Solar Energy Technology
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Lecture - 13
Theory of Flat Plate Collector - Liquid Based (B)

So, we shall continue with, our procedure to obtain the overall loss coefficient. We have excess the loss, from the plate to the cover one, and if you consider the top plate, from which the losses taking place direct to the ambient.

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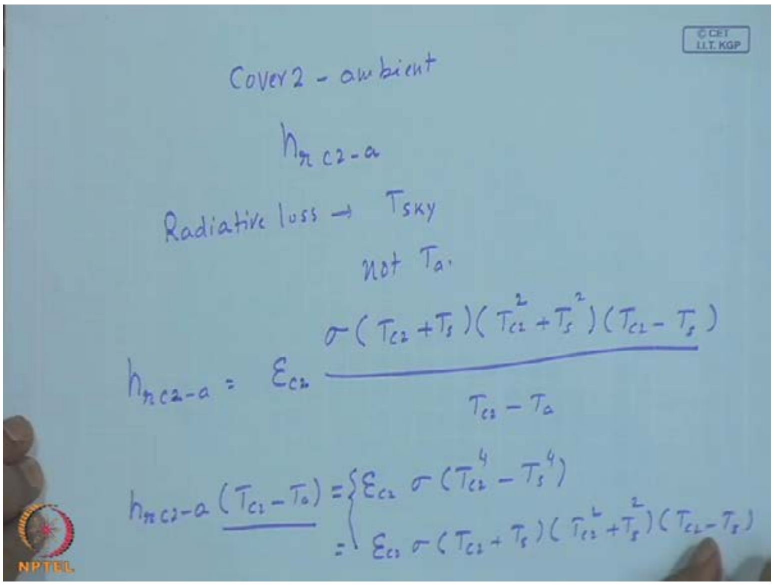

The radiative heat transfer coefficient, from the cover 2 to the ambient, which we shall designate as h r c 2 a. Now we have already expressed, that this radiative loss, is to T sky and not T ambient, but since everything we want to write it as overall loss coefficient, multiplied by T p minus T a, or T f minus T a, or T f I minus T a, we would like to base it with respect to the ambient temperature. So, we can write down h r c 2 a is epsilon c 2 times sigma T c 2 plus T s times T c 2 square plus T s square times T c 2 minus T s upon T c 2 minus T a. Now do not worry about this long expression, you can easily work out this, what all we try to do was, the radiative loss with a heat transfer coefficient of h r c 2 a, from T c 2 to T a, has been expressed as equivalent to epsilon c 2 sigma into T c 2 to the power 4 minus T s or T sky to the power 4, which is nothing, but epsilon c 2 sigma times T c 2 plus T s times T c 2 square plus T s times T c 2 minus T s. So, this is artificially written, as if h r c 2 a, if it is multiplied by T c 2 minus T a, it will give me the actual radiative loss, from T c 2 to T sky. So, that in may overall heat loss coefficient, this is based with reference to a temperature, above the ambient temperature T a.

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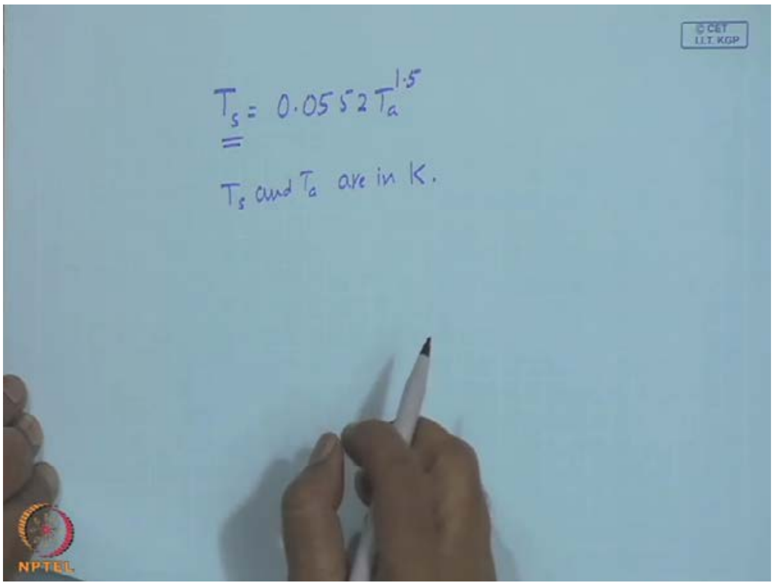

Now T s is given by 0.0552 T a to the power 1.5, where T s and T a are in Kelvin. In other words, this is a estimate, you measure the ambient temperature, and you measure the a temperature of let us say water kept outside, which is cooler than the ambient, and from where you estimate the heat loss, and that should have been lost by infrared radiation, from the body under consideration, thereby giving you an estimate of T sky. In other words, you would have reached a lower temperature for the water in the bucket, in the evening when there is no sunshine, compare it to the ambient temperature, if the sky temperature had been so much, and the loss is given by the amount epsilon c 2 T sky to T water to the power 4 minus T sky to the power 4.

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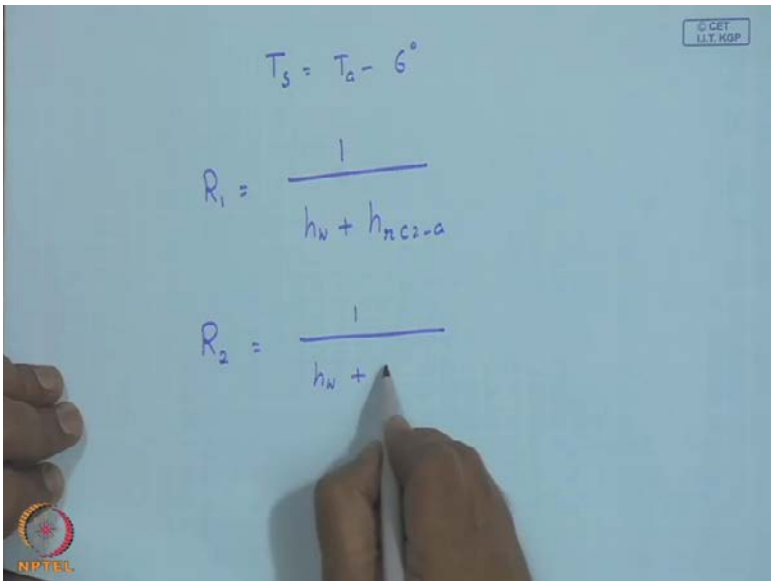

And of course, there is a simpler relation also, simply T sky is nothing, but ambient minus 6 degrees, it works reasonably well. So, my resistance now R 1 1 upon wind heat transfer coefficient already known to us, by the radiative loss coefficient h r c 2 a. h r c 2 a is like exactly h p c 1 or h r c 1 c 2 all the other radiative coefficient. Since the loss takes place to the sky temperature, it has been sort of normalized, with respect to T c 2 minus T a, rather than T c 2 and T square. So, now, we can express R 2 as.

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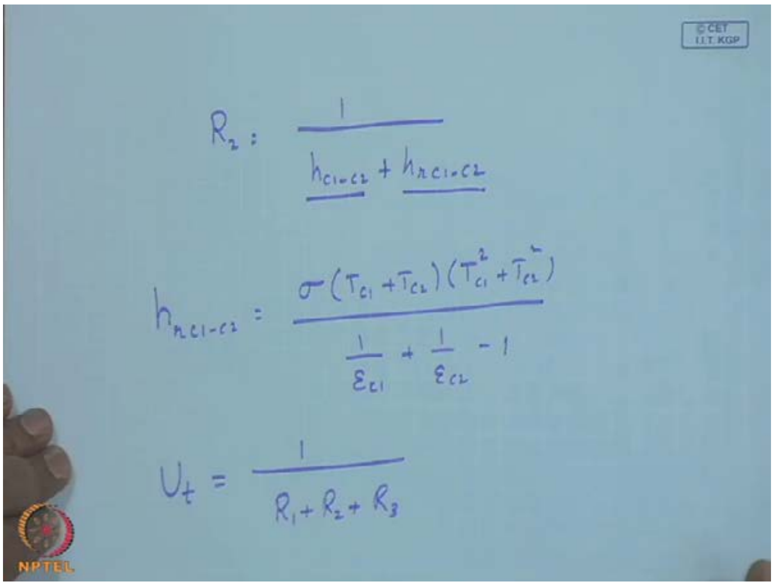

So, it is the summation of the convective heat transfer coefficient between cover 1 and cover 2, and the radiative heat transfer coefficient between cover 1 and cover 2. So, you can again write h r c 1 c 2 as sigma T c 1 plus T c 2 times. This is exactly similar to h r p c 1, where we wrote T p plus T c 1 into T p square plus T c 1 square, upon the corresponding emissivity's. So, now, we are in a position to calculate, the loss coefficient from the top U T, as summation of the resistances R 1 R 2 R 3 inverted; that will be corresponding to the second figure in the thermal network. First one is showing the parallel paths, the second one order them in series, equivalently expressed as R 1 R 2 R 3.

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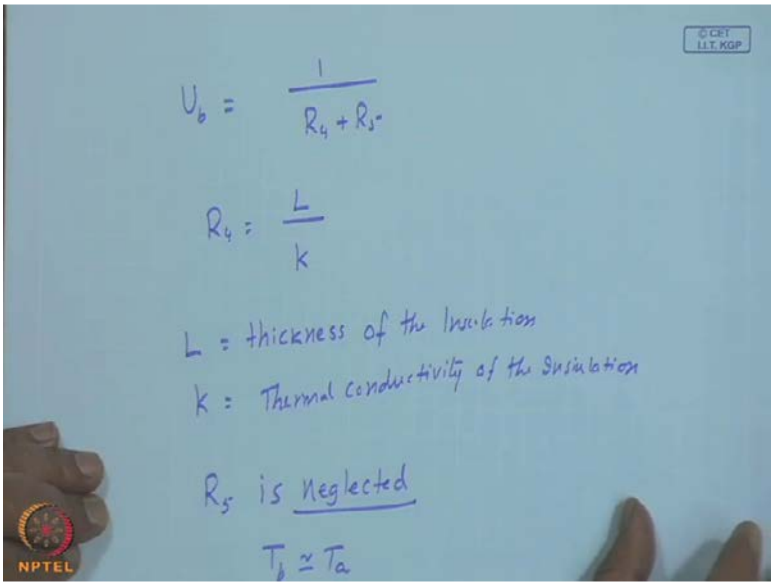

And the bottom loss coefficient U b is straight forward, R 4 plus R 5. R 4 is the conduction resistance, and R 5 is the convective plus radiation resistance, and R 4 is given by L upon K, where L is the thickness of the insulation, and K is the thermal conductivity of the insulation. And usually R 5 is neglected, because T b is almost just pretty close to T a.

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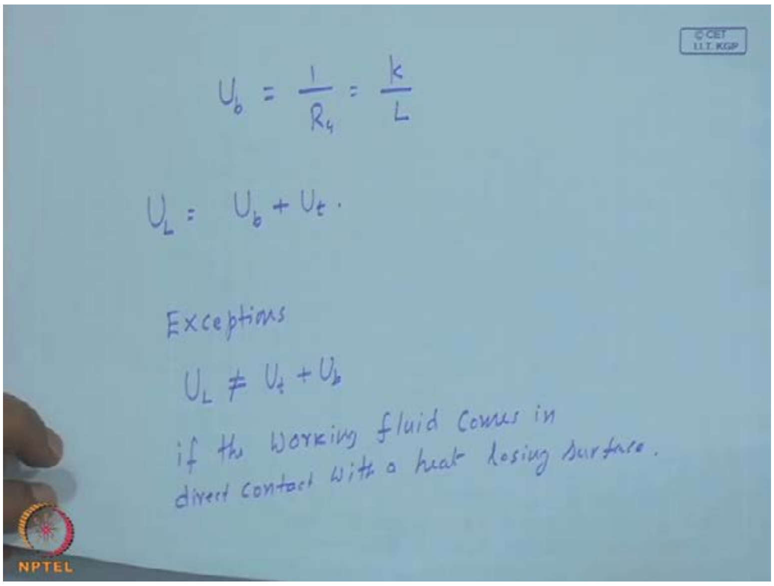

So, back loss coefficient U b, is simply given by 1 upon R 4 equal to K by L. Now my overall loss coefficient U L, equal to U b plus U t. Of course, there will be exceptions, U L will not be equal to U t plus U b, if the working fluid comes in contact, comes in direct contact with a heat losing surface. In other words, all that is being considered as U T, is not a loss, a part of it is going to the fluid also. So, we shall come to that type of collector configurations, a little later, at least one of them.

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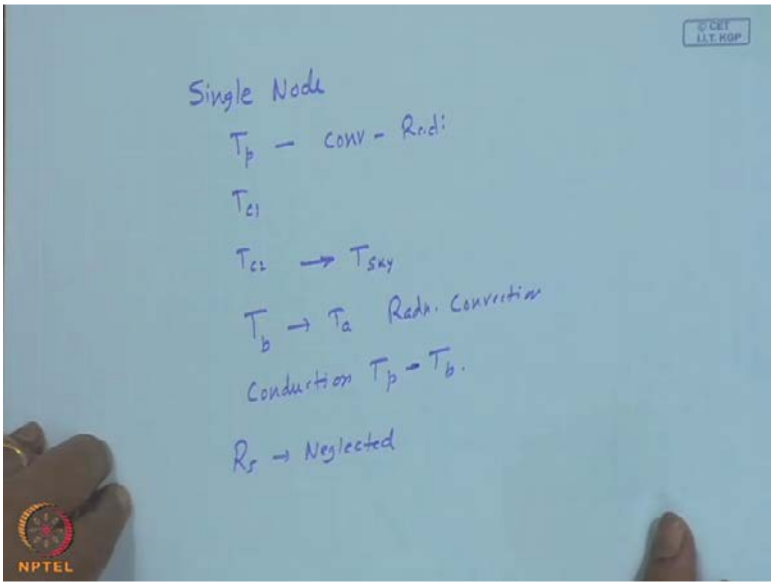

So, having done these things, what did we do. We assumed a single node T p T c 1 T c 2, from here it loses the radiation to T sky, and there are parallel convection and radiation, and then the back temperature T b to T a radiation, and convection, and by conduction, from plate temperature T p to T b. So, we estimated of course this R 5, is neglected; that means, no heat loss by convection radiation from the bottom.

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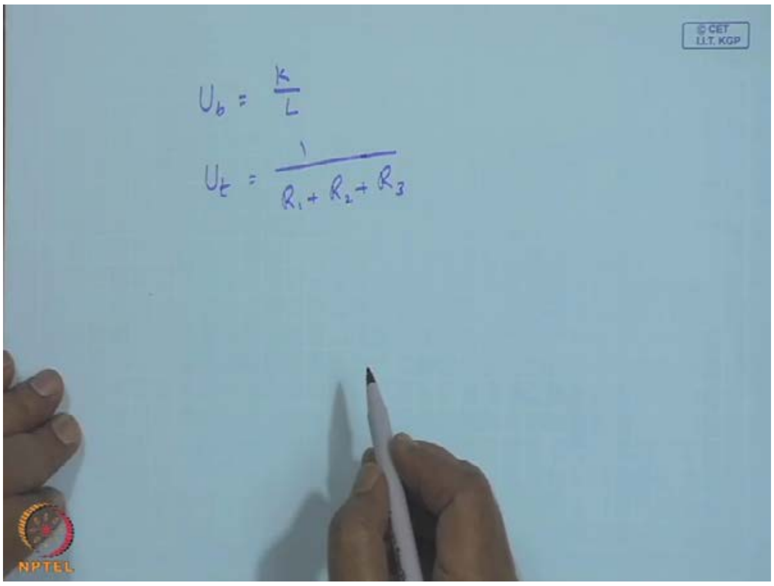

And we found out U b is equal to K upon L, and U t is 1 upon R 1 plus R 2 plus R 3, where the resistances R 1 R 2 R 3, or the convection plus radiation resistances, between the plate, and the cover 1, and between cover 1, and cover 2 and cover 2 to ambient. If there is only one cover, we will not worry about R 1 and it will be only R 2 and R 3. So, how do we estimate now U L; the idea is you need R 1 R 2 R 3 R 4, R 4 is simple it is L by K.

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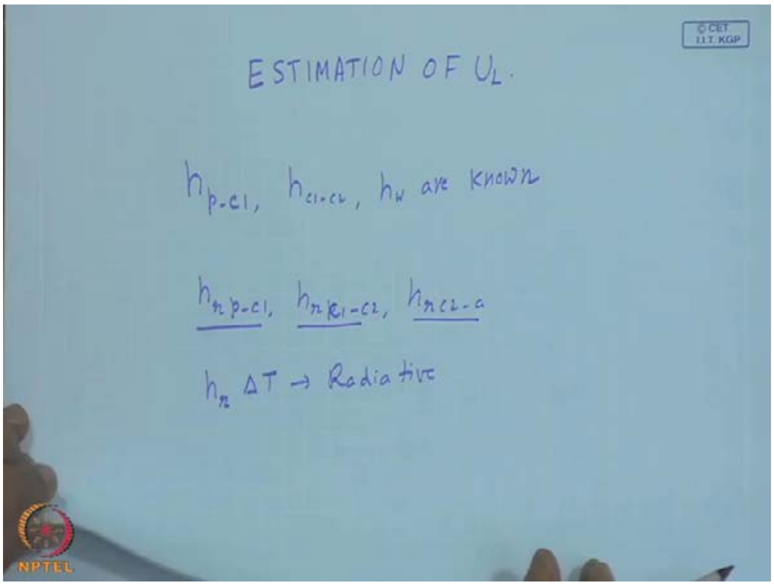

So, we shall gone to estimate this U L. Now, the convective heat transfer coefficient h p to c 1 h c 1 to c 2 and of course, h w are known. Our heat transfer knowledge, gives us to method to calculate the convective heat transport coefficient between the plate, and a parallel cover one, between cover one and cover two, and the wind heat transfer coefficient, which we have already given. Now we also have got, h r p to c 1 h r c 1 to c 2 and h r c 2 to ambient. These are radiative heat transfer coefficient, which in general h r multiplied by some delta T, gives the radiative loss. Though we know, that the radiative loss is proportional to the difference of the fourth power of the temperature. This has been expressed similar to, that of convective loss. Then non-linearity is absorbed, in the definitions of the radiative heat transfer coefficient.

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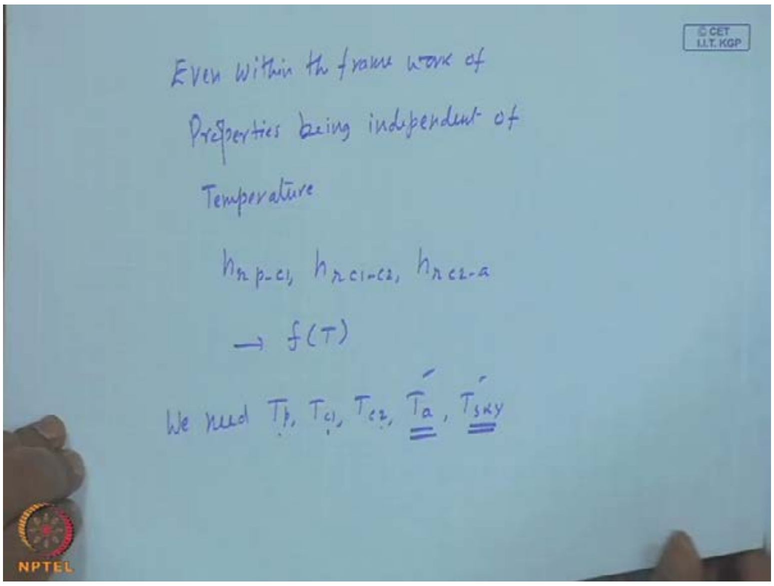

Even within the frame work of, properties being independent of temperature; that means, the thermal convective does not change, heat transfer coefficients we have a method of estimating it, and the viscosity, density or they do not change with temperature, but even with all these assumptions, my h r p to c 1 h r c 1 c 2 and h r c to a they are functions of T. In other words we need T p T c 1 T c 2 T a and of course T sky. This T a is a metallurgical property which we know, T sky can be estimated from the relationship, so these are not an issues. But T p T c 1 T c 2 have to be known in order to calculate my radiative transfer coefficient from the plate to the cover one, or cover one to cover two, or cover two to ambient.

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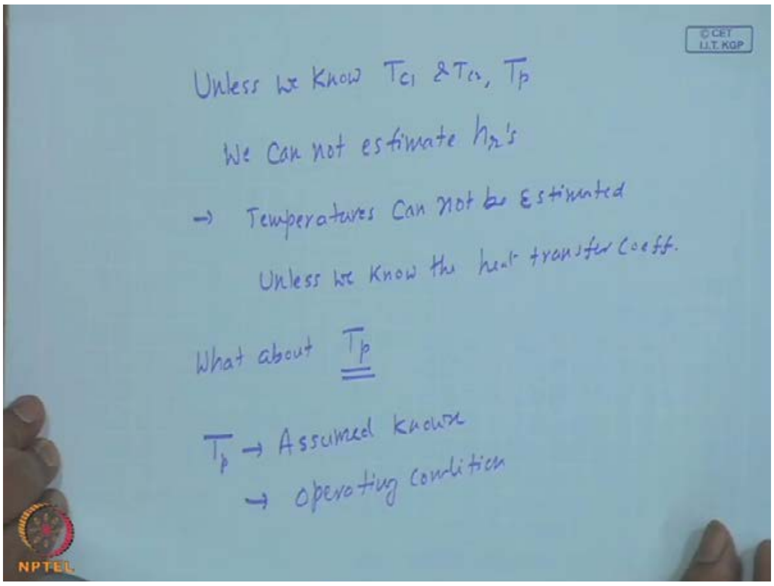

So, now, we have got a interesting problem. Unless we know, T c 1 and T c 2 we cannot estimate h r's in general. Maybe I can add a T p also T a is know. So, radiative heat transfer coefficients cannot be estimated, unless I know the temperatures. And temperatures cannot be estimated, unless we know the heat transfer coefficients. So, it is a tricky situation, what do we know. Of course, you might ask me, what about T a. I am sorry, what about T p; the plate temperature. So, this is assumed known, because this is in operating condition. So, if a solar collector is operating to deliver energy, at a certain temperature, we will be having, that temperature to be the corresponding plate temperature, which will depend upon the internal resistances, which we need to know the theory, so T p is assumed to be known, for a given application. Or alternately we are trying to estimate the overall loss coefficient, for a number of values of T p, and we will pick up the appropriate overall lose coefficient, depending upon the actual operating condition. So, even if T p is assumed as an operating condition, T c 1 and T c 2 are essential to calculate U L through various resistances.

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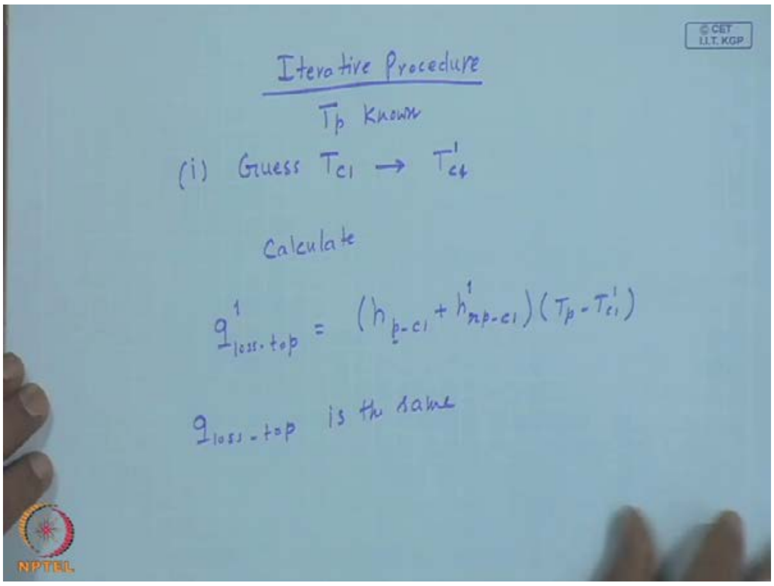

So, this naturally involves a trial and error, or an iterative procedure. So, step number one; we will guess T c 1. Let the first guess be T c 1 1. So, in case my first guess is not correct, I will go to second, if second one is not correct I will go to third and so on and so forth, and we will try to iterate, and we will set up a procedure of the principle behind iterating this exception. So, the first step is, T p is known, let us write it now T p known, so I guess T c 1, and the first guess is designated as T c 1 1. So, calculate q loss top, and this is. Since I am using the guess one, I will qualify it with a superscript of one, which will be h p c 1 plus h r p c 1 1, because it is obtained with T c 1 1 times T p minus T c 1 1. So, let us be very clear, we guessed a value for T c 1 as T c 1 1. The loss that is obtained from plate to cover, because I know the plate temperature, is the conductive heat transfer coefficient, plus the radiative heat transfer coefficient, estimated with a guess temperature of T c 1 1, multiplied by T p times T c 1 1.

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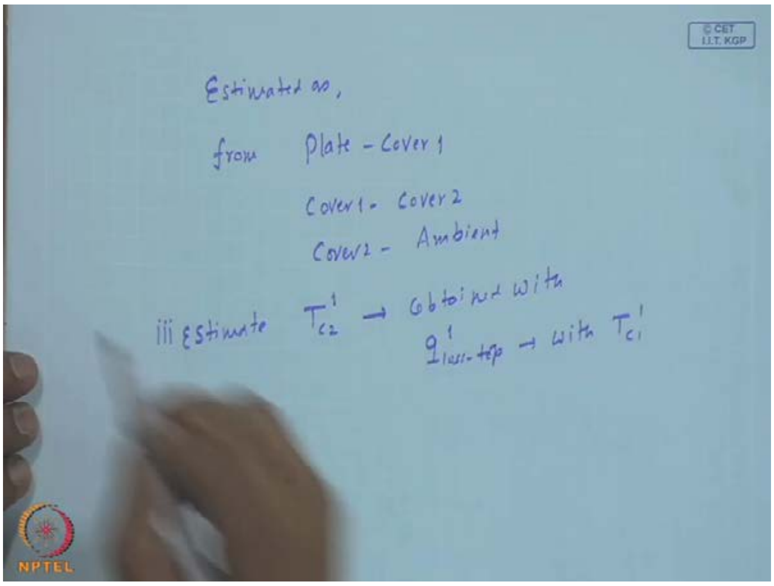

Now having, this q loss top, is the same, estimated as, from plate two cover one or cover one to cover two, or cover two to ambient, and the sky part which we have already taken care of, in defining the radiative heat transfer coefficient. So, let us this q loss top, will help us estimate T c 2. Again I will qualify it with a superscript one; that is T c 2 1 obtained with, q loss top one with T c 1 1.

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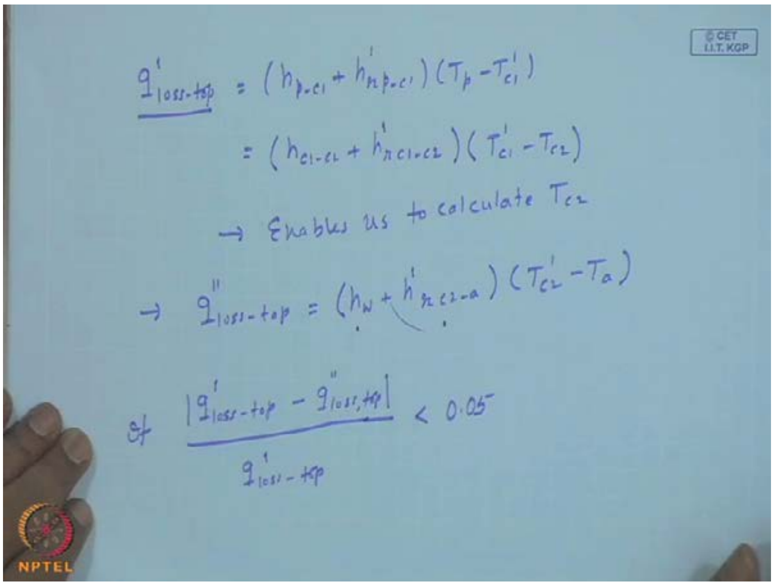

So, this relation will be q loss top 1 should be equal to h p c 1 plus h r p c 1 1 times T p minus T c 1 1, should be the same as h c 1 plus c 2 plus h r c 1 plus c 2. Again I shall qualify it with a superscript 1, because it follows from my first guess times T c 1 1 minus T c 2. Everything is known here except T c 2, so this enables us calculate T c 2. Now from this, q loss top 2, because q loss one is T p to T c 1 1, which we have guessed. The same amount of energy should be going, from cover one to cover two. Now from the top cover I will get a different number, which will be h w plus h r c 2 a times T c 2 1 minus T a if you want to be precise, we can call this h r c 2 a 1, because this has made use of T c 2 1.

So, now, what is the difference between these two, this is the loss taking place from cover one to the plate to the cover one, and that should be the same as cover one to cover two, which enabled us to calculate T c 2, and from the T c 2 my guessing is over, and I should be able to calculate what would be the top loss, from the outer cover to the ambient, by convection and by radiation. And this radiative heat transfer coefficient is calculated based upon, T sky T ambient and T c 2 dashed. So, I compare these two, and if we have q loss top 1 minus q loss top 2 by anyone them, in fact, it does not matter, but they converge. Here I put a modulus sign, so that we really do not know, whether q loss top 2 is higher or less, but our idea is, let us say this should be less than 0.05; that means, the difference in the guest heat transfer from the top, because of a guest cover temperature, should balance with, whatever is the outgoing heat loss, as a consequence these two should not differ by more than 5 percent. I may set it as 2 percent 10 percent, depending upon the accuracy, what we need.

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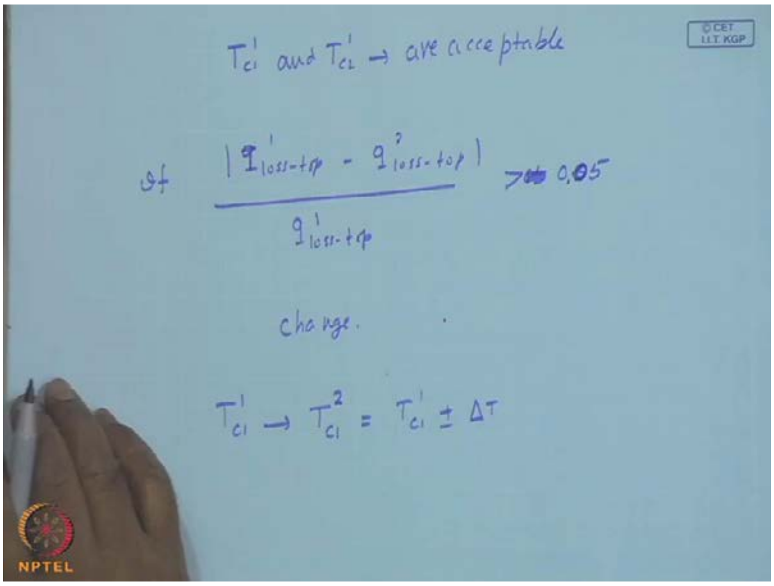

If this is satisfied and T c 1 1, and T c 2 1 are accepted. If not, if it is greater than 5 percent, 0.05 say, then change, instead of T c 1 1, change to T c 1 2, which will be some T c 1 1 plus or minus delta T. So, I change the initial guess by an amount of delta T, and one can physically argue if q loss top is higher than q loss one, whether T c 1 should be increased or decreased, so that physical check one can do, and go on the right direction.

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$$\left(q^{1}_{loss,top} - q^{11}_{loss,top}\right) > 0 \ or \ \left(q^{1}_{loss,top} - q^{11}_{loss,top}\right) < 0$$

CORRELATION FOR U_t

The procedure to evaluate U_t is cumbersome involving an iterative procedure.

So, this is a sort of involved calculation, and it requires iterations, unless you know the guest values and you have to on top of it, most of the time h p c 1 and h c 1 c 2, are the free convective heat transfer coefficients, with themselves will depend upon the temperatures. So, if they start changing, the overall loss coefficient may not be different looking, but my individual component convection and radiation losses, can be quite different. However, this procedure is can be in a routine in a simulation package, and one can calculate the overall loss coefficient, to make our life a bit easier.

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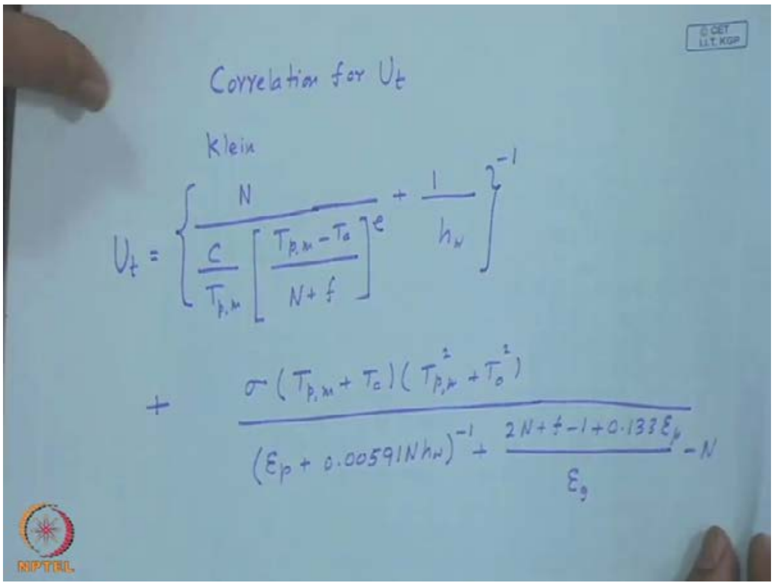

A correlation for U T has been developed, and this is by Klein once again, from the University of Medicine, and so the iterative procedure may be called cumbersome.

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Klein (Solar Energy, Vol.17, p.79, 1975)

developed a correlation for U_t as,

$$U_t = \left\{ \frac{N}{\frac{C}{T_{p,m}}\left[\frac{(T_{p,m}-T_a)}{(N+f)}\right]^e} + \frac{1}{h_w} \right\}^{-1} + \frac{\sigma(T_{p,m}+T_a)(T_{p,m}^2+T_a^2)}{(\varepsilon_p + 0.00591 N h_w)^{-1} + \frac{2N+f-1+0.133\varepsilon_p}{\varepsilon_g} - N}$$

where

N = Number of glass covers

And particularly the heat transfer coefficients involved, are free convective heat transfer coefficients, they also depend on the temperature. Though, in this particular instance, which we assume, we use the current h p c 1 or h c 1 c 2. Though this is a long expression, U t, I shall write it down, so that you can also note down. I shall give a printed hand out, so that the deficiency if any, by hand writing, carefully will not be there in the print out. So, this looks a really difficult relation, I am not sure whether the iterative procedure takes time, less time of this relation, and it is not all, where of course, N is the number of glass covers.

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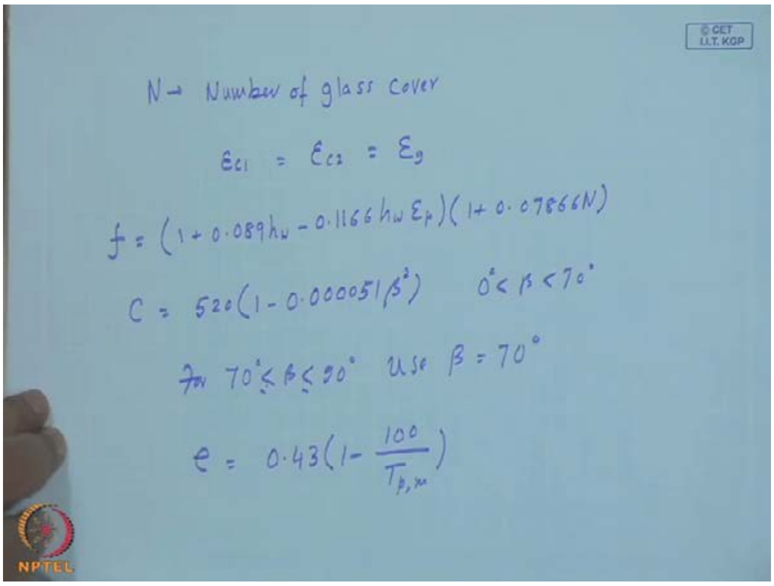

So, you can calculate for one glass cover or two glass covers, but the assumption is, epsilon c 1 is equal to epsilon c 2 equal to epsilon g. The relation you will find is written in terms of epsilon g. Then f is another expression, then the constant c 520 into 1 minus 0.000051 beta square, and this is for 0, less than beta less than 70 degrees. And for 70 degrees less than beta, less than 90 degrees, use beta is equal to 70 degrees. Then e is a 0.43 into 1 minus 100 by T p m, beta of course, is the tilt in degrees.

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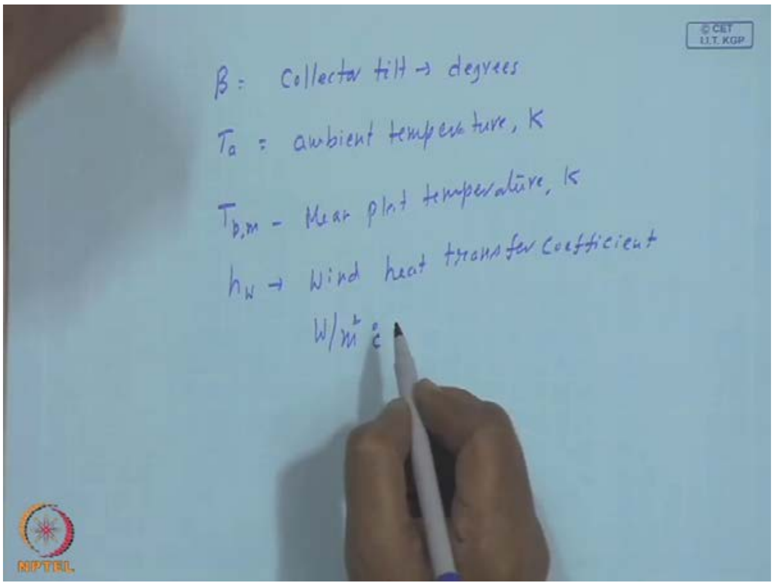

T a, this should be in Kelvin, and T p m mean plate temperature, again in Kelvin, and h w; the wind heat transfer coefficient, in watts per meter square degree centigrade. So, there is a convenient method of calculating the overall loss coefficient, which will involve an iterative procedure, by guessing the cover temperatures for a given operating condition, or the top loss coefficient can be calculated by the correlation due to plane, though it is a long expression, it does not involve iterative calculations.

So, basically what we did today, is gave few configurations for liquid based solar collectors, and qualitatively assessed, what is the temperature variation in the direction of the flow, and in the direction perpendicular to the flow direction, and then we made a number of assumptions in the analysis, most important being steady state, and the properties do not change, and the N number of 14 assumptions, and treating the two dimensional problem as, two one dimensional problems. Then we have expressed the thermal network, to calculate the overall loss coefficient, and then a simple correlation, to calculate the top loss coefficient, has been proposed by Klein, simple in the sense the expression is complicated, but does not require any iterative calculation.

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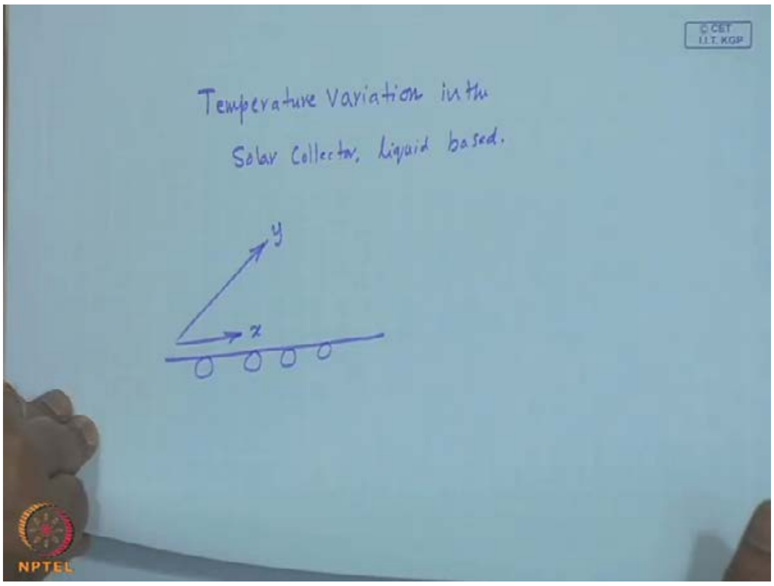

So, what we shall do is, what is the, now temperature variation, in the solar collector liquid based. Now what you have got, is something like this, you say this is x, and this is y.

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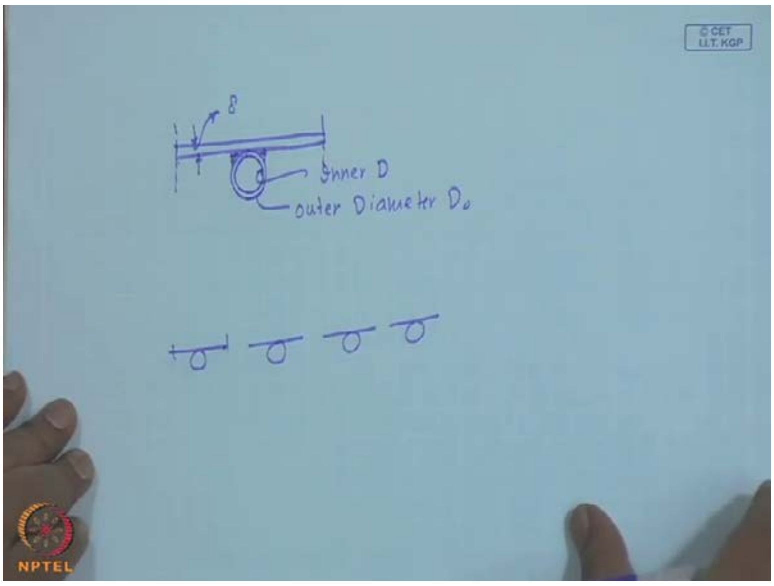

If I take, exaggerated, this is an inner diameter, from D naught, and inner D. Let the thickness of the sheet be delta, and this tube is joined, with a solder or some bondage. So, this is the mid plane, and this is the mid plane. So, I have to have a simple, some W minus D by 2 or W is the distance between the centre to centre, and we have got a number of elements, repeated like this, in the collector, they are all joined, and each one is a half a sheet on this side, and half a sheet on the other side, with a tube in between putting between.

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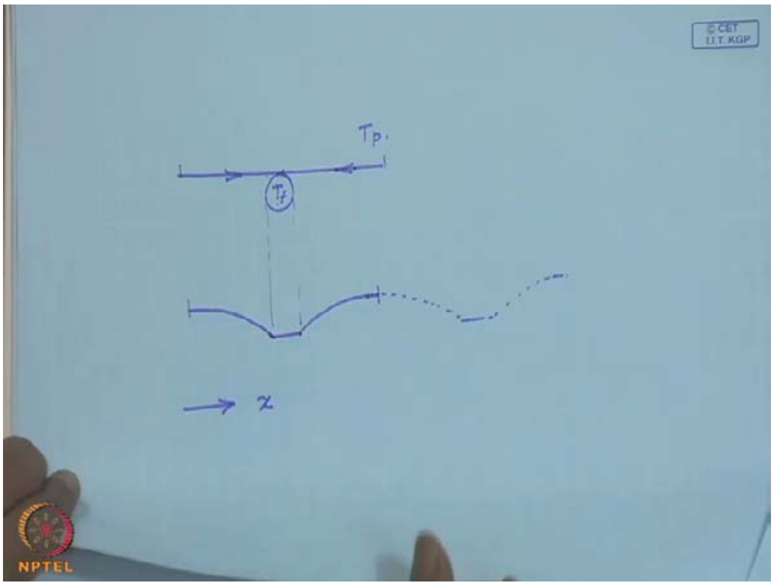

And if we plot the temperature in the x direction, heat is flowing towards the tube, which is at T f, which is lower than T p. So, the temperature is highest, at the symmetric plane, because again it will continue like this, then another tube, again it will continue like that. So, we have taken a half a sheet on this side, and another half a sheet on this side for a particular tube, this is in the x direction. We are saying that the temperature, around the periphery is negligible, so this is n constant temperature.

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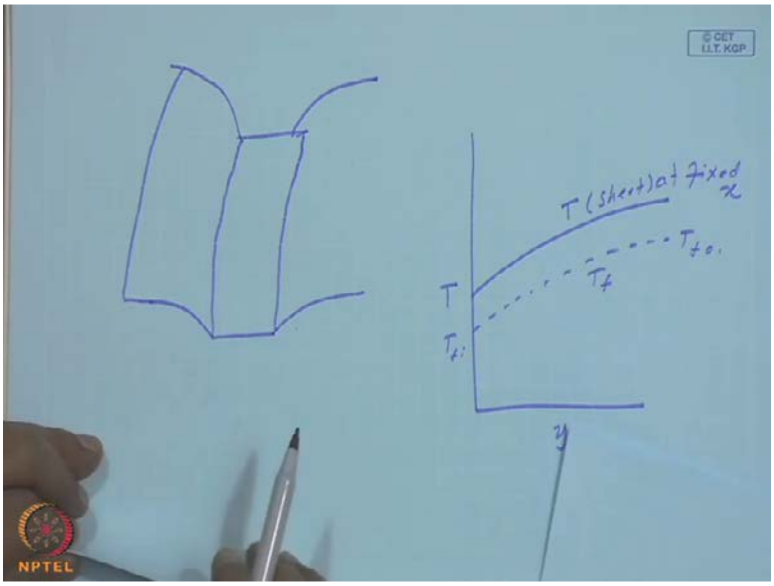

If I take this picture, that seems to be too much, and if I go in the direction of x. So, this will be increasing like this, as I go along L, because it is gaining temperature. So, if I plot it, with respect to y, and T this will be at the beginning of the collector, this may be T on the sheet, at fixed x. Now this will be T f, which will be entering at T f i and exiting at T f o. So, this is the temperature variation in the y direction, at a fixed x, and whether it is high or low depends upon whether it is a fluid region in tube, or the fin region and where we are. And in the x direction it is maximum at the mid plane of the sheet, and uniform temperature across the tube, and again it increases, that element is reproduced.

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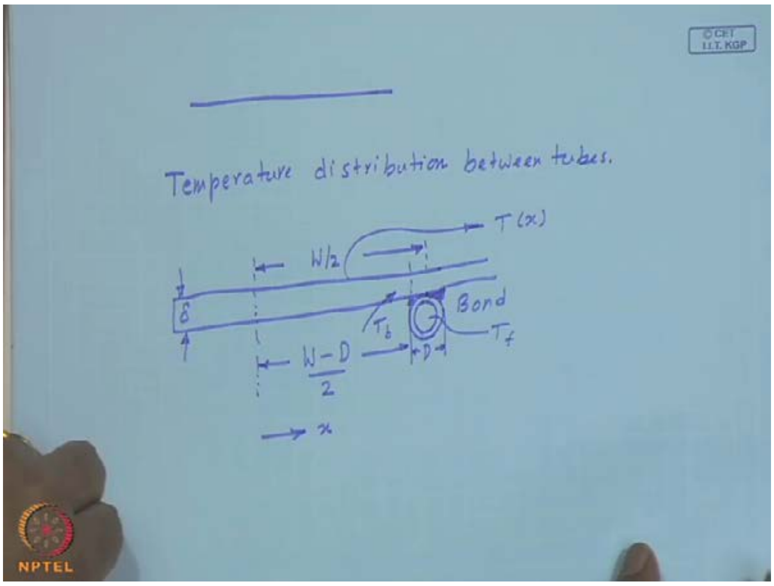

So, we can analyze this particular element. So, temperature distribution between the tubes, and the collector efficiency factor; so, that is what I have already shown the picture. So, that is what the picture is. So, this is one half I will show, of some thickness delta, and here is the tube with the inner diameter of D i and outer diameter D naught, and this continues of course, and here is the bond. Let this thickness be delta, and if we centre of the tube to the centre of the sheet half of it, that will be W by 2, and if this is D capital, this will be W minus D by 2. So, let us see the, this is my x. So, centre of the tube to the symmetric plane is W by 2, the distance between the symmetric plane, and the beginning of the tube is hence W minus D by 2, if capital D is the diameter, it is given by some bond conductance, and then at the base will be a temperature of T b, inside will be T f, over here, it will be a T x.

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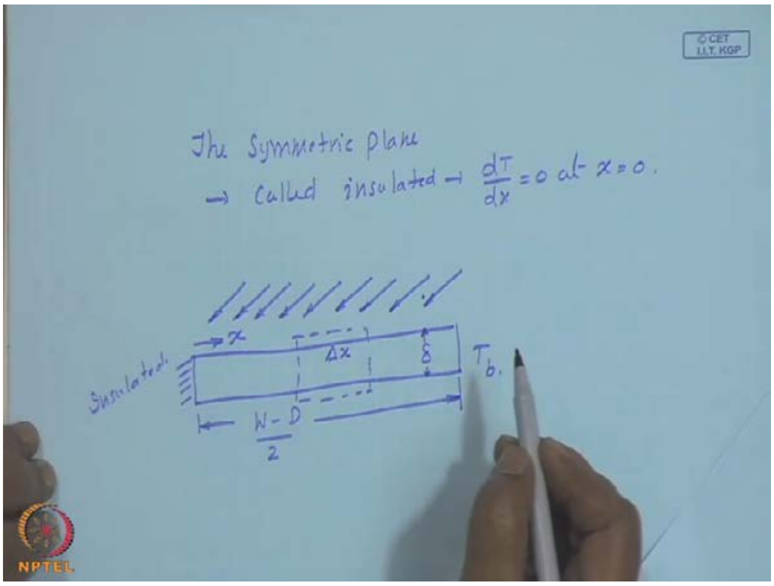

Now if we take this repetitive element, because of symmetry, the also called insulated; that is this d T d x equal to 0 at x is equal to 0 of the previous picture. So, if I just forget about the tube part, this is my base temperature T b, this is the insulated portion, or d T d x 0, and this is my W minus D by 2, and here is the incoming solar radiation, and here I measure my x, I take a elementary volume of width or length or delta x, and of course, this is delta.

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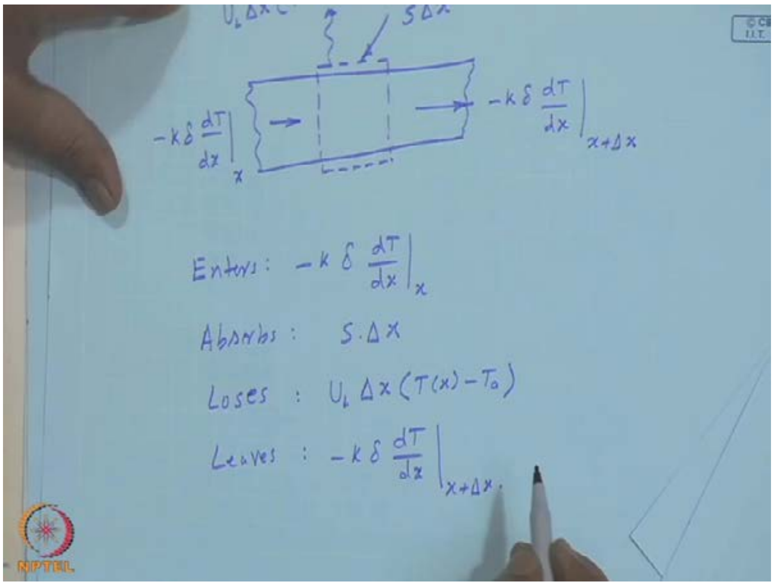

So, I shall just make the energy balance, if I once again, this is the control element. Those of you who are familiar with the heat transfer course, you know that this type of analysis have been done a large number times, and here enters minus K delta d T d x at x, and what goes is minus K delta d T d x at x plus delta x, and what comes on to this is, s times delta x, and what goes out is U L times delta x into T minus T a. So, I can write, enter minus K delta d T d x at x, absorbs s times delta x, loses U L delta x T typically function of x minus T a, and leaves minus K delta d T d x at x plus delta x.

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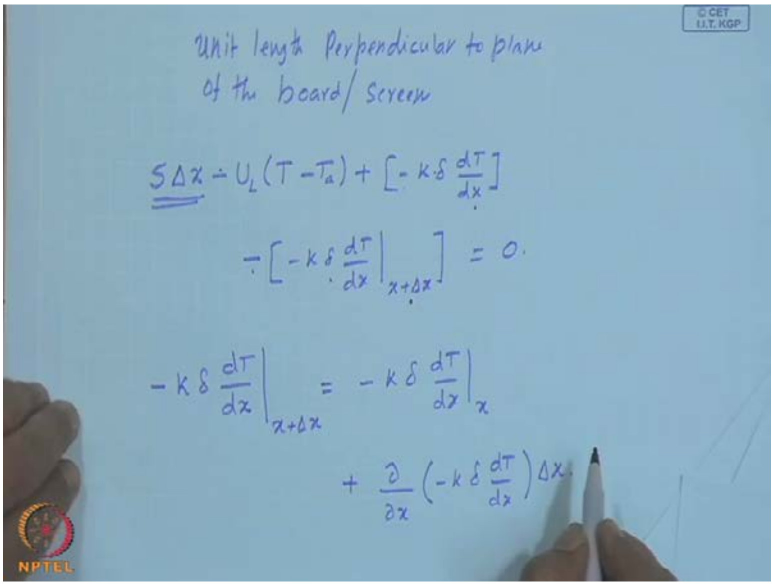

So, this is what we are having, and if you want to make a energy balance, and we are assuming unit length, perpendicular to plane of the board or screen. So, S into delta x enters, plus minus U L into T minus T a, leaves plus minus K delta d T d x enters, and then leaves minus K delta d T d x at x plus delta x, this should be equal to 0. I prefer to write this equation, just not as an equivalent part in terms of physics, this is an entering radiation that is absorbed, and U L into T minus T a leaves it is a loss, and on one side the entering is given by Fourier law of conduction minus K delta d T d x, and what presumably leaves, the control element is, the same thing at x plus delta x, so there is a minus sign. Now the gradient of d T d x is going to take care of, whether this is positive or this is larger, this is smaller etcetera. This minus K delta d T d x, at x plus delta x equal to minus K delta d T d x at x plus d by d x of minus K delta d T d x into delta x.

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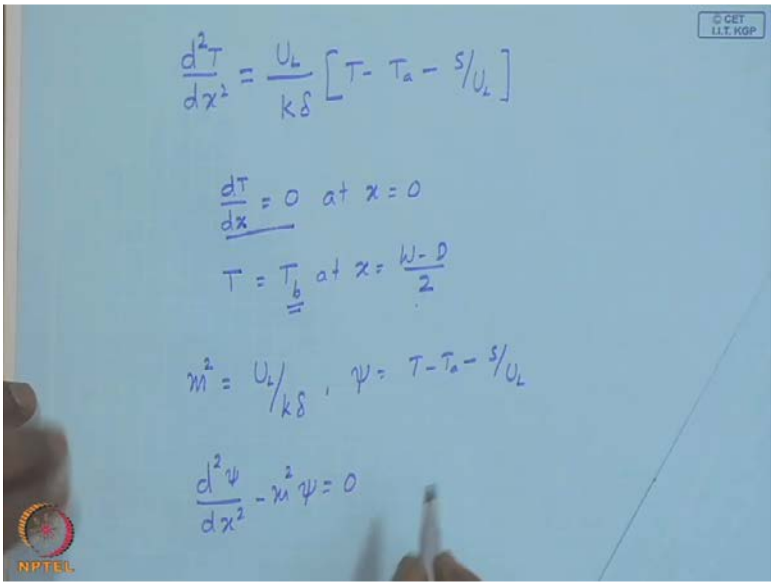

So, if you plug in these values, you will get… So, this second derivative comes from that d d x of minus K delta d T d x, and if I equate that this is what we will have. And this a second order equation requires two boundary conditions which will be, d T by d x is equal to 0 at x is equal to 0; T equal to T b, at x is equal to W minus D by 2, at the base of tube this is given by W minus D by 2, x is equal to and it is T b. And by virtue of symmetry, we call the insulated thing as heat transfer terminology, because the d T d x is 0. And if you introduced m square equal to U L upon K delta, and psi equal to T minus T a minus S upon U L, this becomes, and the boundary conditions in terms of psi be…

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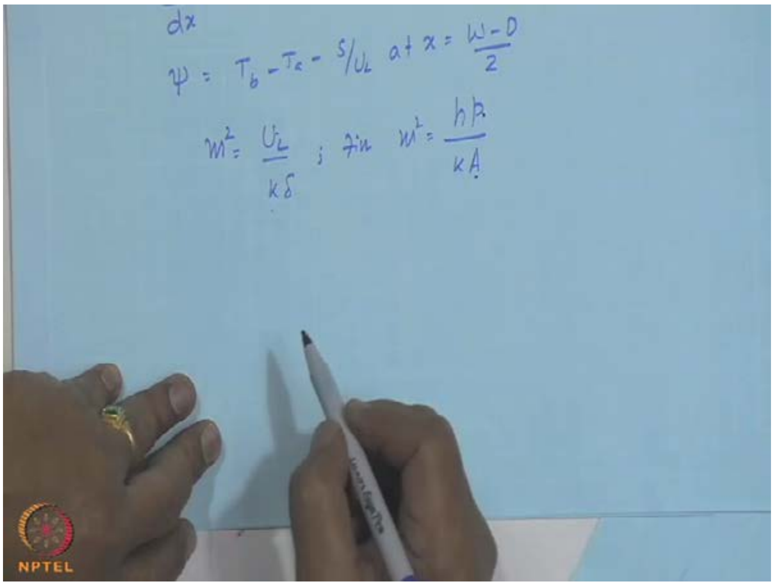

So, we have secondary equation; in the x direction, we had considered half of a repetitive element, from the mid plane of the sheet up to the beginning of the tube; and you should be familiar with this equation in the heat transfer, it is nothing but exactly the fin equation. And you can see that m squared is U L up on k delta, and in fin case m squared is h into p upon K A. So, you have a length dimension extra here, and another length dimension extra here, instead of that you will have U L by K delta in this case, otherwise it is nothing but the fin equation. We will try to get the solution in the next lecture, and until then bye.