Vibrations of Structures

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Lecture No. # 15

Damping in Structures

When you consider a structure, a real life structure, and if you give it some certain disturbance or some initial conditions, so, what you observe is the structure response to this disturbance, to this initial condition and slowly after some time the structure stops vibrating. Now, in our discussions on initial conditions in this course, and in the animations that I have shown, once given the disturbance of the initial condition, you have seen that the structure is continuously vibrating, but that is not observed in nature. We conclude that there is a mechanism inside these structures or because of the interaction of the structure with the external world is a mechanism which drains out the energy of the structure. So, this mechanism of dissipation of mechanical energy is known as damping; and in this lecture, we are going to look at this mechanism and we are going to model, put these damping terms in our models, so that our model looks more realistic.

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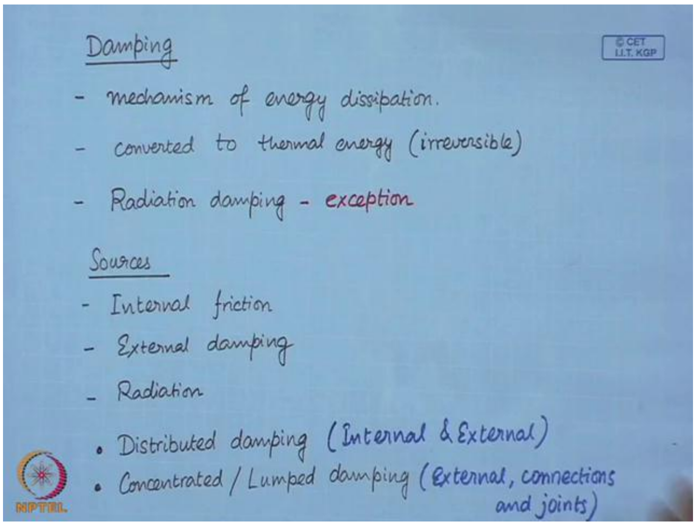

Now, there are various kinds of damping. So first, what I have said this damping is the mechanism of energy dissipation. Now what happens to this energy that is dissipated? It is just converted to thermal energy and this is an irreversible process. This conversion of the mechanical energy to the thermal energy in these cases is an irreversible process. Now, most of the damping mechanism, they work on this. But there is an exception which is known as radiation damping. So here, this is an exception. This damping mechanism, this draining of energy of structure by radiation damping, takes place not by the conversion to the thermal energy, but by radiation of the energy in form of sound in the fluid medium in which the structure is placed, for example air or water or any other liquid or sometimes even from the support points of the structures, where ever the structure is supported; no support is ideal, that means no support is rigid. There will be some flexibility and continuously the energy will get radiated out from the support points. So, we have primarily this energy dissipation by conversion to thermal energy; but there can be another mechanism in which it might also be converted or radiated out from the structure. Now, what are the sources of damping? Now, first is known as the internal friction; internal damping or internal friction. So, whenever the structure is vibrating, between the layers of the structures, molecular layers, because of differential straining, these layers have differential motion and that dissipates energy. Now, one thing to note is that we are considering one dimensional structures. Then there is no question of any layers in one dimensional structures. But then we are going to have a phenomenological model based on the rates of stretching of the structure, so the strain rate, to model this internal friction or internal dissipation. The second source of dissipation is external damping. The structure might be interacting with external fluid. So that provides damping. The third is of course the radiation of energy; that is radiation damping. Now, we have, we can have distributed damping, or concentrated or lumped damping. Now, these internal or external damping because of fluid, so internal dissipation or external damping because of fluid, they are distributed damping, hile concentrated or lumped damping can occur in external damping, when we attach a lumped damper or a dashpot at a particular point. So, this is typically occurs when we have stock-bridge dampers in high-tensioned cables. So, that is a concentrated or lumped damping, which is an external damping; and it can happen at connections and joints. So, we can have concentrated or lumped damping at connections and joints, say for example at riveted connections. So, we have these two kinds of damping. Let us begin with distributed damping. So, let us try to model this internal distributed damping first. Here, we are going to discuss a phenomenological model for internal damping. So, for example in a bar in axial vibration; now when we derive the equation of motion of such a bar, we consider this constitutive relation like this. Now, I mentioned that in order to model the internal damping for one dimensional systems, we must somehow consider the rate at which its straining is taking place. So, we modify our constitutive relation to include the strain rate. Now, this term, this factor is known as a loss factor and the product of these two terms may be represented as d_I, the coefficient of internal damping. So, what we have done is we have introduced in our constitutive relation this additional term which we will see how this works as internal damping. So, therefore, our stress is given by this. Now, when we use this to derive the equation of motion as we have done before, we consider uniform cross-sectional area. So, this term you see is E times del u/del x; this gives this term. Therefore this term will give us… and along with the boundary conditions. So, this is the damping term. So, we have introduced, we have obtained this term by introducing additional strain rate term in our constitutive relation. Similarly with external damping, external distributed damping, we have the term like this; and together with internal and external damping we have this equation of damped vibration of a uniform bar.

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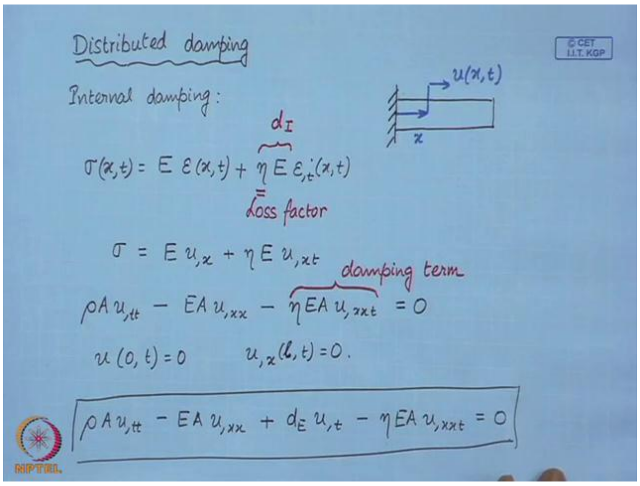

Now, this can be, this equation may be written in a compact form like this, where this damping operator… This is known as the… just like this K is known as the stiffness operator. Now, let us first see these terms, they indeed lead to dissipation of energy; because as yet we have just introduced these terms, but we have not yet checked that they really lead to the dissipation of energy. So, let us start with this equation of motion; and as is the usual way to obtain the energy equation, we multiply this whole equation with the velocity and integrate it over the domain of the bar; so which means what we do is… and this must be zero. So, multiply by velocity and integrate over the domain of the bar. Now, these two terms I will integrate by parts with respect to the space. So, what I obtain… and this term I can write… as can be easily checked. So, this first term can be written like this and this term… Now, there will a space derivative of this term and similarly… Now, if you use the boundary conditions that we have, then these boundary terms, they drop out, because at x equal to zero the velocity is zero; at x equal to l, as it is force free condition, so this is zero. Similarly, this is also zero. So, what we are left with, is this integral. Here, again I can write this term. So, see I can write this also as time derivative, the time derivative; I have taken it out. I will take the other terms to the right hand side. So, you see on the right hand side, we have an integral which is, this integral is always positive provided d_E is positive. So, if this integral is, if this d_E is positive and eta is positive, then this integral is positive. Here there is a negative sign. So, the right hand side is always negative. So, which means this is always less than zero provided d_E is greater than zero and eta is also greater than zero. Now, this is nothing, but the total mechanical energy of the bar. So, this is kinetic energy, this is potential energy, you have already seen these terms. So, the total the rate of change of energy is always negative, which means energy always dissipates or drains out of the systems. So, the mechanical energy is always dissipated by terms, by this damping term that we have considered, so these two terms. So, this shows that, this shows how this energy dissipates with time.

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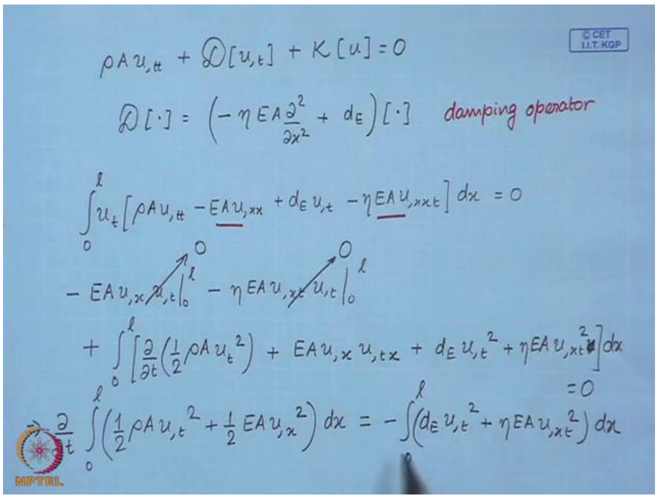

Now, if you integrate… so suppose you have forcing. So in that case, this energy equation I will write as dE/dt… and suppose in addition we have forcing. So, this additional term would come because of forcing. Now, if you… suppose this forcing is something like this, harmonic forcing that you have considered. So, we know already, we have discussed this, that there is a steady state solution and because of this damping now as we have seen here, all the energy from the initial disturbance; suppose, if you have an initial disturbance all the energy must dissipate or drain out of the system. So, what we are left with is the steady state solution. So, any initial condition, any disturbance created from the initial conditions must dissipate out as we know from here. So, if we have the harmonic forcing for example, what will remain at sufficiently large times is the particular solution which is generated only because of this external forcing. So, you can estimate, so from here you can estimate what happens at steady state. So, at steady state, the motion is periodic. Therefore, if I integrate this energy equation over one period, the change of energy must be zero. So, therefore I can write… Here, I am assuming that this forcing is periodic or harmonic. Now at steady state, if there is a steady state solution then at steady state energy change over one period must be zero. Therefore, this is what we have obtained. What this says is the energy provided by the forcing over one cycle is equal to the energy dissipated by the damping term over one cycle.

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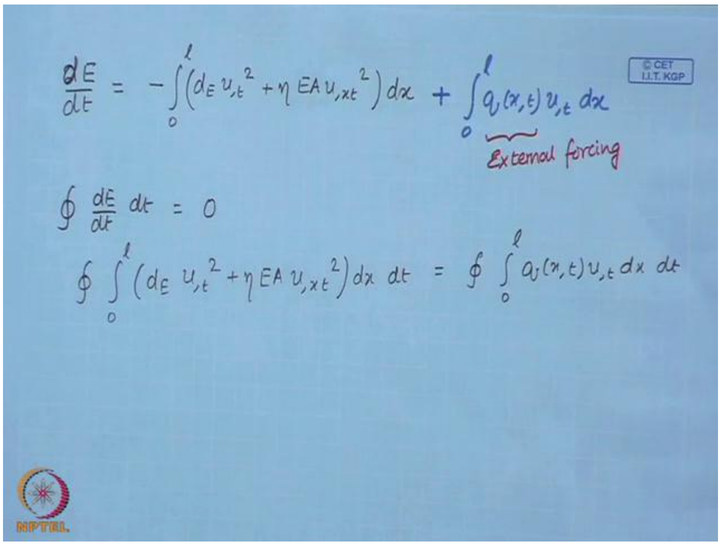

So, from here you can estimate many things. For example, you can estimate the damping. Suppose you give a harmonic input and you record the amount of energy that you are supplying over one period, then you know how much energy is dissipated. So, you calculate this term and that will give you an estimate of energy dissipated by the damping mechanism in the system. So, what we have concluded that any initial condition or any initial disturbance must die out due to this damping terms and what remains is the steady state solution and that can be used to estimate the energy that is dissipated and that can be used to model the internal dissipation as well as, you can estimate the amplitude of motion in certain cases.

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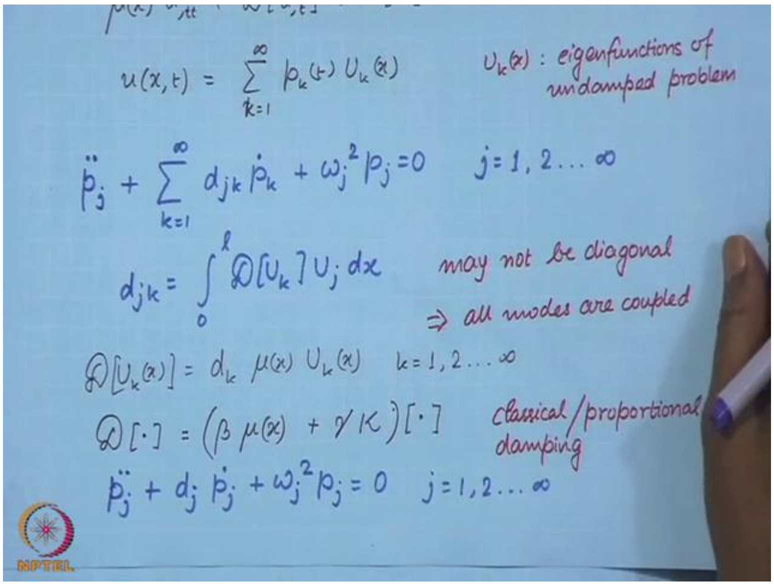

Now, let us once again look at this damped system with certain boundary conditions; so this along with certain boundary conditions. Let us solve, try to find out the solution or let us say let us discretize the system as we have done before using the modal expansion. So, these are the Eigen functions of the undamped Eigen value problem. So, when you substitute in here and finally take inner product with j^{th} Eigen function, what will be obtained… These steps we have done many times before; sp I have directly written the coefficient of the j^{th} term in the solution. So, I can carry this out for all j and get all the equations governing the modal coordinates. Here, this d_{jk} this matrix is given by this integral. Now, in general there is no guaranty that d_{jk} is diagonal. If this is not diagonal, then all the modes are coupled; so all the modes are coupled through these damping terms. Now, under very special situations, this damping matrix, this will be a completely diagonal matrix. So, let us look at that condition. So, that condition follows very easily from here. If $D[U_k]$ is some d_k times mu x times U_k, so all the Eigen functions, operators operating on Eigen functions give this, or in other words Uk is also an Eigen function of the operator D. the damping operator, then the damping matrix is completely diagonal as you can see from here. So, one choice for this damping operator for which this happens is when damping operator is a linear combination of the inertia operator and the stiffness operator; such damping operator is called the classical or proportional damping operator. So, classical or proportional damping, we say that the system is classically or proportionally damped when the damping operator is a linear combination of the inertia and stiffness operators; so in that case, our equations completely decoupled and… So, we have completely decoupled system of differential equations when the damping operator is classical or proportional damping.

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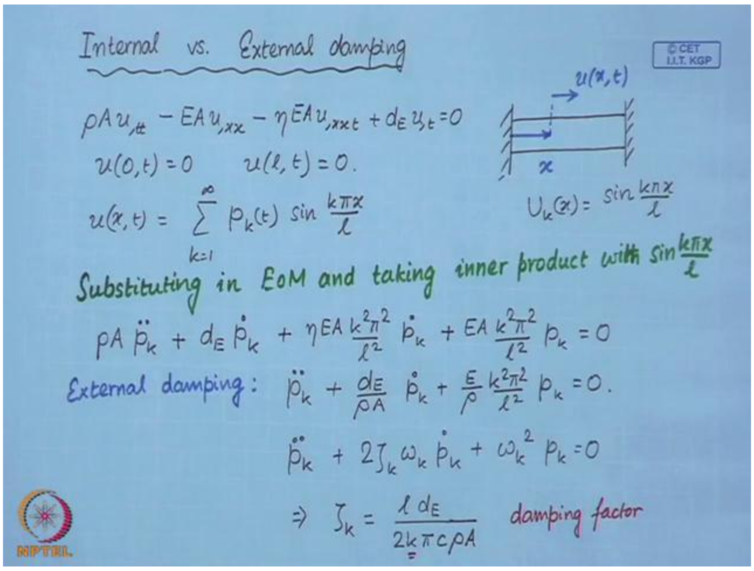

Now so, we have looked at internal and external damping. Now, let us delineate the roles of these internal and external damping terms. So, let us see how they are actually little different. So, for this let us consider, we are going to look at the difference in roles of this internal and external damping. So, to understand this let us consider this simple example of a fixed-fixed bar in axial vibration. So, the equation of motion is this; the boundary conditions are given by this. So, therefore, I can expand as you know that these are the Eigen functions of this fixed-fixed bar; so when you substitute this expansion and take inner product… So, we substitute this expansion in the solution and take the inner product with sine k pi x by l and that filters out the k^{th} coefficient in the expansion. So, what we are going to get… So, this term is for the external damping; this term is for the internal; this is external and this is internal damping. So this is what we are going to get. So, let us look at this equation one by one. Suppose there is no internal damping. So, when we have only internal damping, I can write this as… which can be written as… and if you compare then zeta k can be written as this. So, you see that this k appears in the denominator of the damping factor, so which means for the high values of k this damping factor is very low, which tells us the external damping is inefficient in damping out the higher modes. So, the higher modes will be damped less compared to the lower modes. We will look at the reasons for this.

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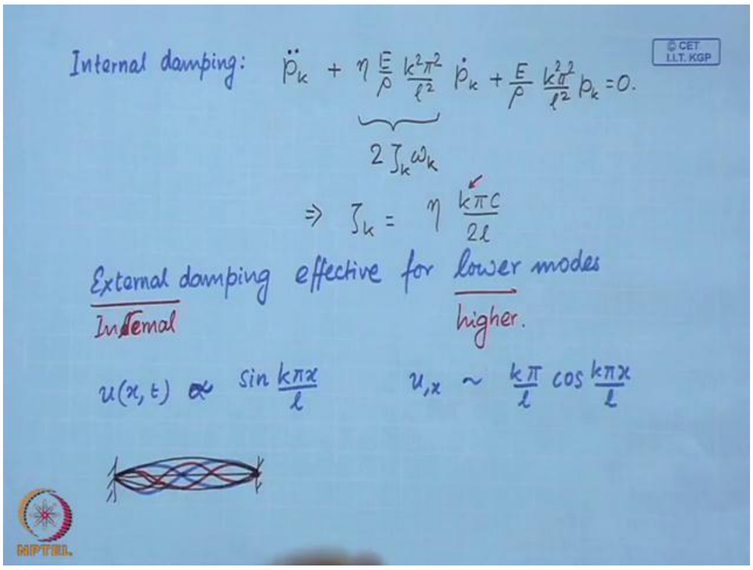

Now, let us consider the internal damping case. So, in that case, I can write… and once again, if you represent this term as two zeta k omega k, then… Now, you see that k appears in the numerator. Therefore, for higher modes higher values of k, zeta k is high, goes higher; so which means the higher modes have better damping for internal dissipation. So, internal damping is more effective for higher modes, while the external damping is more effective for lower modes. So, external damping is effective for lower modes, while internal damping is effective for higher modes. Now, to understand this, you have to look at the expression of say for example the internal damping. So, we have seen already that the way we have introduced, so it is strain rate, dependent on strain rate. Now, as you go to higher modes and if you consider the solution, then del u/ del x, so rate of change of del u/ del x, rate of change of del u/ del x with time; so if you see this solution is, say for example for this bar, so you have terms in the expansion of u like this. So, del u/ del x… So, higher the value of k, higher is the contribution from this strain term, the strain rate term. So for this reason, higher the mode, higher will be effectiveness of the internal damping. On the other hand, if you consider this external damping, if you look at, say for example, the first mode; so the first mode of the bar or the string looks like this; the second mode looks like this; the third mode looks like this. So, higher the mode, higher will be the number of nodes. So, the effective motion of the system in the ambient fluid will be lesser and lesser. So, the effective damping, because of the external damping reduces as you go to higher modes.

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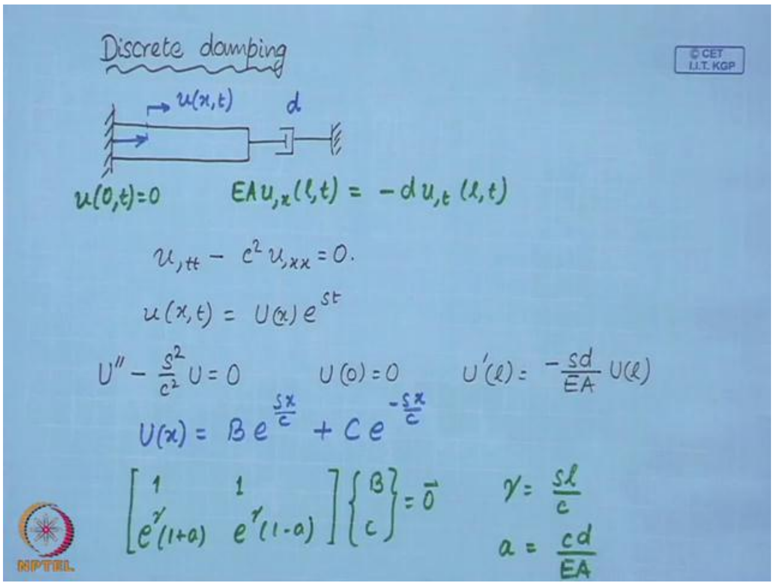

Now, let us look at discrete damping. Now, as I have already mentioned that this discrete damping occurs when we attach, for example an external dashpot to a continuous system at particular points; say for example, a stock-bridge damper is typically used in high tension cables. So, that is attached to a particular point on the cable and that damps the vibrational energy of high tension cables. So, this is a discrete damping or lumped damping. Other than that you can have lumped damping, when you have connections or joins; say for example, you have riveted connection between two structural elements; in that case damping is localized at the connections or at the joints. So, for such cases what you have is the discrete damping. So, let us look at an example. So, this is a bar in axial motion and here we have a dashpot attached to this end of the bar. So, I can write the boundary conditions of this problem; so this is a fixed end; and at this end I have the force because of this damper. The equation of motion is like this. Let us search a solution of the form like this. So, we have discrete damper at this end and we have introduced this damping not in the equation, but in the boundary condition, which we can always do. Now, once you have… once you substitute this solution form in the equation of motion, we obtain an Eigen value problem; we obtain this Eigen value problem; and the solution of this Eigen value problem can be written in this form; and when we use the boundary conditions; so these two boundary conditions can be written in this form, where gamma is s times l over c and this a… Here this small c is of course the speed of axial wave in the bar.

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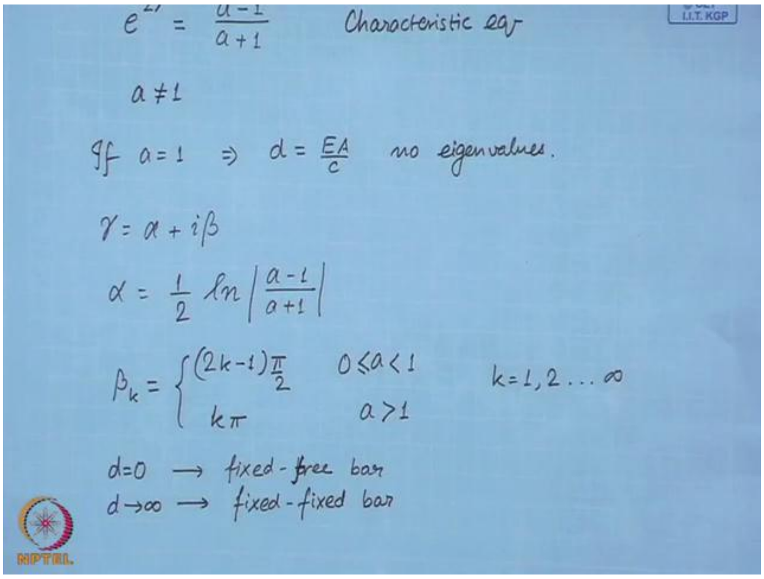

So, here we have to solve this system; the solutions, for non-trivial solutions for B and C, so for non-trivial solutions of B and C, the determinant of this matrix must be zero, which leads us to… as you can very easily find out that this is the equation, which is the characteristic equation, which can be solved for gamma when a is not equal to one. When a is equal to one, there is; so this can be solved when a is not equal to one. When a is equal to one, which implies that the damping, the adjustable parameter of the damping, there are no Eigen values; there are no Eigen values for this problem when the damping is tuned to this value. No Eigen values would imply that there are no solutions; but then we have considered solutions of this very special form. So, there are no solutions of this very special form; but then there are solutions which we have to understand in terms of wave propagation; now, that we will keep for later discussion. Right now let us solve for a equals to one; and for solving that, let us take gamma to be complex number of this form, alpha plus i beta. So, alpha can be easily solved as log natural of… and beta is indexed now, the phase… So, there are infinitely many values of beta, which are now indexed and for various ranges of values of a, we have these solution; and you can check;

if you put d equal to zero, then we can get the Eigen values of a fixed-free bar; and if you put d as infinity then it is for fixed-fixed bar. That can be checked from these solutions of gamma that we have obtained.

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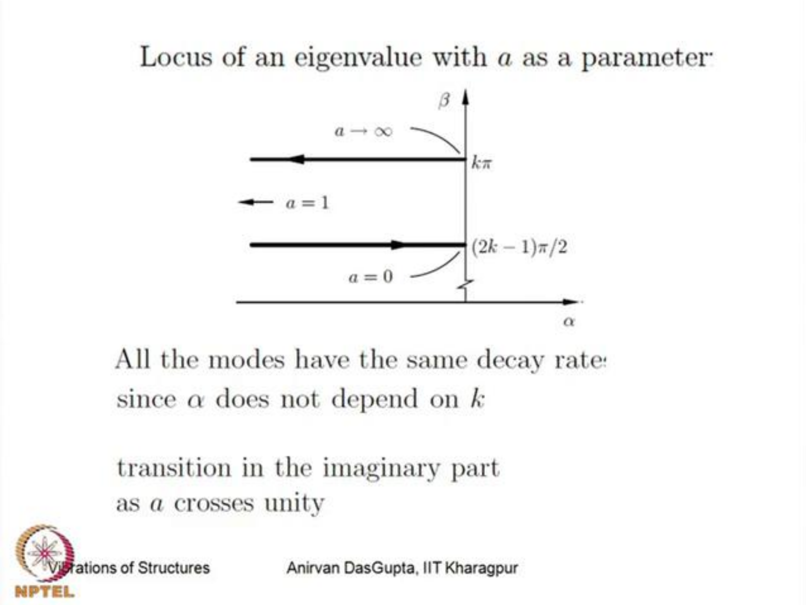

Now here, in this slide, I have plotted the variation of alpha and beta, so the solution of gamma, the real part and imaginary part, as a varies. So, when a is zero, we have a solution here. So, a will be zero, when damping is zero, which means it is a fixed-free bar, while when a is infinity, d is infinity; so it is a fixed-fixed bar; and when a goes to one, this goes to infinity, so this is the locus of the solution a s a varies. So, as you can see, all modes have same decay rate as alpha does not change or does not depend on k; so all the modes will have same decay rate; and there is a jump from this value to this value as a crosses one.

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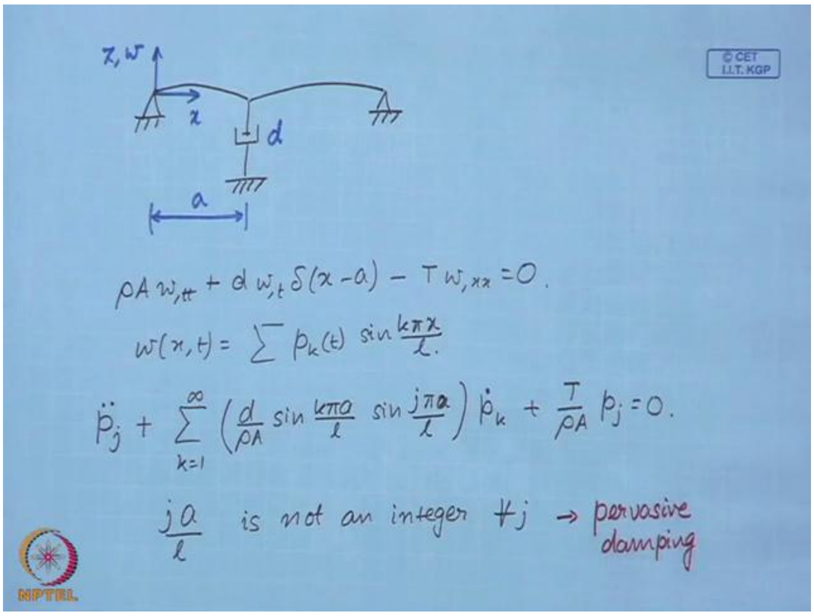

So, we have considered one case of a bar with boundary damping. Let us look at another example of a string with a concentrated damper, a lumped damper at x equal to a. So, in this case, you can write down the equation in this form and if you use the modal expansion, substitute in the equation and take inner product with the j^{th} Eigen function, you obtain this. Now, if you consider that this j times a over l is not an integer, so if you can adjust this a such that this is not an integer for all j, then you will find that all modes are damped, whereas, if this does not happen, then some of the modes will not be damped. So, for this situation when this is not an integer, what we have is known as pervasive damping. So, when the damping is pervasive, all modes will be damped, whereas when you put this damper at the middle of the string, you will find that the second mode is never damped. So, the second mode, the fourth mode, these modes will not be damped. So, the damping is not pervasive.

So, in the today's lecture what we have studied is this. So, we have introduced the damping; and we have seen two forms of the damping, internal and external. We have studied the distributed damping and lumped or concentrated damping; and we have delineated the effects of internal and external damping. So, with that we conclude this lecture.

Keywords: internal and external damping, distributed and discrete damping, classical/proportional damping, pervasive damping.