Solar Energy Technology
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Lecture - 7
Evaluation of the Apparent Sunrise and Sunset Hour Angles

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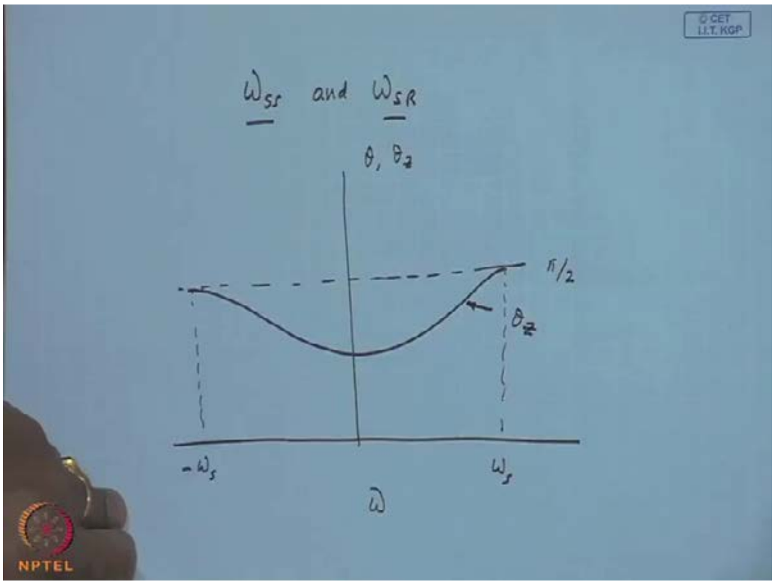

When we were evaluating the daily tilt factor R b bar, we left the apparent sunrise hour angles omega SS and omega SR and determinant. So, the expressions become available for evaluating omega SS and omega SR, when once these values of omega SR and SS could be determined. Now in order to have a clear idea, let us just have an idea. If I plot the hour angle omega verses theta or theta Z in general, does not matter, how it looks like? This dotted line is pi by 2, that is theta will become equal to pi by 2 or theta Z will become equal to pi by 2 and this will be the typical variation of theta Z. That is at the solar noon it will be a minimum angle and at the sunrise it will be pi by 2 at minus omega s and at sunset it will be again pi by 2 at plus omega s. right.

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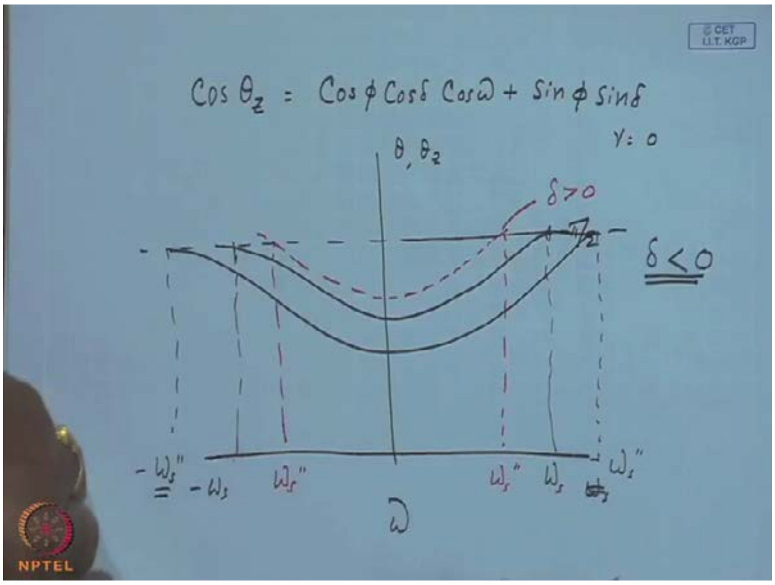

You can take the expression for the cosine theta Z. And then give for a chosen phi and delta, you give omega and plot and you will find that theta Z is equal to pi by 2 at omega s given by cos inverse of minus tan phi by tan delta. And in general it will be decreasing and it will be minimum at the solar noon. Now if I try to plot, again gamma equal to 0 only; because that is easily understood; south facing surface, this is pi by 2 theta or theta Z omega minus omega s plus omega s. Now, if delta is less than 0 I have something like this. You can calculate. This will be my so-called omega s double dash, which we should be correlating with the notation that we introduced.

In other words, the tilted surface when delta is negative will reach theta equal to pi by 2 at a value of omega s double dashed, magnitude of which is higher than omega s. So, of course we will limit all our calculations to omega s. And if I use another color, this may be for delta positive. So, that will be less than the magnitude of omega s. So, you will receive the radiation during this period, not all the time that sun is above the horizon. You can; just from the trigonometry of those two relations you can have these numbers and the figures like this.

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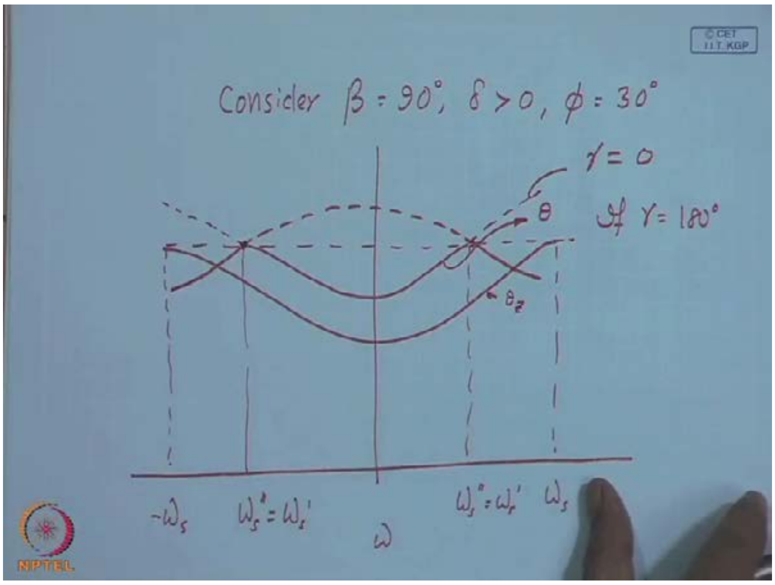

Now, consider a vertical surface beta is equal to 90 degrees and declination some positive values and let say latitude of 30 degrees. Now again if I plot it, so this will be your theta Z and this will be your theta, which is the magnitude of omega s double dashed is less than omega s; that will be also my so-called sunset angles for the tilted surface omega s dashed. Now the same picture if I continue calculating, it keeps on giving you some value of theta which is more than pi by 2, which basically means that the sun is below the horizon or the rays does not grace passed the surface and the surface will not receive any solar radiation. So, by simple optics and common sense if I plot for, this is gamma equal to 0. If gamma is equal to 180 degrees, I should have a mirror image of this. One can set down gamma is equal to 0 and calculate it south facing, then set gamma is equal to 180 degrees. It will be only because gamma, it will become a minus 1 and you will find that.

Again you will get the same number, right, omega s double prime. When you calculate for gamma is equal to 180 with delta same number and with this and gamma 0, you will get the answer as same number, right. But you do not know which is sunrise or sunset as for as the surface is concerned. If you want to have a real inside idea, you plot it in detailed; this theta verses omega. And you will find, upto omega s double dashed the value of theta is less than pi by 2. And from this value to this value, during which time the surface facing south is receiving radiations, it does not and the remaining time it will receive.

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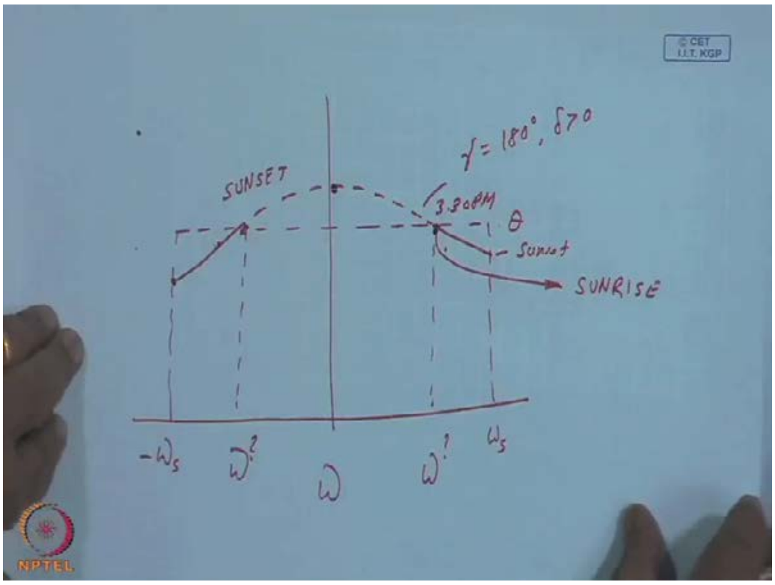

So, if I only amplify this particular thing for the sake of clarity; in other words, this is for more of easy understanding; there is a surface, whatever you like to call. This is of course… Alright. Now, we will ask a question. How do they define the so-called sunrise and sunset?

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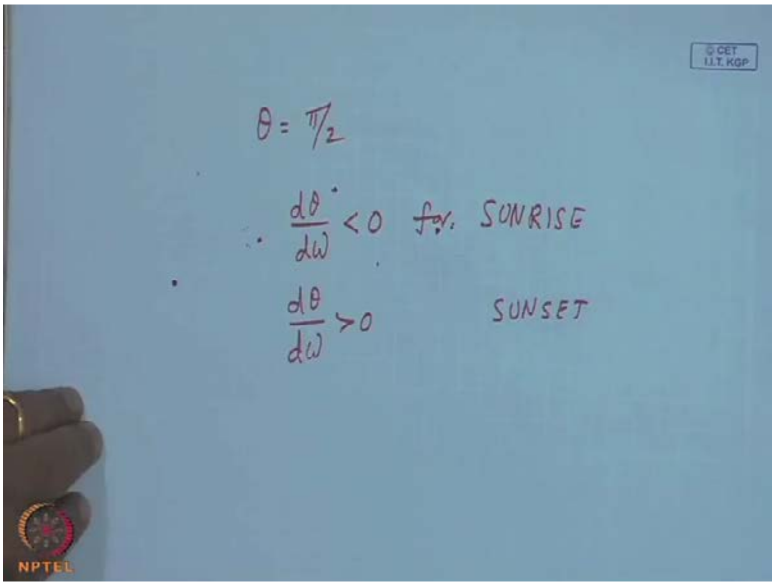

One thing is my theta is equal to pi by 2. So if I look at it, theta is equal to pi by 2; theta equal to pi by 2. So, I can call this is sunrise; this is sunset. Or, you may call this to be sunrise and this to be sunset. So, I will additionally qualify with my experience of general south facing surfaces; not only theta equal to pi by 2, d theta upon d omega should be less than 0 for sunrise and d theta d omega greater than 0 for sunset.

If you look at a normal horizontal surface or a south facing surface, this is your theta equal to pi by 2. And theta is becoming smaller. So, d theta d omega is negative. And then again this is increasing and it will continue to increase or the slope is higher at the sunset. So, mathematically we will define that the sunrise and sunset are not only just theta equal to pi by 2, but the gradient d theta d omega is negative, if I want to call it sunrise and d theta d omega should be positive, if I want to call it sunset.

Now, we will go back to our gamma is equal to 180, delta greater than 0 curve. So, which one will qualify as sunrise? So, this should be the sunrise and this should be the sunset, right, because here onwards; that means, the surface start seeing the sun at let us say 3:30 P M, continuous upto sunset. Again for our own comfortable feeling, next morning it starts seeing from sunrise up to something like this; six or ten 'o' clock or nine thirty A M. Now if I integrate from here to here, this will be cos theta negative; because it is more than 90 degrees, my R b will be negative. So, initially I was telling a blind condition that R b should be positive. Right, because cos theta is negative; when theta is greater than pi by 2, my R b will be written to be negative.

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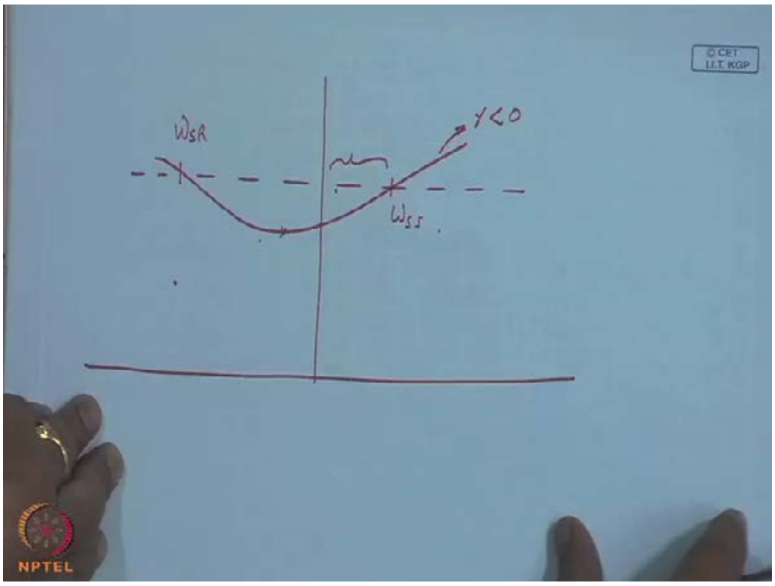

Now if this is the situation, in general if gamma is neither 90, sorry, 0 nor 180, I will not be showing omega s anymore. It is understood it will be limited to omega s, if it is more magnitude wise. I can easily say that this is for gamma less than 0. I have a more time received during the forenoon, than during the afternoon. And the minimum occur sometime in the forenoon itself; something like gamma is equal to minus 10 degrees. So, this satisfies my theta is equal to pi by 2, d theta d omega is negative and this is normal non south facing surface, where our relations directly integrating from omega SR to omega SS or value it.

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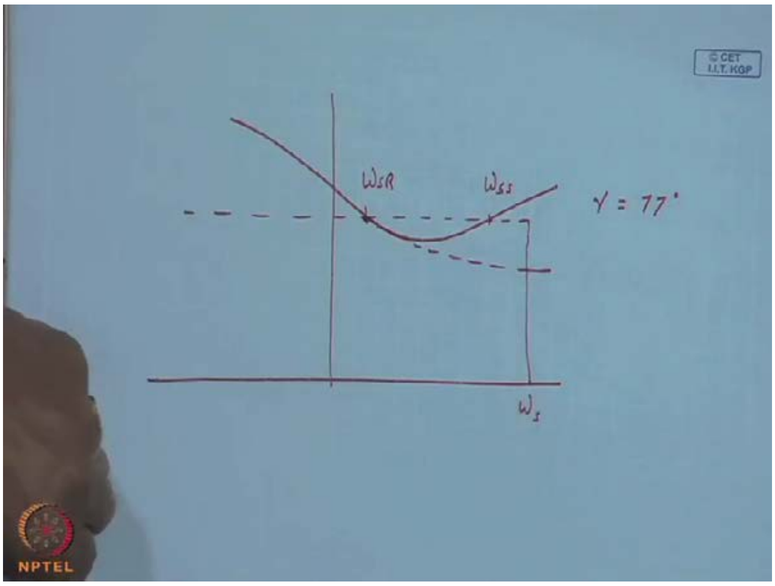

Then, I may have something like this. It starts seeing in the afternoon; stops seeing in the afternoon. This again satisfies my condition. Let us say my, something like gamma equal to 77; almost towards the west. So, it may start seeing around 3 P M and stop seeing at 5 or it may even go like this, wherein it maybe my omega s. In other words, you are towards the west and it stops started seeing after solar noon and continues to see till the sunset; because it is almost towards. If it is a west, as a matter of fact it starts seeing exactly at solar noon time and stop seeing exactly at sunset.

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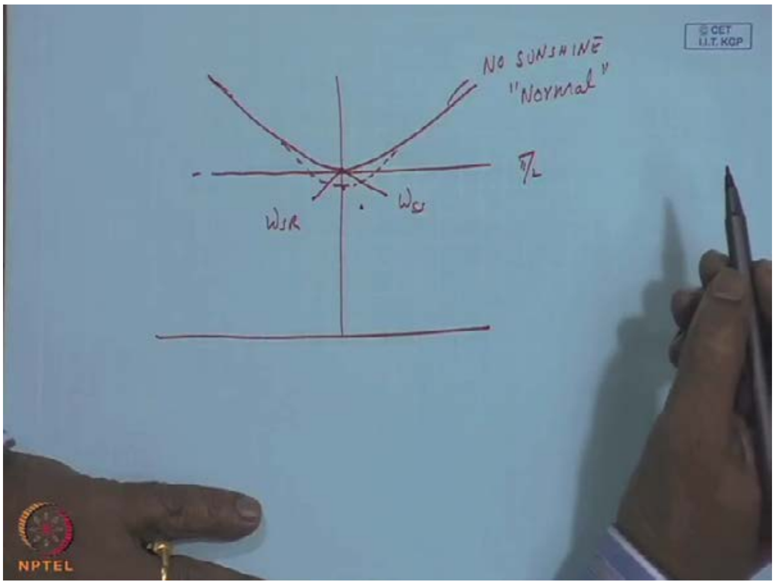

So, you can think of degenerate cases; including, just the minimum is pi by 2 occurring at a solar noon rest of the time. In other words, slightly before this orientation, whatever it is and I might be having something like this and keep on changing my beta. Let us imagine this to be south facing, so that it is symmetric. This is; some beta is equal to 72. I will change it to 78, 80; somewhere it just touches pi by 2.

This is no sunshine, but I would call it normal. In other words, in the limit, this is my omega SR and this is my omega SS, slightly neighboring value of slope beta. I have a omega SR equal to minus 2 degrees and omega SS plus 2 degrees.

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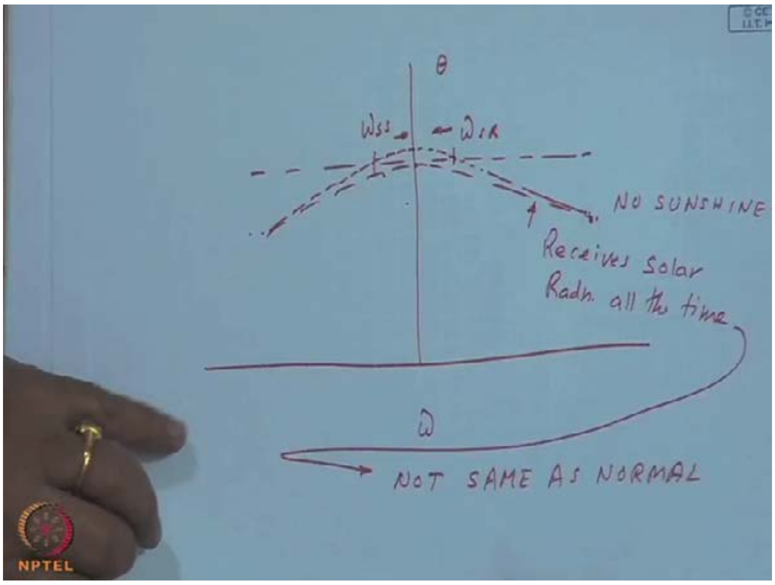

If I take a counter part of this particular, you can say no sunshine period. I may have… This is also no sunshine. Like that, beta is equal to 90 degrees and gamma is equal to 180, I keep on changing beta slightly. And I will just make grace passed it, but the angles are less before that and the angles are again less before this. So technically receives sun, practically all the time, but not same as normal.

In other words, it is a degenerate case of double sunshine; slightly different value of beta. I might have had something like this. So, this is my omega SS; this is my omega SR, which shrunk and tended towards 0. Now if this is like this, I will get a current value; whether no matter how… what is your omega SR and omega SS. It becomes minus omega S to plus omega S. But if I try to do that for this slightly different beta, I have to do from here to here and here to here; not continuously. If I do from here to here, I will get only a negative value. Ok. Now, this is easily understood. I mean, I do not say very easily understood. It can be understood in one sense. In other words, the surface receiving the solar radiations all the time, maybe a degenerate case of double sunshine or a degenerate case of, I mean, a normal case of single sunshine.

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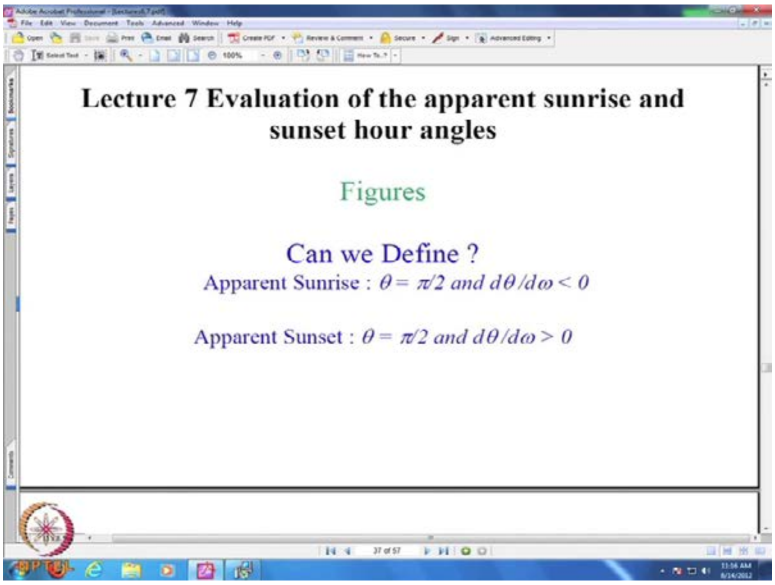

So, these are the two definitions which we have to use; theta equal to pi by 2 and rate of changing is negative according to our convention, as far as d theta d omega is cancelled. How do I evaluate it?

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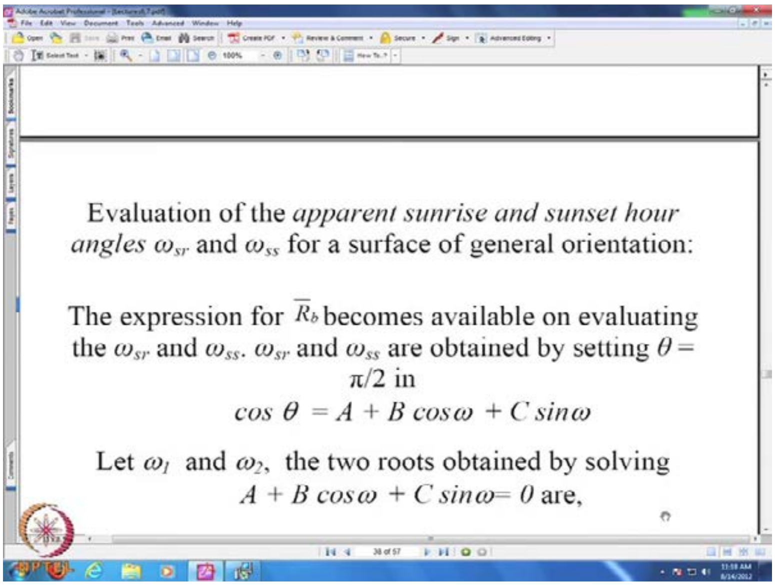

And, why this fallacy? Why for a south facing surface we do not have the problem? But why is it we are having a problem when once gamma is not equal to 0.

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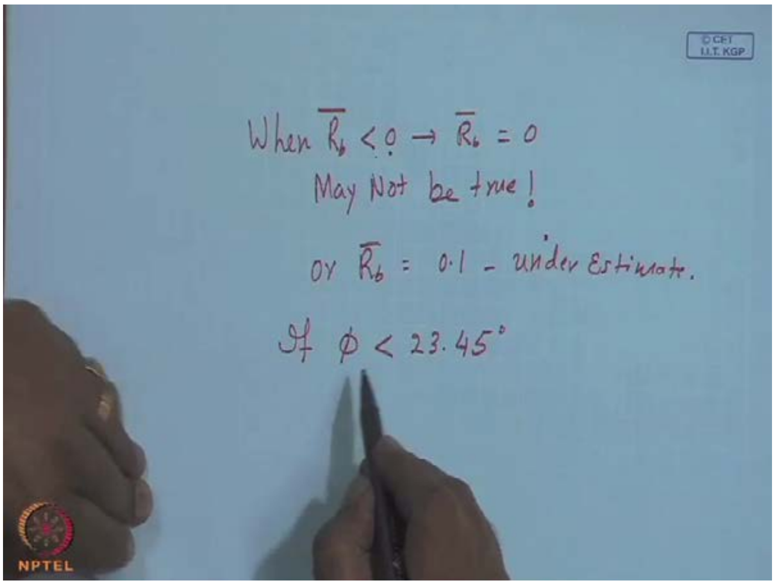

As a matter of fact, quite a few occasions, including in published literature; when… So, open conditions were set; if R b bar is less than 0, set R b equal to 0 is a typical computer statement, which may not be true. Right. You got a R b bar negative because your negative contribution is larger than the positive contribution. Right. Or you might have had R b bar equal to 0.1; where some possible positive contribution, some negative contribution has been subtracted. So, either it may be wrongly shown as negative or underestimate.

So, these are the dangers. Particularly, these things will occur; if your latitudes are lower, the occurrence of this double sunshine is more. Consequently, at low latitudes tropical locations, one has to be little careful in ensuring that theta is equal to pi by 2 and d theta d omega is positive or negative. But then this reads to an elaborate calculation. We have to find out d theta and then differentiate it and find out whether it is positive or negative etcetera.

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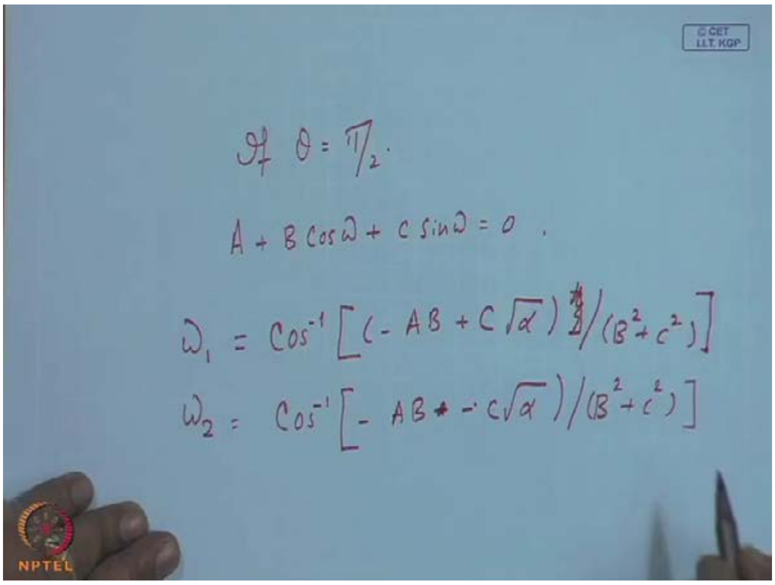

Most of the algorithms that were available in the literature present something like this; where you set theta is equal to pi by 2, then it leads to the equation A plus B cos omega plus C sine omega is equal to 0. It is; for this discussion, it is not necessary to have exact expressions for A, B and C, though the magnitudes will decide ultimately, besides what are the values of your omega SR and omega SS. Now, we can solve this equation. And let us say I do not know whether it is omega SR or omega S S, but omega one will be cos inverse of… right. How do you solve it? You take that c sine omega into the other side, square it up and then write sine squared omega is equal to 1 minus cos squared omega. So, you will get a quadratic equation in cos omega and that is the cause of our problem; because you have squared and taken a root. In that process, somewhere the sine is lost.

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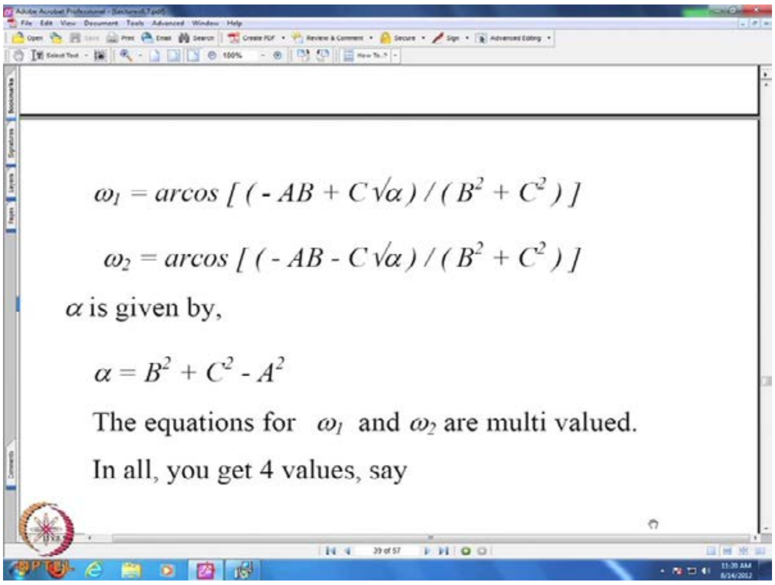

And, you will get omega 2; where, this alpha is B squared plus C squared minus A squared. Now, these are multivalued because any cos inverse will give me plus minus, small value at least. And another two values for omega 1 and two values for omega 2. Earlier, we had two values for omega s or omega s double dashed as cosine inverse of tan phi minus beta tan delta or cosine inverse of tan phi tan delta. And which we according to our notation, identified the negative one as sunrise and positive one as a sunset. But now we have got four and I will just give you arbitrarily some numbers; something like this.

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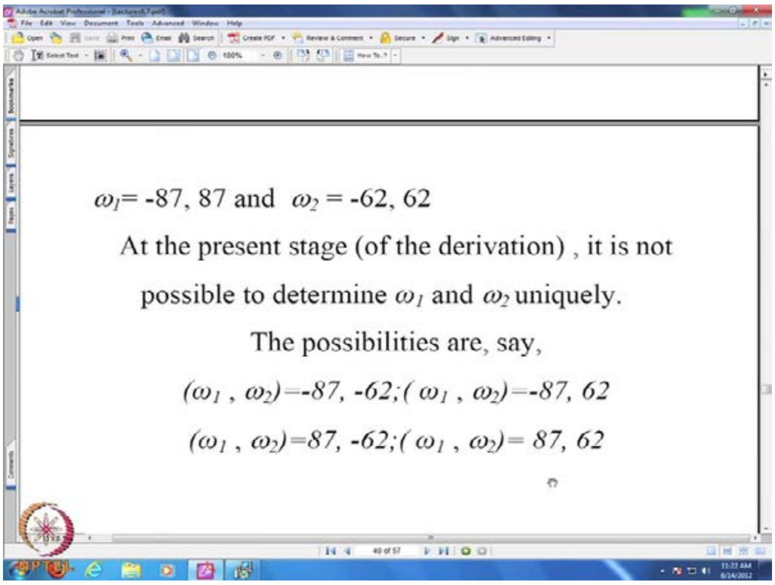

If I solve it, I get a minus 87 and plus 87 from the first one. And second one, minus 62 and plus 62. Nothing, I do not really intend that they should be less than pi by 2 and one can be more than pi by 2, but just to illustrate the point. I really do not know whether my omega SR is minus 87 or 87 or plus 87 and omega SS is minus 62 or plus 62. Also, I do not know whether the magnitude of the omega SR and omega SS are 87 or 62. Though, psychologically I am assuming the first one to be the sunrise. At the present stage of the derivation, not the present stage of knowledge of the humanity, it is not possible to determine omega 1 and omega 2 uniquely. The possibilities are I may have minus 87 minus 62; which means, it starts seeing the sun in the morning and stop seeing the sun in the morning itself; vice versa, like all the four combinations are possible.

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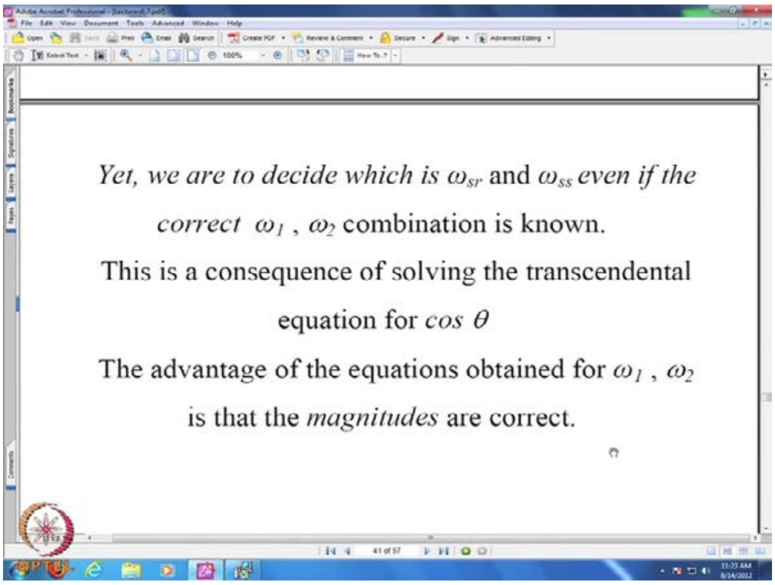

So, yet we are to decide which is omega S R and omega s s, even if the correct omega 1 and omega 2 combination is known. So, this is a consequence of solving the transcendental equation for cos theta.

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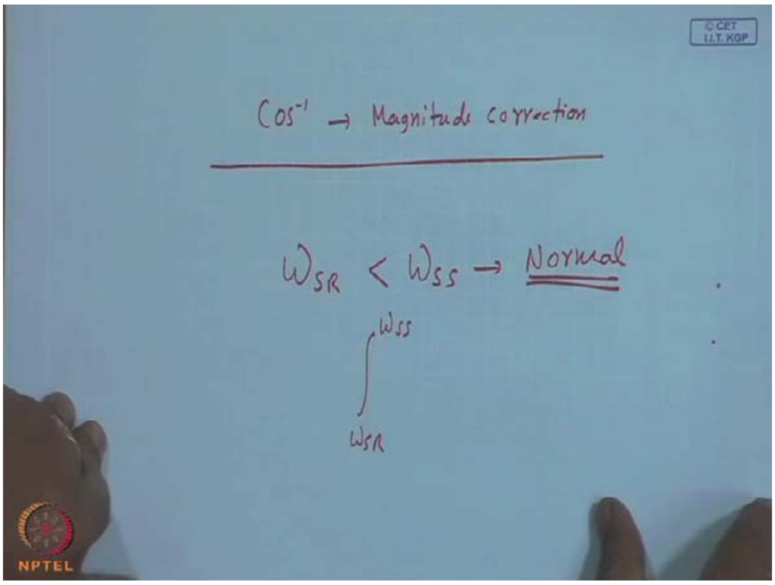

But there is a slight advantage that, this cosine inverse fortunately uses the magnitude correctly. The ambiguity is whether it is minus 87 or plus 87 or plus 62 or minus 62, but 62 and 87 are real.

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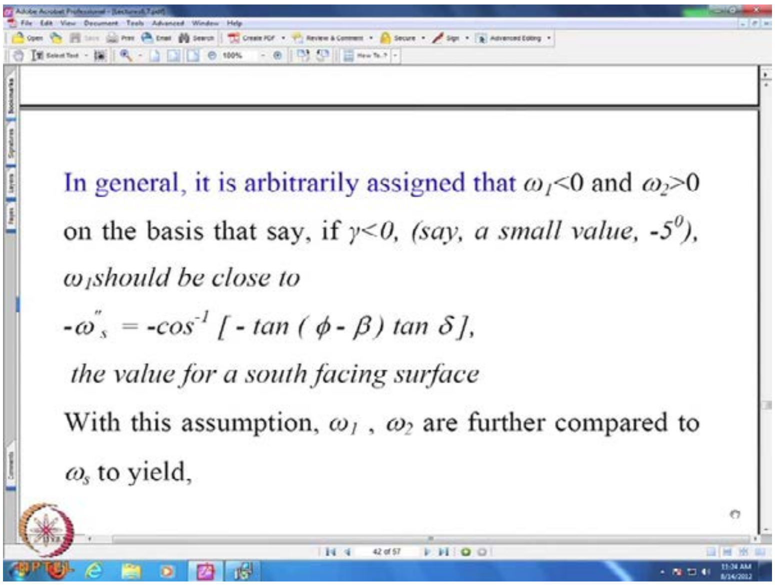

So, what people were doing? They kept on adding one condition after the other. And we can physically think that, you solve for gamma is equal to 0 you will get a omega s dashed minus… for sunrise and sunset normal orientation. And if you set gamma is equal to minus 5 and minus 10, I expect my omega SR magnitude to be larger than omega s dashed. Similarly, omega SS should be less. So, that is satisfied by the first omega 1 will be closer to my omega SR, omega s dashed values and omega 2 will be closer to omega SS or omega s dashed plus values.

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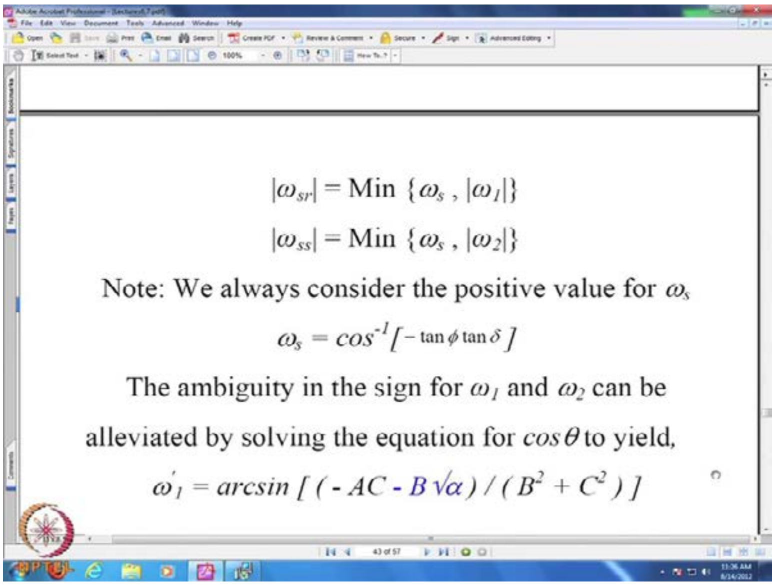

Then, again I will compare with my omega s and then take the magnitude. Actually, this magnitude… written because omega s will always considered to be positive. Right. Otherwise, whatever is the sign of omega 1, will be taken as sign of omega s. It is understood for reasonable values that this is true. And as I pointed out right in the beginning that integration is continuous from omega S R to S S. So, irrespective of this sign, we can say that if omega SR is less than omega SS, normal. You take this sign into account; you take the magnitude also in to account. Right no problem. Minus 82 is less than minus 60. So, I may have a omega SR to be minus 82; omega SS to be plus minus 60. Still my condition is satisfied. Same thing is true, even if they turn out to be both positive; whereas, omega SR is lower than omega S S.

So, as long as this number game is satisfied, it is a normal period. And integration is from omega SR to omega SS. Now, this is where one of our graduate students has come out with an idea. Now, why did not we get into this mess because we are solving it as cosine inverse. We are solving it as a cosine inverse because for gamma equal to 0, it is a cosine inverse and there is no …. So, if you solve it as a sine inverse, take out that B cos omega on to the right hand side, square it and then…as 1 minus sine squared omega. Then I will have a quadratic equation in sine omega and solve it.

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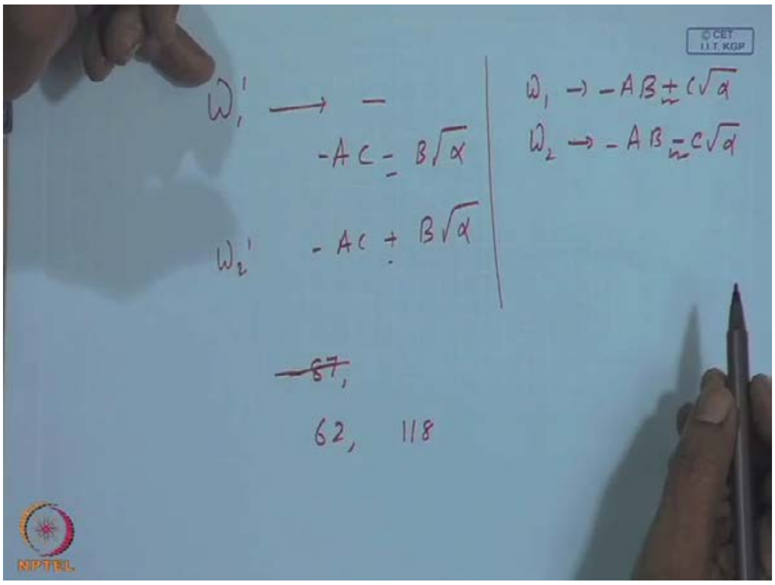

Then, I will get a omega 1 dashed to distinguish from omega 1 and omega 2 dashed to distinguished from your omega 2. You will notice that I played a trick. Omega 1 dash with that minus sign, that is, A C minus B root alpha and omega 2 dashed is with A C minus minus plus B root alpha; whereas your omega 1 will be… In other words, this omega 1 and omega 2, the discriminant sign, plus and minus; they have been deliberately interchanged. This came after considerable trial and errors.

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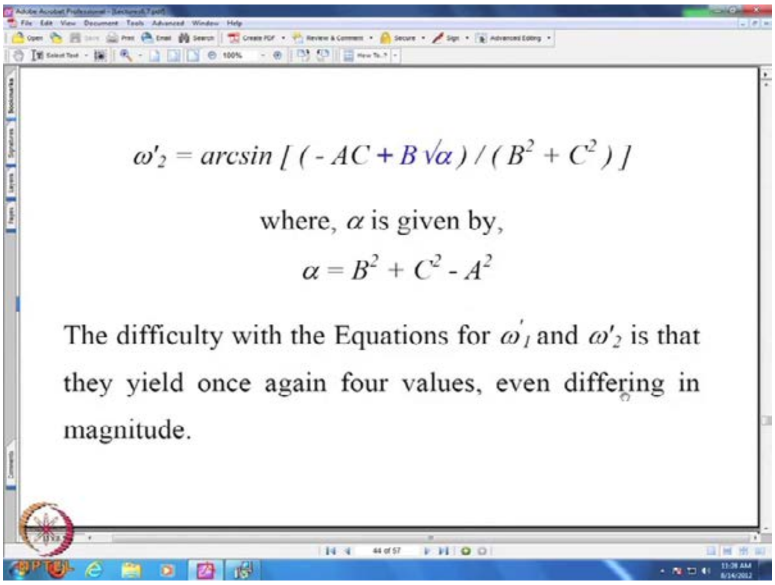

Because, whether you evaluate your omega 1 and omega 2 as cosine inverse or sine inverse, your, in general sine square pi plus cos square pi should be equal to 1, be satisfied analytically. It does so if I choose omega 1 dash to be minus and omega 2 dash to be plus in the discriminant. So, that is the reason why this looks inter changed.

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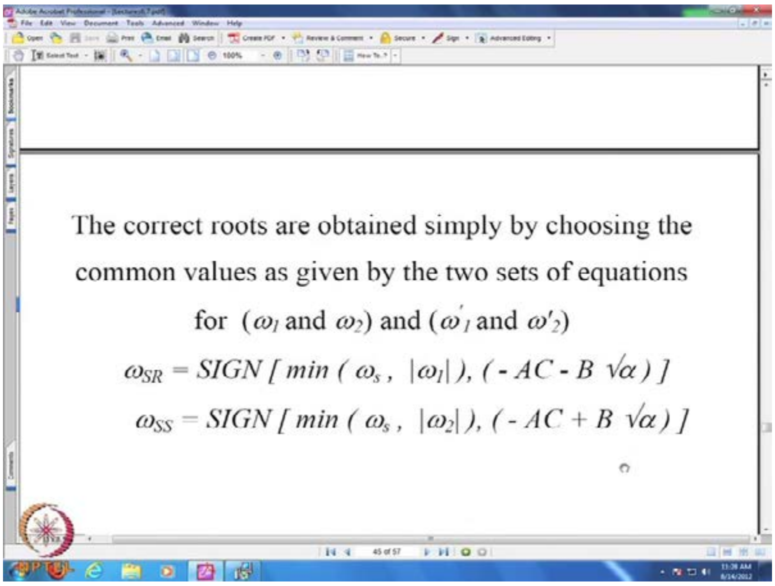

So, now I have got sign; is also not such a innocent function. It will give me; earlier as you have written minus 87 and it will give me how much are 62 and 118. Right. If the principal value is 62, 180 minus also will be satisfied by sign. So, I will have this 62 and

118 or of course if it is minus 62, minus 118. Similarly for 87, whatever … So, now my ambiguity lies in magnitude, whether should I pick up to 62 for omega 2 or 180; similarly, for omega 1 dashed also.

So, the common sense says that, whatever are common between these two are the correct values. So, I may get a minus 62 as cosine inverse, which occurs in omega 2 as minus 62. So, minus 62 is one of the values and that will be my omega SR; the second one will be… omega S S.

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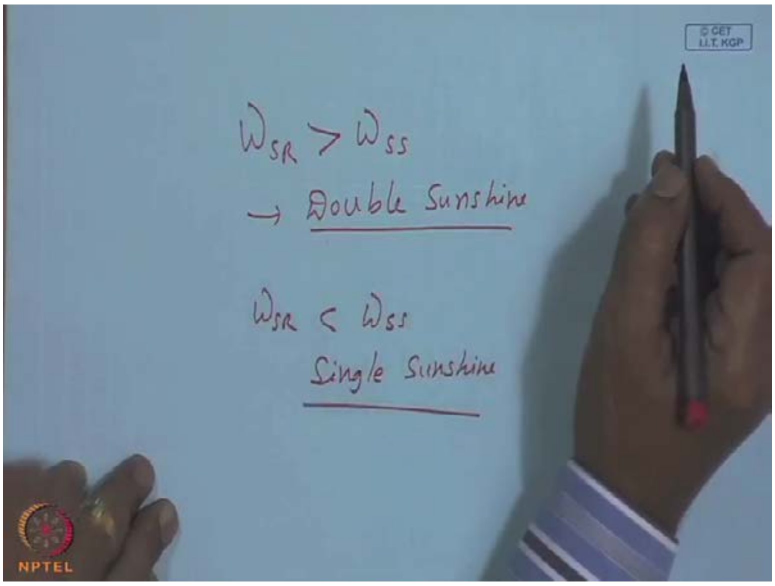

So, then I will apply my criteria that, so we shall now have been correctly determinant and slightly arbitrarily assigning that omega 1 to be omega SR and omega SS. Right. But the numbers magnetically correctly decided and that slight arbitrariness is based upon the physical fact; if gamma deviates slightly from gamma equal to those value, should be closer to the south facing values, right, not that they are completely arbitrary. And still if one bounds, one can verify d theta d omega to be positive or negative from the expressions.

So, this is only mathematical thing. You have to write it in terms of pickup the sign, is decided by minus A C minus B root alpha and minus A C plus B root alpha of the sine inverse functions; because you can see that term and sine will be positive or negative, just depending upon the sine of that discriminant. And you compare the magnitudes with omega s and pick up the sign of this number minus A C minus B root alpha and the magnitude as given by… This is only mathematical way. We understood determining correctly the omega SR and omega SS, magnitude and sin. T hen, we compare with the magnitude of the physical sunset or sunrise. Then we compare with the magnitude of the physical sunset or sunrise hour angles.

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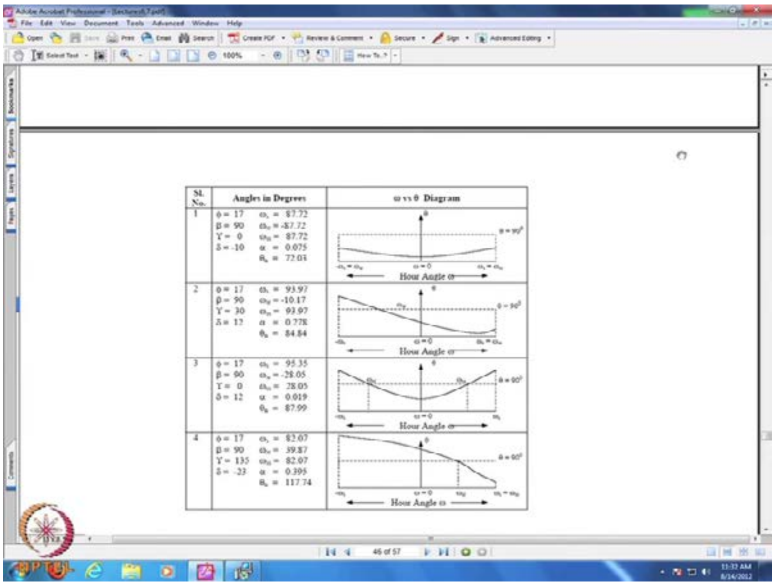

Sl. No.	Angles in Degrees		ω vs θ Diagram
1	ϕ = 17, β = 90, γ = 0, δ = -10	ω_s = 87.72, ω_{sr} = -87.72, ω_{ss} = 87.72, α = 0.075, θ_n = 72.03	θ = 90°; ω = 0; Hour Angle ω
2	ϕ = 17, β = 90, γ = 30, δ = 12	ω_s = 93.97, ω_{sr} = -10.17, ω_{ss} = 93.97, α = 0.778, θ_n = 84.84	θ = 90°; ω = 0; Hour Angle ω
3	ϕ = 17, β = 90, γ = 0, δ = 12	ω_s = 95.35, ω_{sr} = -28.05, ω_{ss} = 28.05, α = 0.019, θ_n = 87.99	θ = 90°; ω = 0; Hour Angle ω
4	ϕ = 17, β = 90, γ = 135, δ = -23	ω_s = 82.07, ω_{sr} = 39.87, ω_{ss} = 82.07, α = 0.395, θ_n = 117.74	θ = 90°; ω = 0; Hour Angle ω

So, I have given examples here, which I shall write down because this print is… small. This first table contains single sunshine orientations with all the types of possibilities, I have shown. First one is quite normal.

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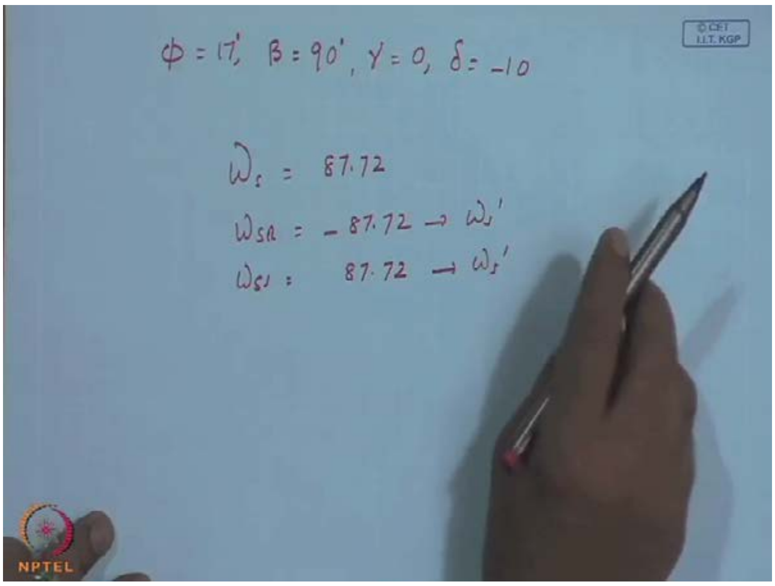

So, latitude 17 degrees; slope 90 degrees; azimuthal angle 0; delta minus 10; with omega s 87.72 and omega SR minus 87.72; omega s dashed 87.72. This is after all a gamma equal to 0 surface. So consequently, though you call it omega SR and omega SS, they are nothing but your omega s dashed. Actually, you have limited to omega S S. If you see the plot of this theta verses omega, it is not equal to pi by 2 at minus omega s or plus omega s. right. At the physical sunrise and sunset in delta negative, it comes out with a fairly good angle of incidence.

That means, lower than ninety. The same thing; a positive delta and a slight positive gamma; that means, towards the west, we can see that omega SR is considerably nearer to the solar noon time away from the sunrise and the minimum occurs almost near the sunset; because my gamma is thirty degrees, which will approximately correspond to the sunset is being normal plane to the surface around three o'clock. So, it will still not have theta is equal to pi by 2 at sunset. So, it continues to see you even at the sunset; which is obviously because it is… west. Then the other one is again, yes, this same thing case number one which is reproduced, accept with a positive declinations.

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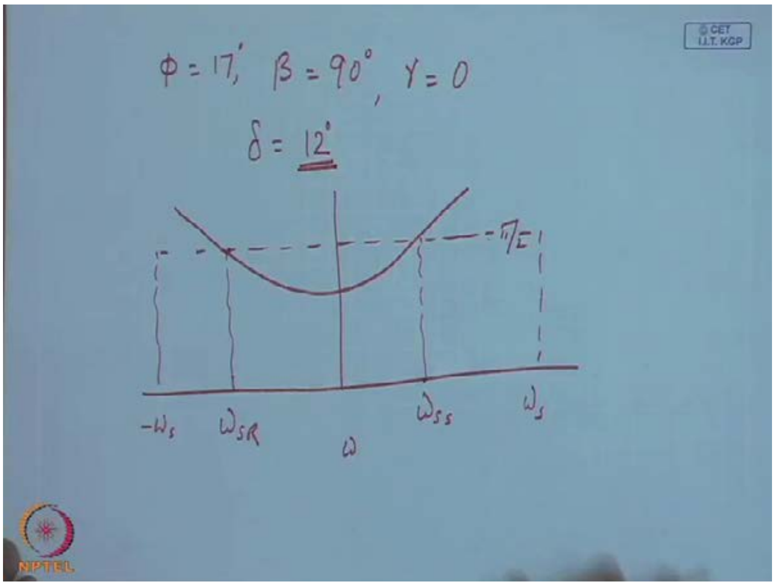

So this, as we have been discussing; for generality, they are being called omega SR and omega S S. But otherwise they are just your omega s dashed because your gamma equal to 0 surface. This demonstrates for positive declination. Theta will be equal to pi by 2 at a value after the sunrise and considerably before the sunset.

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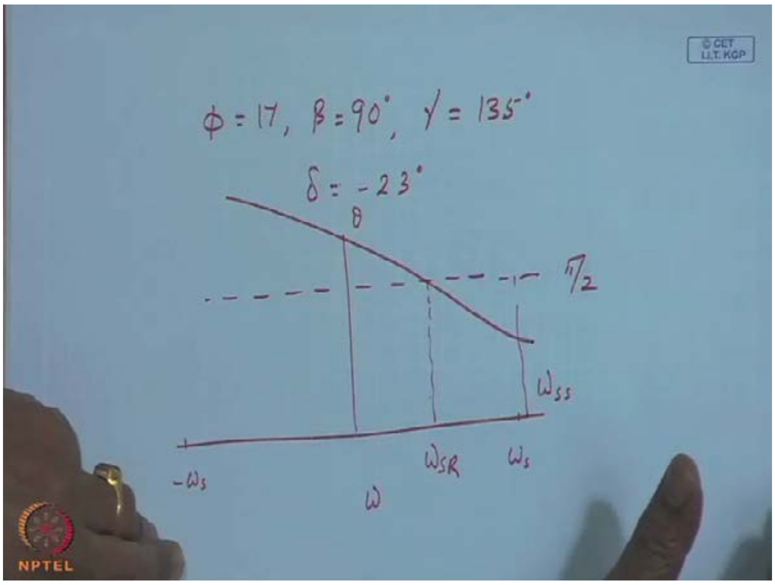

This is quite again gamma equal to 135 degrees; that means, it is in the beyond west; the outer normal is between the west and the north … And delta is minus 23. That means … And you will find, of course this is your omega and you have a minus omega s and plus omega s; this is theta, 90 degree angle. And, what you find is, so it starts seeing and of course it will be limited to omega s; because it is fairly even beyond west. So, it starts seeing quite late in the afternoon; may be four o'clock. These diagrams are not so much to the scale. But illustrate the point that it should be in the afternoon and during the fore noon. It does not receive any solar radiation.

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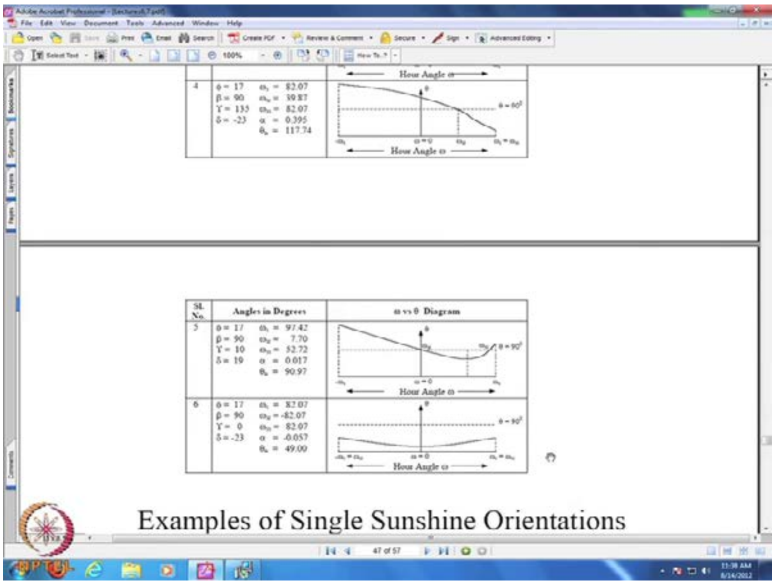

So, this is phi equal to 17; beta 90; gamma 10 and declination 19 degrees. And this one shows both omega SR and omega SS are within the sort of afternoon only; because… only gamma is equal to 10 degrees.

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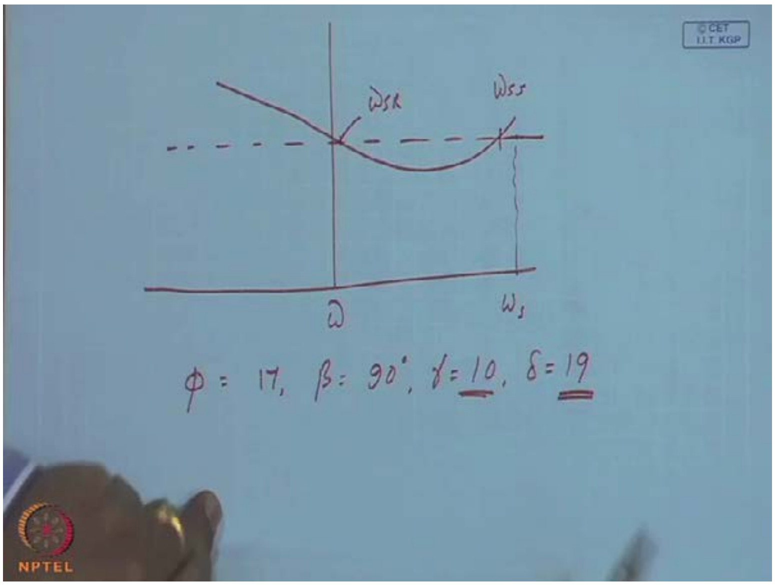

It starts seeing, may be around 12: 30. That figure is not that clear. So, phi is; the one reason why we are sticking to this 17 or low degree and 90 degree slope is that, we will have that more of examples of double sunshine at lower latitudes. And beta is equal to 90 is chosen, if I, we could have chosen 60 or 70 or 40, whatever you are want. But exact symmetry to understand the reversal of the theta variation cannot be seen if you choose … beta. If suppose is 35 degrees, when it is in… south, right, again you have to put beta equal to 35, if it is north, but measured from the behind. So comparative to the previous angle, it will be 165 degrees. You have to turn it towards the south. So, this is to demonstrate mainly that when your declination is positive, the sunset, our apparent sunrise angle for the tilted surface will be within omega s, even if gamma is not equal to 0. That is why low gamma is chosen.

If you choose a gamma equal to 40 degrees or so it might go beyond omega s; right, that means this tilt core, yes, it should be after solar noon and this is omega SS, omega S R because it was started with a very small gamma towards west. So, it starts seeing slightly after the solar noon. And because it is so small and it is a summer or delta 19. So, my in general sunset hour angle should be less than omega s at gamma being equal to 0.

Choose; we choose a small; it goes towards omega s, but still less than omega s. The last one is a typical south facing, maximum negative declination and you will have a fairly low value of angle of incidence at sunrise and sunset; physical sunrise and sunset. That means in the winter time, it will receive all the time. So, and since it is low latitude and high slope, the sun's rays are almost normal to the surface.

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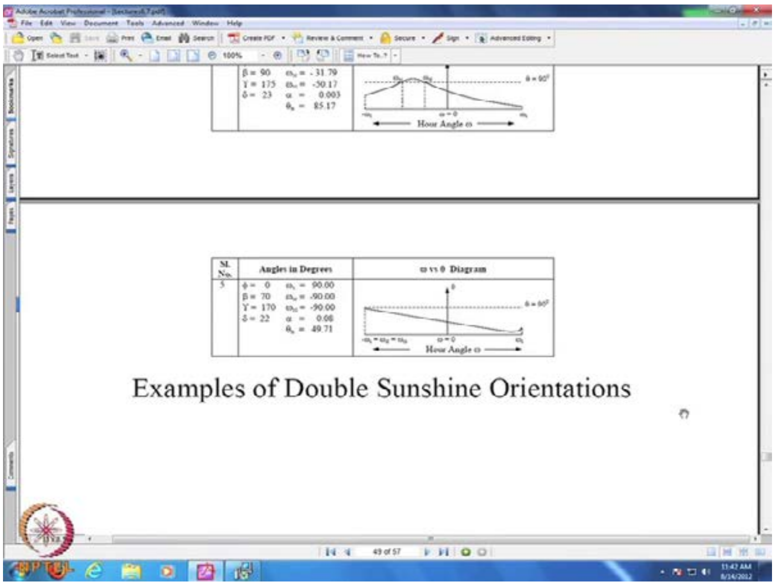

So, these are the examples of single sunshine orientations. Now here are, these are examples of double. Let me go through.

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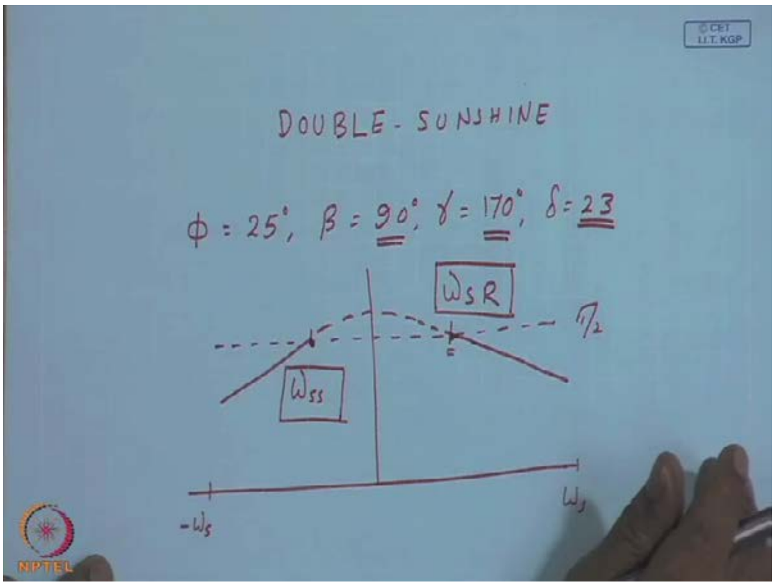

This is somewhat cleverly done in the sense that delta is 23. So, it is a positive declination; wherein a south facing surface we will receive part of the time only, beta is equal to 90. Consequently, again when delta equal to 23 will be a small duration and then instead of gamma equal to 180, 170 is chosen, which is almost 180. And you will find that. This is actually calculated for various values of omega and then plotted, though may not be exactly within the scale.

So this is; it looks little distorting or uncomfortable to say that sunrise is in the afternoon and sunset is in the forenoon. But this is the apparent sunrise as seen by the surface. According to definition, we gave that angle of incidence should be equal to pi by 2 and my angle of incidence should be decreasing, if it started seeing the sun and it should be increasing, if it is about to stop seeing the sun. So, it satisfies that criteria. And if somebody is little uncomfortable saying that it stopped seeing the sun in the morning and started seeing the sun in the afternoon; calling it thereby. What we can imagine? This is when it started today and again it continued to set tomorrow morning at omega SS. In between, this sun was below the horizon and hence the surface did not receive the solar radiation. And we, yes, this is that absolutely degenerate case.

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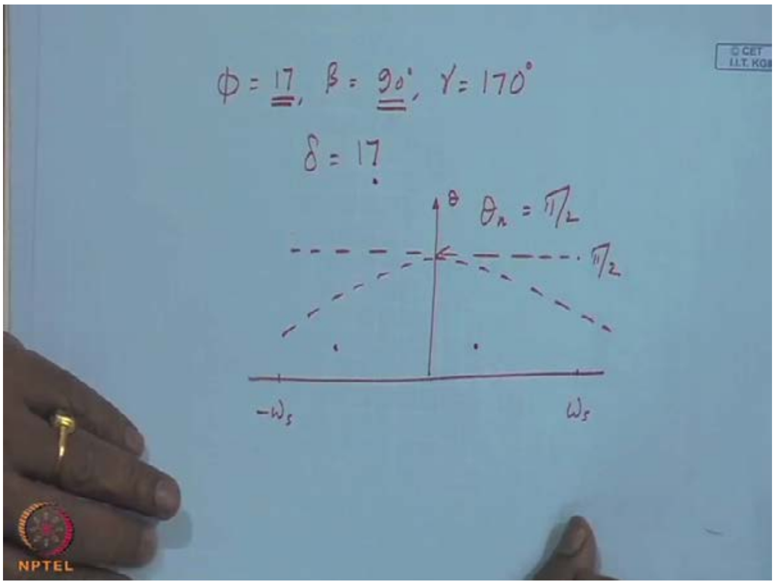

So, phi 17; delta 17; beta 90; gamma is equal to 170; this is very easy to understand, when once you have got beta 90 and delta 17 equal to latitude. And as a matter of fact, one can suspect that this is exactly theta noon equal to pi by 2. And, one can verify that if the surface is vertical and latitude is equal to the declination, at solar noon time the sun is vertically above at that location. So any vertical surface, the rays at solar noon will be raising fast… Ok. So, that is the little trick of choosing declination equal to latitude. That is also the reason for choosing, taking the 17 degrees; because this does not happen at more than 23.45 degrees. In other words, if you look at this, this is theta; noon is pi by 2. But you are receiving solar radiation during this period and this period, theta is less than pi by 2; accept around the noon time.

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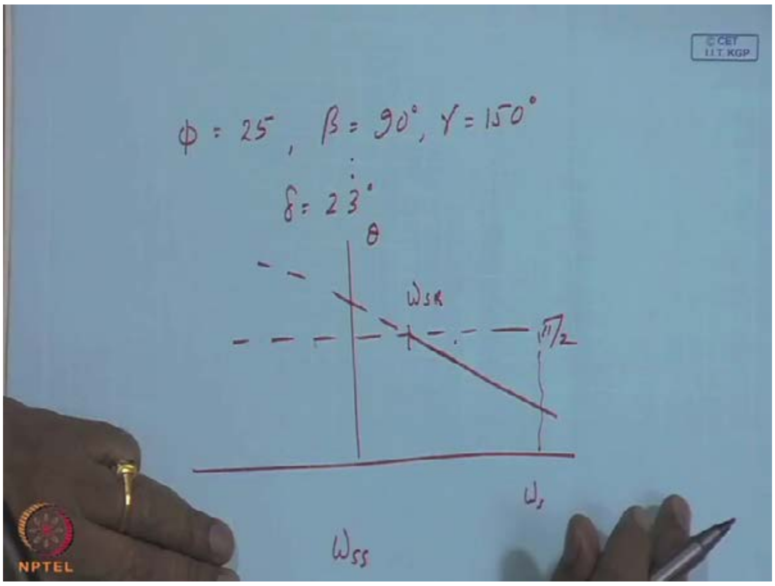

This is again, since normal omega SR; yes, this is exactly not normal. What we will find that is three phi is equal to 25; beta equal to 90; gamma is equal to 150 and delta is 23. If you calculate, and you have got the plot something like this, so you determine according to our… to be omega SR and this is my omega s. What happens was you got a omega SS; pretty large value, which turns into the night. Consequently, you set equal to a negative number of minus omega s in the morning. So, it looks as if it is a single sunshine period, but in reality it must touch the theta equal to pi by 2; sometime between plus omega s and minus omega s. right.

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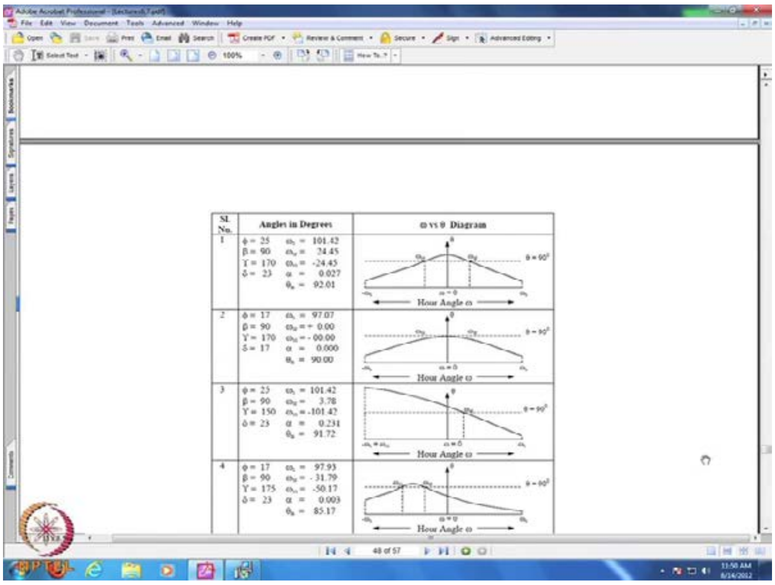

If complete calculations are done, that is why it is written here as minus omega s is equal to omega SS because you are limiting it to that. You got a larger value, so you are limiting it to that.

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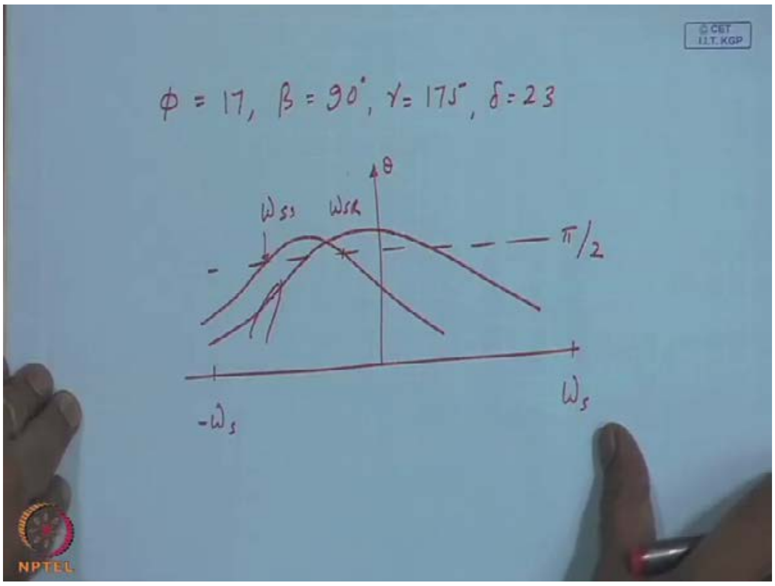

Next example is, of course the whole idea is to show something like normal, but discontinuous, like number one: omega SR is in the afternoon; omega SS in the positive for the morning, forenoon, sorry. And the other cases degenerate that can go to noon time. The last one is that, second one omega SS is beyond simple omega s and the last fourth case is phi 17; beta 90 degrees; gamma 175 and delta 23.

You got; you have something. Sorry, it is, so it starts seeing sometime in the forenoon itself, since all the way upto sunset and again sees from the morning till some omega S S. That means, both omega SR and omega SS occurring only in the forenoon. That is the whole idea compared to… that are being different. So, we shall continue with this in the next class and including few more cases of double sunshine and no sunshine case.