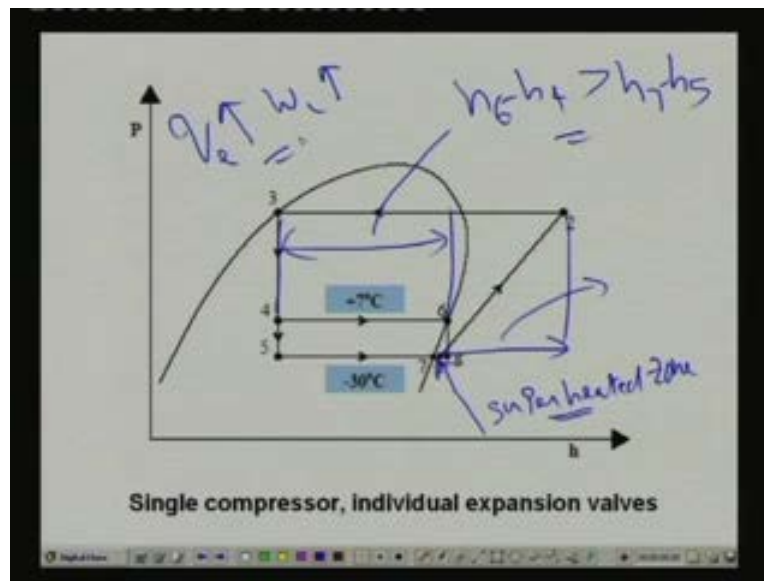
Then since the pressures are different and you are using a single compressor that means at this point the pressure has got to be same. So what are we doing here we are simply reducing pressure of this one from P_e two to P_e one using a pressure reducing valve. That means pressure drop from P_e two to P_e one takes place across the P R V okay. So you are using the pressure reducing valve so that again at the compressor inlet you have a single pressure. So this is the system known as multiple evaporator system with individual expansion valve and a single compressor. Let us look at typical performance of this.

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First let me show the P h diagram, so you can see that, how the P h diagram is different from the earlier one. Since you have two different evaporators operating at different temperatures there are two different pressures and this pressure. As I was mentioning is $P_{e,II}$ and this pressure is $P_{e,I}$ and this pressure is reduced across the PRV okay. So process six to eight is taking place across the PR. So what you are doing is you are simply reducing the pressure from this point to this point. So that ultimately at the inlet to the compressor that is point one same pressure exists and this pressure corresponds to $P_{e,I}$ okay. And compression is taking place from this point to this point. Now you can see the point one point one is a mixture condition because of the mixing of refrigerant point eight with refrigerant at point seven okay.

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So as a result of this mixing you can see that the inlet to the compressor is in a superheated zone okay. Now what are the advantages of this system compare to the earlier system only advantage we see here is the high stage compressor or high temperature compressor I am sorry high temperature evaporator has a larger refrigeration effect. For example h_6 minus h_4 is slightly greater than h_7 minus h_5 . So this is one advantage you get using multiple evaporators but a single compressor but there is a disadvantage here. What is the disadvantage, the disadvantage is you are pushing the suction condition into superheated zone. As a result the work of compression this is the work of compression this also increases. That means your refrigeration effect q_e increases at the same time work of compression also increases. So ultimately you may not get any benefit in terms of COP okay. But still this system is preferred because you are not allowing the warm air or relatively warm air to come into contact with very cold evaporator.

So there by you can prevent the problem of frosting or freezing or very heavy water losses etcetera so this is one advantage. This is, that is why this system is preferred even though you do not really get much benefit in terms of COP now let me give you typical performance equations okay.

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$$\text{COP} = \frac{Q_{e,I} + Q_{e,II}}{W_c} = \frac{m_I(h_7 - h_4) + m_{II}(h_6 - h_4)}{(m_I + m_{II})(h_2 - h_1)}$$

$$m_I = \frac{Q_{e,I}}{(h_7 - h_4)}$$

$$m_{II} = \frac{Q_{e,II}}{(h_6 - h_4)}$$

$$h_2 = \frac{m_I h_7 + m_{II} h_8}{m_I + m_{II}}$$

Know
 $Q_{e,I}, Q_{e,II}$
 $T_{e,I}, T_{e,II}$
 T_c

Mass & energy balance for the mixture

The COP of the system, how the COP of the system is defined here the total useful cooling effect that is you cooling effect at evaporator one cooling effect at evaporator two divided by the total power input in the system. So this is simply written as $Q_{e,1}$ is nothing but mass flow rate through evaporator one into refrigeration effect across evaporator one. And $Q_{e,2}$ is nothing but mass flow rate through evaporator two into refrigeration effect across evaporator two. This is divided by the total work input total work input is nothing but total mass flow rate into work of compression okay.

So this is very simple you can by looking at the P h diagram you can easily arrive at this equation. And how do we find the mass flow rates for example this is known to us, let us say typically in a typical problem the known parameters are something like this or known are specified $Q_{e,1}$ will be specified generally $Q_{e,2}$ will be specified and both the temperatures. For example $T_{e,1}$ $T_{e,2}$ and condenser temperature these things will be generally known. Once these are known we can plot the cycle on P h diagram and we can find out all these state points okay.

And once if you find the state point you can find out mass flow rate through the low temperature evaporator simply by dividing the cooling capacity of the low temperature evaporator divided by the refrigeration effect. Similarly mass flow rate through the high temperature evaporator is the ratio of refrigeration capacity of high temperature evaporator divided by the refrigeration effect okay so this is how you can easily perform the calculations. And as I was telling the condition at

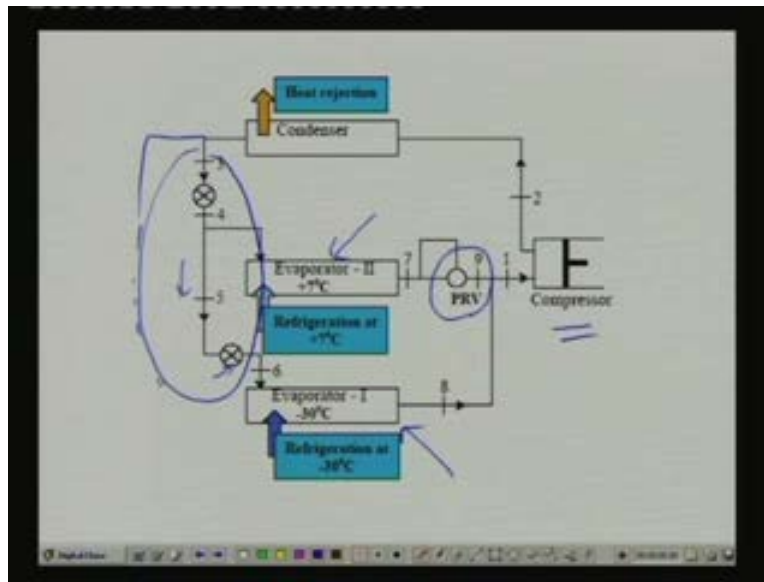
point one, I am sorry this should have been point one h one h one is simply equal to is obtained by simply applying a mass and energy balance across the mixing process so this is from you r mass and energy balance for the mixing okay. So if you are applying mass and energy balance of the mixing you get this equation. So that is how you can get the suction condition of the compressor okay. This is how you have to perform simple calculations.

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So okay, so let me summarize the advantage of this system higher refrigeration effect at high temperature evaporator. But higher specific work input due to operation of compressor in superheated region so you have a benefit and you also have a negative aspect. As a result as I was telling COP may not increase greatly okay. But still this system is better than single stage system due to proper operation of high temperature evaporator okay. So remember that this system uses two evaporators single compressor and individual expansion valves. Now let me describe another system which also use in this single compressors two evaporators but not individual expansion valves. But it uses what is known as multiple expansion valves. So let us see what happens when use a multiple expansion valves in a, in place of individual expansion valves. Okay, so this is system with multiple expansion valves.

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Okay. So the only difference everything else is same here you have the same condenser evaporator high temperature evaporator operating at plus seven low temperature evaporator operating at minus thirty and the pressure reducing valve which reduces the pressure from high temperature evaporator to low temperature evaporator and a single compressor. Only difference is here okay in the earlier case we were using individual expansion valves here you are using a multiple expansion valve. That means what you are doing is the refrigerant that is going to this expansion valve will be saturated liquid in this case whereas earlier case it is directly going from this point okay that is the only difference okay. It will be very clear if you look at the P h diagram so let me show the P h diagram.

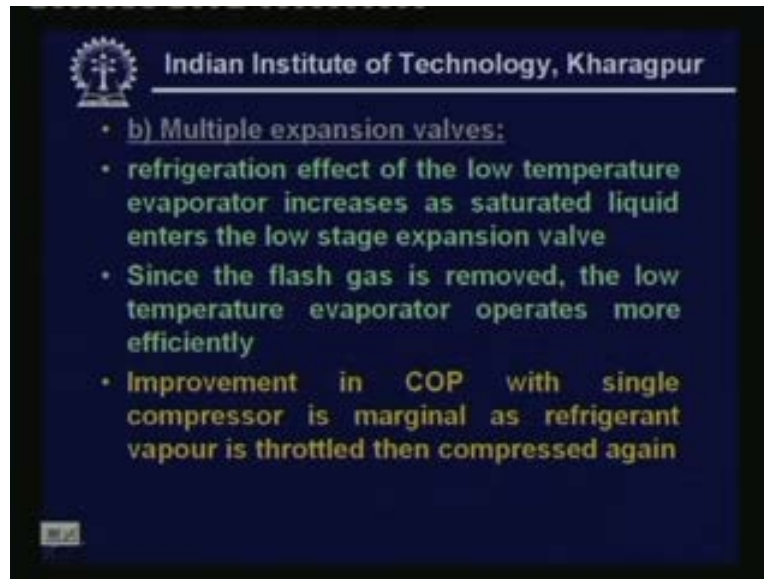
another refrigerant stream at point nine okay. So you can find out what is the resultant condition one and once you know the resultant condition you can find out what is power input to the system okay. So let me give the equations.

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The image shows a slide with handwritten equations. The first equation is
$$\text{COP} = \frac{Q_{e,I} + Q_{e,II}}{W_c} = \frac{m_I(h_8 - h_6) + m_{II}(h_7 - h_4)}{(m_I + m_{II})(h_2 - h_1)}$$
 with blue circles around the numerator and denominator. Below it are two equations for mass flow rates:
$$m_I = \frac{Q_{e,I}}{(h_8 - h_6)}$$
 and
$$m_{II} = \frac{Q_{e,II}}{(h_7 - h_4)}$$
 with blue underlines. The final equation is
$$h_2 = \frac{m_I h_8 + m_{II} h_9}{m_I + m_{II}}$$
 with a blue arrow pointing to the right.

So these are the equations as I was telling you COP is again total refrigeration effect divided by the total power input that is nothing but the refrigerant effect at that is the refrigeration capacity of the low temperature evaporator plus refrigerator capacity of the high temperature evaporator divided by the total power input. Again mass flow rates are obtained as I have already explained from the refrigeration capacity and refrigeration effect okay. And this is the mixture condition so this is how you can easily find out the required performance parameters.

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So this system compare to the individual expansion valve in this system you get higher refrigeration effect at low temperature evaporator because saturated liquid enters the low stage expansion valve. And since the flash gas is removed the low temperature evaporator operates more efficiently one thing you can observe is with these multi evaporator systems you are not really getting much benefit okay. Why are we not getting much benefit as I have already explained? What you are doing in this system is you are using a single compressor okay.

Since you are using single compressor ultimately the compressor has to operate at a single suction pressure and that suction pressure has got to be the pressure corresponding to the lowest temperature evaporator okay.

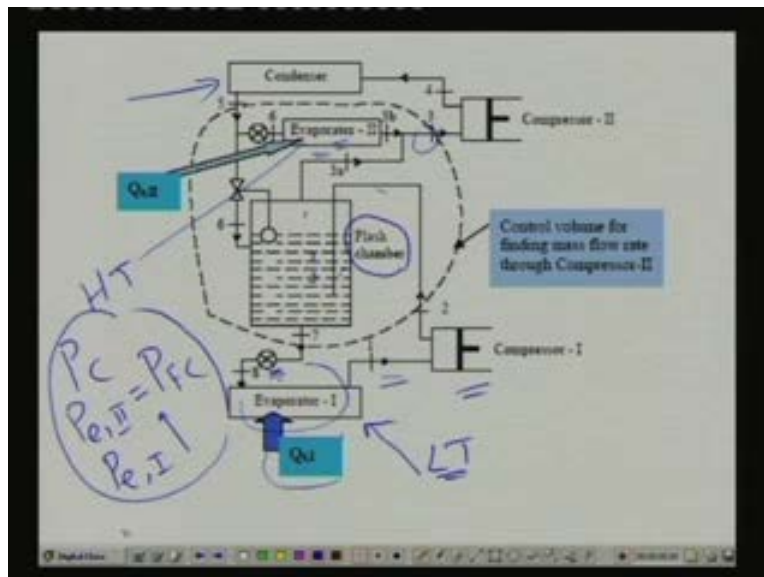
So the evaporator will be always operating between the lowest pressure and the condenser pressure. So what we are doing in these two systems is we are simply throttling the high pressure vapour to the lowest temperature evaporator pressure and then again you are compressing it. That means you are unnecessarily throttling it which is a reversible irreversible process then you are compressing it right. So obviously this is not efficient and you are really wasting energy okay. So as a result of it you are not really getting the benefit of multi stage compression systems okay. But this may have some advantages because you are using the single compressor there are many cases where you can get significant benefits if you are using not one compressor. But multi compressor that means you have multiple evaporators with multiple compressors okay. Let me describe one such system now.

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Okay, a multi evaporator system with multi compression intercooling and flash removal okay. This includes everything okay. Let me explain the system first.

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Okay, so in this system again you have a, this is the two stage system you have condenser here and one evaporator operating at low temperature this is the low temperature evaporator and there is another evaporator which is operating at high temperature okay. So you have two evaporators and we are also using the flash chamber and what we are doing in the flash chamber we are

removing the flash gas okay. And you are sending only refrigerant liquid to the, this low temperature expansion device okay.

So the process is like this at this point you are taking high pressure refrigerant liquid and you are throttling it part of its throttled to the required evaporator pressure two then it performs the required refrigeration here and then it is compressed okay. Then rest of the liquid comes to flash chamber okay. And from the flash chamber you are taking the saturated liquid to the second evaporator through this expansion valve okay. Then this refrigerant liquid refrigerant will perform its refrigeration duty then you have the low pressure, low temperature vapours. Here you are compressing it to an intermediate pressure corresponding to the evaporator two and the flash chamber.

And you are desuperheating this by bubbling it through this flash and flash this thing. And you are mixing whatever vapour generated here with this vapour okay. And ultimately all the vapours are getting mixed at this point okay. That is what is happening so it is quite a complicated system one thing you must keep in mind here is here there are three pressures. One is pressure corresponding to condenser and second one is pressure corresponding to evaporator two which is also equal to pressure of flash chamber okay. And the third pressure is evaporator pressure one okay there are three pressures okay. And this compressor is operating between P_{e1} and P_{e2} and this compressor is operating between P_{e2} and P_c . So you can see that the compressors are operating across the much reduced pressure ratios so they will be performing in an efficient manner okay.

And you can also see that since you are removing the flash gas here this evaporator will be operating very efficiently okay. And for carrying out the performance analysis I have taken a control volume which includes the flash chamber and the high temperature evaporator this expansion valve and the float valve okay. I will let me give the typical governing equations for these kind of a system first let me show.

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$$\text{COP} = \frac{Q_{e,I} + Q_{e,II}}{W_{c,I} + W_{c,II}} = \frac{m_I(h_1 - h_2) + m_{e,II}(h_3 - h_6)}{m_I(h_2 - h_1) + m_{II}(h_4 - h_3)}$$

$\underline{LT} \quad m_I = \frac{Q_{e,I}}{(h_2 - h_6)}$
 $\underline{HT} \rightarrow m_{e,II} = \frac{Q_{e,II}}{(h_3 - h_6)}$
 $m_{e,II} \neq m_{II}$

So the COP again the COP is defined as total refrigeration effect or total refrigeration capacity divided by the total power input. So total refrigeration capacity is a refrigeration capacity of low temperature evaporator which is nothing but mass flow rate through that evaporator into refrigeration effect plus refrigeration capacity of high temperature evaporator which is nothing but mass flow rate through that particular evaporator and its refrigeration effect divided by the total power input. Now the power input consists of two parts power input to the compressor one plus power input to the compressor two. And power input to compressor one is simply equal to, mass flow rate into work of compression and power input to compressor two is mass flow rate into work of compression for that particular compressor.

And how do we get the mass flow rate through low stage low temperature evaporator and high temperature evaporator. This is for the low temperature evaporator. So this simply equal to refrigeration capacity divided by refrigeration effect. Similarly this is for the high temperature evaporator again same definition refrigeration capacity divided by the refrigeration effect there is a small difference here, this, $m_{e,II}$ is not equal to m_{II} that that's what you have to keep in notice okay this is not same as this okay.

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mass flow rate of refrigerant through the high-stage compressor which can be obtained by taking a control volume which includes the flash tank and high temperature evaporator and applying mass and energy balance across the control volume

Mass balance:

$$m_5 + m_2 = m_7 + m_3; \quad m_5 = m_{II} = m_3 \quad \& \quad m_2 = m_1 = m_7$$

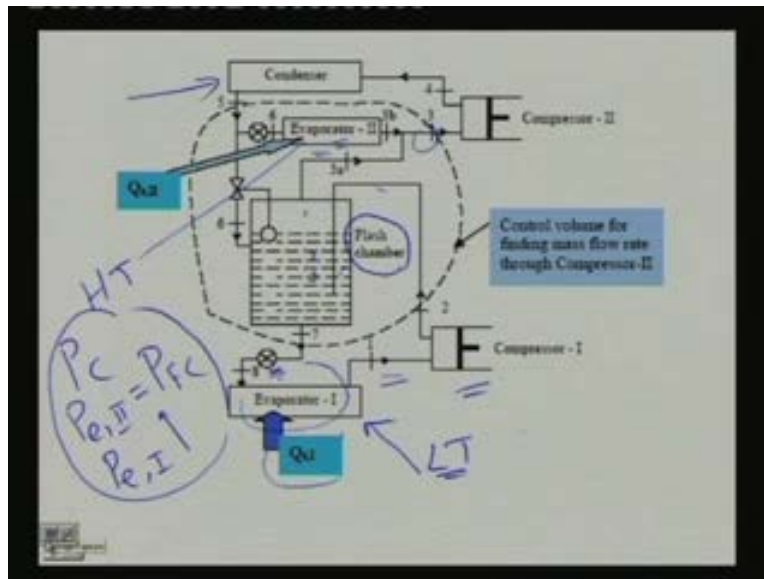
Energy balance:

$$m_5 h_5 + m_2 h_2 + Q_{e,II} = m_7 h_7 + m_3 h_3$$

Now as I was telling you we have to find out mass flow rate of refrigerant through the high stage compressor okay. This you are obtaining by taking a control volume which includes flash tank and high temperature evaporator okay. Then we are applying mass and energy balance across the control volume okay. So simply the total mass coming in is equal to total mass going out and this is the mass flow rate through the condenser okay this is for the condenser. And if you look at the schematic you will find that this is same as the mass flow rate through high stage compressor okay. High pressure compressor so both are same okay. Similarly this is the mass flow rate through the low temperature evaporator which is equal to mass flow rate through the low pressure compressor okay.


So this is simply the mass balance and energy balance if you are applying energy balance you will see that this is the energy coming in this is the energy going out and this is your required refrigeration capacity of the high temperature evaporator okay. So this is how you have to write the equations and we have seen that what are the things known to us typically this is known to us and you can find out this can be obtained. Okay then all the state points can be obtained okay. This state point can be obtained this state point can be obtained. So the only thing that is unknown is this okay. So that can be obtained by substituting this equation in this particular equation okay. So that is how you have to solve the problem.

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Okay. So let me just show the schematic so that you can look at the control volume using which we can find out the mass flow rate through the compressor two okay.

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2. Multi-evaporator system with multi-compression, intercooling and flash gas removal:

- Good for low temperature applications with different refrigeration loads.
- e.g., -40°C for quick freezing and -25°C for storage of frozen food
- The low temperature evaporator operates efficiently as flash gas is removed in the flash tank
- High-stage compressor operates efficiently as the suction vapour is saturated

So what are the advantage of these systems these systems are good for low temperature applications with different refrigeration loads okay. When you have you require very low temperature but different very low temperatures okay, an example is, you may require minus forty degree centigrade for quick freezing and minus twenty-five degree centigrade for storage of frozen food okay. These are typical requirement in a food processing plant. The low temperature

evaporator operates efficiently as flash gas is removed in the flash tank and high stage compressor operates efficiently as the suction vapour is saturated okay. So this is the typical system.

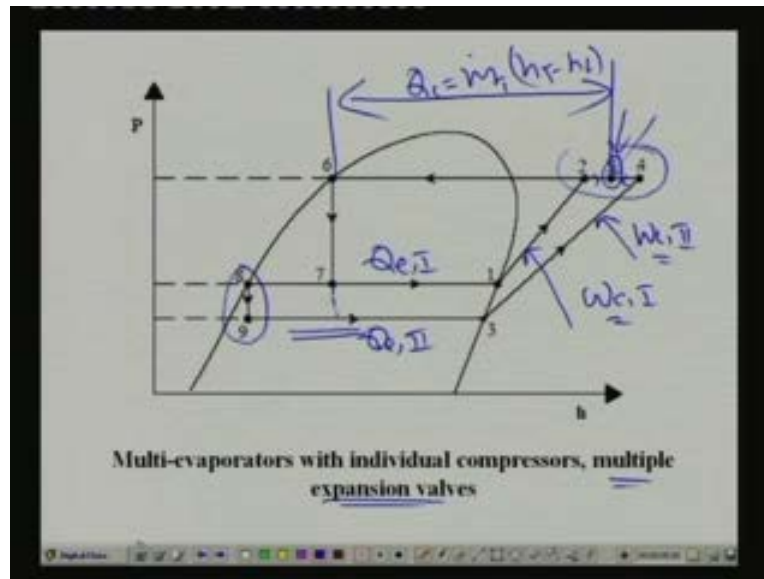
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A good system okay. Now let us look at another system which is multi operator system with individual compressors and multiple expansion valves okay. So you should not get confused by the names okay multi operator systems means you have more than one evaporator. And the multi stage multi operator multi stage system means you have more than one evaporator. And you have more than one low side pressure. That means the evaporator will be operating at different temperatures and different pressures okay.

So this is the meaning of multi evaporator multi stage system and if you add to that multi compression that means you have more than one compressor okay. So that is the meaning of multi compression and I have already shown you what is the difference between individual expansion valves and multiple expansion valves okay. So this terminology you must keep in mind and from the name itself you should be able to draw the schematic and draw the corresponding P h diagram okay. Now let me show the schematic of this particular system okay.

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Okay. So this is the P h diagram with individual compressions. So you can see that since I am using multiple expansion valves for the low stage low temperature evaporators now the condition is like this. That means you are sending saturated liquid that is why you have something like this. If you are using individual expansion valves again I am repeating we will have something like this okay. Since we are using individual compressor. So what you are doing is you are not mixing this vapour and this vapour at this point what you are doing is you are simply compressing this vapour from this pressure to this pressure you are compressing this vapour from this pressure to this pressure and both are getting mixed.

That means this vapour and this vapour they are getting mixed and you are getting the resultant point at point five which is nothing but the inlet condition with the condenser okay. So the heat loss across the condenser is this okay. So Q_c for example is total flow rate into h five minus h six okay. So this is the system with individual compressors now what is the combined COP of this system combined COP of this system you can easily see that, for example, if this has let us say refrigerant capacity of Q_{e1} and this has other refrigerant capacity of Q_{e2} and this compressor. Let us say it consumes a power W_{c1} and this consumes the power W_{c2} so total refrigeration effect is this plus this divided by this plus this. That means total refrigerant capacity divided by total power input right. And as I said this point again you have to obtain point five that means the mix point of mixing has to obtain by applying mass and energy balance across this process okay.

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The image shows a slide with handwritten equations. At the top, the COP is defined as the sum of refrigeration effects divided by the sum of compression work inputs. Below this, two equations show how to find the mass flow rates for each stage based on known temperatures and refrigeration effects.

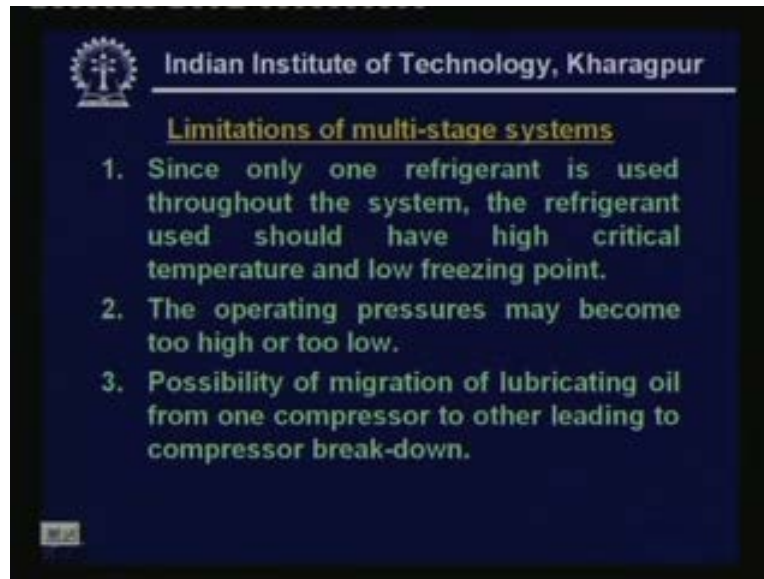
$$\text{COP} = \frac{Q_{e,I} + Q_{e,II}}{W_{c,I} + W_{c,II}} = \frac{m_I(h_3 - h_9) + m_{II}(h_1 - h_7)}{m_I(h_4 - h_3) + m_{II}(h_2 - h_1)}$$

Known T_c , $T_{e,I}$, $T_{e,II}$

$$m_I = \frac{Q_{e,I}}{(h_3 - h_9)}$$
$$m_{II} = \frac{Q_{e,II}}{(h_1 - h_7)}$$

So these are the typical equations as I have already mentioned combined COP is total refrigeration effect divided by the total power input. So total refrigeration effect is simply equal to this mass flow rate through that particular evaporator into refrigeration effect mass flow rate through the evaporator to into the corresponding refrigeration effect divided by work of compression power input to first compressor plus power input to second compressor. And how do we find the mass flow rates normally this is known to us this is known to us and these steps are known if you know the temperatures. Again T_c T_1 T_2 these are known okay. So you can find all these state points on the diagram from that you can find out the required mass flow rates and from that you can find out the COP of the system.

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Now let us look at these are typical multi stage system. As I have mentioned lot of combinations are possible and what we have discussed now in fact in the last lecture as well as this lecture is only two stage system. That means you have only two low side pressures but then theory it is not in not only in theory in practice also stages more than two are very frequently used you can have three stage four stage or any number of stages. The working principle remains same only thing is system schematic becomes more complicated you have more number of components and all. But the principle as long as principle is concerned three stage or four stage we will exactly same principle as that of two stage systems okay.

And you have as I said a different combinations possibility of different combination. That means you can have variety of intercooling and a variety of flash gas removal systems etcetera okay. So actual systems can be different but once you understand the operation of individual components and how to club them together then the analysis should not become too difficult. And as I said once you understand the schematic systems schematic properly and once you can translate that on to a P h diagram then your job becomes easy okay. And there are some slightly difficult systems where you may have take the control volume properly and apply the mass and energy balance equations across the control volume in such a way that you get the required performance parameters okay.

Now this multiple multi evaporator multi stage systems they are used they have been used quite successfully in a wide variety of applications okay. And normally they are used with R twelve R

twenty-two and ammonia okay. So they are commercially available and they have been used and they are being in used. But they have certain limitations okay so let us look at what are the limitations of these systems. First limitation is that since only one refrigerant is used throughout the system the refrigerant used should have high critical temperature and low freezing point. So one thing you must have noticed in all these systems is that you are dealing with a single refrigerant even though you are using multiple compressors multiple evaporators multiple expansion valves and all that a single working fluid flows through the entire system okay. That means single working fluid will be undergoing the large temperature variation okay. Right from, let us say minus forty or minus fifty right up to plus fifty degrees centigrade okay so it undergoes a large temperature variation okay.

So for the refrigerant to work efficiently it should have very low freezing point and very high critical point. Because if you remember from our earlier lectures if you want to have a good efficiency you must operate the condenser away from the critical point okay. That means your critical temperature should be very high. At the same time when you are operating the system at very low temperatures it should not freeze. That means it should have a very low freezing point okay. So this is the first requirement the same refrigerant should have high critical point and low freezing point okay. Which may not be possible in practice or it may be possible with only a few number of refrigerants.

That is the reason why the multiple multi stage systems are used, only with R twelve R twenty-two and ammonia and not with all refrigerants because with other refrigerants you have the temperature problems okay. And the second disadvantage is the operating pressures may become too high or too low. For example if you are used let us say that you are using a refrigerant which has normal boiling point of let us say minus twenty-five degrees centigrade okay. You are using that system for a multi stage multi evaporator application okay. And if your evaporator is operating at let us say minus forty degree centigrade then that means evaporator will be working at very low pressure. That means in vacuum when the system is working in vacuum what are the problems.

You may have first problem is there can be leakage of air from outside into the system which is which will create maintenance problems. And the second problem is that the suction vapour will have very high specific volume. That means you will have big compressors okay. So these are

the two typical problems of operating system under vacuum. So this is one case with not very low normal boiling point working fluid.

On the other hand let us say that you are you have a low temperature of minus forty degree centigrade and you have a working fluid which has a normal boiling point of minus fifty that means your evaporator will be operating above atmospheric pressure. At the same time your condenser will be operating at very high pressures. That means either you may have a problem of very high pressures in the condenser or very low pressures in the lowest temperature evaporator right. You may have this problem or that or you may have both the problems okay. The all this is happening because you are using the single refrigerant okay. And the third practical problem is the possibility of migration of lubricating oil from one compressor to other leading to compressor break down since you are using, let us say this problem is typically with multiple compressor compressors okay.

When you have multi compressor compression system where the same working fluid flows through both the compressors or the working fluid is getting mixed somewhere. Then there is a possibility is that the lubricating oil from compressor one is getting migrated and its finally settling down in compressor two. That means compressor one will be starved of lubricating oil where compressor two has excess lubricating oil so obviously this will lead to the breakdown of the compressor which doesn't have sufficient lubricating oil okay. So this is the typical problem with multi stage compression systems okay.

So multi stage compression systems theoretically may found good they have certain limitations okay and finally whether you have to go for, multi stage system or a single stage system is decided entirely by the economics okay. It has to be economically viable. That is the reason why even though you have a fit case from thermodynamic point of view to use a multi stage evaporator system in domestic refrigerator you still use the single stage system. Because it is not economically viable to use multi stage system in a small capacity system like domestic refrigerator okay.

So economics is the final deciding factor. So now let us look at another type of refrigerant system which is which you may call it as multi stage system but which uses different types of refrigerants okay this system is known as cascade system okay. So let us look at cascade system.

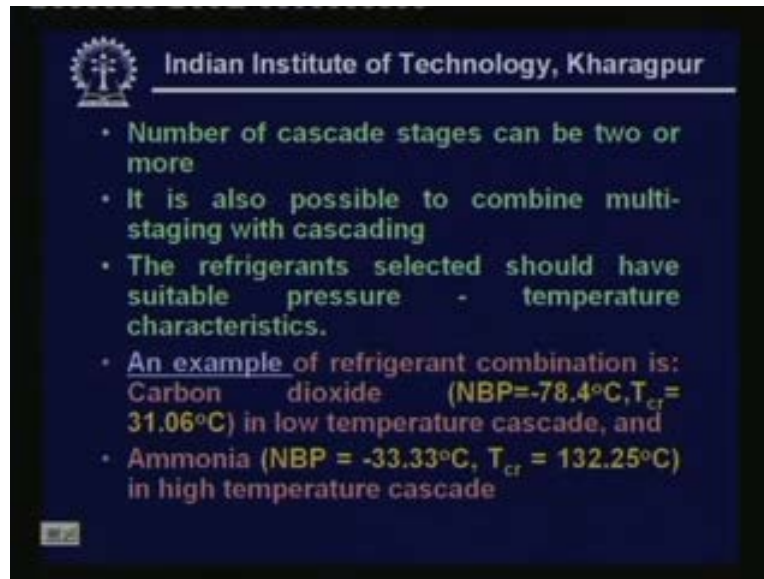
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So what is the cascade system in a cascade system a series of refrigerants with progressively lower boiling points are used in a series of single stage units. That means a cascade system is a combination of many single stage systems okay. So the different single stage system will be using different refrigerants and these refrigerants will not be getting mixed inside the system okay so there is no physical mixing of refrigerants okay. So the only connection between different single stage systems is through thermal contact okay. So that means there will be one of the low temperatures system will be transferring heat to the high temperature system that will be transferring heat to the higher temperature system like that okay. There will be only thermal contact but there is no physical mixing of refrigerants okay. So this is what is known as cascade refrigerant system.

As I said the condenser of lower stage system is coupled to the evaporator of the next higher stage system and so on the component where heat of condensation of lower stage refrigerant is supplied for vaporization of next level refrigerant is called as cascade condensers. So this is a new term that we are introducing here a cascade condenser is one which is the condenser for the low temperature system and at the same time it also acts as evaporator for the next level cascade system okay.

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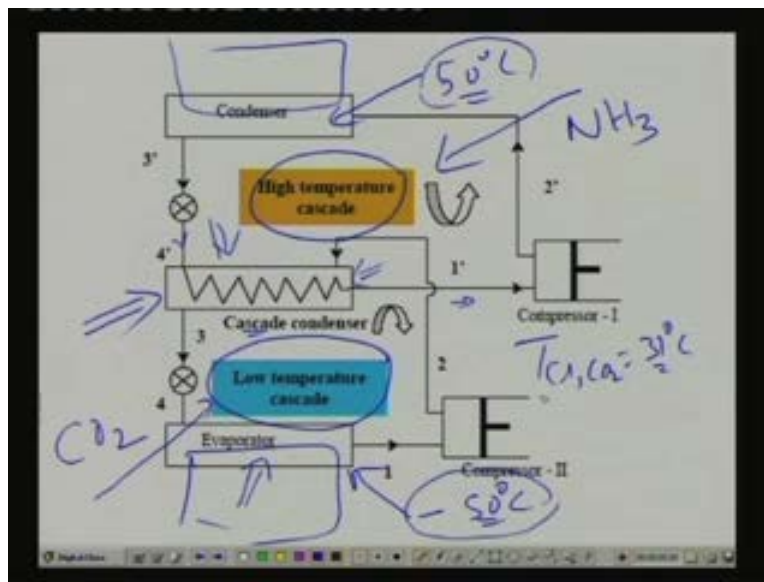


So number of cascade stages can be two or more it is also possible to combine multi staging with cascading. As I said you have lot of combinations possible and the refrigerant selected should have suitable pressure temperature characteristics okay. Since you are it is a nothing but a combination of many single stage systems you have to choose refrigerants for each single stage system properly okay. There must be proper matching between the pressure temperature characteristics of these different refrigerants okay previously people were using R thirteen and R twenty-two as working fluid in a two stage cascade system. Okay. R thirteen is used as a low temperature refrigerant and R twenty-two is used as high temperature refrigerant but R thirteen is banned because of its high ODP okay.

So now a day's people are trying for trying other non ozone depleting refrigerants one typical possibility is use of ammonia and carbon dioxide okay. Let us see the properties of this as I said you can use carbon dioxide in low temperature cascade carbon dioxide has a normal boiling point of minus seventy-eight point four degree centigrade and critical temperature of thirty-one point zero six degree centigrade. So carbon dioxide is good from the low temperature point of view okay because it has very low normal boiling point. So evaporator will be operating at above atmospheric pressures but unfortunately it has very low critical temperature. So you can if you are using carbon dioxide alone then you have to operate the condenser at very low temperatures which may not be possible okay. But if you club this in a cascade then you find that there are lot of advantages what are the advantages.

You find that you can operate the carbon dioxide at low condensing temperatures. Because the condenser of carbon dioxide system will be the evaporator of the next refrigeration system that is nothing but ammonia okay. So you are using carbon dioxide in low temperature cascade and ammonia in high temperature cascade. So you can see the properties ammonia has minus thirty-three point three normal boiling point and critical temperature of one thirty-two point two five degree centigrade okay. So very high critical temperature and reasonably high normal boiling point compare to carbon dioxide. Let me show this cycle the schematic of this.

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Okay. So this shows schematic of a two stage cascade system as I said you have high temperature cascade here and the low temperature cascade. Here as I said an example is in low temperature cascade you can use. For example carbon dioxide and here you can use ammonia okay. So they are not physically getting mixed the only coupling with these two systems is in this cascade condenser. The cascade condenser as I said what is cascade condenser cascade condenser is nothing.

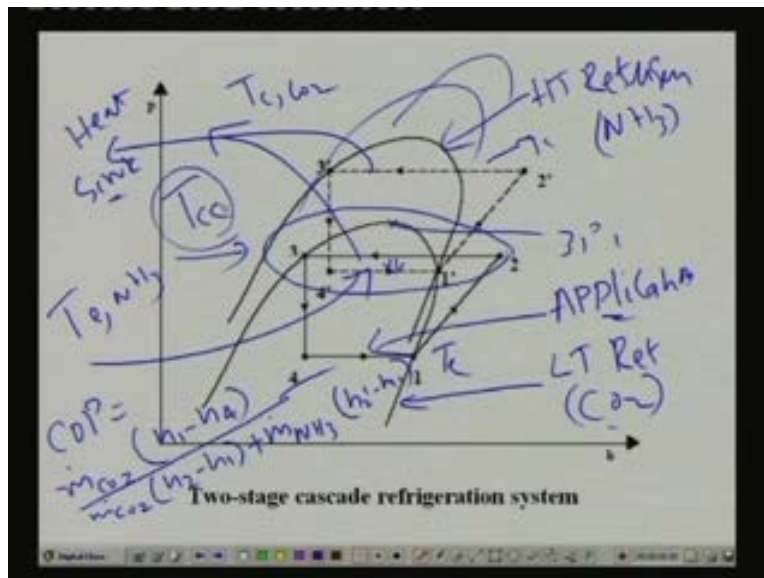
But the condenser of this low temperature system and the evaporator of this high temperature system so both are clubbed together in a single component and that component is called as cascade condenser. So carbon dioxide if i am if you are taking the example of carbon dioxide carbon dioxide evaporates at very low temperature here it is compressed okay. And it condenses

in this cascade condenser and it rejects the heat of condensation to the refrigerant liquid that is ammonia of the high temperature cascade okay.

In this process ammonia picks the heat from the carbon dioxide and it becomes vapour that is compressed and it is again condensed in the condenser and expanded and the cycle continues. So you in principle you can have many stages that means you can have another stage here okay or you can have another stage here you can add go on adding stages you can have multiple cascade systems okay. So because of by using this combination you can achieve a very low temperature here. For example here you may get temperatures as low as minus fifty degree centigrade here at the same time you can operate the condenser at high temperatures may be fifty degree centigrade okay.

That means you have a system where you are extracting heat at minus fifty degree centigrade and you are rejecting heat at plus fifty degree centigrade okay. This would not have been possible with carbon dioxide alone. Because as I said critical temperature of carbon dioxide is thirty-one degrees okay. So you it cannot reject heat to at this temperature okay if you are using the phase change right. So this is an example of a typical cascade two stage cascade system let me show the P h diagram of this.

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Okay, so this is the P h diagram of a two stage cascade refrigeration system this P h this vapour dome corresponds to the high temperature refrigerant okay. So example is for example ammonia

okay and this particular P-h is for the low temperature refrigerant okay that is example is carbon dioxide okay. As I said carbon dioxide has a critical point of about thirty-one degrees right so what you can see that the condensing temperature this is the condensing temperature of carbon dioxide okay. So this is condensing temperature of carbon dioxide okay and this is your evaporating temperature of ammonia okay. In practical system there must be some temperature difference so that heat transfer can take place from carbon dioxide to ammonia okay. And how do you find the total how do you find the COP of the system COP of the system again for this is nothing but what is the useful refrigeration of a useful refrigeration effect this. That means if I am taking the example of carbon dioxide this into $h_1 - h_4$ okay divided by total power input.

Total power input is mass flow rate of carbon dioxide okay into $h_2 - h_1$ plus mass flow rate of ammonia into $h_2 - h_1$ okay. So they are two individual as far as analysis concerned they are just like two individual systems only coupling is happening here.

And you are not getting any refrigeration effect from the ammonia system because that refrigeration effect is used for condensing the carbon dioxide vapour okay. And if you have multiple cascade system you have again P-h diagram increasing in this direction right. So this is the typical example of a two stage cascade refrigeration system and one design option you have is to choose this temperature normally this temperature is fixed by the application okay. What is the required low temperature that is fixed by the application and this is typically fixed by the available heat sink where you are rejecting heat okay.

So that is fixed by your available heat sink and the refrigeration temperature is fixed by your application. So only flexibility you have is in deciding this cascade temperature okay, let me call this is T_c okay so you have a flexibility of choosing this temperature. So that I am being if, I am assuming that here this heat transfer is reversible. That means there is no temperature difference then you have only three temperatures. One is low temperature this thing and the other one is condenser temperature and the other one is cascade temperature T_c . So how do you select the cascade temperature and normally what is the requirement you select the cascade temperature in such a way that that will give you the maximum COP okay. That means you optimize the temperature, so as to get the maximum COP from the system. So how do we get the expression for this optimum temperature there are again different methods okay. If you are assuming that this cycle is operating on a Carnot cycle okay.

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The slide contains the following text and equations:

- Optimum cascade condenser temperature for maximum COP of a two-stage cascade with Carnot cycles:**

$$T_{cc,opt} = \sqrt{T_e \cdot T_c}$$
- Optimum cascade condenser temperature for maximum COP of a two-stage cascade with actual cycles:**

$$T_{cc,opt} = \left(\frac{b_1 + b_2}{\frac{b_2}{T_c} + \frac{b_1}{T_e}} \right)^{\frac{1}{2}}$$
- b1, b2 are constants of Clausius-Clayperon equation**

Handwritten notes on the slide include:

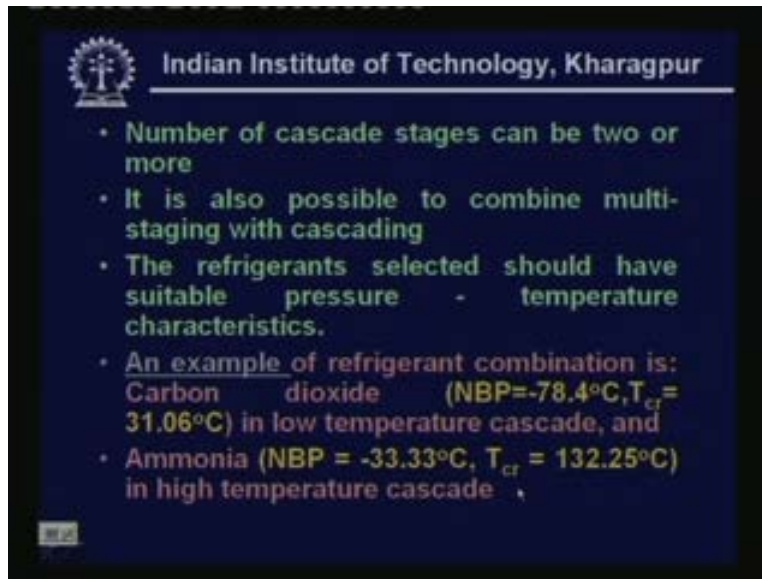
- $\ln P = a - \frac{b}{T}$
- $\Rightarrow C-C$
- $\left\{ \begin{matrix} \text{evap} \\ \text{cond} \end{matrix} \right.$
- Arrows pointing to T_c and T_e in the second equation.
- A handwritten $\frac{1}{2}$ next to the exponent in the second equation.

Okay. So as I mentioned if you are this optimum cascade condenser temperature for maximum COP of a two stage cascade with Carnot cycle okay. If you are talking about Carnot cycle then the expression is nothing but geometric mean of okay, geometric mean of evaporator temperature and condenser temperature okay. So this is the optimum cascade temperature with Carnot cycle okay. But for actual vapour compression refrigeration cycle that is reverse Rankine cycle and all that is it has been shown that the optimum cascade condenser temperature is given by this expression where again T_c and T_e are your condenser and evaporator temperatures and b_1 and b_2 are the expressions of, are nothing but the constants in your Clausius Clayperon equation. That means if you remember the Clausius Clayperon equation is given by like this something like this okay.

So this b corresponds to this b okay. So this is your c equation right and b_1 is for the high temperature refrigerant and b_2 is for the low temperature refrigerant okay. And this expression is based on the assumption that you have the same pressure ratios okay. Pressure ratio for high temperature and low temperature are so under that assumption this equation has been derived okay. So as I said b_1 and b_2 are constants of Clausius Clayperon equation okay.

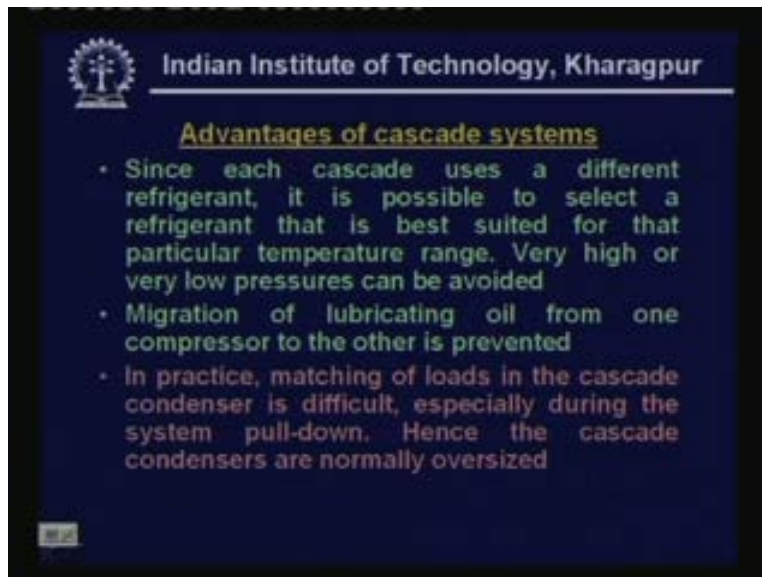
So once you have this equation with you then you can find out b_1 and b_2 and normally T_c and T_e are specified in a given problem. So we can find out what is the optimum cascade condenser temperature then on you can perform the required calculation using your $P-h$ diagram or refrigerant property tables.

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Okay, so this is about cascade refrigeration systems so what are the advantages of cascade systems. Since each cascade uses a different refrigerants.

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It is possible to select a refrigerant that is best suited for that particular temperature range okay. So now you can select the refrigerant suited for that particular temperature range because you are not really operating the same refrigerant across the entire temperature range okay. So you can optimize the selection of refrigerant this is first advantage okay. And you can also avoid very high or very low pressures in the system okay. And the second advantage is you can avoid the

migration of lubricating oil from one compressor to the other how this is avoided. Because there is no physical mixing of refrigerants and each system is operating individually. So the lubricating oil of that particular system stays in that system only okay. So the compressors will be operating properly okay. So you do not have the problem of compressor break down due to lack of lubrication oil okay. Of course this system has some practical problems one practical problem is the matching of loads in the cascade condenser.

What do you mean the matching of loads you might have noticed from your P h diagram in the system schematic that the evaporator duty of the high temperature cascade should be same as the condenser heat rejected by the low temperature cascade okay. They have to match okay this is what is known as load matching you may design the condenser cascaded condenser in such a way that the loads match at designed conditions. But when you are pulling down the system, that means when you are starting the system for the first time what is known as typical pull down period the loads may not match okay. So load matching under all conditions is very difficult. So to overcome this problem what is normally done is the cascade condenser is typically over designed. That means if you are required size is x you take the size as one point five x that will give you fifty percent extra area okay. So that even under different load conditions the load matching can take place okay. So this is one typical problem right. Now what are the applications of cascade refrigerant system cascade refrigerant systems are used in.

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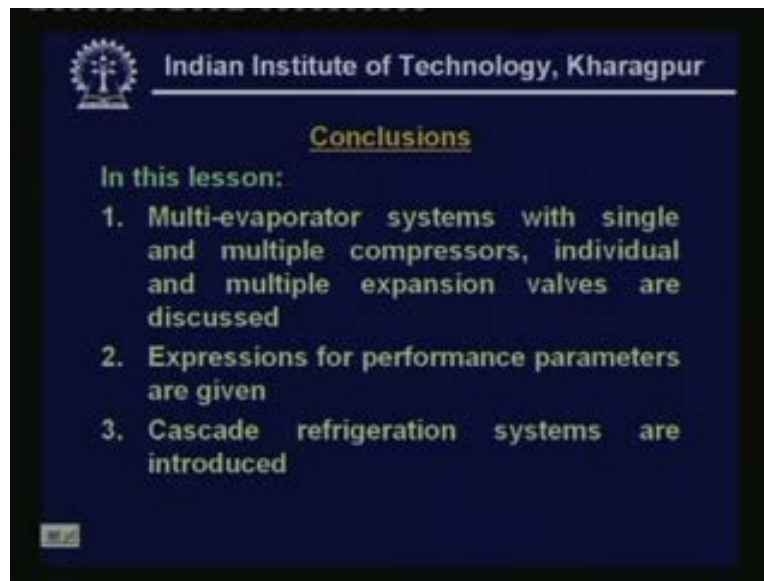


Liquefaction of petroleum vapours very widely used for liquefaction of petroleum vapours and liquefaction of industrial gases. Because liquefaction requires very low temperatures minus one fifty minus one sixty degrees centigrade and all and you have to reject the heat of condensation ultimately to the ambient. That means you may have a temperature lift as high as about two hundred degrees centigrade okay. When you have such a high temperature lift obviously you cannot use the single stage system. So you have to use either multi stage system or a cascade system okay.

If you are using a multi stage system finding a right refrigerant which for sufficiently in this large temperature range is difficult okay. So normally you cannot use multi stage system for this kind of very large temperature lift applications. So only option left is the use of cascade system okay typically a number of cascade stages will be more than two you have three four like that right. So this is the second application and the third application very popular application is in the manufacturing of dry ice okay. So dry ice is nothing but carbon dioxide solid carbon dioxide. So in the manufacture of solid carbon dioxide typically a cascade refrigerant system is used this is very popular application of cascade refrigerant system.

And nowadays cascade refrigeration systems are also being used in food processing application for defreezing okay. For example you want freezing at minus fifty degree centigrade and you want to reject the heat at plus forty degree centigrade okay. Then people use cascade systems and one of the reason advance is to use carbon dioxide ammonia cascades in cold storage applications requiring very low temperatures okay. So this is the fourth application of cascade systems right. So let us conclude what we have learned in this lesson.

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In this lesson we have discussed multi evaporator systems with single and multiple compressors individual and multiple expansion valves. And we have given expression for typical performance parameters and we have also discussed very briefly cascade refrigeration systems okay. The working principle and what are the typical problem and how do we find the optimum cascade condenser temperature etcetera okay. With this we come to we conclude the lectures on vapour compression refrigeration systems okay. And from next class onwards we will discuss vapour absorption refrigeration systems.

Thank you.