# Solar Energy Technology
**Prof. V.V. Satyamurty**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kharagpur**

## Lecture - 14
## Theory of Flat Plate Collector-Liquid Based (C)

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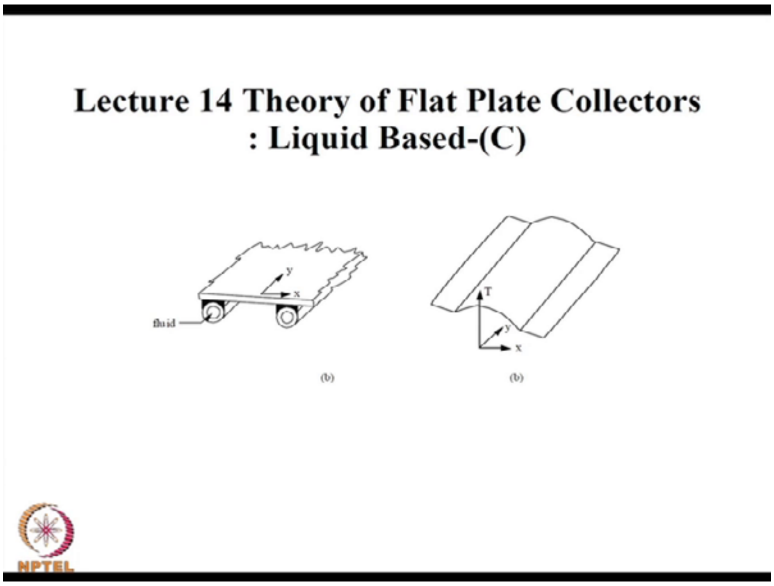


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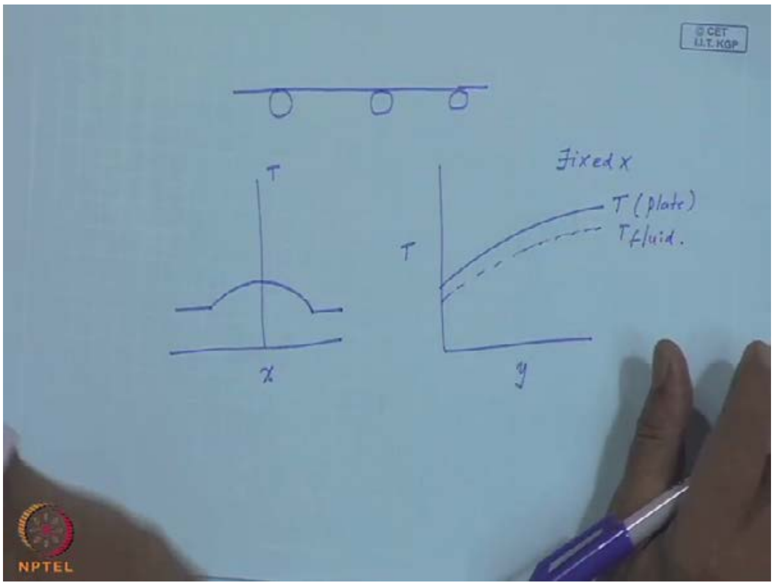


So last time, we have considered the theory of flat plate collectors, particularly the liquid heaters, which will consists of an absorber, with tubes attached at the bottom.

Subsequently we have shown that the temperature distributions, in the direction of x, a constant over the region of the tube, and with a maximum at the middle, of the plate in between the two tubes. This is temperature axis, this is the x axis. Similarly, if you want to have temperature, at any fixed x, it changes with respect to y, fixed x and this is T. This may be in general T anywhere on the plate, and the corresponding T fluid temperature, will be a bit lower than that, depending upon the how efficient the heat transfer mechanism is.

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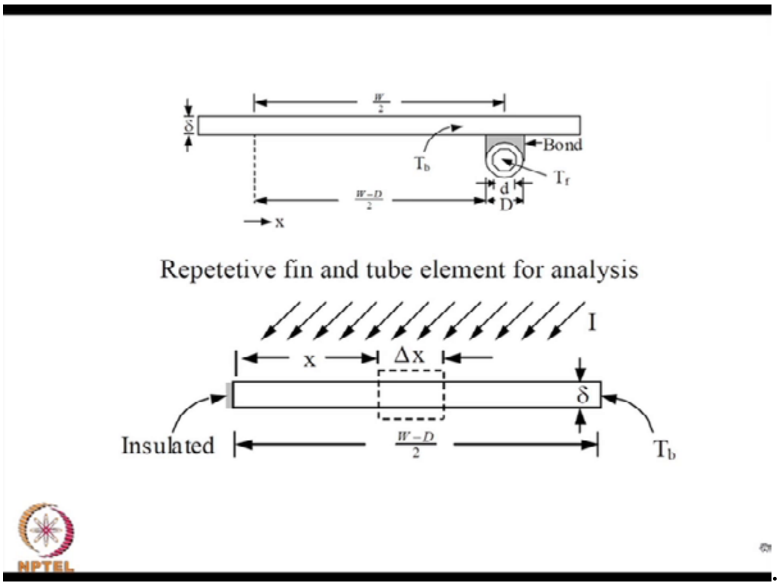


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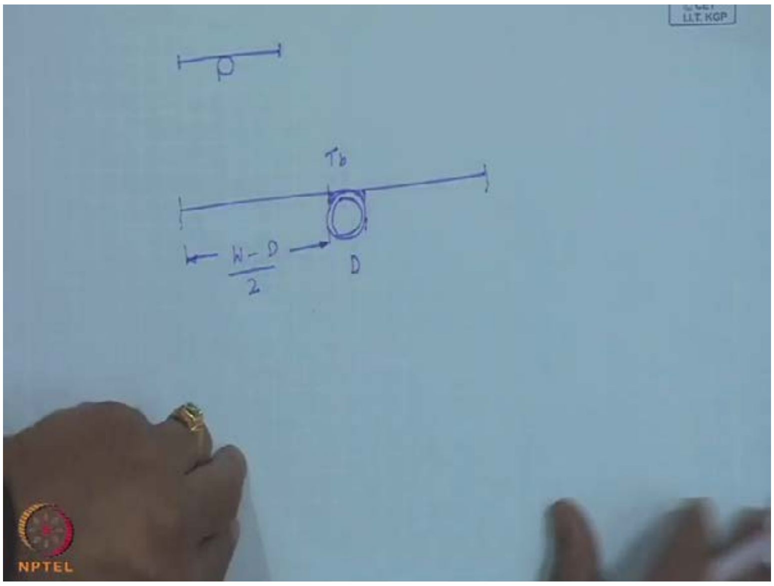

Then we have gone through the analysis, considering a repetitive element of a tube, and half a plate on either side, with the distance being W. Let me draw it bigger. There will be inside diameter D i, and outside diameter D. This is the repetitive element, and this distance is W minus D by 2, if the diameter is D, and this is at a temperature of T b, and this is bounded by soldering, or welding to be tube. We have subsequently considered an elemental volume.

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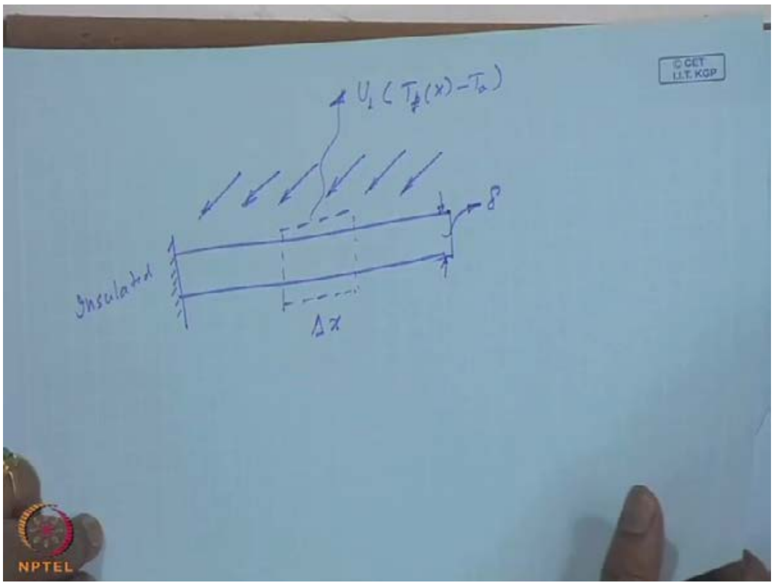


In the let us say the fin area, the mid plane choose the temperature is maximum, can be considered as insulated, with the element being shown, of elemental length delta x. In general this is receiving the solar radiation, and it will be losing also heat, by a loss coefficient, times T p, function of x minus T a, this p is unnecessary, it is a function of s and the thickness of this, is delta; not to be confused with the declination, they do not appear at any time, the same time.

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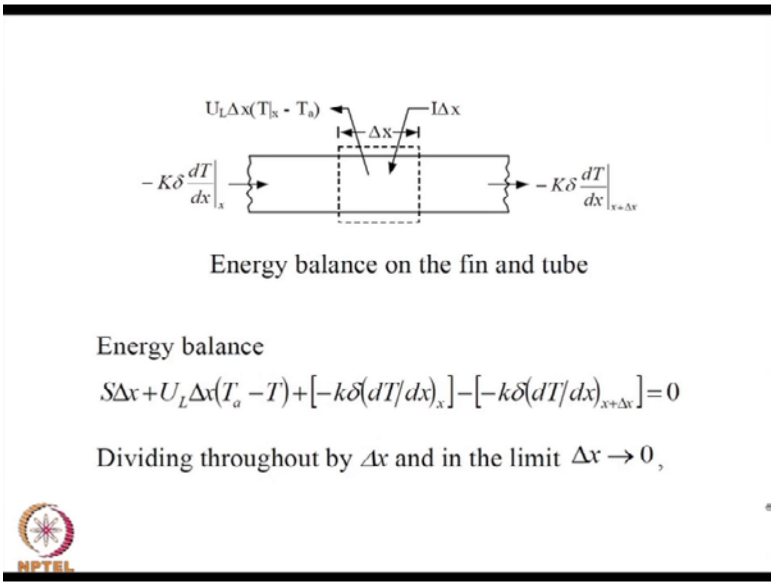


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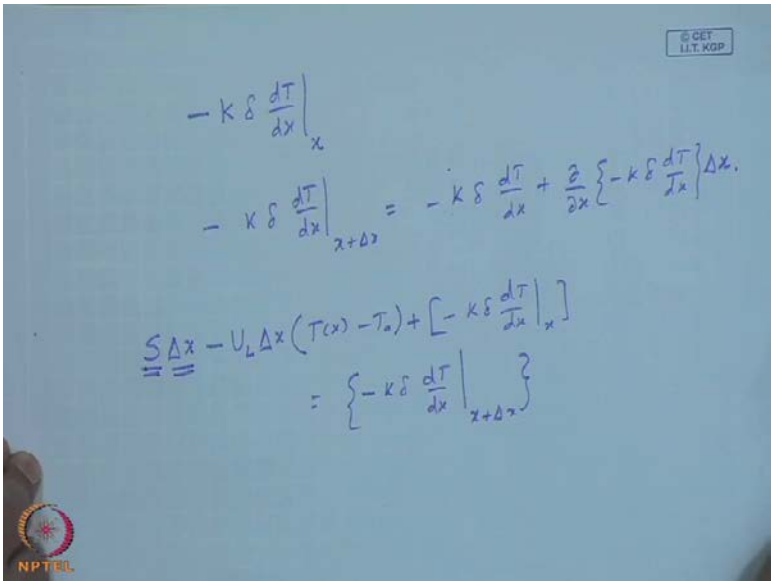


So we have set up that a conduction heat transfer of k delta d T d x, enters at x, and leaves as k delta d T d x at x plus delta x, which is nothing but, minus k delta d T d x plus d by d x of minus k delta d T d x evaluated at multiplied by delta x. In the limit, we made an energy balance, of the incoming radiation delta x, and then a loss U L times delta x into T function of x minus T a plus minus k delta d T d x at x, should be equal to minus k delta d T d x evaluated at x plus delta x. So this is written as a balance equation. Incoming radiation is the absorbed energy, multiplied by delta x, and a unit length perpendicular to the plane of the paper, minus the losses over the same area, and what enters is minus k delta d T d x, and what finally, goes out is minus k delta d T d x, at x plus delta x.

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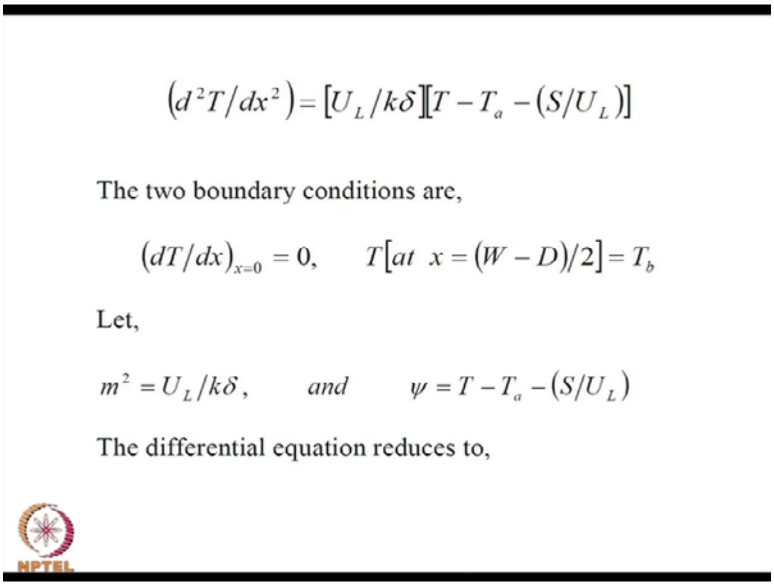


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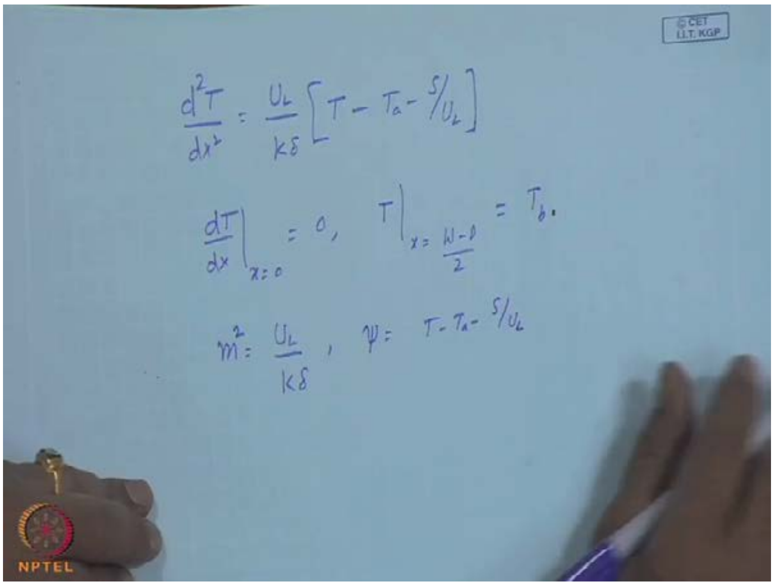


This quantity, if you take the limit and x plus k delta d T d x at x plus delta x. You will get an equation, d squared t by d x squared equal to u l upon k delta times T minus T a minus s upon U L. You can check all of them have the same dimensions of temperature, and the two boundary conditions should be d T d x at x is equal to 0 0, and T at x is equal to W minus d by 2; that is at the base of the pipe, or the tube will be equal to T b. And of course, we define the parameter U L upon k delta as m squared, and psi as T minus T a minus s by U L.

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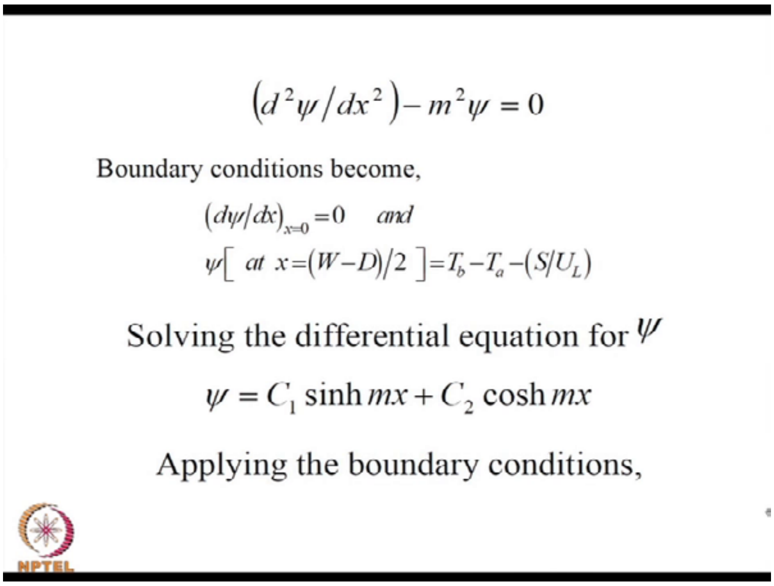


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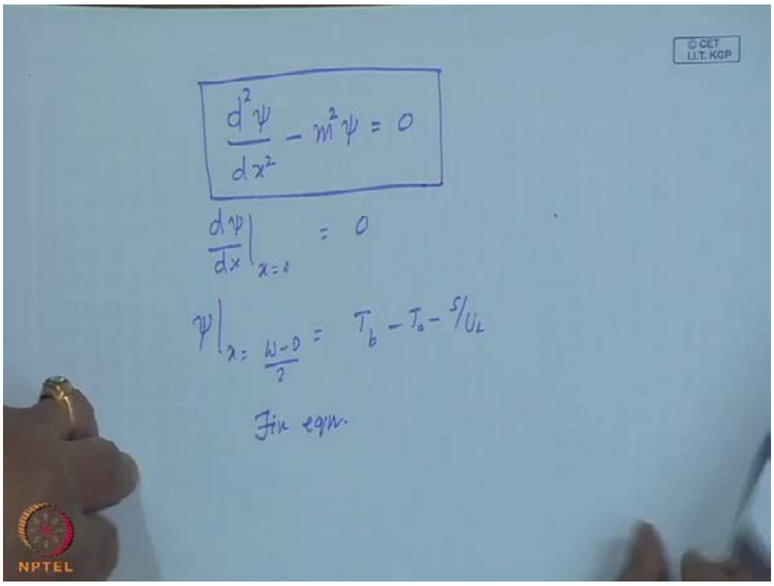

So this simplifies to an equation d square psi by d x square, minus m square psi equal to 0, with d psi d x at x equal to 0 as 0, and psi at x is equal to W minus D by 2, should be equal to T at the base minus T a minus s upon U L. Now this equation, should (( )), you must come across, in your heat transfer course, this is nothing, but a fin equation.

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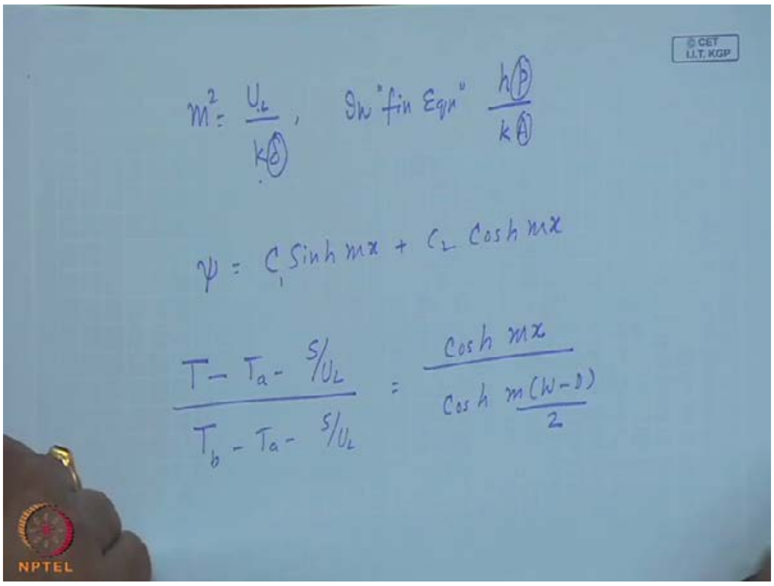


And you will find that the parameter m squared equal to U L by k delta, where as in the conventional fin equation, it is h perimeter upon k a. You can see both of them are non dimensional, h corresponds to U L and k is k, instead of one length dimensional the denominator, you have a length, and square of the length, in the denominator, in the conventional fin equation. This is absolutely the same as the fin equation, which you must have gone through, you are in your heat transfer course. The general solution for psi will be c sin hyperbolic m x plus c 2 cos hyperbolic m x.

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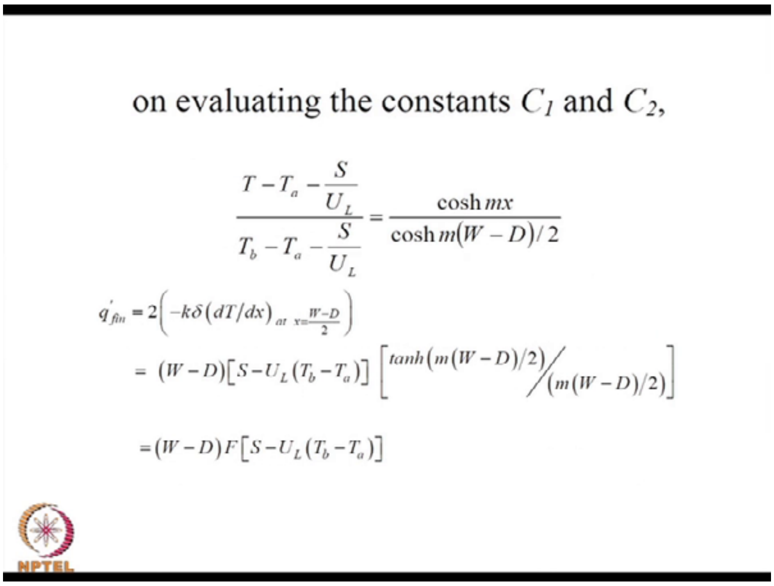

And if you apply the boundary conditions; that d T d x is equal to 0 or d psi d x is equal to 0, or psi equal to T b minus T a minus S by U L, you will obtain the temperature solution as T minus T a minus S by U L by T at the base, minus T a minus s upon U L as cosine hyperbolic m into x by cosine hyperbolic m into W minus D upon 2. Now we got the temperature distribution in the x direction at a fixed y.

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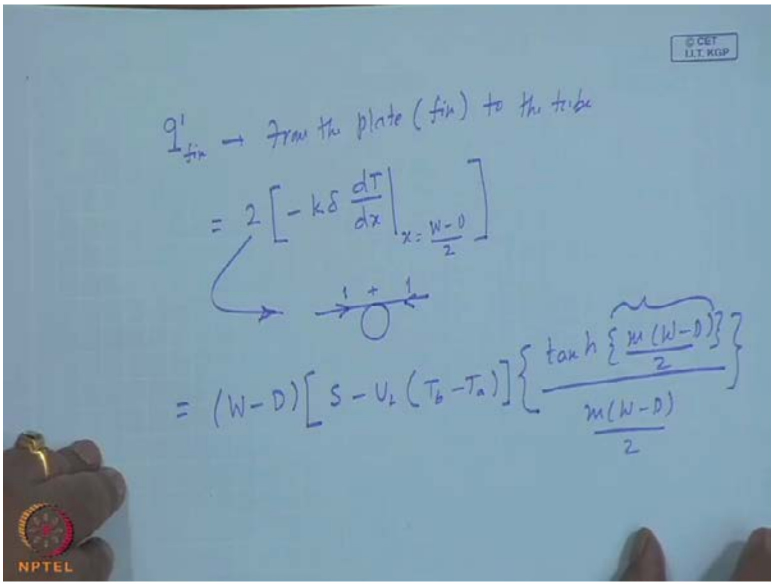


And if you look at the contribution from the fin; that is from the plate, or fin, to the tube, should be equal to, two times minus k delta, multiplied by unit length perpendicular to the plane of the board or paper, evaluated at x is equal to w minus d upon 2, and this 2 comes from. There are two half's; one plus one, that becomes two. So that if you evaluate you will have a W minus D times S minus U L times T b minus T a multiplied by tan hyperbolic m into W minus D upon 2 whole thing by m into W minus D upon 2, so m being non dimensional, so this entire quantity will have a length dimensional.

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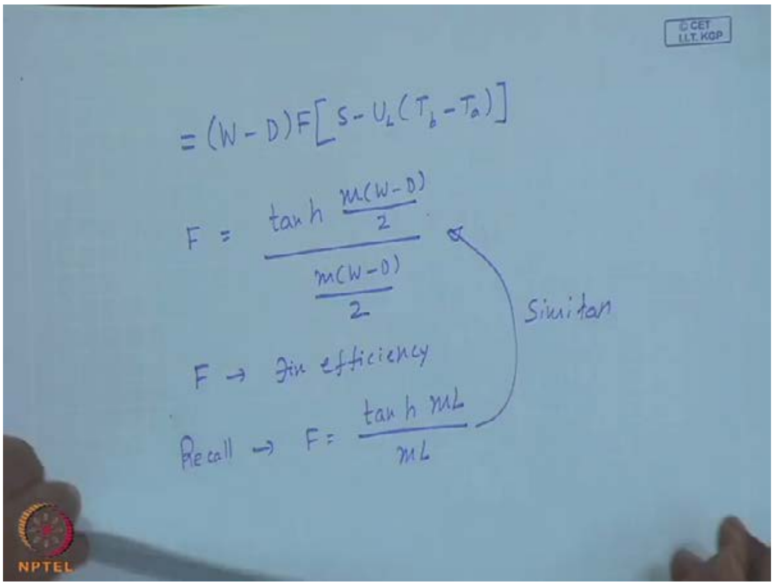


So this can be rewritten, as W minus D times F times S minus U L into T b minus T a. Now you will find that F is, tan hyperbolic m times W minus D by 2 upon m into W minus D by 2.

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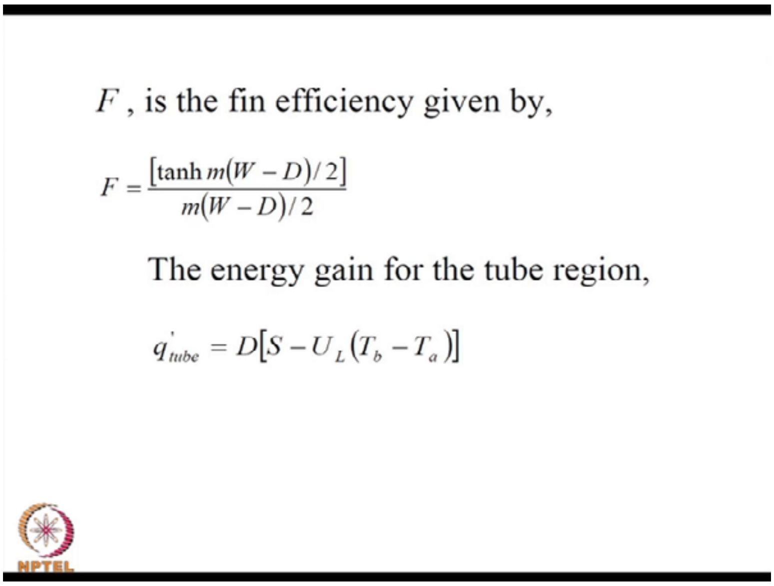


And this F, now we call it the fin efficiency. So it depends upon m, which contains the loss coefficient, thermal conductivity, and the distance between the plates W minus D by 2. Now recall, if there is a length of fin of length L, the fin efficiency is defined by tan h m L upon m L, for a certain boundary condition of negligible heat transfer from the tip, F is equal to tan h m L by m L, which is exactly similar to, what we got for the fin and tube type of absorber.

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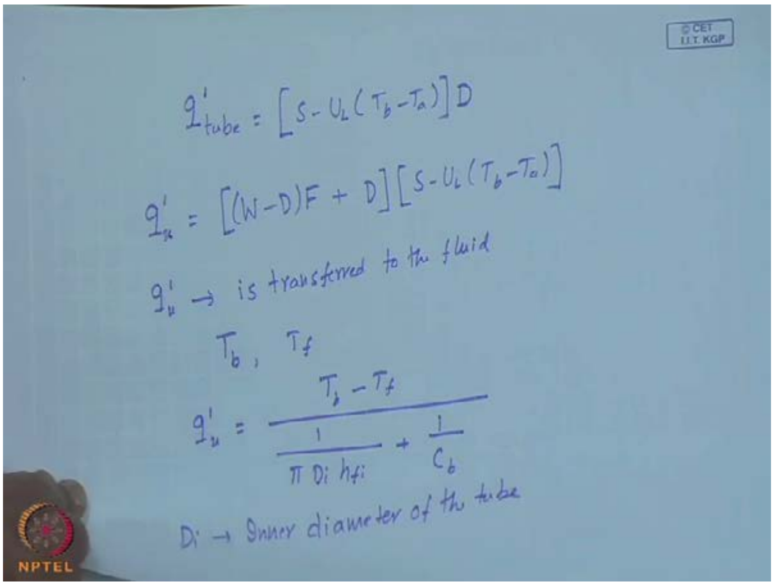


Now, the total energy gain, also will comprises of, the energy gained from the tube region, which will be S minus U L into T b minus T a. We assumed that there is no temperature variation on the periphery of the tube, so that will be the same as the base temperature T b, and D is the projected length, and multiplied by the unit length, perpendicular to the plane of the paper, will be the corresponding area. So q dash tube per unit length, of the collector will be S minus U L into T b minus T a times D. Now useful energy gain, should be comprising of, what came from the plate region, plus what is gain from the tube region, which when combined, will give me nothing but, W minus D times F plus D times S minus U L times T b minus T a.

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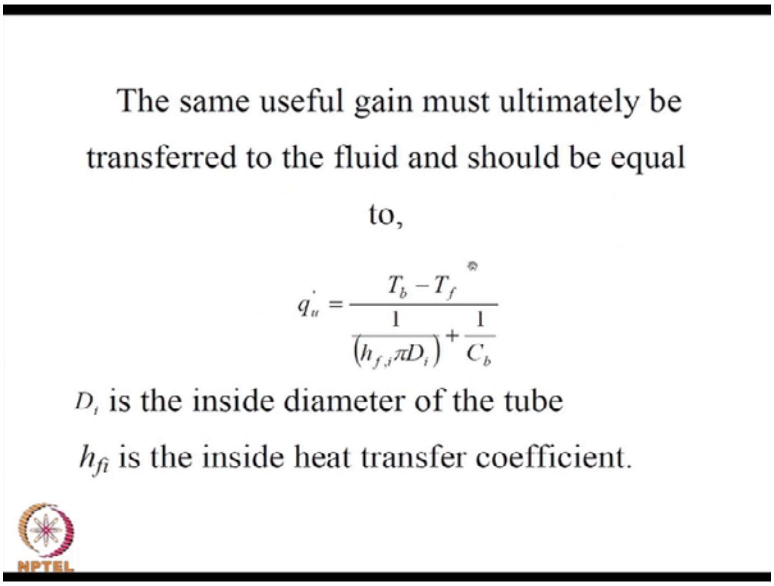


Now the same q u dashed, is transferred to the fluid. So T b, if it is the tube temperature, and T f is the fluid temperature, by q u dashed, should be transferred, with a potential of temperature difference of T b minus T f, upon the convective resistance pi D i h f i plus a resistance, due to bonding between the tube and the plate, D i is the inside diameter of the tube, and h f i is the heat transfer coefficient in the tube.

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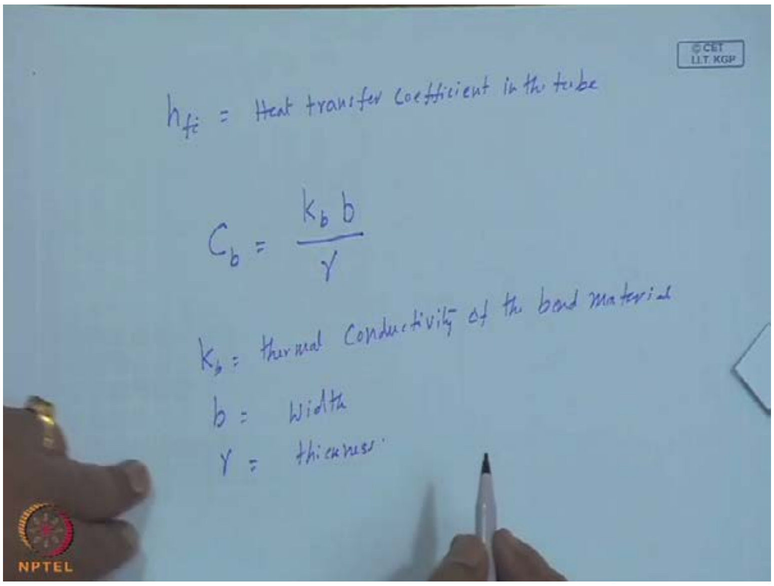

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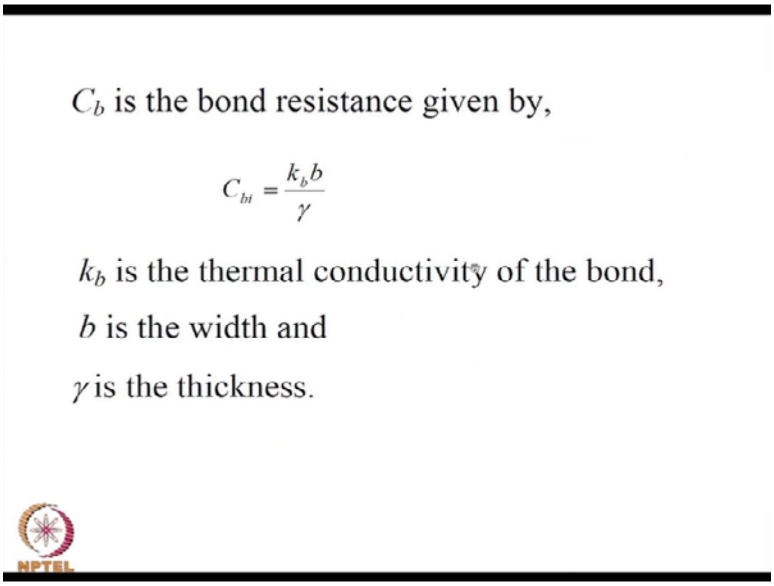


If the flow is laminar and fully developed, you have a 3.66 for constant temperature, and about 4.12 for constant heat flux. But the situation is neither constant heat temperature, nor constant heat flux, so it should be something in between. And the bond conductance C b is given by, thermal conductivity bond material, by the width of the, thickness of the bond material, and gamma is the thickness. k b thermal conductivity of the bond material, and b is the width, and gamma is the thickness. Once again, this gamma is not to be confused with the azimuthally angle. Of course, they do not appear side by side simultaneously.

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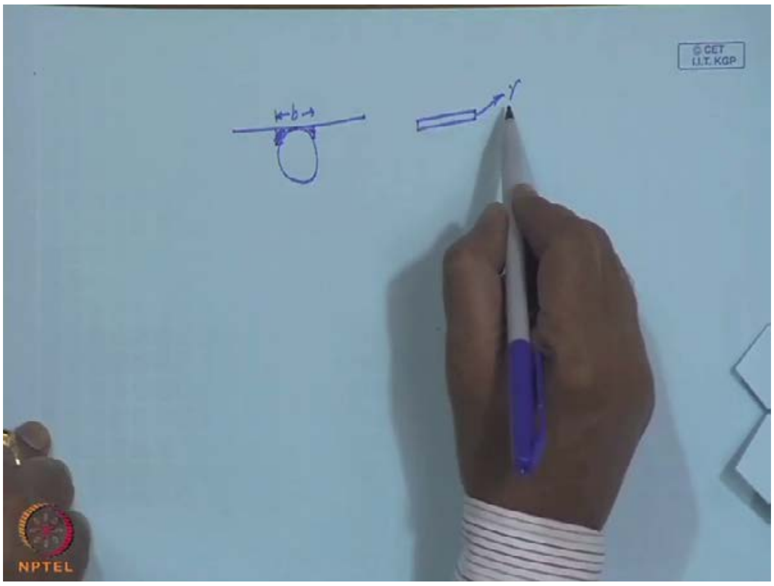

Now, let me explain a little bit, about this width and the thickness. If this is the tube, and I may put it like this, which may be equal to the diameter, or it may be less than the diameter. So this is my sort of width b, may or may be less than diameter, or greater than diameter, and thickness one way to define it is, we can equivalent material, this thickness will be gamma. In other words, if you how use this certain amount of material, over a width of b, with varying thickness depending upon the contour, of the tube. It may be equivalent to the same material of width b, and thickness gamma. So we equivalently evaluate the bond resistance by this method.

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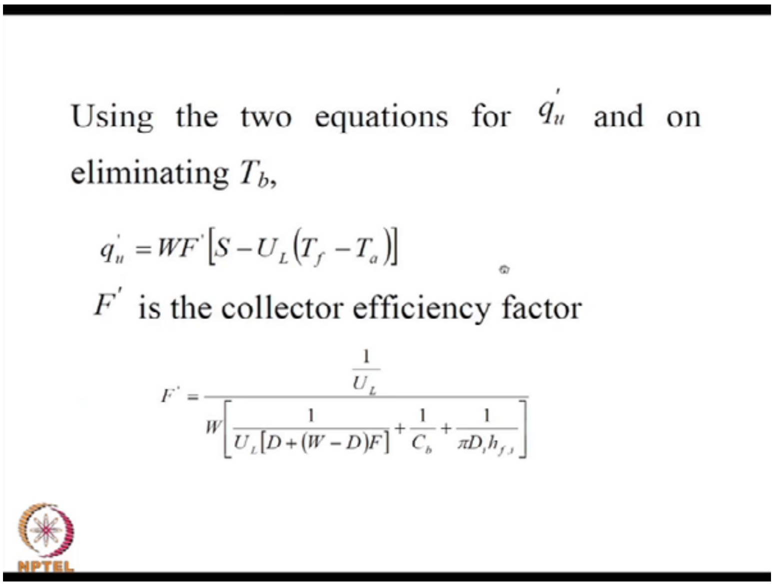


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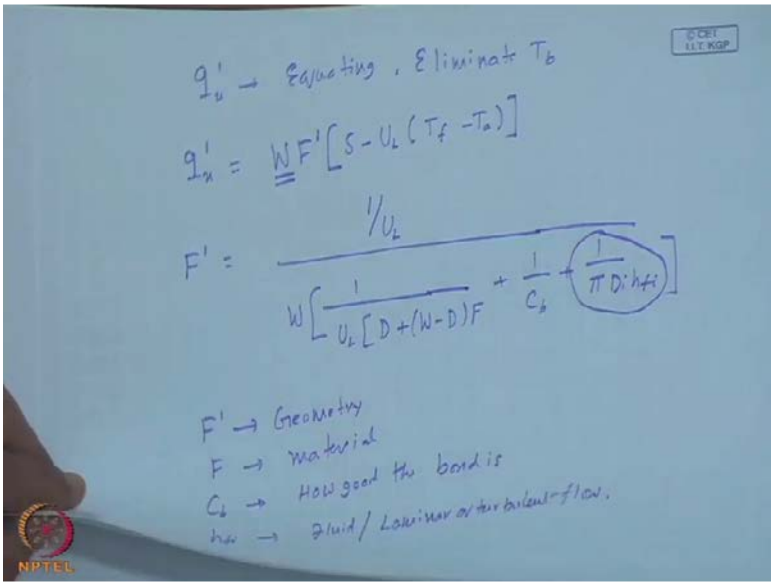

So if we equate q u dashed, what is transferred to the fluid, and what is coming from the plate, plus the tube region then, and eliminate T b; we do not want to have T b. Now I have something like an area, W multiplied by the unit length perpendicular to the paper will be the area, and that F dashed, whatever we are defining earlier as the collector efficiency factor, multiplied by absorbed radiation minus the losses, taking place from the local fluid temperature to the ambient. And here if you equate those two equations, F dashed turns out to be 1 upon U L, by W 1 by U L times D plus W minus D into F plus 1 upon the bond conductance plus one upon the convective conductance. Now you will find F dash, blinks in geometry by W, W minus D etcetera, and through f, it is the material plus the heat transfer coefficients, C b how good, good the bond is. And of course, this h f i, is an indicator of fluid, and laminar generally laminar, or turbulent flow, or at times, we may do something to enhance the heat transfer coefficient inside, so that this pi D i h f i will be a large and one by pi D i by h f i will be small, the resistance due to convection will be small.

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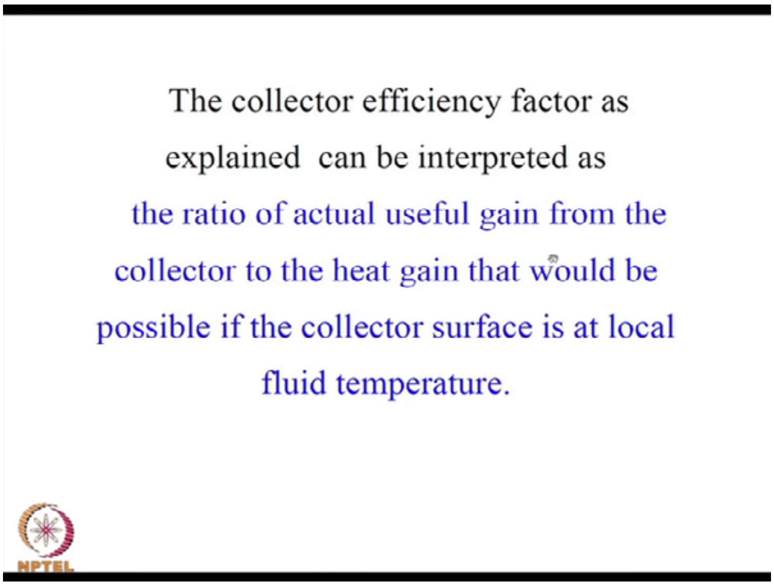

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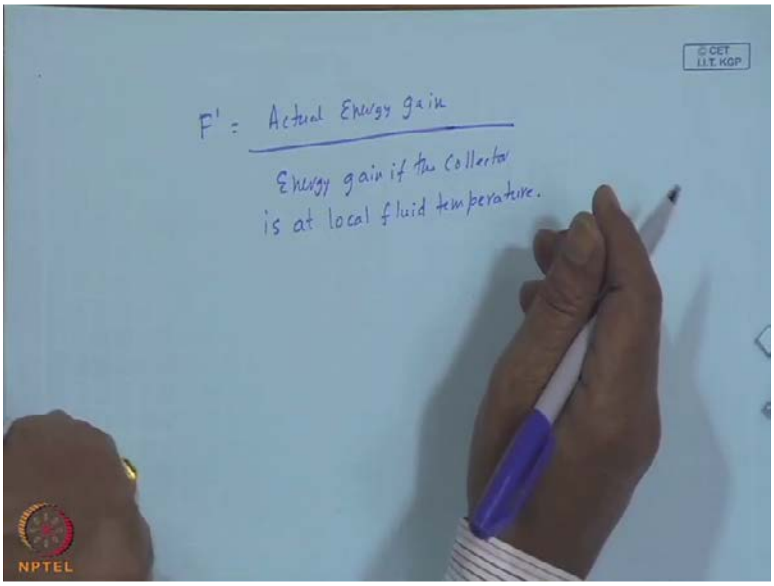


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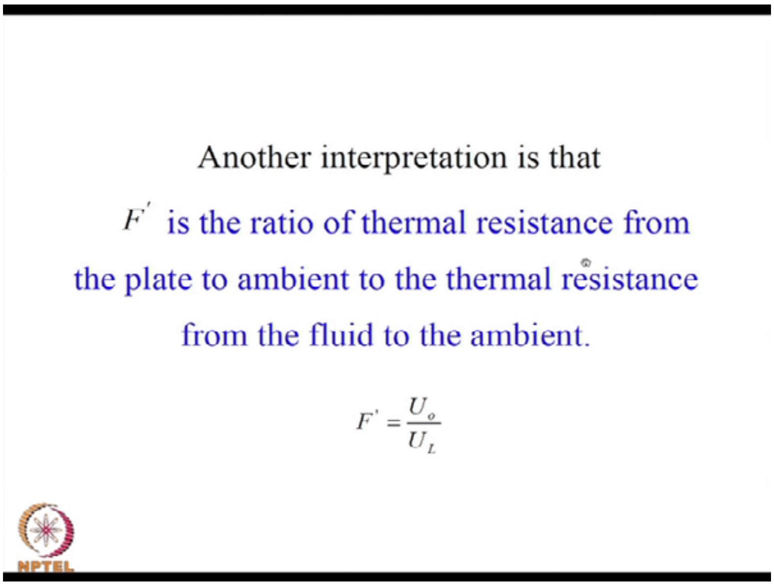


So, now when can interpret thus, actual useful energy gain, so F dashed, by energy gain, if the collector is at local fluid temperature. Now, another interpretation is possible.

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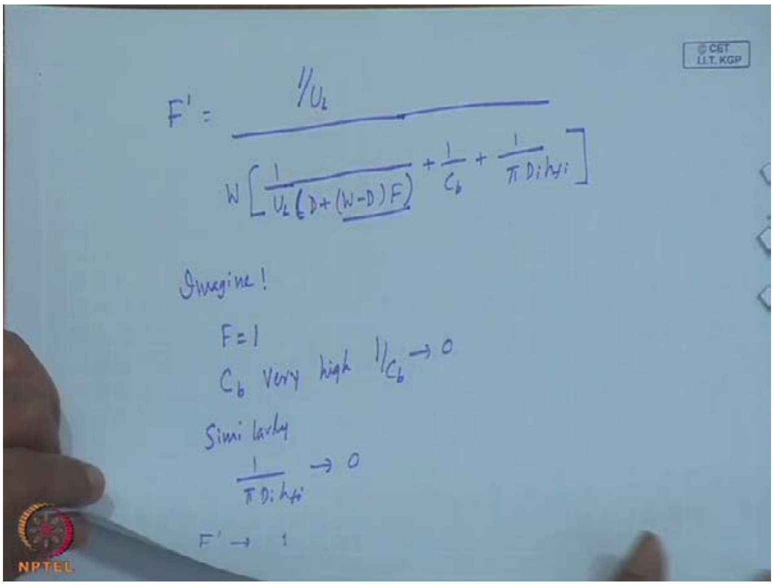


Now we have got 1 upon U L by this quantity; that is w times 1 upon u l into d plus w minus d times f plus 1 upon c b plus 1 by pi D i h f i. Let us for a moment imagine, F is equal to 1, it is a 100 percent fin efficiency, and C b is bond conductors very high, so that 1 by C b tends to 0. Similarly, 1 by pi d i h f i also goes to 0. If you have a very large heat transfer coefficient, the converter is being negligible. So my F dashed tends to 1, so this term is 0, U L into. I think this should be w. So you will have F dash tending to 1, if this bond conductance is infinity.

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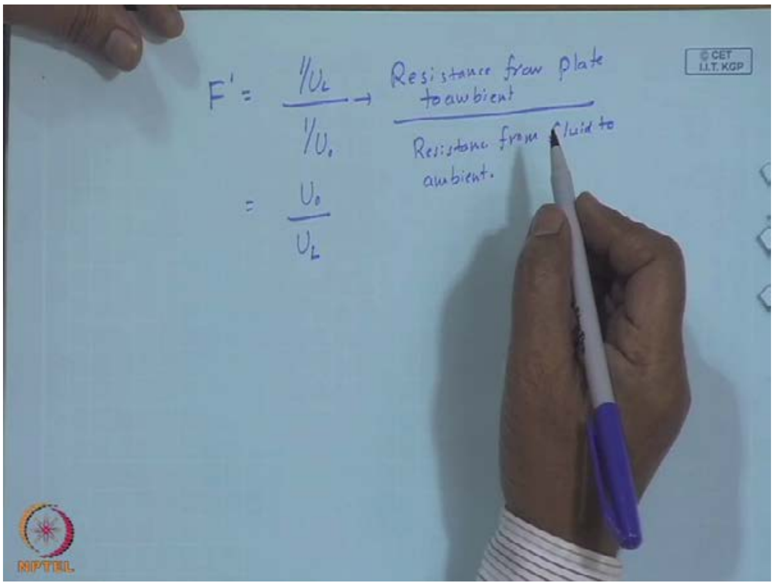

If the converter resistance is 0, so this U L U L gets cancelled and F dashed will turn out equal to be 1. Or one can write down 1 upon U L divided by 1 upon U naught, which can be rewritten as U naught upon U L. This is resistance from plate to ambient, by resistance from fluid to ambient. So F dash is not only the actual energy gain, upon the possible energy gain, if the entire collector had been at the local fluid temperature, but it also can be defined as, the resistance from plate to the ambient, to the resistance from the fluid to the ambient.

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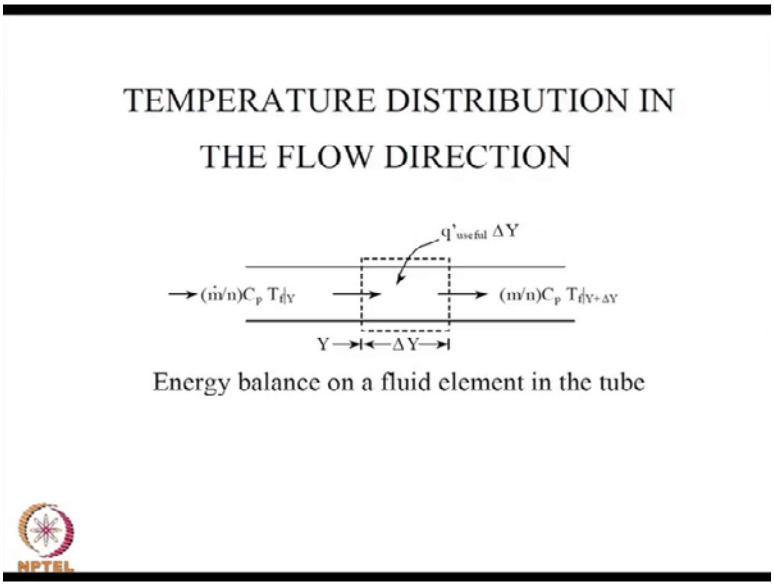


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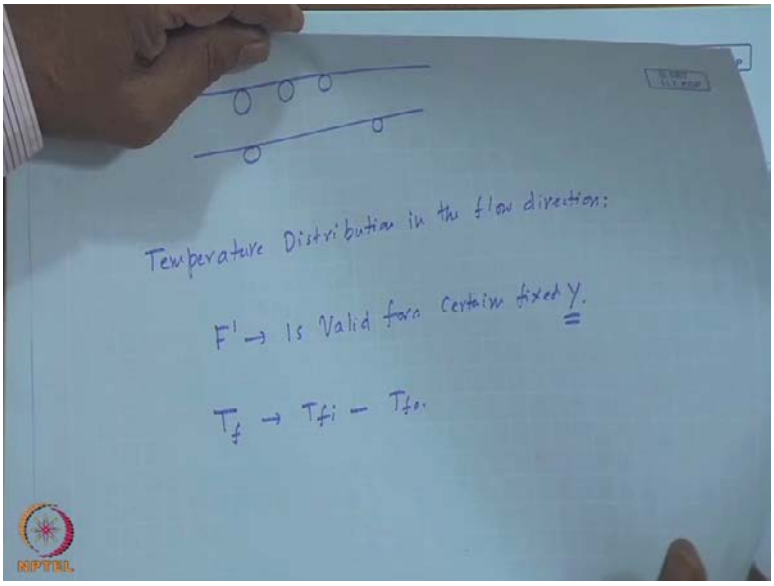

So now we got, out of the two one dimensional equations, temperature distribution along the x direction has been obtained, from that the fin efficiency comes into the picture, which will tell you, whether this tubes are pretty close to each other, or far off, so that my fin efficiency will be lower, because w minus b will be larger, and of course, the material comes to the picture the thermal conductivity. And finally, the energy transfer to the fluid will take into account the heat transfer coefficient h f i. So at a particular y, fixed y we know how the temperatures varies, on the plate, in the solar collector. Now we will go for temperature distribution, in the flow direction. One of the important things that we have to remember, before we going into this, f dashed is valid for a certain fixed y. So in principle it can be changing with respect to y, or one may define a particular average sort of f dashed, depending upon y chosen, but now if we find the temperature distribution in the flow direction, first only we may concentrate T f changing from initial temperature T f i, with which is enters to the exit temperature T f o. Since we know the fin efficiency, bond conductance, and the convective resistance T p or T b can be calculated, from T b T at any other x can be calculated. So we shall concentrate on the energy balance, on the fluid element, inside the tube.

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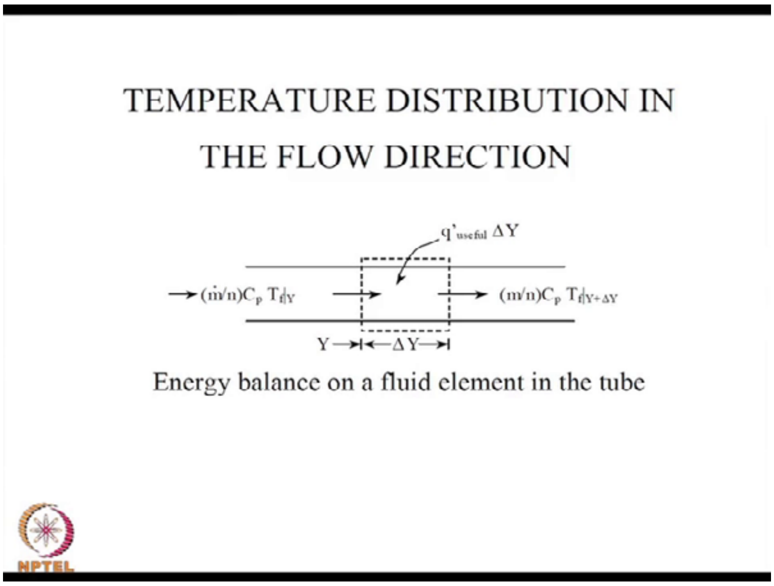

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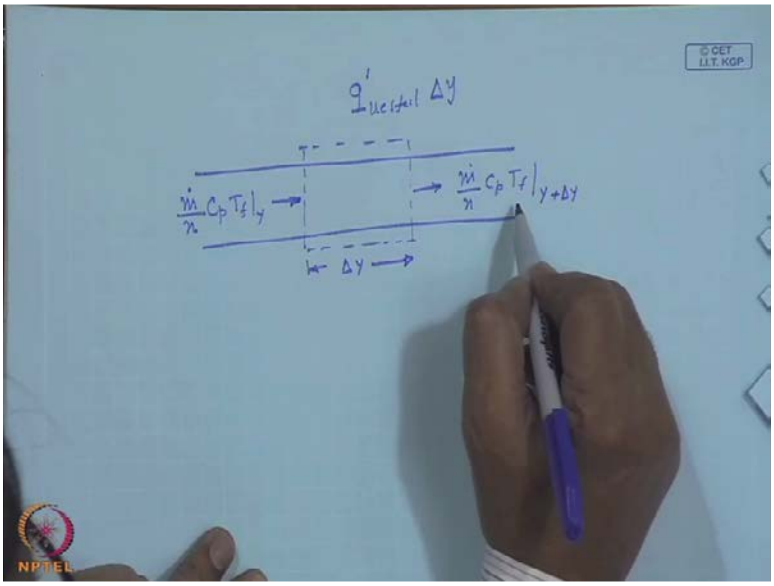


So, we consider the tube. Again take a control element, of length delta y now. So this will receive, q dashed useful times delta y. What gets in is, mass flow rate divided by the number of tubes, multiplied by the specific heat, and the fluid at y. What gets out, is m dot by n C p and T f, at y plus delta y, and this is the delta y elemental length in the direction of the flow, inside the tube. So it is receiving a q dash useful times delta y, and energy at any particular y if it is entering at the temperature of T f, the temperature is one can imagine to have been augmented to T f at y plus delta y, indicating the energy gain from the solar energy.

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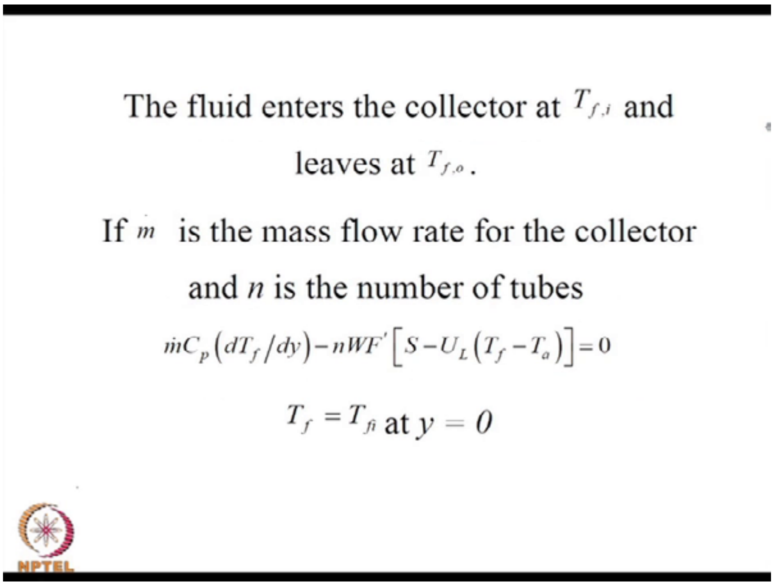

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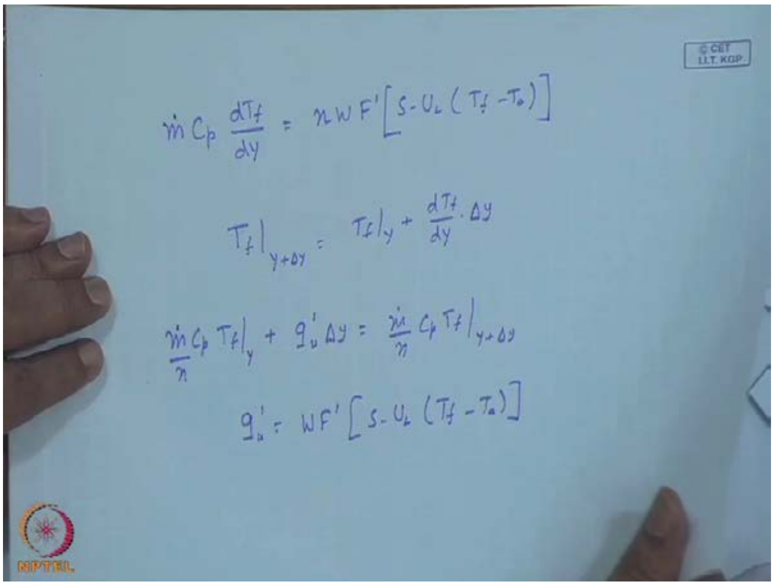


So if you make the energy balance, you will have a simple first order differential equation. We have used the equation q u equation, and we also have used T f at y plus delta y will be T f at y plus d T f by d y times delta y in writing the values. In other words, we wrote m dot C p T f at y plus this q u dashed delta y, should be equal to m dot by. Of course, there is an n C p T f at y plus delta y, where q u dashed we also did we know, as equal to w times F dashed, S minus U L into T f minus T a. So this we know if we substitute in that equation, we have the first order equation m dot C p.

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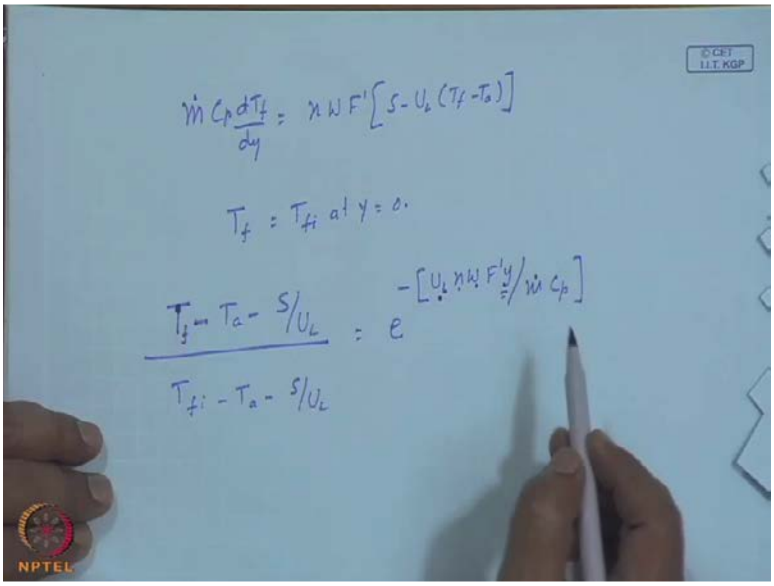

Again let me repeat; the one boundary condition that you will be requiring, will be T f equal to T f initial, at y equal to 0. The temperature at which the fluid enters is the boundary condition needed here. So it is simple to integrate, it is a first order equation only. So you can and T a and s are assumed to be constant, along with your U L T minus T a minus s upon U L by T f i minus T a minus S upon U L, will be exponentially decaying U L n W F dashed y by m dot C p. So, the temperature as the matter fact it is T f, so T f changes according to y, depending upon the overall loss coefficient, spacing, number of tubes, and the mass flow rate m dot by n is nothing, but the mass flow rate per tube, and the C p.

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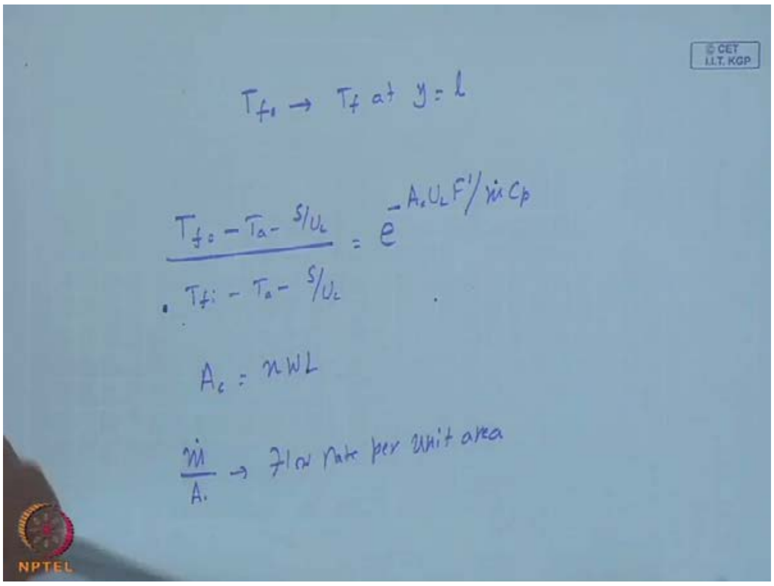


So if I want to estimate, my exit temperature T f o, is nothing but, T f at y is equal to L. If L is the length of the solar collector tube, then at y is 2 to L T f will be exit temperature T f o, so that is given by T f o minus T a minus s upon U L upon T f i minus T a minus s upon U L, should be equal to e to the power minus A c U L F dashed upon m dot c p. A c we will realize is nothing but, n w into L, if L is the length of the tube, and w is the width times n that will be the total collector area. Now you may have this A c by m dot is nothing but, in the denominator m dot by A c, so you may have m dot by A c; the flow rate per unit area. Some books can may write it like that at g, instead of A c in the nominator and m dot in the denominator. Now, what do I have, I know how the fluid temperature varies from inlet to the outlet, and I can estimate the outlet temperature, and we have already found the variation in the x direction, through the fin concept.

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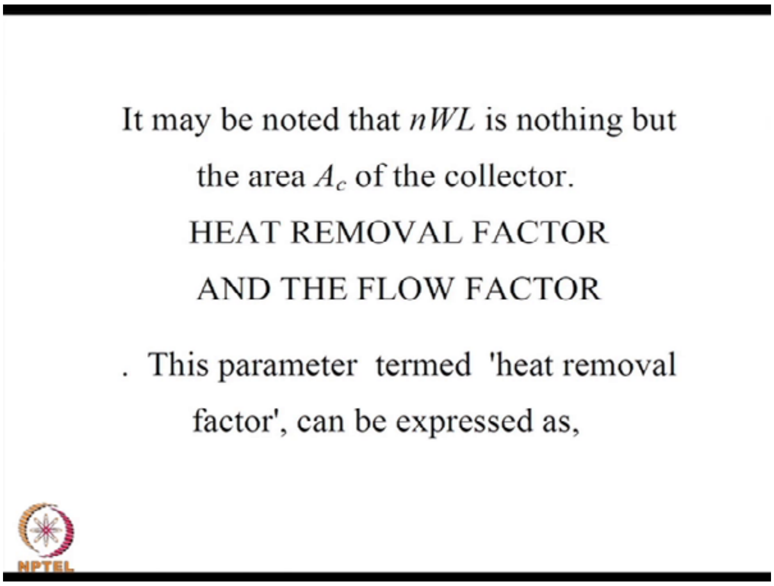


Now, having done this, we shall now estimate, heat removal factor F R, and to one more factor, that so called flow factor.

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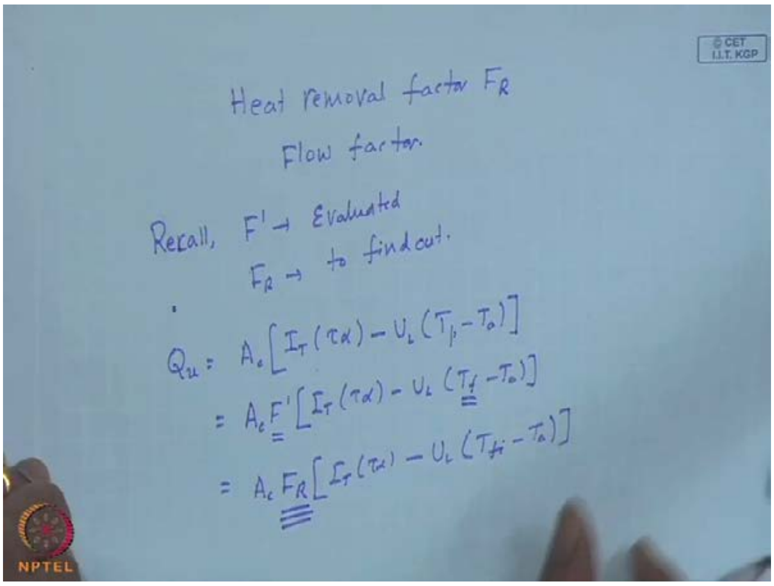


You again remember F dash is evaluated, the collector efficiency factor. Now we are about to find out, the heat removal factor F R, that originated from our basic equation q u, useful energy gain, from a collector of area A c, as incoming radiation, multiplied by an effective transmittance absorbance product, minus the losses taking place from the plate temperature to the ambient, which has been successfully rather subsequently modified as A c times collector efficiency factor F dashed into I T tau alpha minus U L into T f minus T a; that means as if the losses are taking place from the fluid temperature, rather than the plate temperature, to compensate this under estimation, we have put a factor F dashed which should be less than one. Subsequently we said even the uncertainty of this T f, whether it is the mean temperature or the temperature fluid of the fluid at a mean location, a heat removable factor has been introduced F R minus U L, the minimum possible loss from the collector will be at the minimum temperature T f i, so you compensate by again the second factor, the heat removal factor F R.

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$$F_R = \frac{\dot{m}C_p\left(T_{f,o} - T_{f,i}\right)}{A_c\left[S - U_L\left(T_{f,i} - T_a\right)\right]}$$

$$F_R = \frac{\dot{m}C_p\left(T_{f,o} - T_{f,i}\right)}{A_c U_L\left[\frac{S}{U_L} - \left(T_{f,i} - T_a\right)\right]}$$

$$= \frac{\dot{m}C_p\left(T_{f,o} - T_a - \frac{S}{U_L}\right) - \left(T_{f,i} - T_a - \frac{S}{U_L}\right)}{A_c U_L\left[\frac{S}{U_L} - \left(T_{f,i} - T_a\right)\right]}$$

So now if i try to express, heat removal factor F R. Irrespective of collector energy gain equation, the actual energy gain by the collector, is truly mass flow rate, multiplied by this specific heat, multiplied by temperature difference between the exit, and the inlet, upon the energy gain, if the entire collector had been at the inlet temperature of the fluid minus T. In other words, this is a maximum possible energy gain by a collector, compared to the actual one m dot c p into T f o minus T f i. So this can be rewritten as m dot c p by a c times U L T f o minus T f i to bring it to the standard form S by U L minus T f i minus T a. So, this can be again rewritten, this is simple algebra, but so add and subtract T a minus S upon U L minus T f i minus T a minus S upon U L, because we know that ratio; that is exactly the reason why we are, trying to express in this fashion, upon a C U L times S upon U L minus T f i minus T a.

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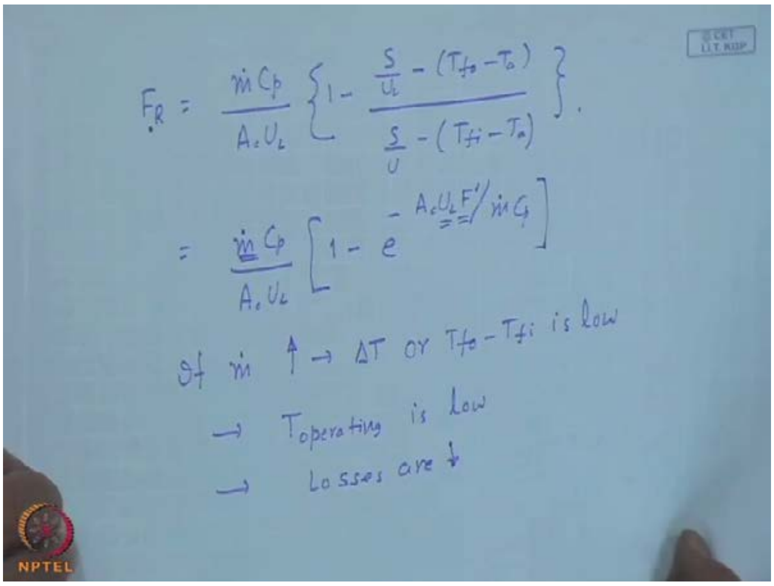


So F R will be, if I take out the common m dot C p by A c U L times 1 minus S by U L minus T f o minus T a by S by U L minus T f i minus T a. So this is simply m dot C p A c U L times, from the relation which we have got for T f o minus T a minus S by U L by T f i minus T a minus S by U L 1 minus e to the power minus A c U L F dashed by m dot C p. So if you look at this particular equation, heat removal factor, in addition to containing f dashed, and the loss coefficient. F dashed contains the thermal conductivity of the material, spacing of the tube geometry in general, it contains an additional factor m dot. So the flow rate, indirectly is going to influence your, heat transfer coefficient. Even if that is not very significant unless the reign, changes laminar flow to turbulent flow, you have got the effect of higher m dot to reflect it in F R; that is if indirectly, m dot is high, by delta t or t f o minus t f i is low. If you send a large flow rate to the collector, you will have the temperature increase to be smaller. So the t operating, if I try to call it that, is low, so losses are less.

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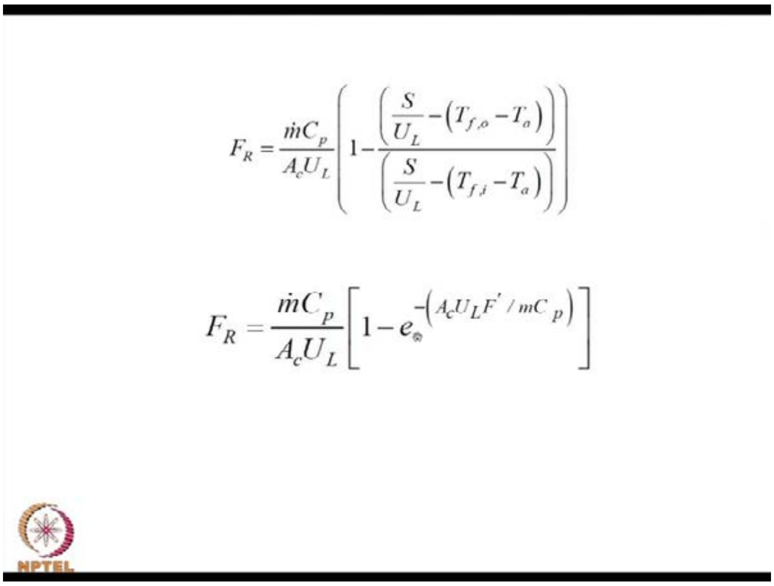


So this implies, my heat removal factor will be higher, if the flow rate is higher; that may be due to an improvement, in the heat transfer coefficient, or a direct effect of higher m dot, is to have a lower delta t, and hence lower losses.

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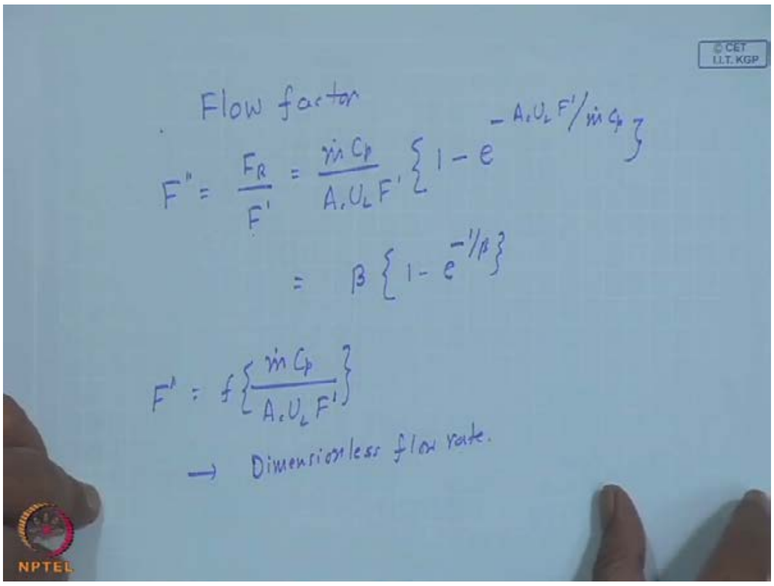


Now, people have introduced another factor; flow factor, which is F double prime equal to F R divided by F dashed, which will be m dot C p A c U L F dashed times 1 minus e to the power minus A c U L F dashed by m dot C p. So if you look at this, this is nothing but, some sort of a beta 1 minus e to the power minus 1 by beta. So obviously this should be a non dimensional quantity. So the flow factor of any collector, is simply the mass flow rate, overall coefficient area, and the collector efficiency factor, F double prime is a function of only this quantity. For lack of a better name, we will call it the dimensionless flow rate. So the flow factor, is a sort of non dimensional flow rate function.

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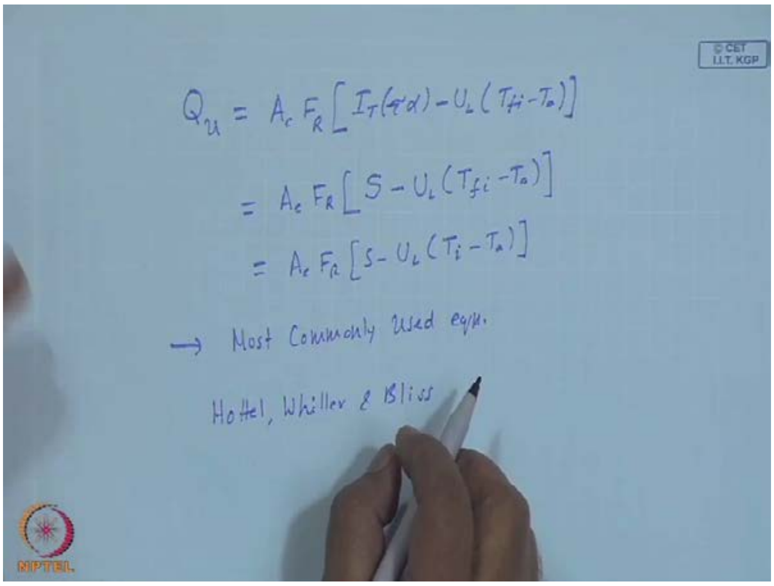


Now if we look at finally, Q u has been expressed as A c F R times I T tau alpha minus U L into T f i minus T a, where F R is the heat removal factor. In another notation, we may write it as F R into S is absorbable energy minus U L into we will drop that fluid T f i minus T a, which simply is written A c F R S minus U L into T i minus T a. In other words, there is no confusion about the inlet temperature, a fluid inlet temperature is unnecessary, most of the time T i is, this is most commonly used equation. It is originally due to Hottel Whiller and bliss of m I T, and the interesting feature is, this is based upon simple to one dimensional equations, and there have been comparisons, with two dimensional approaches for the collector, using finite difference method, finite element method, and you will find that the difference has not been, more than five to five percent or so, comparative to this simple equation.

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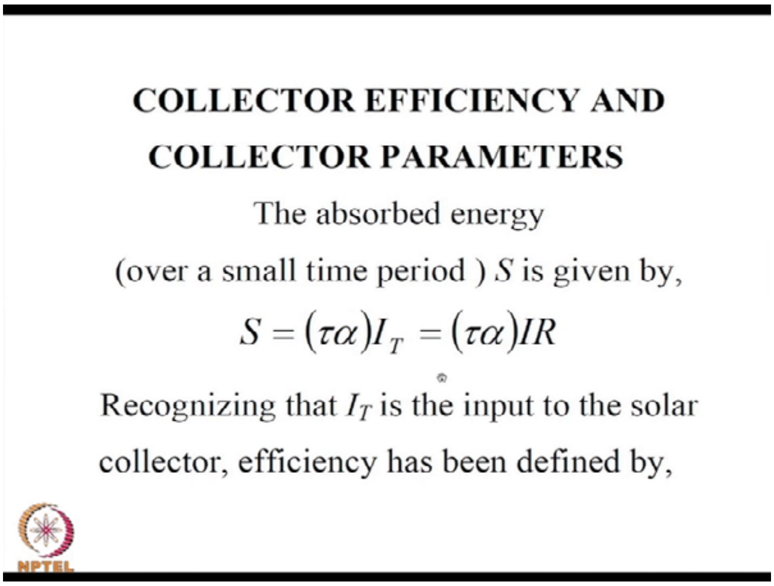


So this remained a very popular equation, still being used, in spite of the assumptions that we made, in calculating or expressing this quantities.

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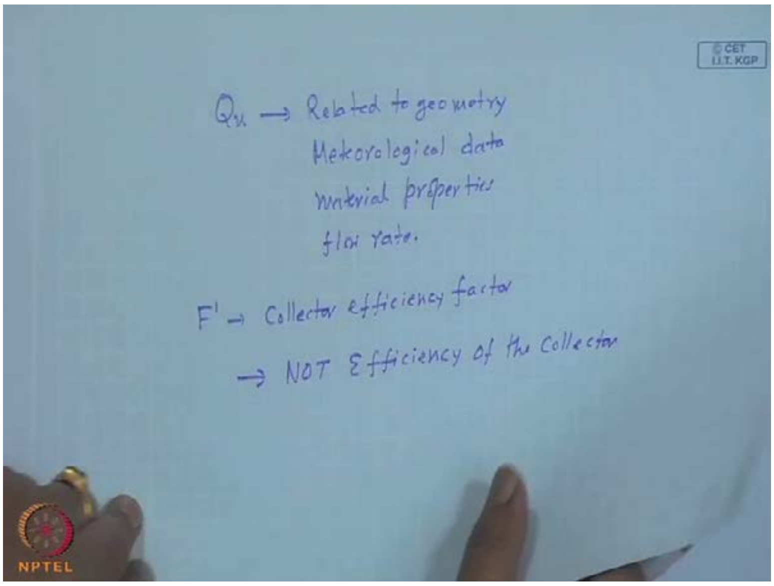


Now, so far we found the useful energy gain, which to our satisfaction, related to the geometry, meteorological data, material properties, including thermal conductivity and the transmittance of certain product. Flow rate, more importantly, equally importantly, we got it. Now, there is a efficiency f dashed, is the collector efficiency factor, but this is not efficiency of the collector.

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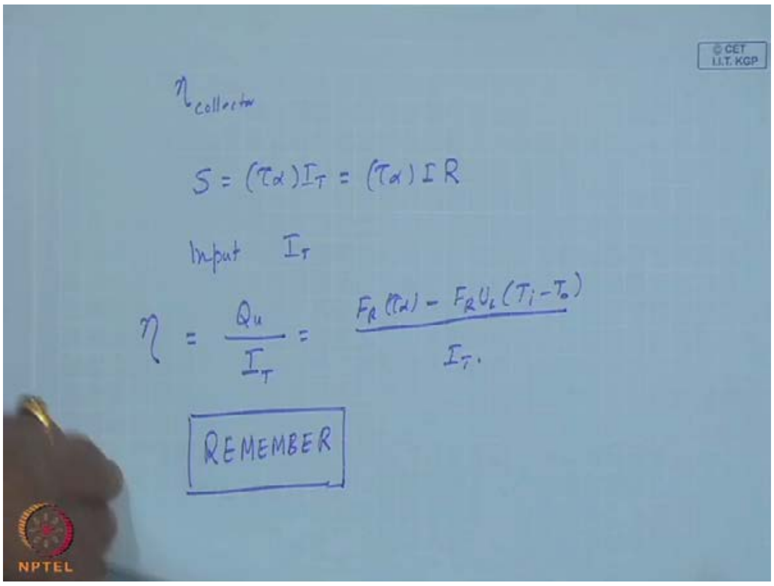


In principle; one can make F dashed, can go to one; for example, if I have tube to tube, large number of tubes, this F dash will tend to one, or for let us say air heating collector. This may be only a duct, with the flow, perpendicular to the plane of the paper, and here also the fin efficiency is 1 F dashed. sorry this will be fin efficiency equal to one, consequently your F dashed also will be higher. So this is only to emphasize, that your collector efficiency factor, is not an efficiency. So we will try to find out, what is efficiency of the collector, and how do we do find, and absorbed energy is, tau alpha into I T, which can be rewritten as tau alpha into i into that global conversion factor R, what is the input, is I T. So I may define, efficiency to be a useful energy gain, upon the input I T .So this will be F R tau alpha minus F R U L times T i minus T a upon I T. So this can be written already, It is in the same form. We should only remember, in calculation of efficiency also.

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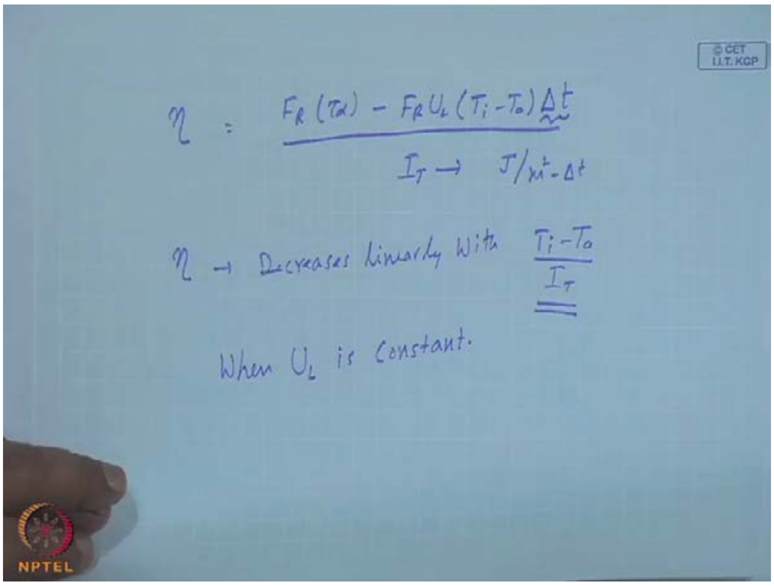


This should be interpreted as minus, that hidden time factor, comes into the picture, depending upon the time. This is joules per meter square joule in delta t, otherwise this will be watts per meter square, this will be joules per meter square integrated, over a time delta t. So this is one mistake, that is commonly done, do not do that.

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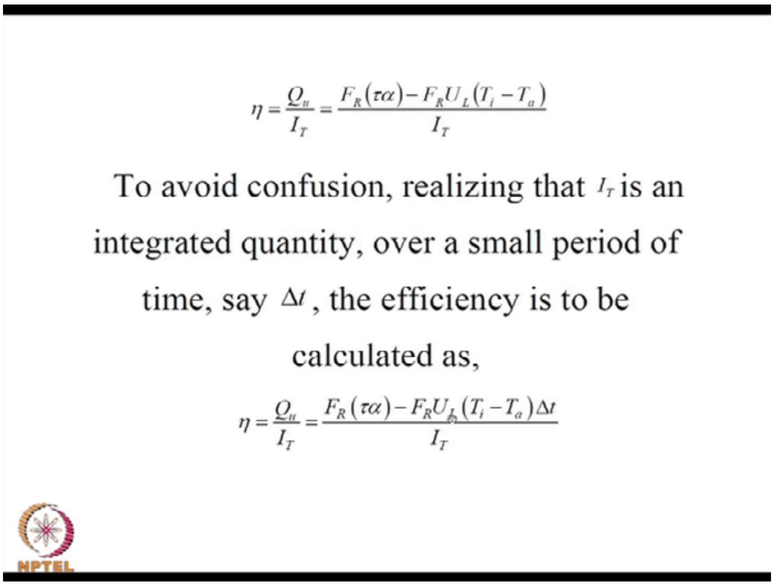

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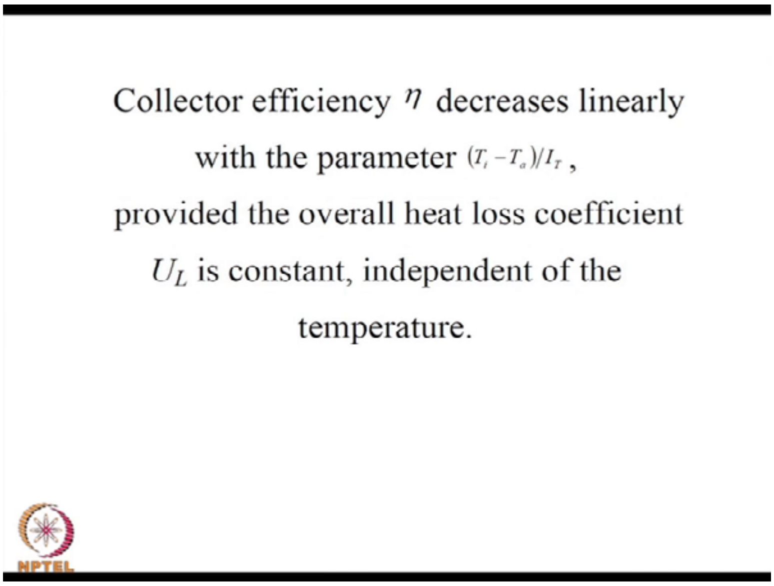


Now if you look at it, eta decreases linearly with T i minus T a upon I T. If you look at this parameter, if the temperature difference is large, my efficiency will be lower, which means the operating temperature is higher, than if the incoming radiation is high, the losses will be a part of the incoming all radiation, so my efficiency should be high. So this seems to be a good parameter, which of course, the equations give. Of course, this is true when, U L is constant. So eta decreases with the parameter T i minus T a upon I T linearly, if the overall loss coefficient is small.

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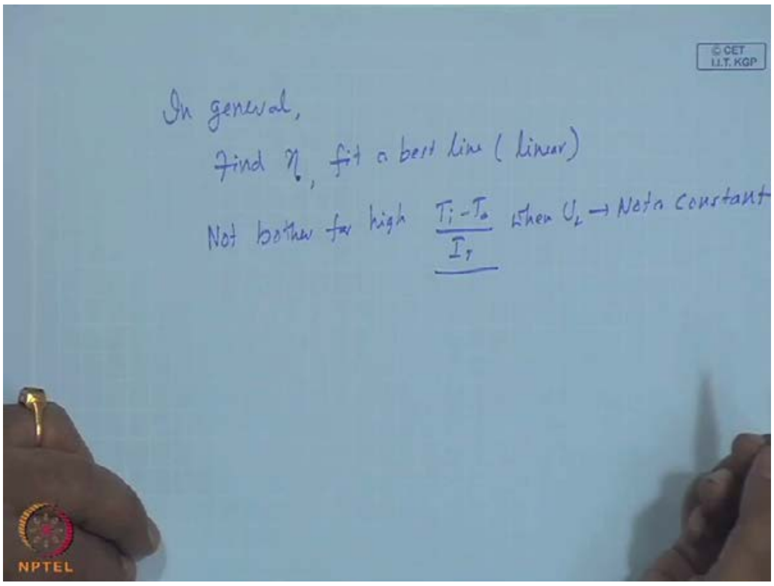

In general, by experiments, find efficiency, fit a best line, a linear variation, and not bother much for high T i minus T a by I T, when U L is not a constant. So we assume that the collector will be operating, at reasonable values of T i minus T a upon I T, one can have an idea of the numbers, once we solve some exercises, and you do not go, to a very large value of T i by T a by I T, so that variation of U L makes the efficiency variation non-linear, not because of you are afraid of the non-linearity, but the efficiency will be low, consequently effectiveness of the solar collector will be small.