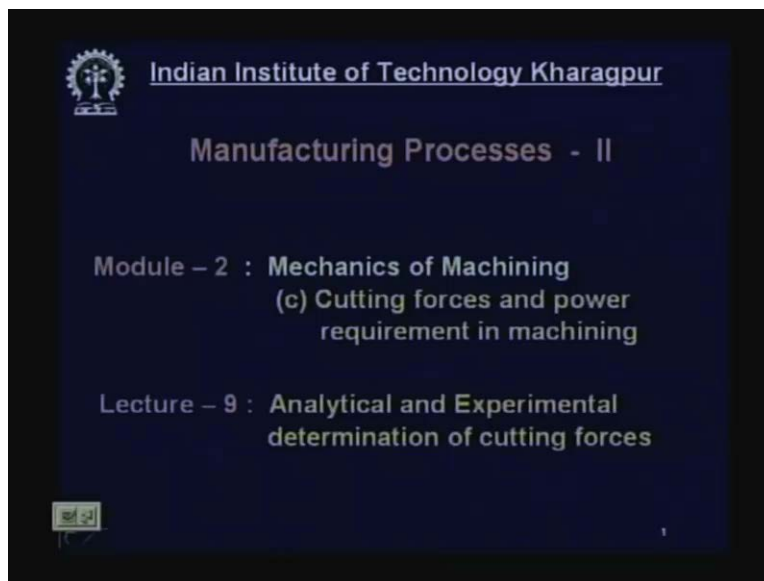


Manufacturing Process II
Professor A.B. Chattopadhyay
Department of Mechanical Engineering
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Lecture No. 9
Analytical and Experimental

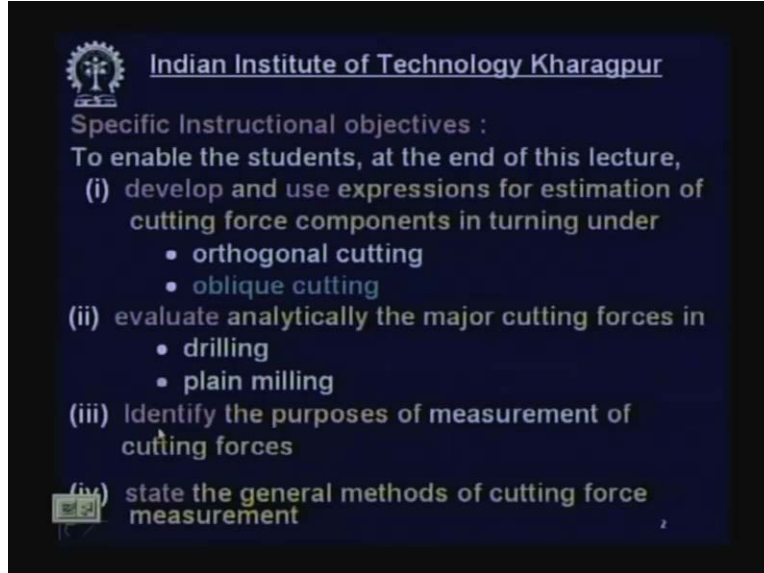
Good afternoon!


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Our subject is Manufacturing Process II and Module II is going on Mechanics of Machining and today our area of discussion is the cutting forces and cutting power requirement in machining and the topic of the lecture - 9 today is analytical and experimental determination of cutting forces. Now what are the content of the lectures?

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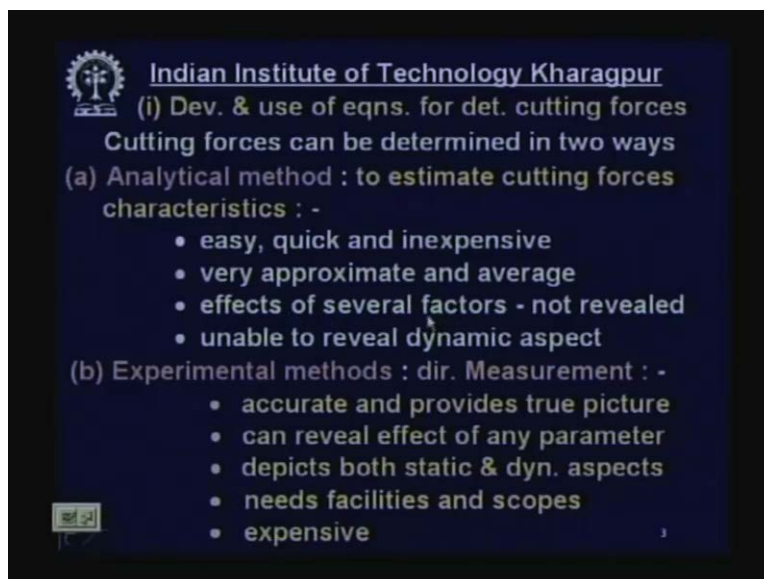
Specific Instructional objectives :


To enable the students, at the end of this lecture,

- (i) develop and use expressions for estimation of cutting force components in turning under
 - orthogonal cutting
 - oblique cutting
- (ii) evaluate analytically the major cutting forces in
 - drilling
 - plain milling
- (iii) Identify the purposes of measurement of cutting forces
- (iv) state the general methods of cutting force measurement

The specific instructional objectives of today's lecture: This will to enable the students at the end of this lecture, (i) develop and use expressions or equations for estimation of cutting force components in turning under orthogonal cutting as well as oblique cutting. (ii) evaluate analytically the major cutting forces in drilling and milling especially plain milling. (iii) to identify the purposes of measurement of cutting force. Next and last (iv) state the general methods of cutting force measurement. So these are the topics to be discussed under this lecture.

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(i) Dev. & use of eqns. for det. cutting forces

Cutting forces can be determined in two ways

(a) Analytical method : to estimate cutting forces characteristics : -

- easy, quick and inexpensive
- very approximate and average
- effects of several factors - not revealed
- unable to reveal dynamic aspect

(b) Experimental methods : dir. Measurement : -

- accurate and provides true picture
- can reveal effect of any parameter
- depicts both static & dyn. aspects
- needs facilities and scopes
- expensive

Now (i) development and use of equations for determining cutting forces being target is to determine the magnitude of the cutting force. Cutting forces can be determined in two ways;

Basically one method is (a) Analytical method: This is to estimate the cutting forces, these are characterized by this process is easy quick and inexpensive, low cost but very approximate and average it does not give the picture in detail. Effects of several factors or parameters are not revealed. In machining there are number of process parameters, environmental parameters, tool parameters but in equations all these parameters cannot be incorporated. There is the limitation and last unable to reveal the dynamic aspect. Every cutting force in machining has got two components static and dynamic. One may be static may be more predominant but the dynamic aspect also need to be studied in cases but by equations it is not possible to get the idea or information about the dynamic characteristics.

The other method is (b) Experimental methods: This is a direct measurement process and is very good. This gives accurate measurement and provides true picture, the distribution of the force location of the force and all these things are revealed and the static and dynamic aspects, this can reveal effect of any parameter. If you can if you change any parameter the effect of that parameter change on cutting force can be easily and immediately determined by experimental method. This depicts both static and dynamic aspects but this experimental method needs facilities like dynamometer charge amplifier, oscilloscope then PC data virtual system and so on and also scopes in industries normally this scope does not exist but in R & D and institutions, we have scope to do experiments and lastly the experimental methods are very expensive. So many people try to avoid.

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Equations for salient forces in turning

- Under orthogonal cutting

- Tangential or main component, P_z

From the MCD

$$P_z = R \cos(\eta - \gamma_o) \quad (1)$$

$$P_s = R \cos(\beta_o + \eta - \gamma_o) \quad (2)$$

From (1) and (2)

$$P_z = \frac{P_s \cos(\eta - \gamma_o)}{\cos(\beta_o + \eta - \gamma_o)} \quad (3)$$

Now the equations for salient forces in turning that we are going to develop equations that are expressions relating the forces and the parameters. First we shall consider under orthogonal cutting. You know orthogonal cutting again I remind you is a process where the chips flow along the orthogonal plane. The chip does not deviate from the orthogonal plane and all the forces are contained within the orthogonal plane where Merchant Circle Diagram is valid. Now let us start with tangential or main component P_z . You know that the P the single point cutting tool turning

tool produces only one force, for convenience of analysis and use it is resolved into three components: P_x , P_y , P_z and P_z is the largest component and most significant. So we shall discuss about first P_z how to estimate P_z analytically without doing experiment. Now here you see the merchant circle diagram. This merchant circle diagram displays all the cutting forces in orthogonal plane, this is the main force P_z which we are going to formulate. P_{xy} acting in the horizontal plane, AF is the friction force at the chip tool surface, N is the force normal to the rake surface P_s is the force acting along the shear plane causing the shear of the chip from the parent body and P_n is another force acting normal to P_s .

So resultant of P_{xy} and P_z or P_s and P_n or F and N is R . So this angle is rake angle and this is the shear angle. So what we are going to do? We are going to develop an equation for determination of or estimation of the force P_z . Now from the merchant circle diagram, we find that this P_z this angle is η . So this angle is γ , so this angle here is η minus γ . Therefore P_z is equal to $R \cos(\eta - \gamma)$. P_z is equal to $R \cos(\eta - \gamma)$. Now P_s this is the force what is the angle here η minus γ plus this angle β so β plus η minus γ . So P_s is equal to $R \cos(\beta + \eta - \gamma)$. Now this second equation now combining these two equations what we get P_z is equal to $P_s \cos(\eta - \gamma) / \cos(\beta + \eta - \gamma)$. What is η ? The friction angle and what is γ ? Orthogonal rake and β is the shear angle in orthogonal plane.

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 P_z in orthogonal turning continued

$$P_z = \frac{P_s \cos(\eta - \gamma_o)}{\cos(\beta_o + \eta - \gamma_o)} \quad (3)$$

where, $P_s = A_s \tau_s$, $A_s = (t s_o) / \sin \beta_o$

$$\text{So, } P_z = \frac{t s_o \tau_s \cos(\eta - \gamma_o)}{\sin \beta_o \cos(\beta_o + \eta - \gamma_o)} \quad (4)$$

Angle relationships (commonly used)

$2\beta_o + \eta - \gamma_o = 90^\circ$ for brittle materials
 $\beta_o + \eta - \gamma_o = 45^\circ$ for ductile materials

Using $2\beta_o + \eta - \gamma_o = 90^\circ$ equation (4) becomes

$$P_z = 2 t s_o \tau_s \cot \beta_o \quad P_z = 2 t s_o \tau_s (\zeta - \tan \gamma_o) \text{ for brittle materials (5)}$$

So what do we get? The P_z in orthogonal cutting is expressed by this equation that is a shear force multiplied by this angle now what is P_s ? The P_s has to be broken P_s is equal to A_s into τ_s what is A_s ? A_s is the shear area, this is the shear area and τ_s is the yield shear strength of the work material under the cutting condition or dynamic yield shear strength. So area multiplied by strength or stress that is the force P_s , what is A_s again? The shear area that we derived previously that is the chip cross section area t depth of cut into feed divided by $\sin \beta$ now

putting this values P_s in this form into this equation three we get P_z tso tau s because P_s is equal to tso tau s and $\sin \beta$ goes at the denominator cosine η minus gamma remains and cosine β plus η minus gamma remains at the bottom. So this expression is very important. This equation (4) that gives a general equation of P_z and this is applicable for any single point cutting tool action. Now angle relationship; Now in industry, the operators of the machines or this low level people who are not that knowledgeable about theory of machining may not understand these values and hence may not be able to utilize this equation for estimation of the cutting force because they have no idea about tau s eta beta and so on.

So what has to be done. It has to simplified how you can simplify. There are relationships commonly used there are two angle relationships which are commonly used there are number of angle relations but two of them are very common. One of them is twice beta plus eta minus gamma is equal to 90 degree. This has been evaluated by Ernst and Merchant from minimum energy principle and this is applicable for brittle materials only. Another one from slip line theory, Leon Shaffer developed this equation beta o plus eta minus gamma is 45 degree and this is applicable for ductile materials. Again I remind you that most of the engineering materials are ductile in nature and even some say pseudo or semi brittle materials may be ductile under the cutting condition.

Now using the first equation for the brittle material, two beta plus eta minus gamma 90 degree in equation (4) if you put it what we get P_z is equal to ts. So this eta minus gamma and beta plus eta minus gamma they will be replaced by this equation then we get P_z is equal to twice depth of cut feet tau s is the dynamic strength of the work material and cot beta were beta is the shear angle. So P_z now cot beta is again zeta minus ten gamma. So P_z is equal to twice tso tau s zeta minus tan gamma. Now this is applicable for brittle materials, but the question is if the work material is very brittle like cast iron grey cast iron then, we do not get continuous chip. It is very difficult to measure the chip thickness and zeta it is very difficult and tau s dynamic initial strength that is also not that applicable. But however in the work material is semi brittle or ductile in cutting condition and produces some continuous chip or semi continuous chip in a small form, we can utilize this equation otherwise it is difficult to utilize this equation for very brittle material like cast iron.

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 P_z in orthogonal turning of ductile materials
 Using, $\beta_o + \eta - \gamma_o = 45^\circ$ in

$$P_z = \frac{t s_o \tau_s \cos(\eta - \gamma_o)}{\sin \beta_o \cos(\beta_o + \eta - \gamma_o)}$$

 We get, $P_z = t s_o \tau_s (\cot \beta_o + 1)$ where, $\cot \beta_o \cong \zeta - \tan \gamma_o$

$$P_z = t s_o \tau_s (\zeta - \tan \gamma_o + 1) \quad (6)$$


 The value of τ_s can be obtained from
 $\tau_s = 0.175 \text{ BHN}$ - for brittle materials
 0.186 BHN - for ductile materials
 or $= 0.74 \sigma_u \epsilon^{0.6\Delta}$ - for ductile materials
 where $\epsilon = \zeta - \tan \gamma_o$ and $\Delta = \% \text{ elong. of the WM}$

Now the continuation; Now for P_z in orthogonal cutting of ductile materials: Now we shall discuss about the ductile material. Most of the materials are ductile. Here this relation has to be utilized beta plus eta minus gamma 45 degree into this expression here eta minus gamma. So beta plus eta minus gamma we replace by 45 degree and eta minus gamma by 45 degree minus beta. So what we get, if you solve it simplify then we get P_z is equal to $t s_o \tau_s$ depth of cut feed that is the chip load multiplied by shear strength and this is a found factor cot beta plus one were cot and then cotangent of beta is equal to zeta minus tan gamma that was done perviously you can see your previous note and previous lectures.

Now putting this value into this expression, we get P_z is equal to $t s_o \tau_s$ zeta minus tan gamma plus one. Now remember this expression is very very important expression. First of all, this equation enables analytical estimation of the most important force P_z in turning under orthogonal cutting this expression. It is very very close to accurate not accurate like experimental but it is a reliable equation. Now the question is, what about τ_s ? This dynamic initial strength how can we determine this? This is not available in the handbook because it depends upon the cutting condition, condition to condition of machining this varies.

The value of τ_s can be obtained from; there are large number of expression or equations available. But these equations are very useful. Number 1. τ_s is equal to 0.175 multiplied by BHN. What is BHN? Brinell hardness number in kg per millimeter square and that kind for brittle material. This is applicable for brittle material and is very approximate 0.186 into Brinells number. This is applicable for ductile materials but this is also not very accurate but τ_s is equal to $0.74 \sigma_u \epsilon^{0.6\Delta}$. These expression developed by **abulogy** is very common and this gives more accurate results and this is applicable only for ductile materials because here we consider cutting strain and percentage elongation, now what is σ_u ? σ_u is a percentage elongation because it is the ductile material. What is epsilon? Cutting strength that is approximately equal to zeta minus tan gamma o where zeta is the chip reduction coefficient a2 by a1. So this is valid for ductile materials.

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• Equations for P_x and P_y in orth. turning

From the MCD,

$$P_{xy} = P_z \tan(\eta - \gamma_o) \quad (7)$$

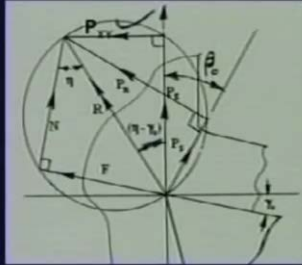
$$P_{xy} = \frac{ts_o \tau_s \sin(\eta - \gamma_o)}{\sin \beta_o \cos(\beta_o + \eta - \gamma_o)} \quad (8)$$

Using $\beta_o + \eta - \gamma_o = 45^\circ$
for ductile metals

$$P_{xy} = ts_o \tau_s (\zeta - \tan \gamma_o - 1) \quad (9)$$

$P_x = P_{xy} \sin \phi$ and
 $P_y = P_{xy} \cos \phi$


We get, $P_x = ts_o \tau_s (\zeta - \tan \gamma_o - 1) \sin \phi \quad (10)$
and $P_y = ts_o \tau_s (\zeta - \tan \gamma_o - 1) \cos \phi \quad (11)$



Now come to the equation. There are other forces P_x and P_y those have to be determined. Now P_x and P_y are not situated in the orthogonal plane or in the merchant circle diagram. But P_x and P_y are the components of P_{xy} . This P_{xy} has got two components P_x , $P_{xy} \sin \phi$ and P_y , $P_{xy} \cos \phi$. These two components have been done earlier. Therefore if we know P_{xy} , then P_x and P_y can be easily determined by multiplying that by $\sin \phi$ and $\cos \phi$. Now come to P_{xy} . From the merchant circle diagram, we find that P_{xy} is equal to P_z multiplied by tangent of eta minus gamma **tangent of eta minus gamma** that is P_{xy} , then what is P_z ? We already determined that was $ts_o \tau_s \cos \eta - \gamma_o$, $\sin \beta_o \cos \beta_o + \eta - \gamma_o$.

So this was $\cos \eta - \gamma_o$ when this is multiplied by tangent of eta minus gamma this becomes \sin . So that is the major difference this becomes \sin , in P_z this is \cos in P_{xy} this is \sin otherwise it remain same. Now again using this angle relationship valid for ductile material we get, P_{xy} is equal to $ts_o \tau_s (\zeta - \tan \gamma_o - 1)$, this expression is very similar to the previous expression of P_z . You see the P_z , this is plus one but in case of P_{xy} this is minus one. So that is the only difference, **this is the only difference line**. So this is minus instead of plus for P_z and for P_{xy} minus. So from here we get P_x P_y and finally this is multiplied by $\sin \phi$ and this is multiplied by $\cos \phi$. So P_x is equal to is given by this expression and P_y by this expression very this part is very similar to P_z on these two signs are minus and these two components are by $\sin \phi$ and $\cos \phi$.

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Friction force, F , normal force N and μ_a in Orthogonal turning

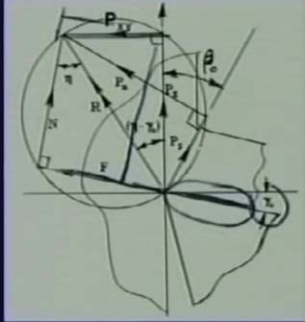
From the MCD,

$$F = P_z \sin \gamma_o + P_{xy} \cos \gamma_o \quad (12)$$

$$N = P_z \cos \gamma_o - P_{xy} \sin \gamma_o \quad (13)$$

$$\mu_a = \frac{F}{N} = \frac{P_z \sin \gamma_o + P_{xy} \cos \gamma_o}{P_z \cos \gamma_o - P_{xy} \sin \gamma_o}$$

or
$$\mu_a = \frac{P_z \tan \gamma_o + P_{xy}}{P_z - P_{xy} \tan \gamma_o} \quad (14)$$



Now the friction force, this is the friction force and this is a force normal to the friction force acting at the chip tool interface. These are acting at the chip tool interface here this friction force and normal force and the ratio between these two is the apparent coefficient of friction at the chip tool interface. This is also important information to be developed for research and other purposes to study the condition etcetera.

Now friction force F and N and μ apparent coefficient of friction in orthogonal turning. So this merchant circle diagram is valid. So from this diagram what we get that F **F is this one**. F is equal to this F and N has to be determined expressed in terms of P_z and P_{xy} why because P_z and P_{xy} are obtained generally by experiment. So if these two values are known and their expressions are already developed also there we know already what is the expression equation for P_z and P_{xy} . Therefore F and N have to be developed in terms of P_z and P_{xy} which have already been derived. F is equal to from this diagram this F is equal to this F is equal to P_z , so if you draw a perpendicular this one this is equal to $P_z \sin \gamma$ and this is equal to this P_{xy} multiplied by $\cos \gamma$ where γ is a rake angle. What is N ? N is along P_z so $P_z \cos \gamma$ minus $P_{xy} \sin \gamma$. So what is μ then apparent coefficient of friction is the ratio of F and N that is the coefficient of friction but apparent.

Now F and N will be taken from equation (12) and (13) so you put it here. Now if we divide the numerator and denominator by $\cos \gamma$ then this becomes $\tan \gamma$ this $\cos \gamma$ goes away, this $\cos \gamma$ goes away. Again this $\sin \gamma$ this becomes $\tan \gamma$. So this expression is very common for apparent coefficient of friction under orthogonal cutting that has to be remembered for which this rake angles are taken as orthogonal rake and P_z and P_{xy} which can be taken only in orthogonal system.

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Equations for P_s , P_n and τ_s in orthogonal turning

From the MCD,

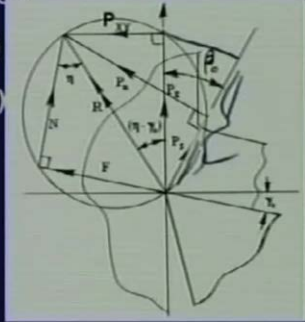
$$P_s = P_z \cos \beta_o - P_{xy} \sin \beta_o \quad (15)$$

$$P_n = P_z \sin \beta_o + P_{xy} \cos \beta_o \quad (16)$$

Again, $P_s = A_s \tau_s$

$$A_s = (t s_o) / \sin \beta_o$$

$$\tau_s = \frac{P_s \sin \beta_o}{t s_o}$$

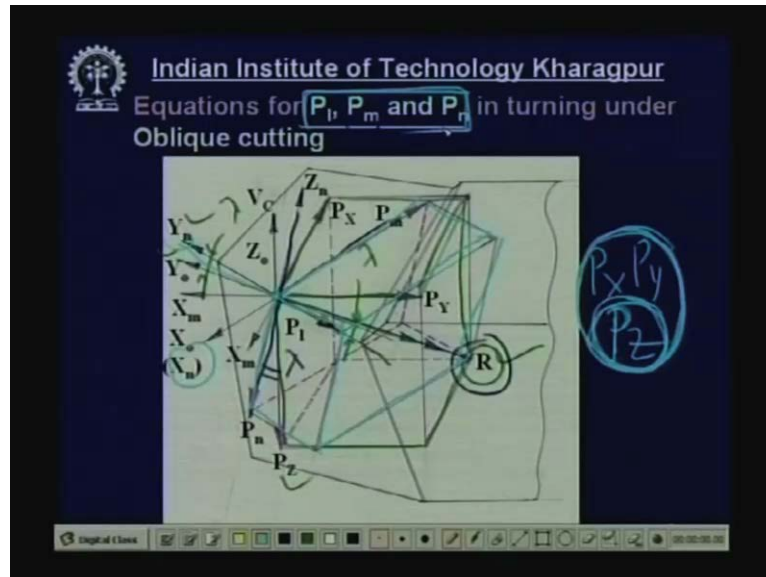
$$= \frac{(P_z \cos \beta_o - P_{xy} \sin \beta_o) \sin \beta_o}{t s_o} \quad (17)$$


Now equation for the shear force P_s is then P_n normal to that and τ_s dynamic initial strength of work material under orthogonal turning. So this has to be developed to study the characteristics or behaviour of the work material sometime we did determination of τ_s . What is the value of the shear strength of the work material under the working condition and how much shear force will be required how much will be P_n etcetera. How can we do that? From the merchant cycle diagram again is orthogonal plane this P_s is equal to R or this P_s is equal to in terms of P_z and P_{xy} because P_z and P_{xy} have already been derived. So P_s is equal to $P_z \cos$ of this angle. **So this much minus this much how much is** $P_{xy} \sin \beta$ then what is P_n this one. This is equal to $P_z \sin \beta$ plus $P_{xy} \cos \beta$. So this plus this, so this equations we get for determining P_s and P_n where P_z and P_{xy} are already derived they have got equations already. So multiply by $\sin \beta \cos \beta$ we get but what about this P_s ? This P_s is equal to **again you remember that** As shears area multiplied by the shear strength of the work material.

Now our target is to determine the value of this τ_s dynamic initial strength of work material under the cutting condition. We already know P_s from this expression in addition to the equations for P_z and P_{xy} . Now from here we also know that A_s shear area is equal to chip load divided by $\sin \beta$ is already derived. So τ_s from this expression we get τ_s is equal to $P_s \sin \beta$ by $t s_o$. Now in this equation we want to know τ_s P_s is replaced by this expression. So $P_z \cos \beta$ minus $P_{xy} \sin \beta$ that divided by $t s_o$ and multiplied by $\sin \beta$ so utilizing these things we can easily determine τ_s P_z and P_{xy} equations are already derived for the material t depth of cut feed are known and shear angle β is also known from ζ chip reduction coefficient or the chip thickness which can be known or which can measured easily. So far we discussed about orthogonal cutting. Orthogonal cutting is very ideal very good it is very easy to understand, realize and analyze utilize very good but in reality in most of the situations the cutting is not orthogonal cutting. It can be oblique cutting because of three reasons that restricted cutting effect and the nose radius effect and the inclination angle λ .

So in most of the cases it can be oblique cutting. If it is oblique cutting then what is oblique cutting? The chip will not flow along the orthogonal plane that is the forces will not remain within the orthogonal plane, so the merchant cycle diagram which is drawn in orthogonal plane is not valid. So orthogonal plane is not valid then in that situation what shall we do?

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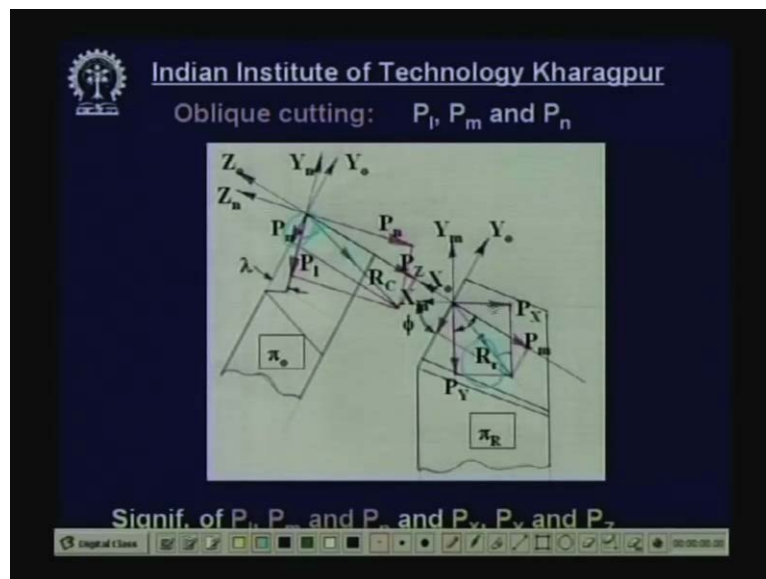
You just look in to his diagram. We are going to develop expressions for P_l , P_m and P_n , I am coming to that what are those forces in turning under oblique cutting. Where from they develop? Where do they act? Basically in turning by a single point tool, there is only one force this force R this from here to here the resultant force. This force can be resolved into P_z , P_x and P_y . So this forms a box like. So R is resolved into P_x , P_y and P_z which are acting along x , y , z directions and x , y directions are the axis of the machine tool. Now if this oblique cutting that is the λ exist this angle this is the ϕ axis and this is the η and this angle is λ that is the cutting edge this is the cutting edge is not situated along the reference plane it is deviated by an angle λ that causes oblique cutting. Now in oblique cutting three forces are assumed P_l , P_m , P_n . P_l will be acting along the cutting edge, this will be acting along the cutting edge but in orthogonal cutting it does not happen.

In orthogonal cutting there is no force along the cutting edge. Another force P_m that is along the orthogonal plane, this component is taken along the orthogonal plane and P_n is taken along not this velocity vector like P_z it is taken along this z_n direction and you understand that this angle is λ . Therefore P_l acts along the cutting edge, P_m acts along the orthogonal plane or reference plane and P_n along the normal plane or z_n axis and P_l , P_m , P_n are right angle to each other. Now the same force R can be resolved into three directions P_l , P_m and P_n you complete the box. So this force R is resolved into P_l , P_m and P_n . Earlier it was resolved into P_x and P_y into P_z . Now what are the significance of P_x , P_y and P_z ? These are forces which can be measured by dynamometer directly. When we measure force, we normally measure P_x , P_y , P_z .

There is one importance. P_z is the most significant force which is largest in magnitude and this decides the magnitude of the power consumption.

If somebody wants to say analyze the stresses thermal mechanical thermomechanical and then optimize according by then by find element methods or bond element technique then for that kind of stress analysis the forces side P_l P_m P_n need to be utilized not P_x P_y P_z . Therefore for these purposes for design of the cutting tool and stress analysis an optimization of the tool geometry P_l P_m P_n will be referred but how to determine this three forces analytically without going into experiment P_x P_l P_z can be determined analytically to some approximately from there it has to be determined P_l P_m P_n . Now these forces are shown again in a more convenient manner in it. The previous one was 3D now we have shown 2D.

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In machining there is only one force acting that is, one cutting force R . Now that cutting force R has been resolved for convenience of understanding in to two components. In this case one is this R_c . This R_c is one component which acts along the cutting plane. Now this is the plane along which it cuts so this is the cutting plane and this is the reference plane. This is the reference plane and this is cutting plane. So this R that is force acting on the tool is resolved in to two components first. One component is acting in the reference plane and another component R_c in the cutting plane. Now this R_c in this cutting plane, which forces are working? This P_n force and P_l acting in the cutting edge and P_n along the Z_n axis and they are right angle to each other. **These are right angle to each other.** Here P_z and P_x also acts in the same plane. We assume another virtual force P_h along Y_o axis therefore this R_c can be resolved into either P_z and P_h or P_n and P_l . So P_n and P_l belong to the P_l P_n group that is oblique cutting and P_z and P_x are the measurement by dynamometer.

Similarly this component R_r another component of the cutting force in taken in the plane this force ϕ this R_r can be resolved into forces in two ways, either into P_x and P_y or in the

direction of the P_m and P_h the resultant is same. So what we get that resultant the P_z and P_h summation is equal to R is equal to so we P_n P_l . Similarly here this P_x plus P_y is equal to R_r which is again equal to P_m and P_h force. Now P_h is a common thing which has to be eliminated later on.

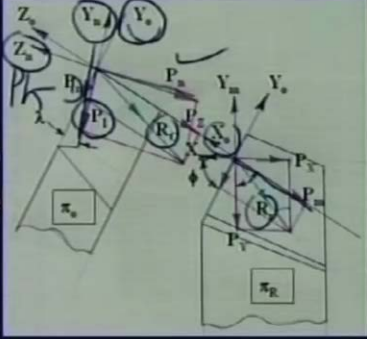
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Equations for P_l , P_m and P_n in oblique cutting

$$\bar{R} = \bar{R}_C + \bar{R}_T$$

where,
 $R_C = P_z + P_h = P_n + P_l$
 and $R_T = P_x + P_y = P_m + P_h$


P_l – along the cut. edge
 (Y_n axis)
 P_m – along π_o (X_n axis)
 P_n – along π_n (Z_n axis)



(contd.)

Now see from the diagram what we get R that is the only cutting force in single point turning is given by two forces R_c and R_r . Now from this diagram this R_c is equal to summation of vector summation of P_z and P_h . This is a P_h along acting along y_o axis that is reference plane Ph . Now this force R can also be resolved into P_n and P_l and this R_r can be resolved into P_x and P_y as I said here or P_m and P_h . Again I remind you that P_l this force P_l acts along the cutting edge that is Y_n axis, P_m along orthogonal plane or X_o axis or X_n axis and P_n is acting in the normal plane and Z_n axis.

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P_l, P_m and $P_n = f(P_x, P_y \text{ and } P_z)$

From the forces in π_C

$$P_n = P_z \cos \lambda - P_h \sin \lambda \quad (18)$$

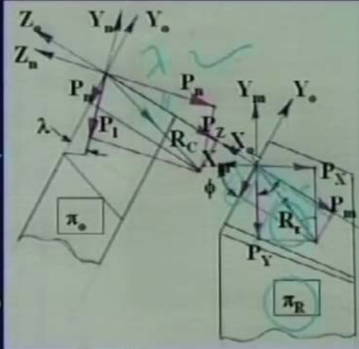
$$P_l = P_z \sin \lambda + P_h \cos \lambda \quad (19)$$

From the forces in π_R

$$P_m = P_x \sin \phi + P_y \cos \phi \quad (20)$$


$$P_h = -P_x \cos \phi + P_y \sin \phi \quad (21)$$

Contd.



Now come to the equations from here this P_n is equal to, it takes component from P_z and P_h . So this angle is λ . So P_n is equal to $P_z \cos \lambda$ minus $P_h \sin \lambda$. Similarly P_l is equal to $P_z \sin \lambda$ and $P_h \cos \lambda$. So these two equations will relate P_n and P_l with P_z and P_h . Now from the forces, these forces come here acting on ϕ . This P_m is equal to this angle is ϕ this is ϕ this is also ϕ . So P_m is equal to $P_x \sin \phi$ plus $P_y \cos \phi$ whereas this P_h is equal to $P_x \cos \phi$ minus $P_y \sin \phi$. This will be minus because this has got opposite component in this direction minus $\cos \phi$ plus $P_y \sin \phi$. You can easily try this one and find out this is true.

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P_l, P_m, P_n – contd.

Combining equations 18 – 21,

$$P_l = -P_x \cos \phi \cos \lambda + P_y \sin \phi \cos \lambda + P_z \sin \lambda \quad (22)$$

$$P_m = P_x \sin \phi + P_y \cos \phi \quad (23)$$

$$P_n = P_x \cos \phi \sin \lambda - P_y \sin \phi \sin \lambda + P_z \cos \lambda \quad (24)$$

or

$$\begin{bmatrix} P_l \\ P_m \\ P_n \end{bmatrix} = \begin{bmatrix} -\cos \phi \cos \lambda & \sin \phi \cos \lambda & \sin \lambda \\ \sin \phi & \cos \phi & 0 \\ \cos \phi \sin \lambda & -\sin \phi \sin \lambda & \cos \lambda \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \quad (25)$$

How do you get about it? Combining equations (18),(21). (18),(19),(20),(21) we get four equations. One P_n P_l another P_m P_h . P_h is there so we have to eliminate this P_h from all these four equations and ultimately express P_n P_l P_m and P_h in terms of P_x P_y P_z that has to be done because P_x P_y P_z have already been derived, equations have been made available. So if this P_l P_m P_n can be correlated with that then theoretically P_l P_m P_n can be derived from the theoretical values of P_x P_y P_z . Now that has to be same.

Now combining those three equation, four combining equations (18),(19),(20),(21) we find out these three expressions. P_l is equal to minus $P_x \cos \phi \cos \lambda$ plus $P_y \sin \phi \cos \lambda$ P_z **none of these** these three equations. Therefore what we learn from this that if we know, P_x analytically or experimentally P_y and P_z we can easily determine P_l . Our ultimate target is to develop analytical model. Analytical models are already developed for equations P_x P_y and P_z . Put those equations here and then multiply it by $\cos \phi \cos \lambda$ as and when required. What is ϕ ? Principle cutting edge angle and λ is the inclination angle of the cutting tool both of which are known.

Similarly P_m it depends upon P_x P_y but no P_z similarly P_n is a function of P_x P_y and P_z . Now these equations can be utilized for analytical estimation of P_l P_m P_n but for convenience of understanding and remembering these three equations can be written in a combined form of a matrix. P_l P_m P_n in the form of a column matrix and its coefficient as it contains or the elements of a square matrix and then P_x P_y P_z in another column matrix that is P_l P_m P_n are function of P_x P_y P_z and connected by a square matrix called transformation matrix and the value of the determinant of this transition matrix will be always one because P_x P_y P_z are normal to each other, P_l P_m P_n are normal to each other and origin of these three force and these three forces are same. So this value of the determinant will be same. If by chance if you want to determine P_x P_y P_z from P_l P_m P_n then you have to take the inversion of this matrix or you can derive it directly from the geometry. This is how you can determine the forces in oblique cutting.

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μ_a under oblique cutting

$$\mu_a = \frac{F'}{N'} = \frac{\cos \rho_c}{\cos \rho_c} \frac{P_n \sin \gamma_n + P_m \cos \gamma_n}{P_n \cos \gamma_n - P_m \sin \gamma_n}$$

$\rho_c \equiv \lambda$

$$\mu_a = \frac{P_n \tan \gamma_n + P_m}{\cos \lambda (P_n - P_m \tan \gamma_n)} \quad (26)$$

Now the question is apparent coefficient of friction under oblique cutting. Now this question comes, in oblique cutting the chip does not flow along the orthogonal plane, it also does not flow along the normal plane; it flows along the different direction. If you remember that is the tool that is the orthogonal plane, the chip is supposed to flow in this direction. If it is a normal cutting say normal plane, oblique cutting because of lambda this is supposed to flow in this direction but chip flows in another direction and you have to make this is the effective rake and this angle is chip flow. Ideal direction is this one, but it flows along this one really. So this angle is called rho c, chip flow deviation angle and the entire friction force etcetera will be taken into consideration under the direction of chip flow. Wherever the chip flows about the rake surface, the friction will take place there and coefficient of friction is also concerned with that. Therefore this mu a apparent coefficient of friction has to be determined from this ratio F N that the friction force along the direction of chip flow divide by N prime the force acting in the normal plane in this direction N prime.

Now what is a prime' is this direction the force in this direction if and this rho c is this angle. So if the force in this direction is a prime' then in that direction force will be F divided by cosine rho c. So this divided by the force acting normal to the surface that is N prime. Now this N prime is acting in the normal plane a prime' and N prime' both are acting in the normal plane. If it is a normal plane so the rake angle has to be considered normal rake always. Now this mu a apparent coefficient of friction is equal to a prime and N prime is a friction force and normal force measured on normal plane divided by cosine rho c. You remember that in case of orthogonal cutting mu a was P z instead of P n, it was P z because there lambda was zero. If lambda is zero then it becomes orthogonal cutting and P n becomes equal to P z P n becomes equal to P xy. So this will be P z sin gamma n plus P xy cosine gamma n in case of orthogonal cutting and here P z cosine gamma o minus P xy sin gamma o and rho c is zero, so this is one. But since it is oblique cutting we have to take this P n and P m in case of P z and P n and gamma n in place of gamma o orthogonal plane because it is oblique cutting and at the top of that this whole thing has to be divided by cosine rho c where rho c is the chip flow deviation angle.

So μ_a ultimately if you divide both numerator and denominator by $\cos \gamma_n$ so this becomes $\tan \gamma_n$. This is eliminated and this becomes $\tan \gamma_n$. So final expression is this μ is equal to $P_n \tan \gamma_n$ plus $P_m \cos \lambda$. Why λ ? because this ρ_c is very close to λ that is called Stabler's rule practically. So we get P_n , the equation of P_n is known equation of P_m is already known in terms of P_x P_y P_z . Put those values here λ is also known. So you can easily get the value of μ_a without going into experimental measurement that is the uniqueness of this thing.

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(ii) Analytical estimat. of drilling and milling forces

(a) Drilling forces, (Shaw & Oxford)

Axial force ($P_x = K_{x1} \cdot H_B \cdot S_0^{0.8} d^{0.8} + K_{x2} \cdot H_B \cdot d^2$) kg

And torque ($T = K_T H_B S_0^{0.8} d^{1.8}$) kg-mm

where $H_B = \text{Brinell Hardness}$

and K_{x1} , K_{x2} and $K_T = \dots$

For steels of $H_B < 250$ and $d_c (d) = 0.18$

$P_x = 0.195 H_B S_0^{0.8} d^{0.8} + 0.0022 H_B d^2$ kg

$T = 0.087 H_B S_0^{0.8} d^{1.8}$ kg-mm

Also may $(T = C_1 d^x S_0^y)$ C_1, C_2, x, y etc are available in Handbook

be used : - $(P = C_2 H^x S^y)$

Now come to other machining process. Say analytical estimation of drilling and milling forces

Analytical models: Many people have attempted and developed many equations some of them are quite accurate, some of them are not very accurate, some of them are very approximate and some of them are not applicable that way or versatile any way. We shall give you only one set of equations developed by Mc Shaw and Oxford, well known scientist in this area who carried out research and then ultimately established two expressions. One for total axial force in drilling and another force torque. You remember that if this be the drill, this is the drill axis. One force developed that is called torque and another force developed in this direction that is called P_x . So the P_x is very large in case of drilling and torque is utilized for determining the power and design.

Now Mc Shaw and Oxford have developed P_x is equal to K_{x1} constant into H_B . H_B means Brinell hardness number of the work material to the power 0.8 d is the diameter of the drill to the power 0.8 plus another factor. This is mainly for this chisel edge because the chisel edge is very small compared to the cutting edges but it cost a lot of axial force P_{xe} . So this part is also added but in case of torque T , this is another constant K_t depends on the work material Brinell hardness number of the work material feed and diameter. Here you see this is one point. It close to two but in case of P_x , it is close to one because here the torque has to be multiplied by the diameter.

HB is Brinell hardness number BHN and what is $Kx1$ $Kx2$ Kt ? These are all constants and it depends upon mostly on the work material and also on the cutting condition. For example for steels say low carbon or medium carbon steel or low alloy steel were the hardness is within 250 say Brinell hardness number and dc by d ratio **what is dc ?** dc means the diameter of the chisel edge and d is the diameter of the drill which is approximately 0.18 it is 0.18 to 0.2 it varies like this. If we assume this then this two equations simplify to this form. These two forms that is 0.195 approximately into Brinell hardness number of the work material feed to the power of 0.8 diameter of the drill 0.8 and 0.0022 HB hardness of the work material, d is a diameter of drill square. So this will be in kg kilogram and torque 0.087 Brinell's number d to the So to the 0.8 that remains same but d to the power 1.8 and this is in kg millimeter.

Now there are many other equations similarly developed. Say for example, another set of equations. The torque is equal to a constant $C1$ d to the power x . So to the power y is like index 0.8. Similarly the force P x is equal to Cx dx prime S to the power y prime. Here this $C1$ $C2$ x y x prime y prime, these are available in machining handbook. So you can utilize this handbook for putting the values here and for a given work material and feed or diameter of the drill. You can estimate approximately the torque and thrust, but this is more approximate than Mc Shaw and Oxford's model.

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(b) Cutting forces in Milling (plain) (Vulf)

$$P_{Tavg} = \frac{C_p B S V_c Z_c 4}{\pi} \text{ kg}$$

$$T = P_{Tavg} \times \frac{D_c}{2} \text{ Kg - mm}$$

$$P_G = P_{Tavg} \times V_c \text{ Kg - m / min}$$

$$= \frac{9.81 \cdot P_{Tavg} \times V_c}{60 \times 1000} \text{ kW}$$

Get X, Y, Z and C_p from Handbook

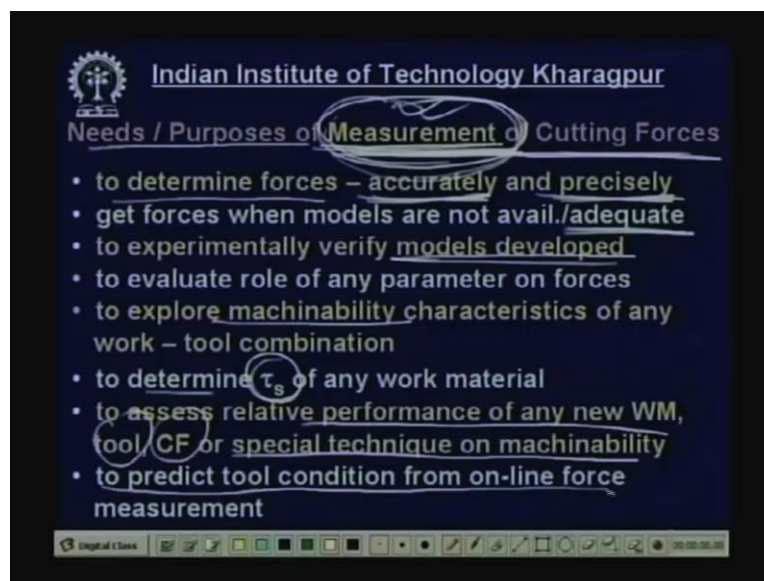
Cutting forces in milling process; Now there are different types of milling process as I told you the form milling or profile sharpen milling then again within form milling, plain milling plain milling is one plain milling end milling phase milling and other millings but our concern is only plain milling which is very similar to slab milling as well as end milling process. Here we are not going into much detail about the geometry or configuration that you can see from book but what my point is here and asking you to remember these equations and show you that this equations are available and one can utilize somebody has developed this expressions which can be utilized

to determine or estimate approximately the value of the average tangential force the torque on the milling cutter or the arbor and P_c is the amount of power consumption.

These equations are PT average. This has been developed by a scientist Vulf and this expression is very novel though it is approximate but it is independent of helix angle, whether it is a helical milling cutter or milling cutter and irrespective of the helix angle, it is also irrespective of the fact that how many cutting edges are involved in machining at a time that is called multi tooth engagement, single tooth engagement. There are various operations but this equation is valid for any situation of course at the cost of accuracy. Accuracy is not that high but this is more universal. What are there in this expression? The average tangential force. So this is the milling cutter suppose and this is the work piece this is the work piece rotating in this direction. So the torque will develop this is the average tangential force PT average. C_p a constant material properties by ϕ , B is the width of the job. S_o is the feed for tooth of the milling cutter and d is the depth of cut and Z_c is the number of teeth in the milling cutters and D_c is the diameter of the milling cutter and this is two to the power z .

So x , y and z these are three indices of S_o feed for tooth depth of cut and diameter of the cutter. T the torque will be this force PT average multiplied by this distance R that is D_c by two and what about power component. The cutting power consumption is equal to PT average multiplied by the cutting velocity. Cutting velocity is equal to ϕDN by 1000 meter per minute. So this comes into kg meter per minute and torque kg millimeter. This is utilized for design purpose. This is for power estimation and here this can be used in this form also to get the value of power consumption in kilowatt. Here the constant C_p , the index x , y , z etcetera can be made available from handbook. So these are all available in handbook or Bhattacharya's book.

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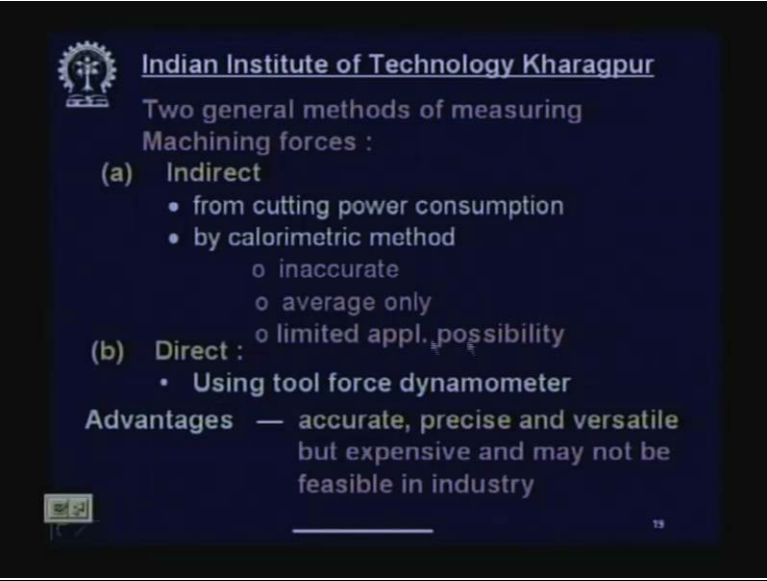
Now comes to measurement. You remember that cutting forces can be determined in two ways. Two means one is analytical method very easy, quick and inexpensive but not very accurate.


Other one is a direct measurement with the help of instruments and this is expensive. This is not feasible in industry always but this can be done in institution, R & D organizations and gives very accurate and reliable results in detail. So the needs and purposes of measurement of cutting force not say analytical method you determine the force by measurement accurately why to determine the forces accurately. So measurement will give you accurate values not say within plus minus 5 percent accuracy but analytical method will give you in the order of plus minus 20 percent and precisely in detail. So what is the location, pulsation, variation with time all these things will be depicted by measurement?

To get forces when models are not available, when equations are available or you can develop you can utilize but if it is not available then experiment is a must measurement has to be done or if the equations are not adequate or proper you do not believe or trust or reliable, then you have to go for measurement by instruments. To experimentally verify models developed suppose this Ernst and Merchant or Oxford and M C Shaw developed number of analytical models but to what extent those models are valid or accurate, how you will know? You have to carry out experiment and by experiment; you get the values and compare. So this way to evaluate role of any parameters on forces like cutting velocity, built up edge friction etcetera to explore a machine ability characteristics of any work tool combination.

Machine ability is an index of judging the quality of machining and machine ability is judged by cutting force etcetera. So cutting forces has to be measured to determine the shear strength of the work material, or the work material if required to access relative performances of any new work material cutting tool cutting fluid or any special technique. If you want to know to what extent they are good or bad compare to the existing systems, then you have to measure the force and compare and also to predicate the tool condition from online force measurement. So if the cutting tool undergoes you know wear and tear damage the cutting force will keep on increasing and increasing, so from the pattern of increase and extended increase you can get the values.

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Two general methods of measuring
Machining forces :

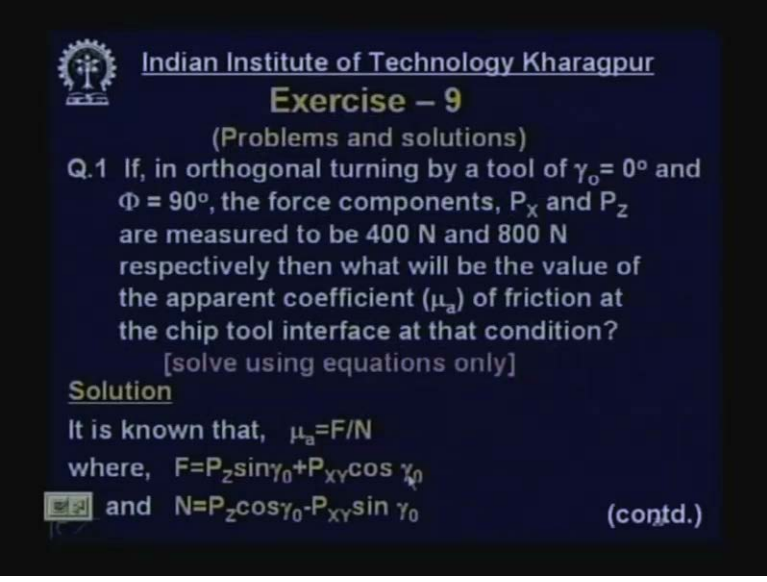
- (a) Indirect
 - from cutting power consumption
 - by calorimetric method
 - inaccurate
 - average only
 - limited appl. possibility
- (b) Direct :
 - Using tool force dynamometer


Advantages — accurate, precise and versatile
but expensive and may not be
feasible in industry

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Last here, two general methods of measurement. These are indirect method from cutting power consumption or by calorimetric method. These methods are inaccurate average only and limited application possibility. But there is another method direct method that is very a good using tool force dynamometer instruments these advantageous accurate precise and versatile but it is very expensive these instruments expensive and may not be feasible in the industry because industry may not afford this kind of facilities or they may not have that much time to do this measurement practice. So these are the basic methods and in the next lecture all these things the measurement we discussed in detail and this dynamometer design application construction we discussed in detail.

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Exercise – 9
 (Problems and solutions)

Q.1 If, in orthogonal turning by a tool of $\gamma_o = 0^\circ$ and $\Phi = 90^\circ$, the force components, P_x and P_z are measured to be 400 N and 800 N respectively then what will be the value of the apparent coefficient (μ_a) of friction at the chip tool interface at that condition?
 [solve using equations only]

Solution
 It is known that, $\mu_a = F/N$
 where, $F = P_z \sin \gamma_o + P_{xy} \cos \gamma_o$
 and $N = P_z \cos \gamma_o - P_{xy} \sin \gamma_o$ (contd.)

Now you will find after that there are exercises given four x problems and the solutions are given. So you can go through that here.

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Solution of Prob. 1 – contd.


Now, $P_{XY} = P_X / \sin \phi = 400 / \sin 90^\circ = 400 \text{ N}$.

$\sin \gamma_0 = \sin 0^\circ = 0$

$\cos \gamma_0 = \cos 0^\circ = 1$.


$\mu_a = P_{XY} / P_Z = 400 / 800 = 0.5$

Ans.



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Q.2 Determine without using MCD, the values of P_s (shear force) and P_n using the following given values associated with a turning operation : $P_z = 1000 \text{ N}$, $P_x = 400 \text{ N}$, $P_y = 200 \text{ N}$, $\gamma_0 = 15^\circ$ and $\zeta = 2.0$

Solution

The known relations are:


$$P_s = P_z \cos \beta_0 - P_{XY} \sin \beta_0$$
$$P_n = P_z \sin \beta_0 + P_{XY} \cos \beta_0$$

Get β_0 (shear angle) from

$$\tan \beta_0 = \cos \gamma_0 / (\zeta - \sin \gamma_0)$$
$$= \cos 15^\circ / (2.0 - \sin 15^\circ) = 0.554 \quad (\text{contd.})$$


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Solution of Problem 2 continued

$\therefore \beta_0 = 29^\circ; \cos \beta_0 = 0.875$
and $\sin \beta_0 = 0.485$

$P_{XY} = \sqrt{P_X^2 + P_Y^2} = \sqrt{(400)^2 + (200)^2} = 445\text{N.}$


So, $P_s = 1000 \times 0.875 - 445 \times 0.485 = 659\text{N}$
and $P_n = 1000 \times 0.485 + 445 \times 0.875 = 874\text{N}$

Ans.



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Q. 3 During turning a steel rod of diameter 150 mm by a carbide tool of geometry; $0^\circ, -12^\circ, 8^\circ, 6^\circ, 15^\circ, 60^\circ, 0$ (mm) at speed 560 rpm, feed 0.32 mm/rev. and depth of cut 4.0 mm the followings were observed :


$P_z = 1000$ N, $P_y = 200$ N, $a_2 = 0.8$ mm

Determine, without using MCD, the expected values of F , N , μ , P_s , P_n , τ_s , cutting power and specific energy requirement for the above mentioned machining operation.

Solution followsgt




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Solution of Prob. 3

- $P_{XY} = P_X / \sin \Phi = 200 / \cos 60^\circ = 400 \text{ N}$
- $F = P_Z \sin \gamma_o + P_{XY} \cos \gamma_o$;
Here $\gamma_o = -12^\circ \therefore \sin \gamma_o = -0.208$ & $\cos \gamma_o = 0.978$
 $\therefore F = 1000(-0.208) + 400(0.978) = 600 \text{ N}$ ans.
- and $N = P_Z \cos \gamma_o - P_{XY} \sin \gamma_o$
 $= 1000(0.978) - 400(-0.208)$
 $= 1060 \text{ N}$ answer
- So, $\mu_a = F/N = 600/1060 = 0.566$ answer
- $P_S = P_Z \cos \beta_o - P_{XY} \cos \beta_o$
where $\beta_o = \tan^{-1}(\cos \gamma_o / (\xi - \sin \gamma_o))$

Contd.
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Here, $\zeta = a_z / (s_o \sin \Phi) = 0.8 / (0.32 \times \sin 60^\circ) = 2.88$
 $\therefore \beta_o = \tan^{-1}\{(0.978 / (2.88 + 0.208))\} = 17.6^\circ$
So, $P_S = 1000 \times \cos(17.6^\circ) - 400 \times \sin(17.6^\circ)$
 $= 832 \text{ N}$ answer
and $P_N = 1000 \sin(17.6^\circ) + 400 \cos(17.6^\circ)$
 $= 683 \text{ N}$ answer


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Solution of Prob. 3 - continued

- $P_s = (t_s \tau_s) / \sin \beta_o$
 $\therefore \tau_s = P_s \sin \beta_o / (t_s) = 832 \sin(17.6^\circ) / (4 \times 0.32)$
 $= 200 \text{ N/mm}^2$ answer
- Cutting power, $P_c = P_z \cdot V_c$
where $V_c = \pi D N / 1000 = \pi \times 150 \times 560 / 1000$
 $= 263 \text{ m/min}$
 $\therefore P_c = 1000 \times 263 \text{ N-m/min} = 4.33 \text{ KW}$ answer
- Specific energy consumption, E_c
 $E_c = \text{power/MRR} = (P_z \cdot V_c) / (V_c \cdot s_o \cdot t) \text{ N-m/m-mm}^2$
 $= 1000 \times 263 \text{ (J/min)} / \{263 \times 0.32 \times 4 \times 1000 \text{ (mm}^3/\text{min)}\}$
 $= 0.78 \text{ Joules/mm}^3$ answer



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Solution of Problem 4 continued


where, $\zeta = a_2/a_1 = a_2/\sin \phi = 1.0 / (0.36 \times \sin 75^\circ)$
 $= 2.87$

$\therefore \beta_o = \tan^{-1} \{ \cos(-10^\circ) / (2.87 - \sin(-10^\circ)) \} = 17.9^\circ$

\therefore Shear strength, $\tau_s = 0.186 \text{ BHN}$
 $= 0.186 \times 240 \times 9.81 \text{ N/mm}^2$
 $= 424 \text{ N/mm}^2$

So, $P_z = 2 \times 5 \times 0.36 \times 424 \times \cot(17.9^\circ)$
 $= 4697 \text{ N.}$ Ans.

— End —



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So this is the end of this lecture.

Thank you.