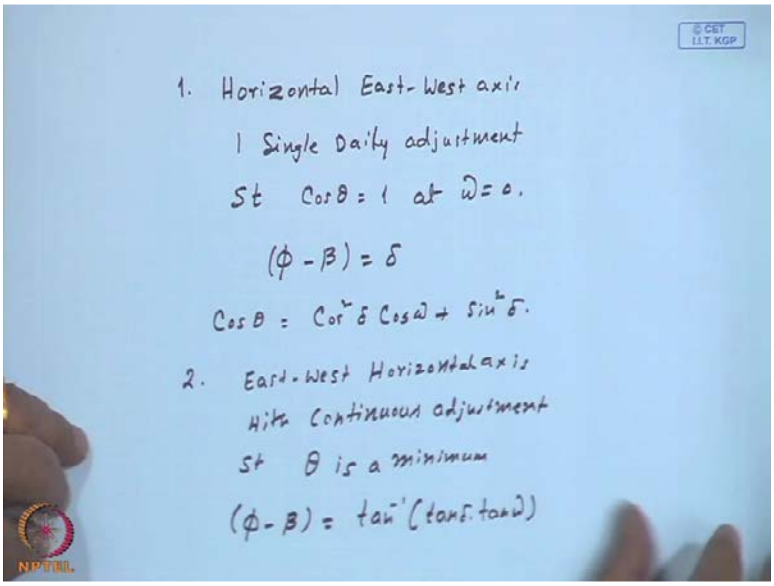
Solar Energy Technology
Prof. V.V. Satyamurty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 20
Concentrating Collectors (Contd.)

Last time, we were considering concentrating collectors, and we found that in order that concentration is achieved, the solar radiation received over a larger aperture area either through reflection or refraction has to be focused on to a smaller receiver area. This requires tracking. So, quite often tracking collectors, concentrating collectors, focusing collectors these names are synonymously used though even a flat plate conductor can be tracked.

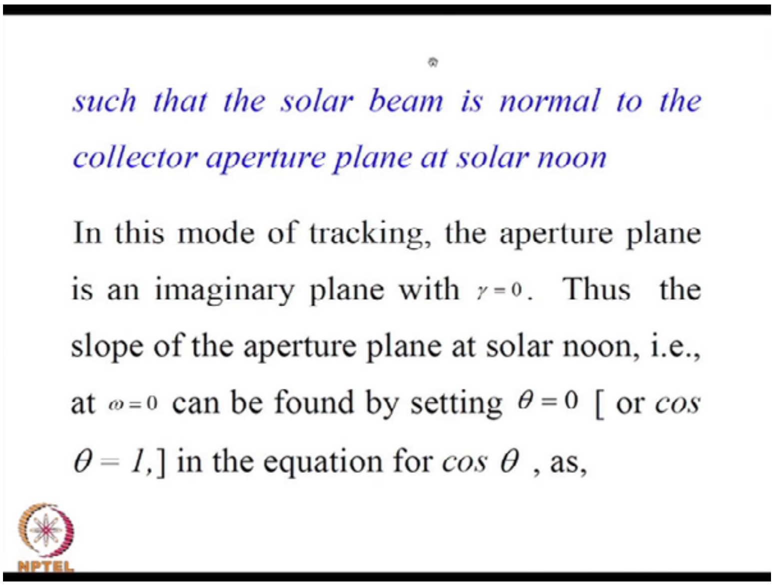
So, in order that concentration is achieved, focusing is as necessary which calls for tracking; that is the way I remember the distinction, when to call a concentrating collector, focusing collector and or tracking is necessary thing to achieve focus and concentration.

(Refer Slide Time: 01:27)



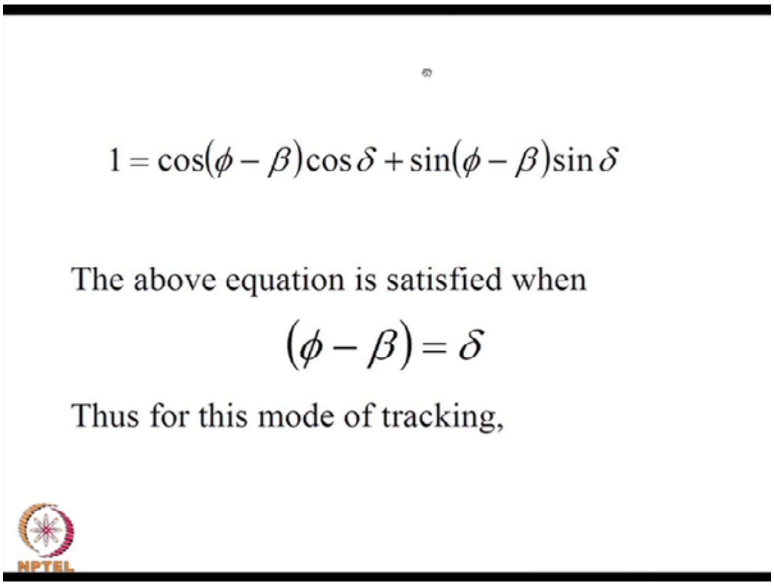
In that, we consider two of the simpler tracking modes; with one, a horizontal east west axis with one single daily adjustment, such that cos theta is 1 at the noon time equal to 0.

(Refer Slide Time: 02:00)



So, this gives us the condition to be satisfied phi minus beta equal to delta.

(Refer Slide Time: 02:03)



So, that cosine theta is given by cos square delta cos omega plus sin square delta.

(Refer Slide Time: 02:17)

$$\cos\theta = \cos^2\delta\cos\omega + \sin^2\delta \quad \text{(Tr. 1)}$$

2. A plane (aperture) is rotated about a horizontal east-west axis with continuous adjustment to minimize the angle of incidence. Since the aperture plane is facing south ,

Eq. (a)

Then the second mode, again a east west horizontal axis with continuous adjustment such that theta is a minimum.

(Refer Slide Time: 03:13)

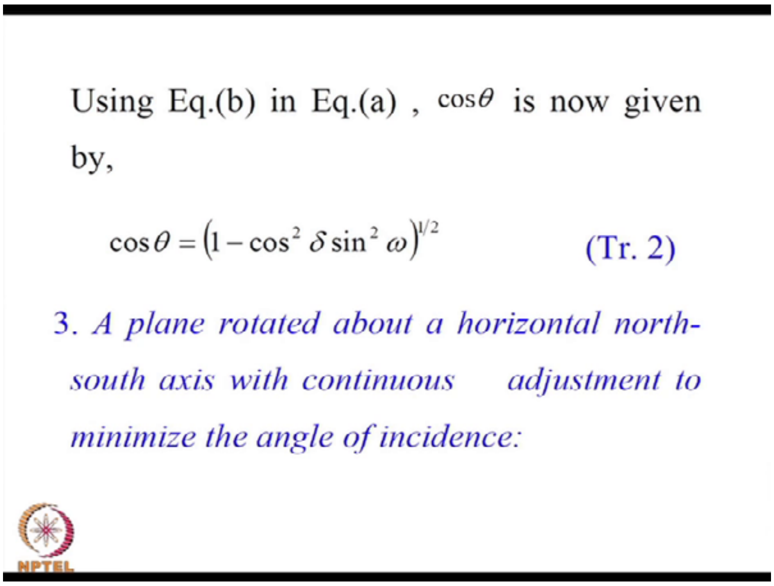
$$\cos\theta = \cos(\phi-\beta)\cos\delta\cos\omega + \sin(\phi-\beta)\sin\delta$$

In order to find the condition to be satisfied for θ to be a minimum, RHS of the above equation is differentiated with respect to β and on equating to zero, it follows,

$$(\phi-\beta) = \tan^{-1}(\tan\delta/\cos\omega) \quad \text{(b)}$$

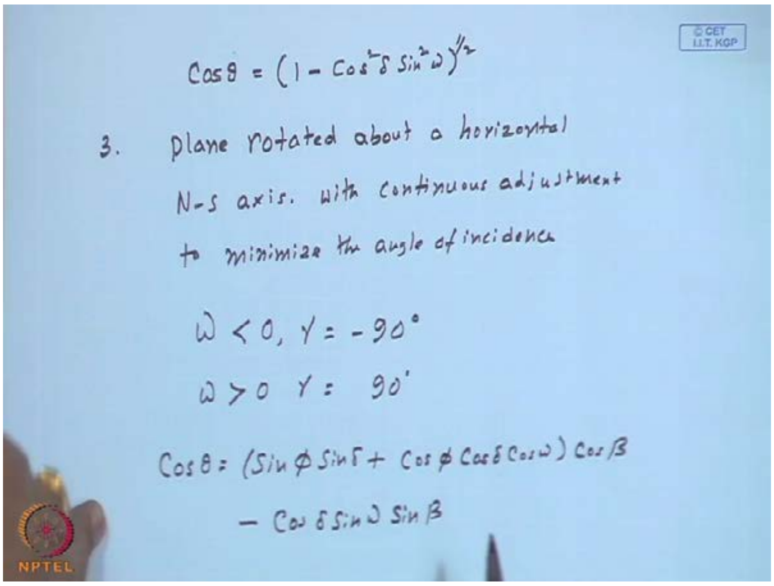
So, we consider the equation for the cosine theta for a south facing surface, because it is a east west axis, then differentiate it with respect to beta and obtained that phi minus beta at any hour angle should satisfy tan inverse tan delta tan omega, which gives you the same condition as phi minus beta is equal to delta, at the time omega equal to 0.

(Refer Slide Time: 03:45)



And for this the angle of incidence at any instance is given by 1 minus cos square delta sin squared omega to the power 1 half.

(Refer Slide Time: 03:46)



Now, tracking mode three is what we are going to consider in detail now in this class. It is a plane rotated about a horizontal north south axis with continuous adjustment to minimize the angle of incidence. So, in order to distinguish between the two modes we described earlier and the third mode that we are talking about now. Let us say this is the horizontal north south axis and I turn towards east right in the morning, so that the Sun's rays are near normal to the aperture plane and I keep rotating, becomes horizontal around noon time and turns towards west as the Sun goes to set in the west.

So, the horizontal south, north south axis about which we will be rotating the collector if this is a parabolic collector facing east to my left and then turns like this faces west around the sunset time which is to my right.

(Refer Slide Time: 06:17)

In this case, the surface azimuthal angle (γ), is -90^{o} before noon and $+90^{o}$ after noon. Thus, the general equation for $\cos\theta$ before noon becomes,

Eq. (a)

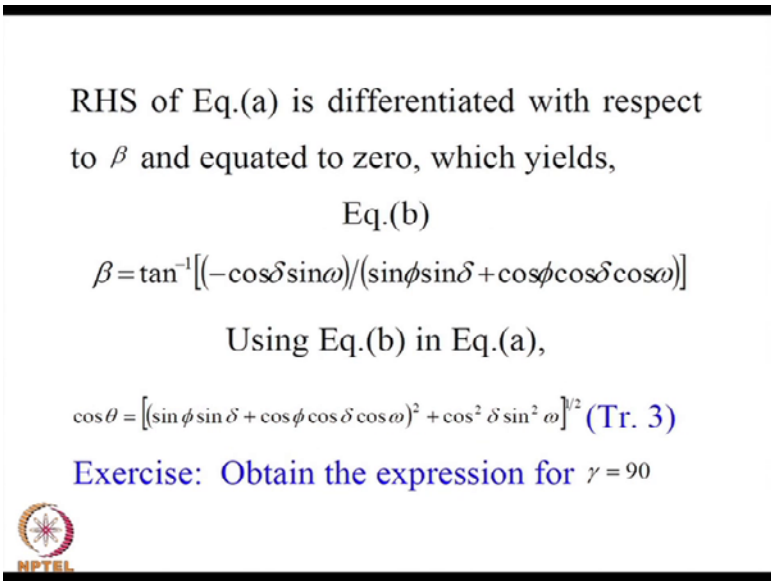
$$\cos\theta = (\sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega)\cos\beta - \cos\delta\sin\omega\sin\beta$$

To find the condition to be satisfied such that the angle of incidence is a minimum,

So, this is such that we should choose the cosine theta for this. What we realize is for omega less than 0 gamma will be minus 90 degrees, for omega greater than 0 gamma will be plus 90 degrees. There are only two states if you consider this is the plane, this is the horizontal at noon time then there is no azimuthal angle because the projection of the outer normal will be a point only and towards the east or any time before solar noon, you have the outer normal of the surface facing towards east which indicates gamma minus 90 as per our notation.

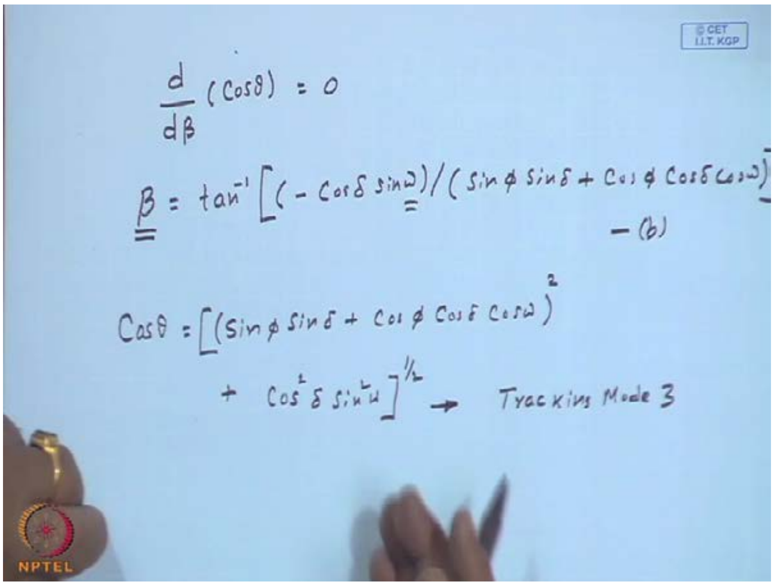
Similarly, in the afternoon it will be gamma is equal to plus 90. So, for this either you choose minus 90, so cosine theta gets simplified from our general equation that cos theta is equal to a plus b cos omega plus c sin omega that simplifies to whatever I am writing down over here times cos beta minus cos delta sin omega sin beta.

(Refer Slide Time: 07:56)



To find this we differentiate this with respect to beta and equate it to 0.

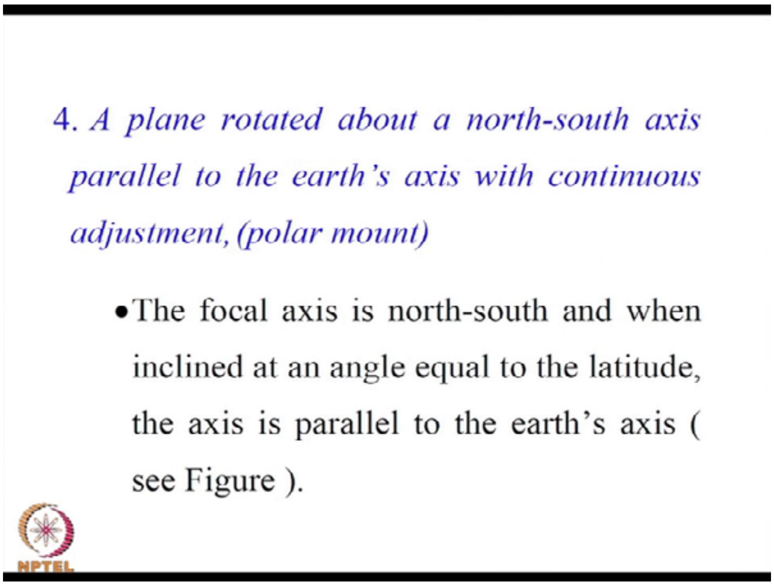
(Refer Slide Time: 08:05)



And you will get the condition that beta should be tan inverse of minus cos delta sin omega upon sin phi sin delta plus cos phi cos delta cos omega bracket closed bracket closed. So, if we use, call this b. In the previous equation for cos theta we get cos theta longer expression sin phi sin delta plus cos phi cos delta cos omega whole square plus cos squared delta sin squared omega whole to the power one half. So, this is the equals angle of incidence for tracking mode three.

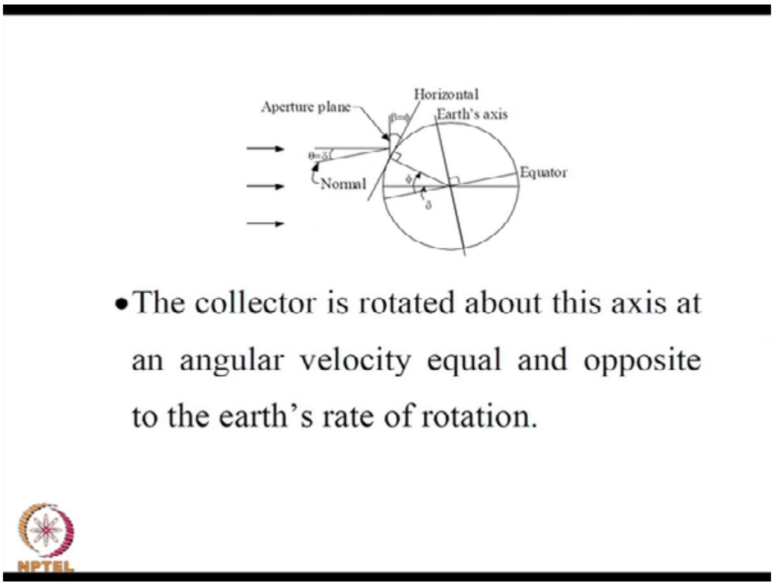
Tracking mode three recall it is a horizontal north south axis turned from east to west to minimize the angle of incidence for which we get the condition that beta at any instance as given by omega is governed by this and then you can have a similar exercise put in gamma equal to 0 and you will get a cosine theta angle similar.

(Refer Slide Time: 10:11)



Now, fourth mode of tracking, it is called a polar mount or a plane rotated about a north south axis parallel to Earth's axis with continuous adjustment. So, we have got a north south axis, but tilted parallel to the earth surface and rotate the plane under consideration in the direction opposite to that of the rotation of the earth at the same speed as the rotation of the earth.

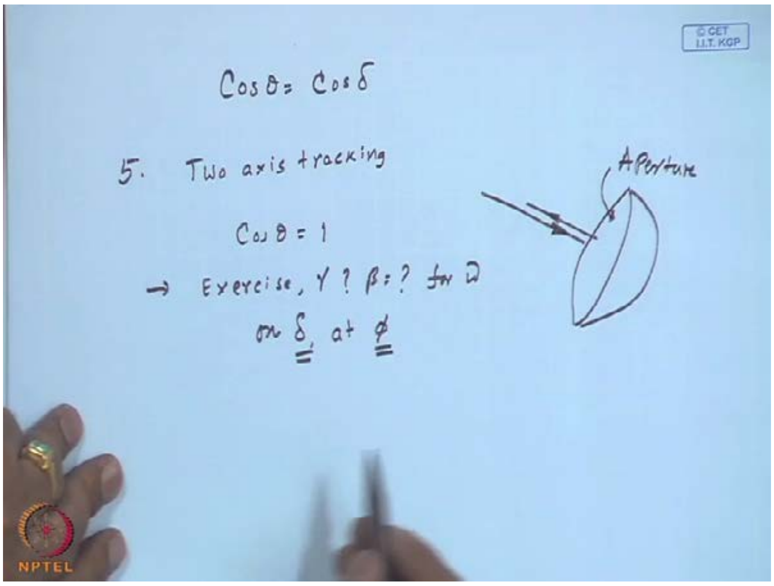
(Refer Slide Time: 11:33)



So, you can see the picture over here. Equatorial plane is the mid line to which the inclination, this is the equator and this is your Earth's axis and this will be your delta, the tilt or the declination and if I am having at any location of latitude phi and you have got a horizontal and this is your aperture plane with a slope beta being continuously adjusted then if this is the Sun's ray and this is the outer normal you have got the angle of theta should be equal to delta.

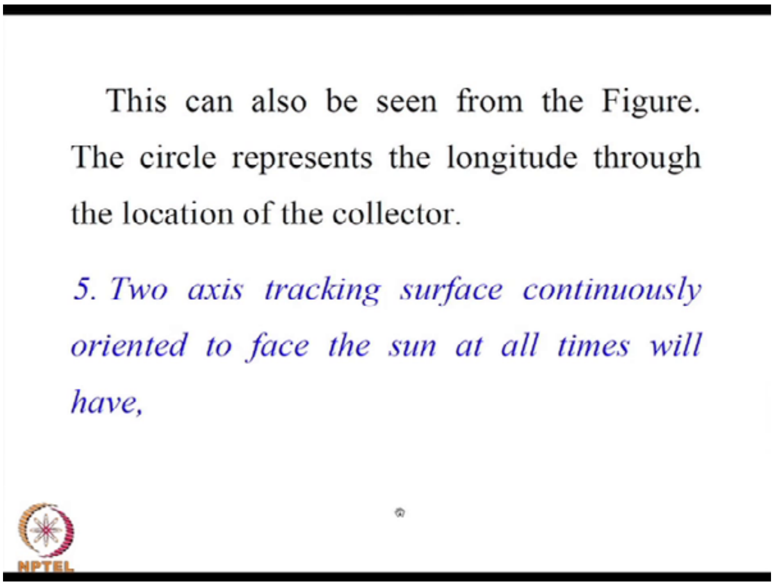
This is your Earth's axis, this is horizontal or you can focus this figure a little longer. This is the x axis which is shown over here and if you look at carefully the angles will be clear and consequently your cosine theta will simply be equal to cos delta.

(Refer Slide Time: 13:12)



So, this is also a single axis tracking a north south axis parallel to the Earth's axis, the collector being rotated in the direction opposite to that of Earth's rotation, at the same speed as Earth. So, it feels like the location with the tilt delta on that particular day consequently the angle of incidence will be cos theta is given by cos delta. So, this is also single axis tracking.

(Refer Slide Time: 13:51)



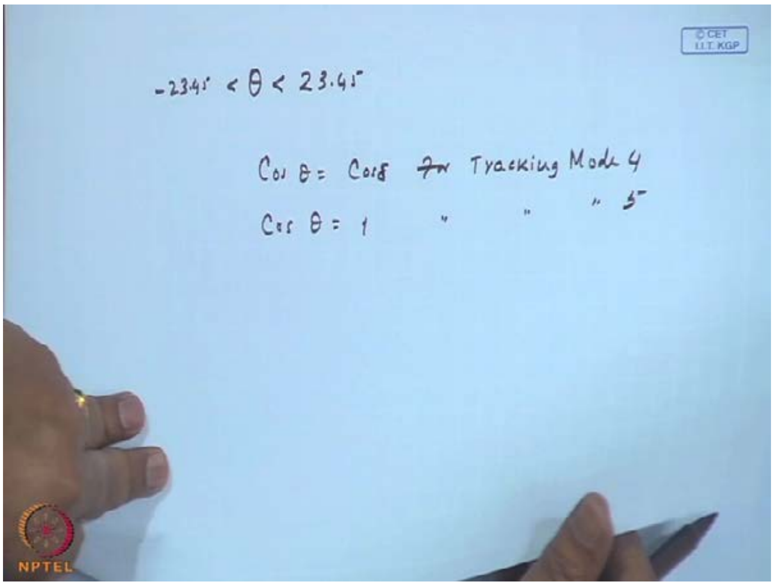
Then last is two axis tracking. So, you do whatever you are doing in the case of tracking four. In addition you further see that the Sun's ray is normal to the outer surface under consideration. So, that cos theta will be equal to your 1 all the time, that means theta is equal to 0 degrees which means coincides Sun's rays will coincide with the outer normal to the surface. So, you can have a little exercise that means you need a gamma how much? Beta how much? For a given omega on a delta day at a location of phi.

So, if you can find out what should be the azimuthal angle and the slope at a given instance of time given by the hour angle for a particular day characterized by the declination delta at a location of latitude phi you can find out cos theta is equal to 1. In other words if you are having let us say a paraboloidal dish, it will be moving from east to west in so doing the north south axis will be at different tilts. Always, the Sun's rays are parallel to the outer normal to the aperture plane which may be called this is the aperture. So, cos theta is equal to 1. So, let us recapitulate five modes of tracking we have considered.

One is a east west axis tracking horizontal with one single daily adjustment which gives the condition phi minus beta equal to delta. The second one is again a east west horizontal axis with continuous adjustment so that angle of incidence is a minimum, that gave a condition from which we found phi minus beta should be something which will be obviously better than the first mode of tracking. The third mode is a horizontal north south axis. The aperture plane being rotated facing east in the morning to facing west in the afternoon for which the condition will be, your azimuthal angle should be minus 90 in the forenoon and should be equal to plus 90 as per notation in the afternoon.

And if you set the general equation for cosine theta or the angle of incidence and differentiate with respect to beta that is the slope you will find, what should be the slope at different instance and it gives the long expression for the cosine angle of incidence for the surface tracked along a horizontal north south axis. The last one, fourth one is a polar mount. It is as if the surface is rotating parallel to the Earth's axis. So, consequently cosine theta should be equal to cos delta or theta is equal to delta.

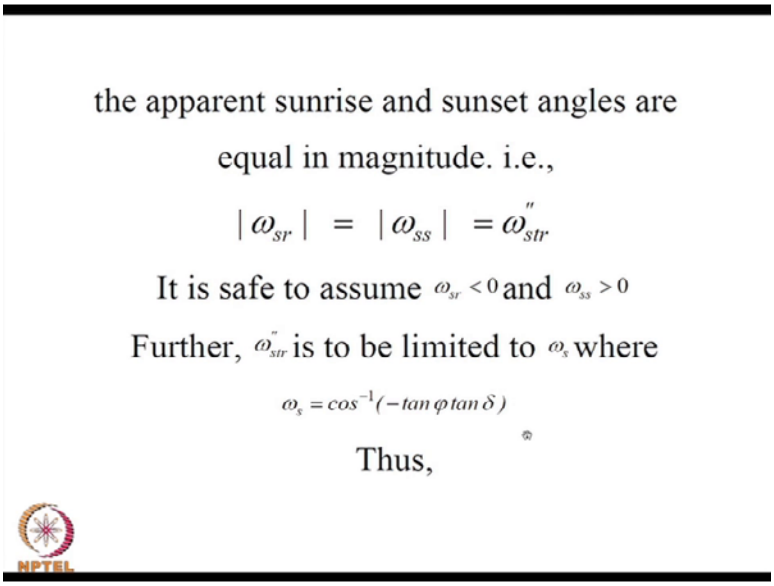
(Refer Slide Time: 17:35)



So, a word about this is theta will change from plus 23.45 to minus 23.45. In other words plus or minus it really does not matter because it is a cosine theta equal to cos delta. So, your theta will have the largest value of 23.45 magnitude, cosine theta will be almost 0.92 or 0.95, not much different from cos theta is equal to 1 though it is a single axis tracking compared to double axis tracking where cos theta compare cos theta is equal to cos delta for tracking mode four and cos theta is equal to 1 for tracking mode five.

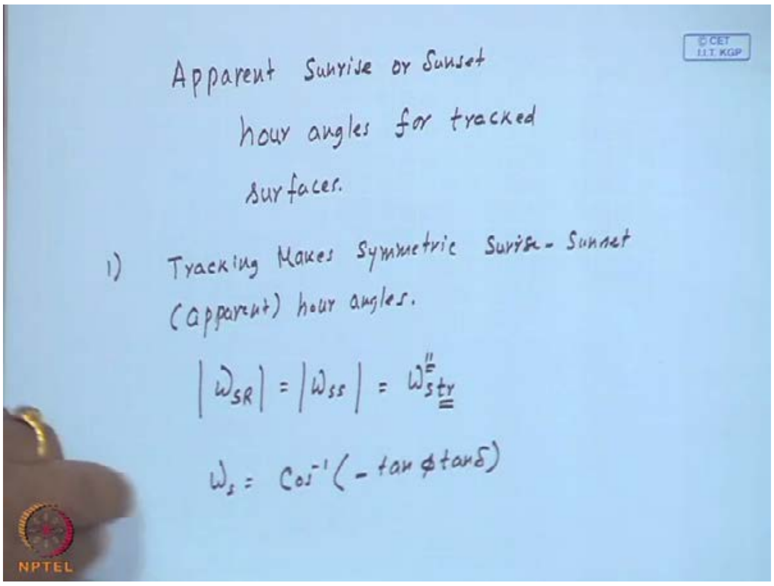
Two axis tracking, so whereas this is a single axis tracking. So, this does not differ by more than 10 percent compared to cos theta being equal to 1. Now, we obviously, this will result in a similar change in the R b factor for the tracking surfaces.

(Refer Slide Time: 19:01)



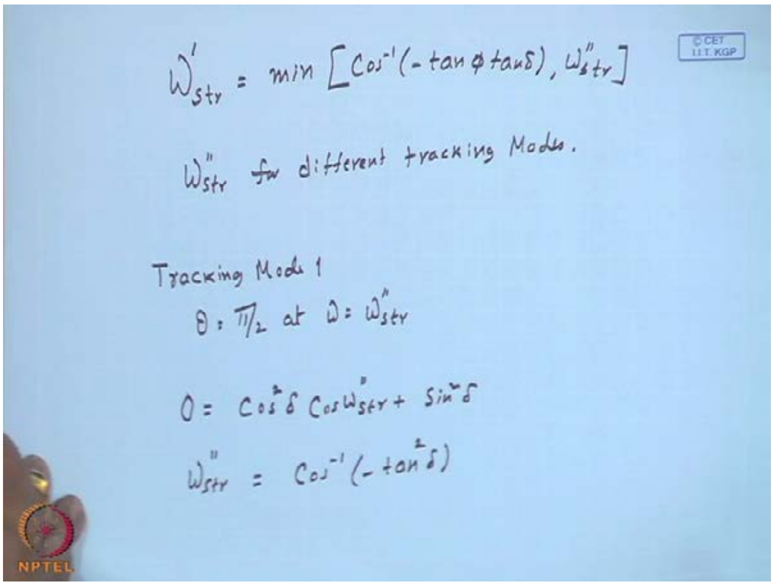
Now, apparent sunrise or sunset hour angles for tracked surfaces.

(Refer Slide Time: 19:04)



First thing, we realize is tracking makes symmetric sunrise and sunset apparent hour angles. So, if I designate it with our conventional notation of omega S R modulus should be equal to modulus of omega S S which I shall call it simply omega S tracking double dashed because we are considering omega S dashed from the south facing fixed surface. So, this t r indicates that it is tracking and we have used omega S double dashed as the hour angle as seen by the tracking surfaces without or before emitting it to the physical sunrise or sunset hour angle. So, physical sunrise sunset is given by omega S cos inverse of minus tan phi tan delta, you can recall one of our very early equations.

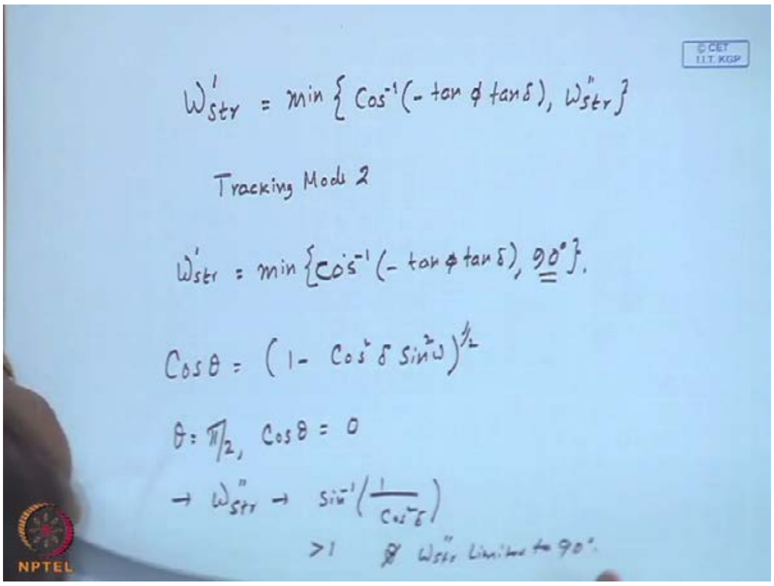
(Refer Slide Time: 21:13)



So, in our standard notation omega S dashed, but for a tracking surface it will be the minimum of cos inverse minus tan phi tan delta and this number omega S t r double prime. Now, we need to find out omega S t r double prime for different tracking modes. So, tracking mode one, how do we do? Theta should be equal to pi by 2 at omega equal to omega S t r double prime giving rise to 0 should be equal to cos square delta cos omega S t r double prime plus sin squared delta.

This I am, we have done it, I have written it elaborately by setting cos theta the simple equation for the tracking mode 1 equal to 0 because theta is equal to pi by 2 the hour angle at which that occurs is the apparent sunrise sunset hour angle for the plane rotated about a horizontal east west axis with one single daily adjustment. Omega S t r double prime thus will be equal to cos inverse tan square delta. This coincides with our south facing surface because phi minus beta is equal to delta, cos inverse of tan phi minus beta tan delta now becomes tan squared delta.

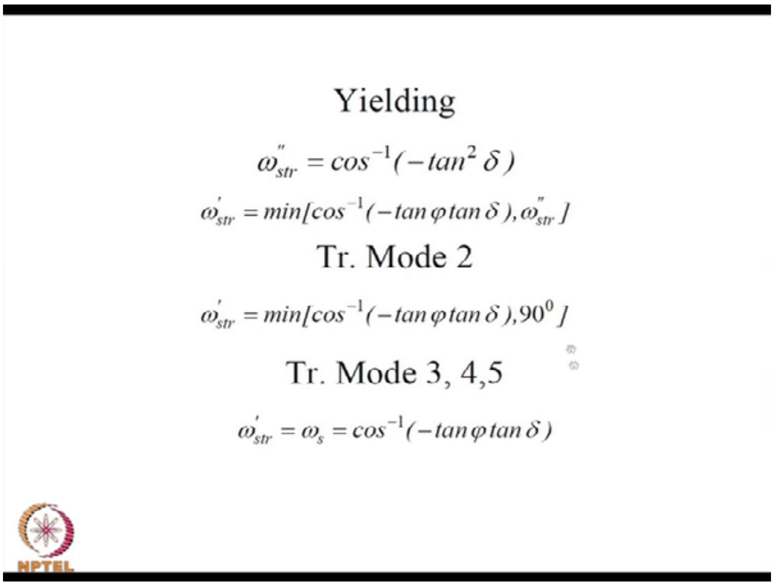
(Refer Slide Time: 23:32)



So, you have got finally, omega S t r dashed should be equal to minimum of cos inverse minus tan phi tan delta and omega S t r double prime. So, if you do the same thing for tracking mode two set theta is equal to pi by 2 omega S t r dashed will be again minimum of physical sunset hour angle which is cos inverse minus tan phi tan delta and 90 degrees. Now, you will find that this 90 degrees come from, if you look at your tracking mode two, this is the one, cos theta is 1 minus cos squared delta sin squared omega to the power one half.

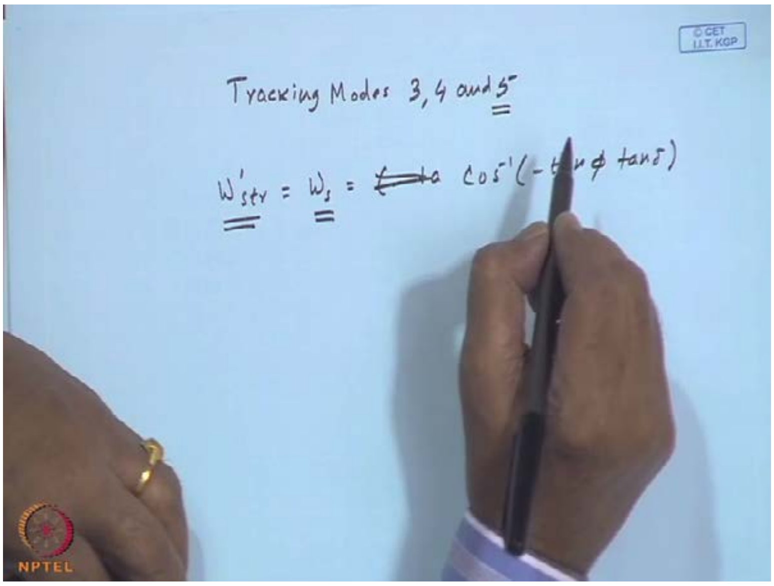
So, if you put theta is equal to pi by 2 and cos theta hence equal to 0 solve for omega S t r double prime, this will be something like sin inverse 1 upon cos squared delta, is it right? If I, this is 0 so I can square it up, sin squared omega will be equal to 1 by cos squared delta. So, sin inverse of 1 by cos squared delta. So, this is going to be greater than 1. So, theta should be sorry omega S t r double prime be limited to 90 degrees. The logic being it will pass through 90 and beyond that there is no solar radiation received by the tracking mode two.

(Refer Slide Time: 26:36)



Let us get back to our other things, tracking mode two is over.

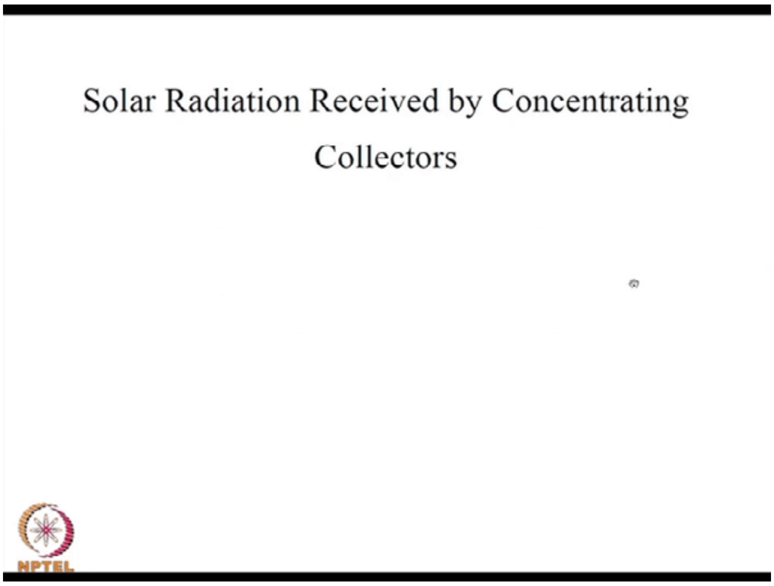
(Refer Slide Time: 26:40)



For tracking modes three, four and five, three is a horizontal north south axis facing east in the sunrise time and facing west in the sunset time consequently my omega S t r single dashed should be the same as omega s should be equal to minus cos inverse minus tan phi tan delta. Similarly, this is for tracking mode four. It is a polar mode again a north south axis tilted parallel to the Earth's axis. So, it will see this on all the time the sun is above the horizon.

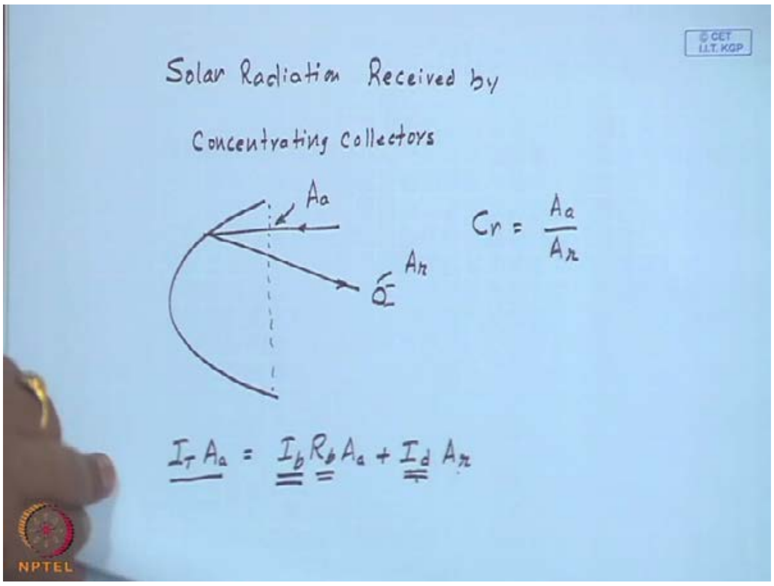
Consequently, again omega S t r dashed will be the physical sunrise or sunset hour angle given by cos inverse minus tan phi tan delta. Of course, there is no ambiguity as far as the mode five of tracking is considered that is because it is a double axis at every instant the Sun's ray is normal to the aperture plane so it will see the sun all the time, the sun is above the horizon.

(Refer Slide Time; 29:10)



Now, why we are doing it is, parallel to the fixed surfaces we will try to estimate solar radiation received by concentrating collectors. So, now we know the time duration over which the rotating or tracking plane receives the solar radiation.

(Refer Slide Time: 29:37)

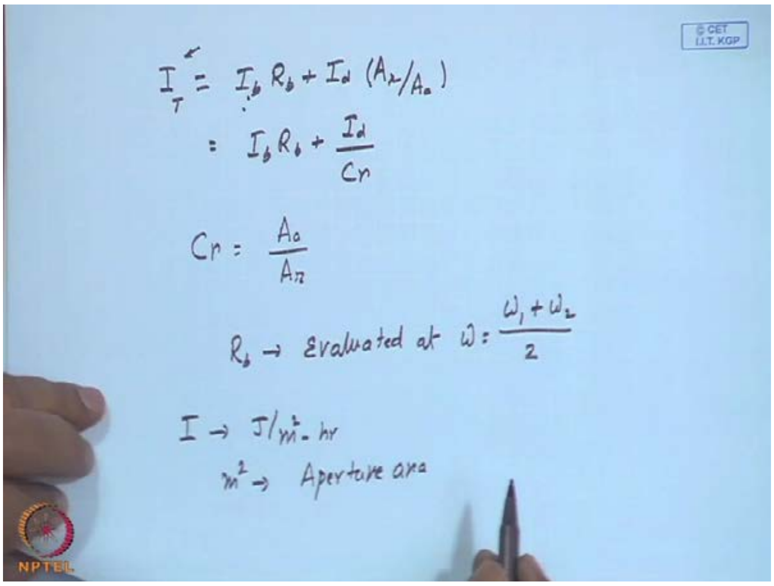


So, if to get back this is a parabolic reflector of aperture area A a reflected on to the receiver of receiver area A r. So, per unit area of the aperture if I T is the solar radiation received per meter square multiplied by A a is the total radiation received by the aperture should be equal to I b R b, whatever is the radiation of the horizontal plane multiplied by the R b factor for the aperture which is been tracked times A a plus I d times A r.

So, this is the total amount of solar radiation received by the aperture which gets reflected according to some optical efficiency, that should be equal to whatever is the beam radiation received plus whatever is the diffused radiation directly received over the projected area of the receiver, because the diffused radiation that passes through the aperture will not be reflected according to the loss of optics and it may not be captured by A r.

Nevertheless, whatever is the diffused radiation that is directly captured by the receiver will be I d into A r. This is all the difference between the flat plate collector and the tracking collector. And we deliberately write it in terms of the area of the aperture because that is what is going to indicate how large the collector is and depending upon the area of the A r the ratio is going to give us what is called the geometric concentration ratio C r is A a upon A r.

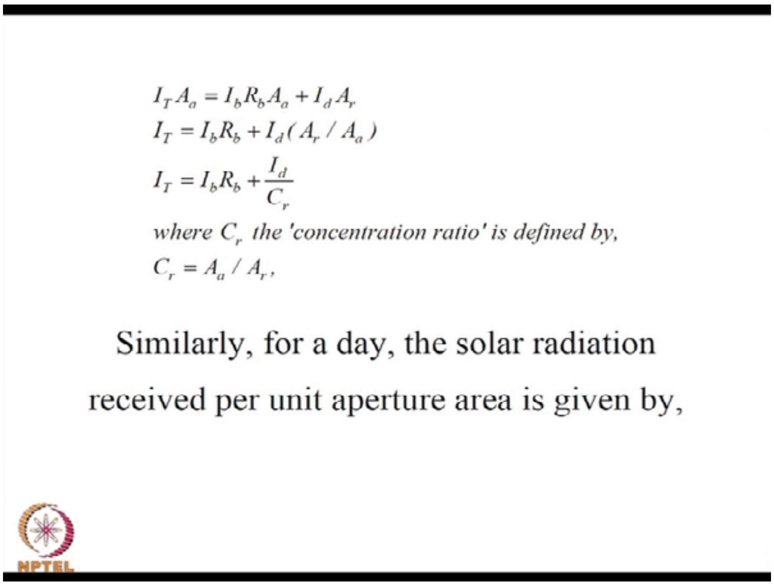
(Refer Slide Time: 32:38)



So, now you will find I t equal to I b R b plus I d times A r by A a which is I b R b plus I d by the geometric concentration ratio C r. So, if C r is very large this will be a pretty small a component where C r once again is defined as A a upon A r. So, this is for any instance which we normally use it for a period of one hour and R b being evaluated as we have done for flat plate collectors at the midpoint of the hour characterized by the two hour angles omega 1 and omega 2 by 2.

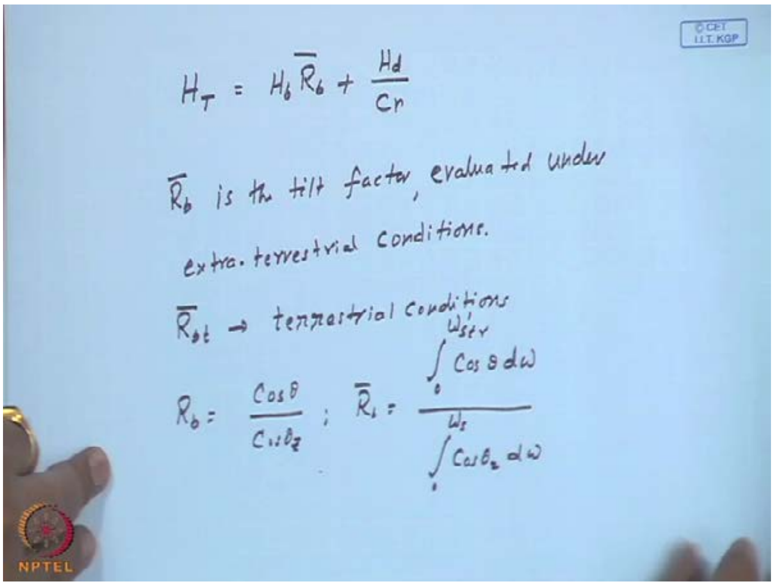
And I b is the solar direct radiation on a horizontal surface and I d is the diffused radiation on a horizontal surface. So, this I t we should say is so many joules per meter square hour let us say and this m squared is pertains to aperture. In other words this is a radiation received per unit area of the aperture.

(Refer Slide Time: 34:28)



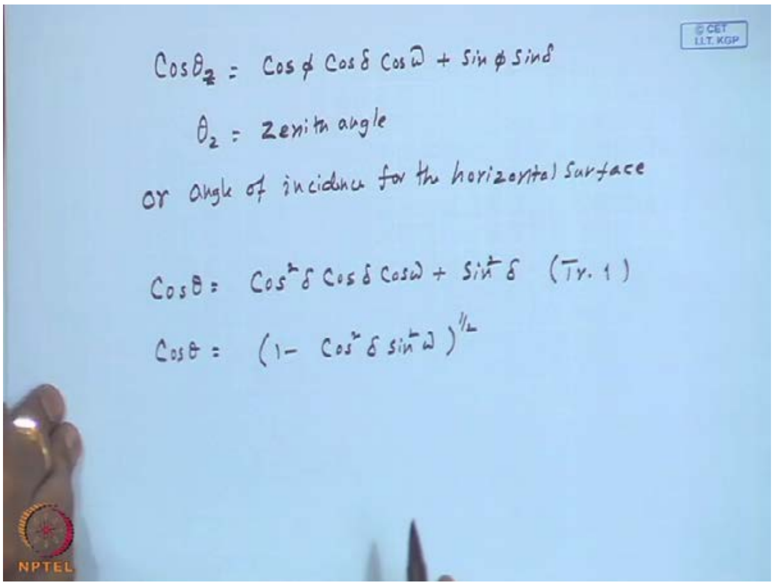
Similarly, for a day you can sum up and you will get…

(Refer Slide Time: 34:32)



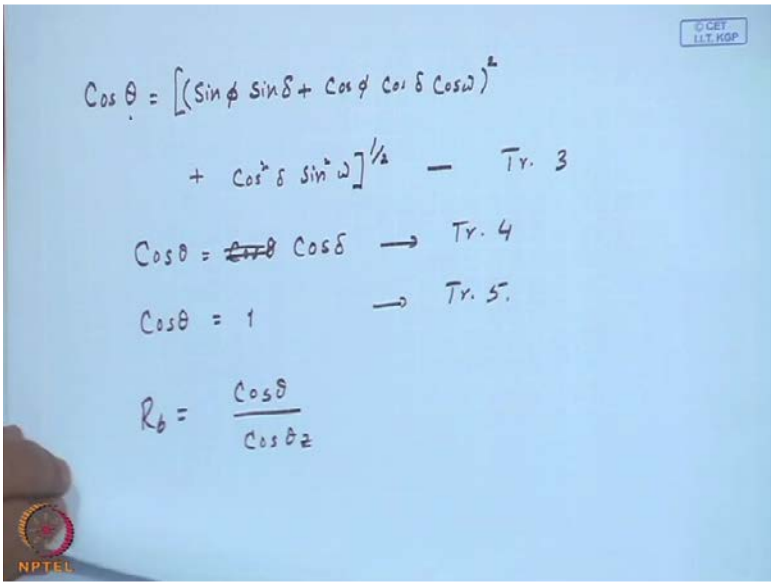
According to our notation H T will be H b times R b bar plus a fraction due to the diffused radiation caught by the receiver. Now, R b bar is the tilt factor for the day evaluated under extra terrestrial conditions according to our notation because if you recall R b bar t is evaluated under terrestrial conditions. Mind you, your R b at any instance is cos theta by cos theta z and R b bar is under extra terrestrial conditions integral 0 to omega S t r dashed cos theta d omega upon integral 0 to omega S cos theta z d omega… Where cos theta z is given by cos phi cos delta cos omega plus sin phi sin delta where this is nothing but the theta z the zenith angle or the angle of incidence further horizontal surface.

(Refer Slide Time: 36:27)



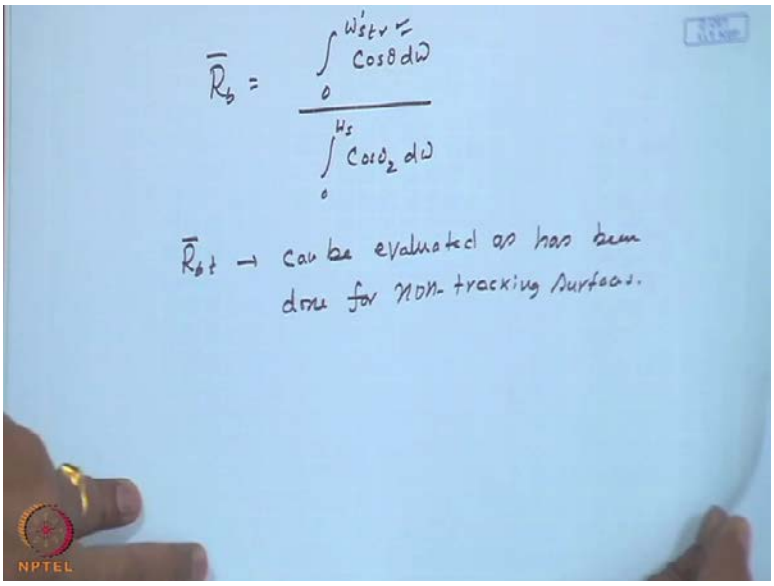
So, this is a equation that we already knew and used a large number of times. Now, cos theta for the purpose of calculating R b or R b bar for focusing collectors is given by let me repeat, so that it will be with you, you remember that the appropriate cos theta expression needs to be used for the tracking collectors. So, this is tracking mode one and for mode two, to the power one half.

(Refer Slide Time: 38:13)



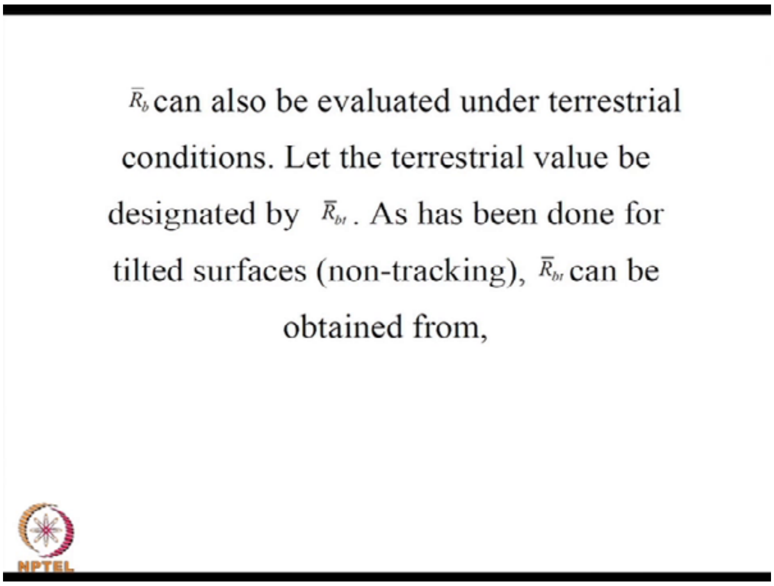
For tracking mode three, sin phi sin delta plus cos phi cos delta cos omega whole squared plus cos squared delta sin squared omega whole to the power one half. So, you use this expression in calculating R b bar for tracking mode three or R b for tracking mode three. And of course, cos theta is simply, this is for tracking mode three is simply cos delta sorry for tracking mode four and cos theta is 1 for tracking mode five. So, I can now evaluate R b as in general cos theta by cos theta z depending upon the tracking mode.

(Refer Slide Time: 39:53)



And R b bar integral 0 omega S t r dashed cosine theta d omega upon 0 to omega s cos theta z d omega. We already know the expressions for omega S dashed t r and theta z is the zenith angle which we already know.

(Refer Slide Time: 40:31)



Now, we have done this calculation under terrestrial conditions for fixed orientation surfaces which we called it R b bar t. We can do, similar can be evaluated as has been done for non tracking surfaces. You should realize that the definition in terms of the radiation components remains the same. What differs is the R b factor or specifically cosine theta expression that needs to be used depending upon the tracking mode.

(Refer Slide Time: 41:32)

$$\overline{R}_{bt} = \frac{\int_0^{\omega_{str}} \left(a + b\cos\omega - D_f\right)\cos\theta \, d\omega}{\cos\phi\cos\delta\int_0^{\omega_s} \left(a + b\cos\omega - D_f\right)(\cos\omega - \cos\omega_s)\, d\omega}$$

$cos\,\theta$ is one of the expressions obtained depending on the tracking mode.

Expressions for $\overline{R}_b$

So, I can start with a slightly second or third step as we have already done this in the case of fixed surfaces a plus b cos omega minus D f times cosine theta d omega upon cos phi cos delta integral 0 to omega s a plus b cos omega minus D f times cos omega minus cos omega s d omega. Let me quickly recapitulate, what we did in the case of fixed collectors or non tracking surfaces. We expressed I b as I minus I d as I into 1 minus…