Machinery Fault Diagnosis and Signal Processing

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Lecture -09
Frequency Domain Signal Analysis

This lecture is on Frequency Domain Signal Analysis like as I told you before, in condition based monitoring, Signal analysis is very, very important. And that to every machinery has a distinct frequency of operation. So, through the signal which has been obtained through the transducers, we need to analyze them to find out the characteristics frequencies of the signal.

Or in other words the signature of this machinery because every machinery has a distinct frequency. So, how do we find out the frequency of that machine in a particular signal which we have acquired? So, in this class, we will be going briefly into, the mathematics of such signals; how through Fourier analysis, you know, people do determine the frequency components of a signal.

For example, you all would recall in your undergraduate math class on Fourier transform, the teacher would have given you, many waveforms. It could have been saw tooth and square wave etcetera triangular and told you find out the A, B and C coefficients. And then, expand this waveform as a series of, sums of, sines and cosines. And in fact, this is the very starting point of frequency domain of signal analysis.

And in this class, we will be focusing our attention into such analysis. And later on, we will see how such analysis can be used to analyze real world mechanical signals.

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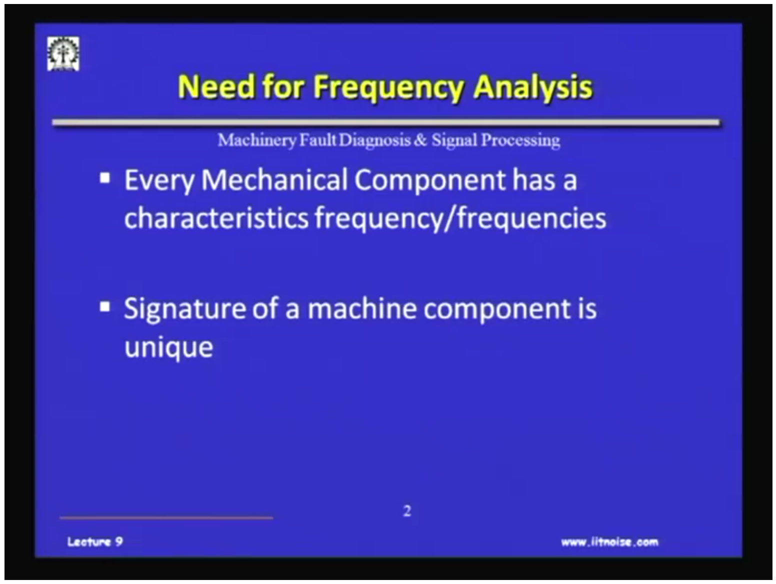

Well, you, as you all know, the need for frequency analysis is because of the fact that every mechanical component has a characteristic defect, characteristic frequency or frequencies which could be related to the defect in the machinery.

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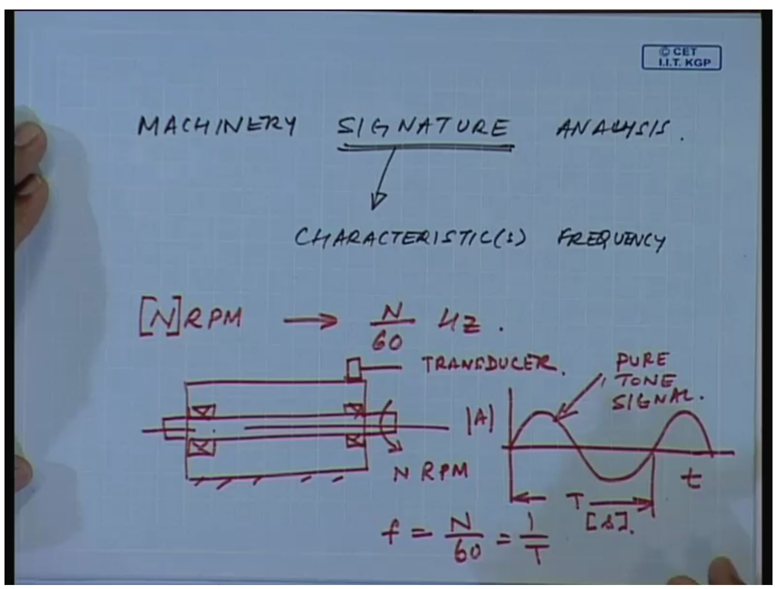

So, we will be having a distinct signature out of this machinery. This signature is its characteristics frequency. For example, if a machine is rotating at, say, N RPM, its rotational frequency is nothing but N by 60 Hertz. So, if I have say just a machine, may be shaft supported enough bearing and this was rotating at N RPM and if I had put an transducer here, okay.

We all can appreciate the fact that, in the time domain, the signal obtained by such a transducer would somehow look like this. And the inverse of its time period is a frequency f is equal to nothing but N by 60 is equal to 1 by T where T is in seconds, okay. So, this is a simple signal coming out of machinery.

Well, if the signal is of this form, I can very easily tell, you know, the frequency is the inverse of the time period. And such a signal having just one frequency is usually known as a pure tone signal. But you will realize later on, you will see actual signals, coming out of machinery. No such signal from an actual machine is of such a single pure tone.

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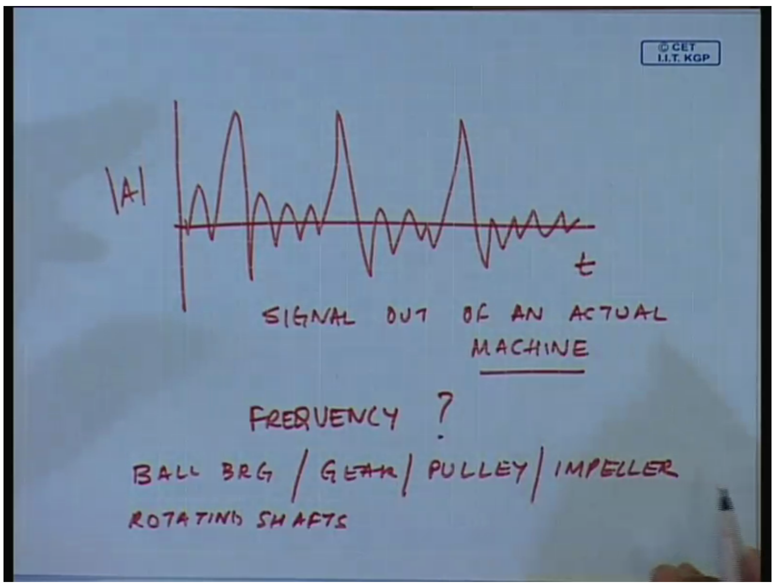

In fact, if you are to measure or record, the signal sort of real machineries, this could be of this nature; signal out of an actual machine. So, if I asked you the frequency of the signal, you all will be lost as to, well, how do I find out the frequency of such signals because I do not see a distinct time period? They are randomly varying. So, this is the problem we have in front of us. How do we estimate frequencies of the signal and obviously this is not a pure tone signal.

This is not a single frequency signal. There are many frequencies, the problem before us is, how do you, I found out, how do I find out if frequency contains in the signal. So that is what we are going to look into, in this frequency domain signal analysis and machineries. And we will see how we can do that, okay. So, because of the fact, why again frequency domain signal analysis is important because signature of a machine component is unique; when I am talking about, say, in a machinery ball-bearing, gear, pulley, impeller, etcetera, a rotating shaft.

Rotating shaft itself can manifest in many ways as an unbalanced, as a crack shaft, as a misaligned shaft, as a loose shaft, as a board shaft. So, there are many ways so each of these mechanical conditions meets a defective bearing, defective gear, defective pulley, unbalanced impeller, impeller with a blade not present. So, they all will have different characteristic frequencies.

So, if I got such a signal from a machinery where all these components were there, and if I can find out the characteristic frequencies of the defects I can pretty well, say, well, that defect has occurred. Because I see that frequency in the signature obtained from this machinery. Very simple to know, you have the student's row list and you ask everybody to sign those who are present; they will sign. So, you know, in this class, these, these students are present.

Similar to that, if I know everybody's signature, before hand, and if I do a signature analysis and if I see the frequency components showing up, I can pretty well, say, well perhaps, this, this, these defects occur; because their characteristic frequencies have come up in the analysis which I have done, okay. Now, let us see, how we do such analysis.

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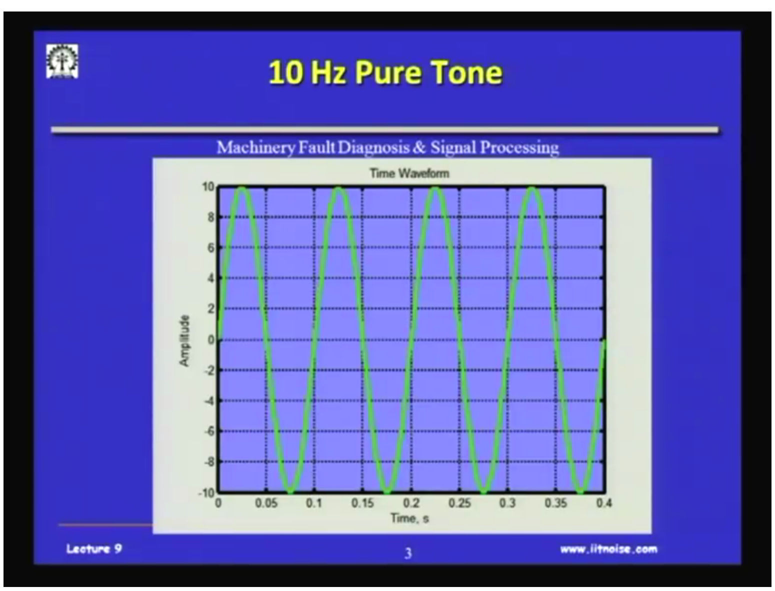

For example, I just explained to you before, that this is a 10 Hertz pure tone, because the time taken from 1, 0, 1 minimum to the next minimum is about 0.1 seconds; exactly, point one seconds, okay. And if I take the inverse of this, I will get its frequency 10 Hertz, pure tone. In Oslo scope, if I get such a signal, I can just take the inverse of the time period and then find out the frequency contained in another signal.

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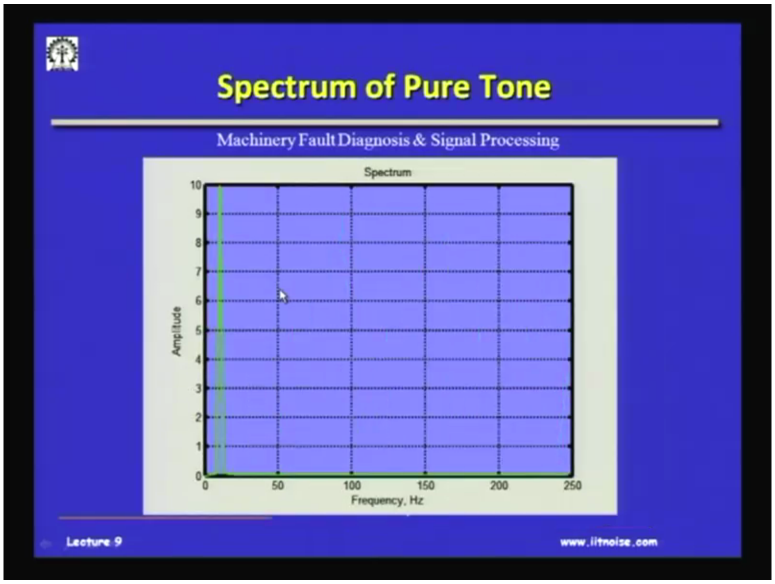

But, and if I do this spectrum of this pure tone, I will get what is known; and this is in frequency so I will get a peak at 10 Hertz, okay. I notice this, this had an amplitude of 10 you know whatever the unit 10 volts or 10 whatever mechanical units. So, this is its amplitude. I know it's time period. So, in the frequency domain once the analysis has been done, I also see 10, amplitude and also a frequency of 10 because this is 50 and if I can divide it by 5 times this comes to 10, 10 Hertz.

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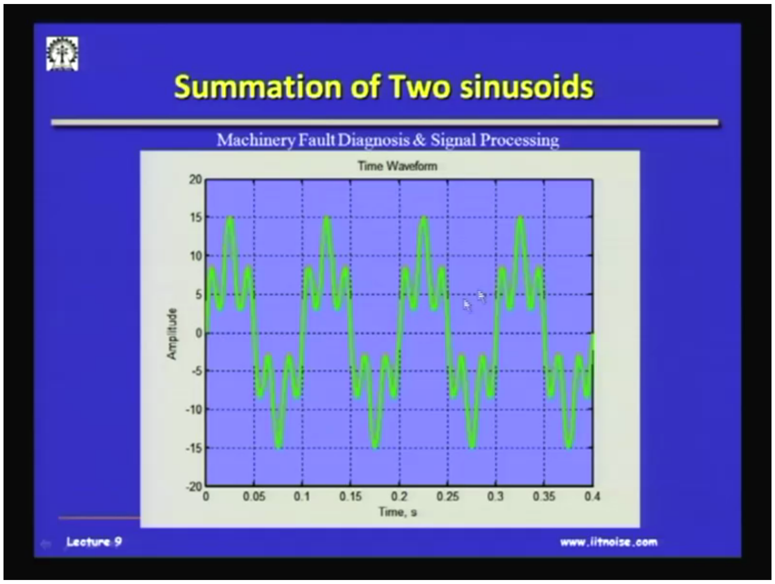

Now, we will take it a little forward, in the sense, if I have just added two sinusoids, it will be very difficult. If I asked you, what are the two frequencies present in this signal? If some of you may estimate, you know, and do a correct estimation; but now, imagine, today it is two sinusoids; tomorrow it could be 30 sinusoids, 50 sinusoids. If I sum them up together, you will have a signal, we are just described like the signal of an, a real world machinery signal.

And this is the problem we have in hand. So, a real world signal looks, in fact even worse than this. It is not even as uniform and simple like this. Those, if you have done experiments in the lab must have seen this; in the, on the oscilloscope or on your analyzer, okay. So, I have just done the analysis of this signal. And I will show you this is what in fact it is.

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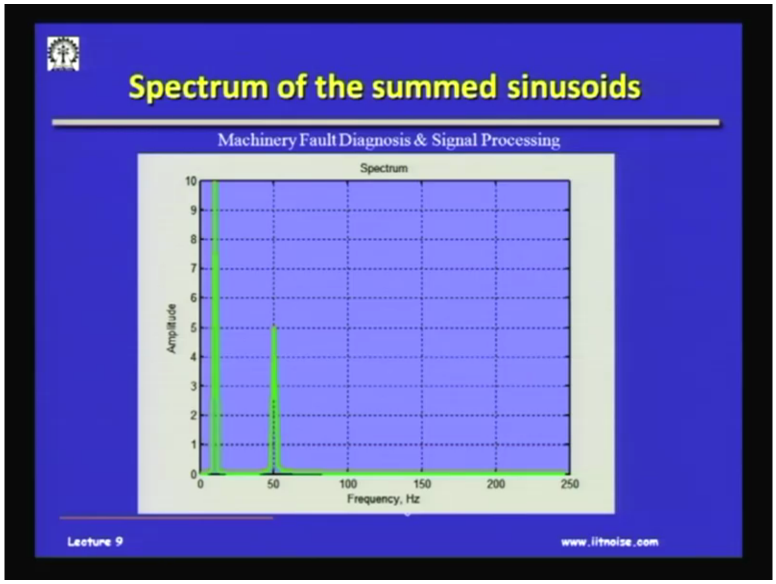

It is the summation of a sine wave of 10 Hertz and at the amplitude 10 and another with a n amplitude of 5; and then, frequency of 50 Hertz.

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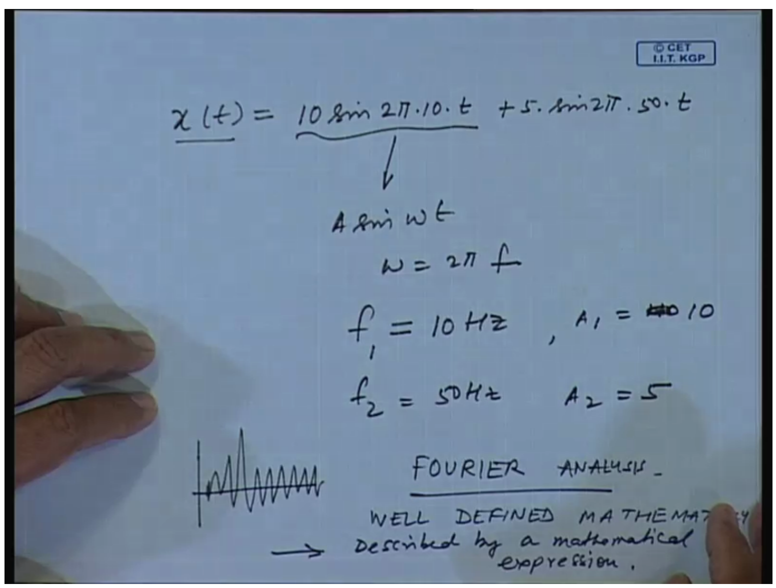

So, if I have to write this signal so the composite signal x (t) is nothing but 10 sine 2 pi times 10 t + 5 sine 2 pi 50 t ,ok. So, if I just replace this as A sine Omega t, ok. So, Omega is equal to 2 pi f. So, in this problem, we have f in one case, f 1 is 10 Hertz; other case f 2 is 50 Hertz; A1 is 10, sorry, 10 and A2 is 5, okay. Just by comparison, I have such a signal. Now, I could mathematically add many functions. And then, get up to this signal.

But, in the real world signal, I have signal like this. If I can break it up into such components, I will know, yes, this frequency is f 1, f 2, f 3. And that is what we will be using the concept of Fourier analysis to find out the frequency components, okay. We will first see, for the case of signals, which are well defined mathematically. That means, they can be described by mathematical expression or a function, okay.

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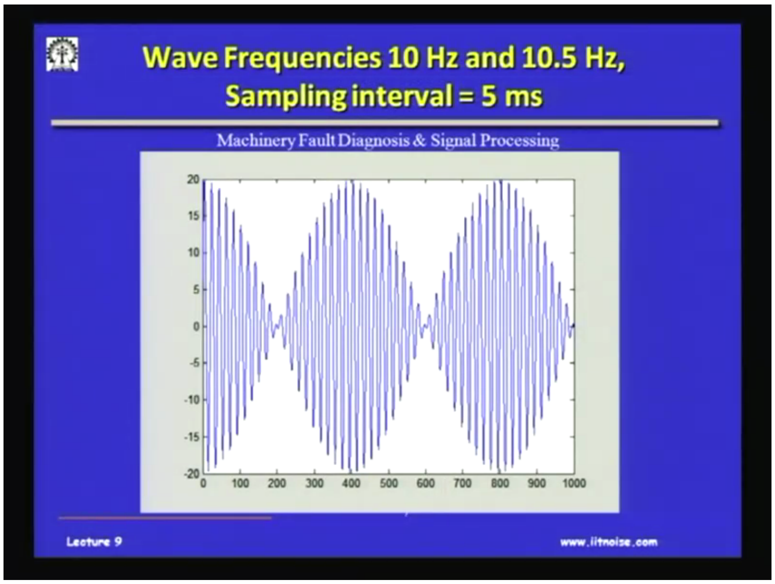

Now, I had shown you the case, when one was 10 Hertz and other, another was, another was 50 Hertz, okay; but if I add two signals 10, 10 Hertz plus a signal, having 10.5 Hertz, that means, these frequencies are very close, very close.

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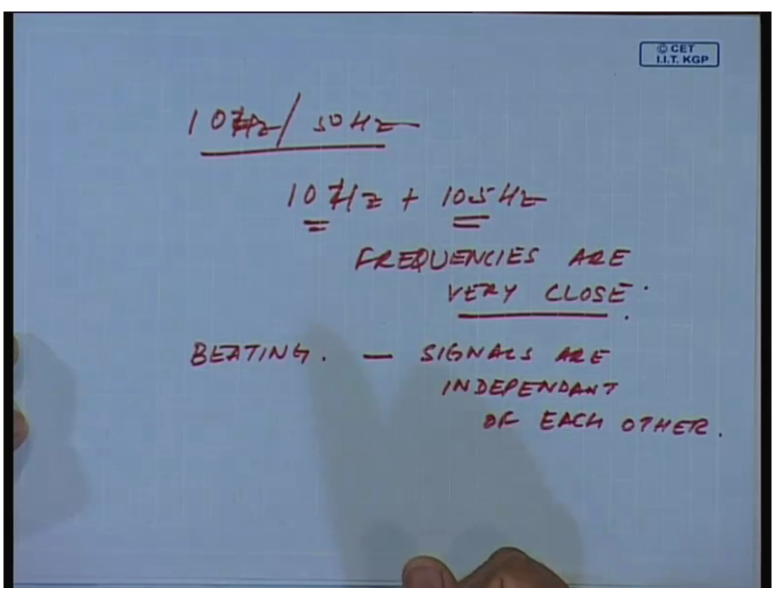

Then what happens? When the frequencies are closed, a phenomena, the resulting amplitude looks something like this. That is the amplitude. Suddenly increase, decrease, okay. And this is known as the signals are beating or beat a phenomenon has occurred, where the signals are independent of each other, okay. Now, I will give you an example. There are ways, you know, Fourier series is one method of estimating the frequencies of the signal.

But there are other methods to find out the frequency of the signals, okay. One such method is what is known as by signal heterodyning. I will pose you a problem, in the case, in the sense, suppose in this signal, one is 10 Hertz and another is 10.5 Hertz. I slowly bring by a frequency, where I have a provision that I will have a frequency oscillator or a generator wherein I can change the frequencies.

And I have a meter which is observing the amplitude of this meter, of the signal, resultant signal. So, when there are frequencies of these matches with this 10 Hertz, this amplitude will is going to be steady, is it not t? So, then I will say that the frequency of the unknown signal is equal to the frequency of my known signal. Let me explain to you in this method.

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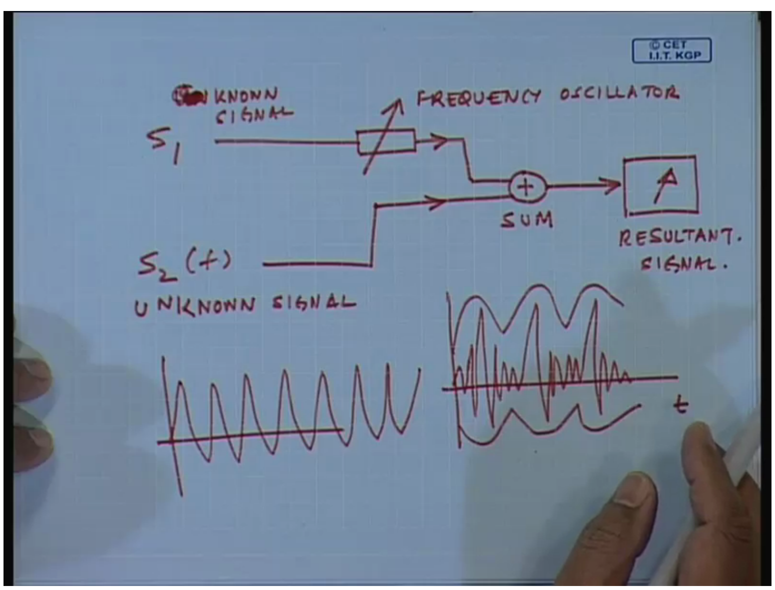

Suppose, I have a signal s1; suppose, I have a signal s2 where is the s2 frequency is known to me, ok. And I have, well this is the unknown signal. Now, my problem here is to know the frequency of this unknown signal, okay. I have another signal which is my reference signal whose frequency is known and whose frequency is set by me, all right. So, if I was to these, these two frequencies were well apart, I will be having certain frequencies here.

But once they come close to the, unknown signal is close to this frequency, what is going to happen? I just told you, this beating is going to happen, right, okay. Signal as, is going to reduce in amplitude with time, okay. Now, once the unknown signals frequency matches with the known signals frequency, this is going to be steady, amplitude, okay.

So, this will be a steady thing. I will relate this to, you know, in the earlier days, you know, people use transistor radios, okay. When they tune the transistor radios and when the change the frequency setting of the station, when they came close to the frequency of the station, you will hear a kind of a noise like waning and waxing, okay wow, wow.

And then, finally it will catch on to ye Akashvani hai, okay; because it has last on to the, because the frequency which you are setting is equal to the frequency of the received signal. And then, they will match. And then, you will hear the clear voice. But once you are really very close to this station, while tuning it manually, you will hear this signal which is wanning, waxing because of the beating, okay.

And earlier days, there is, you know, when, you know, people were not having Fourier systems, when they are not having fast computers, they were tuning the frequency of the; because one frequency is known to you; so the unknown frequency if it matches, there will be a steady amplitude, okay. And this is what is known as signal heterodyning. And I will show you the math behind this.

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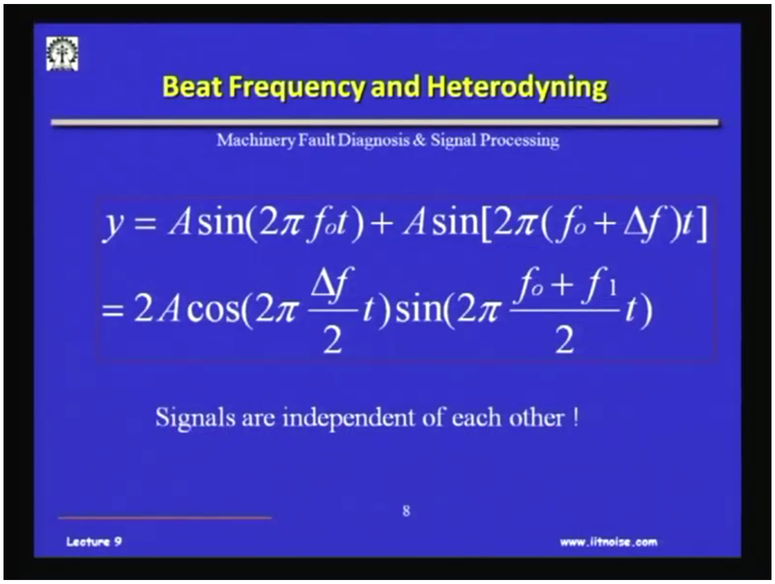

And this is what happens. For example, when I have the signal 2 pi f naught t, and it is very close to f naught plus delta f, okay and once this two frequencies match, delta f becomes zero. It is, okay and once this becomes zero, I will have because delta f is nothing but f1 minus f naught. So, this will become 0 and whole things will disappear and then, I will have a steady amplitude, okay.

And this is not a signal heterodyne. To know the signal of an unknown signal by matching it with the signal of a known signal, I hope this is clear to all of you. And this is just from the trigonometric functions. But mind you, the signals are independent of each other; if they are dependent, they will modulate. And that is something we will discuss later on, in the classes.

But beating signals because, you know, the known signal and unknown signal, they are independent of each other, ok. I have a set of known signal, some unknown signal is coming I am summing it up. And then, seeing this resultant signal, ok; so, if I just add two signals, this is from the trigonometry. This is the sine A + sine B formula, has been used here and I will get this relationship.

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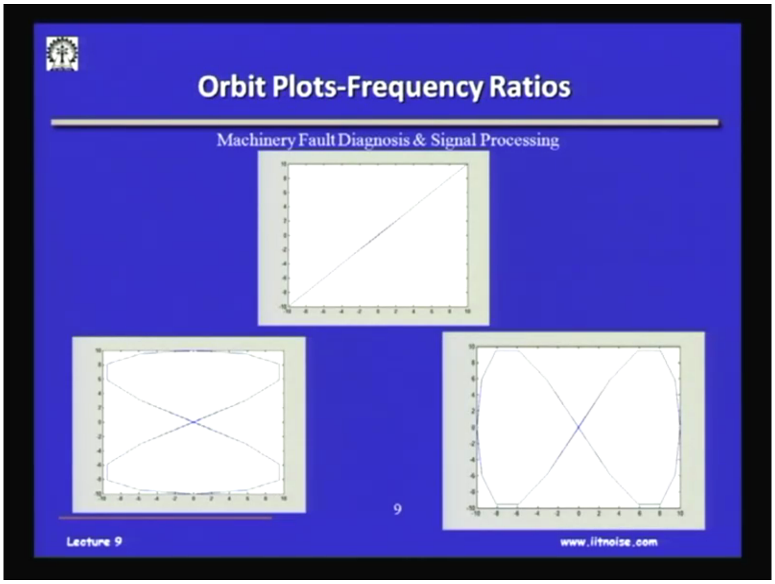

Now, another way of finding out the frequency, because I am telling you, certain tricks which are used without even using Fourier analysis. And one such is heterodyning; another one is comparing the signals. If signals, you know, if you do this, if the signal is the two signals are the same frequency, this is, if one is in the x axis, other in the y axis. I will get a plot of like this. If one is the twice of the other I will get such figures and these are known as Lissajou's figures.

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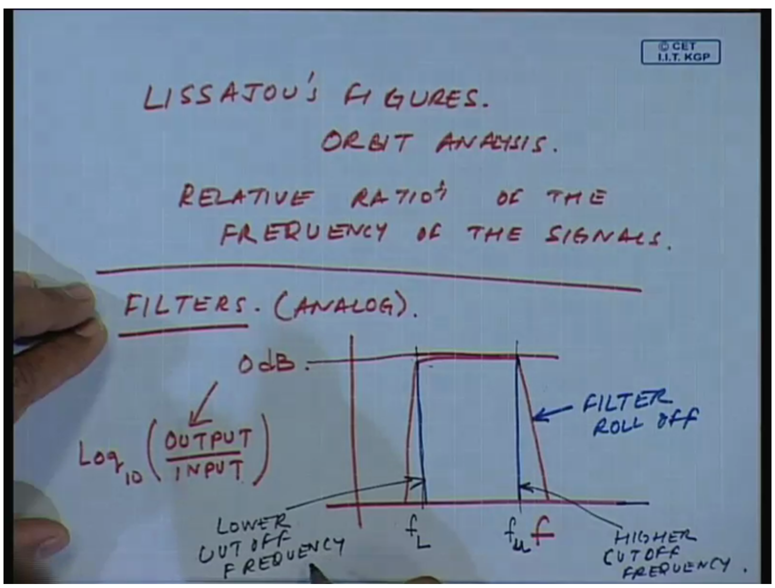

And these are mostly used in orbit analysis. By this, we can find out the relative ratios of the frequency of the signals, ok. Now, before I come to Fourier series, there is another method which we can do is, set up filters. This could be analog in nature, in the sense, what is a filter? Filter means it will allow signals of certain frequencies to pass through and not pass through.

For example, if I do the frequency response in the frequency domain, this is how typically the response of a filter looks like. And this is known as 0 dB. This means logarithm of output by input, okay. It could be log 10, when output is equal to input, what happens? It becomes 1. And log of 1 to the base 10 is zero, so, this is known as zero decibel.

And this is the typical frequency response of a filter. This is the band pass filter wherein it has a lower limit f of L and there is an upper limit f of C or F of u. And this is known as a lower cutoff frequency. And this is known as a higher cutoff frequency, okay. Now, an ideal filter, what would happen in an ideal filter? This should be very, very sharp, okay.

But this does not happen it takes certain other frequencies because this is known as a filter roll-off. And this depends on the number the order of the filter. I mean, I can have a sharp filter, sharp band or steep, depending on the order of the filter. I will not discuss this in details. But such filters are available in the market, wherein I know the lower frequency and the higher frequency.

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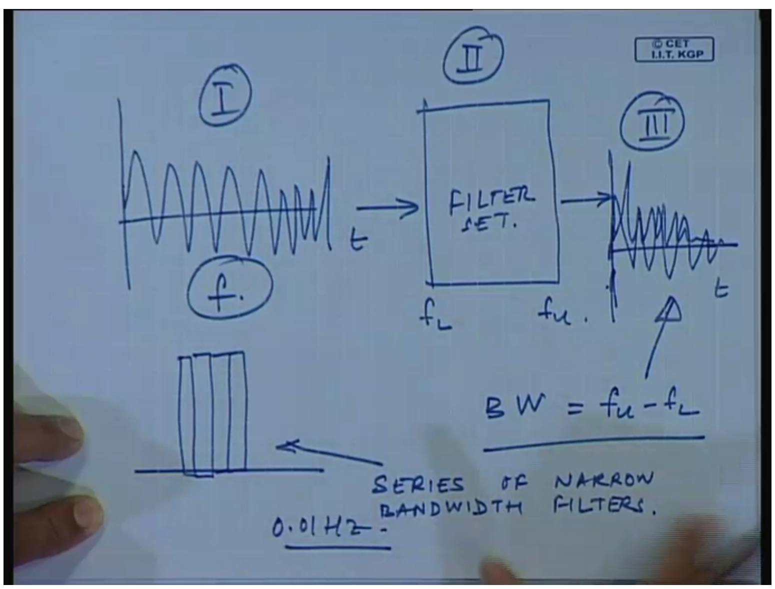

So, if there is an unknown signal frequency is not known, I will pass it through a filter set wherein, I know, the f lower and f upper. And if I get some signal, I know that this signal is devoid of or this signal has only the signals in, or again, I can look, look into them also in time domain, this is all in time domain.

This, means that the frequency and this is between f upper minus f lower; and this is the frequency bandwidth. Here, all frequencies are present. I do not know what they are. In stage one, I pass them through your filter in stage two, in stage three, if I get certain signal, I know, the frequency of this signal is only between f upper and f lower, right. No problem? All, in time domain, I need not know what the exact frequencies are.

Well, question is, now, if I want to know what the frequencies are, I can make this bandwidth very, very small. I can put a series of filters with very narrow bandwidth, is it not? So, I can put a series of narrow band width filters. And if I know each one's bandwidth, I will see what are the frequencies which are present or not, just by monitoring the amplitude coming out of this frequency of this filter set.

So, I can have filter set, wherein, these bandwidths could be tuned, they could be digitally programmed, they could be operated through a software, they could be set manually. So, there are many ways by which we can do that. Traditionally, when computers were not available, people use such analog filters. Very broadly, the analog filters were used; they would manually tune the frequency of this filter set; see, the amplitude at the output of the filter sent.

But, nowadays, there are computers, where we can very; in fact, this bandwidth could be as low as .01 Hertz. Very fine resolution of 0.1, 0.1 into .0, 1 + 2, into 2, into 3, into 4, into 5 and then, I can have series of such digital filters, ok. And this could be done very serially one after, the other. But, this as you will see, will be very time consuming.

And it cannot be done in real time and there are problems associated with it, ok. Nevertheless, filters can be used analog filters, digital filters to do such analysis. And while I am on the subject of filters I should tell you about few other filters and how their responses looks like in the frequency domain.

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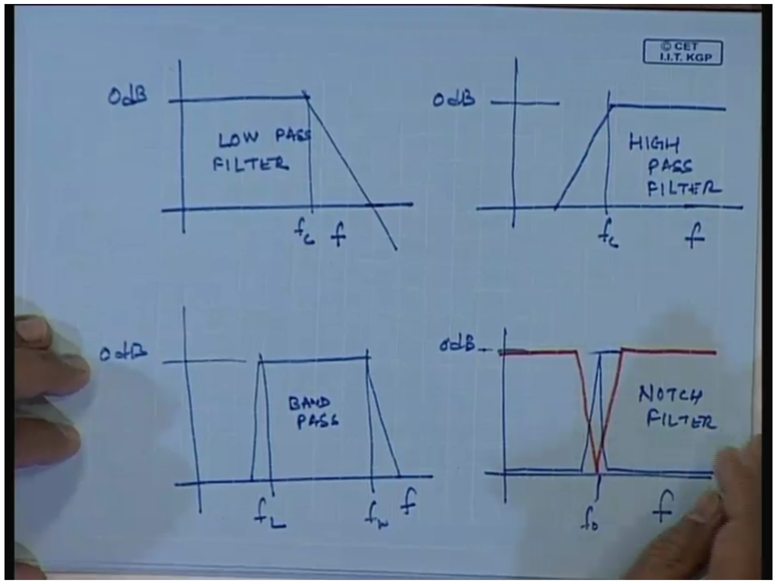

Traditionally, we will be using about 4 types of signals. Of signal of filter of this sort, this is the response of the filter. Other could be another, we just talked about. And another is or I am sorry, this one is the other way, in fact, please make a correction. So, this is, as you will see, is known as a low pass filter. That means this allows the low pass frequency signal to go up and then after certain frequencies, they are cut down, okay.

Because by using such a low pass filter, I can know, that no frequency is beyond this cutoff frequencies are present in the resulted signal, low-pass filter. And opposed to this, this is known as the high pass filter. That means, every frequency beyond the high pass frequency are available, are allowed to pass through. And this is, as you know, the band pass. And this is, it will kill a particular frequency and this is known as the Notch filter.

Notch filter, I will just give an example. For example, in many of the electrical circuits we have a predominant frequency of 50 Hertz. So, if I put a notch filter of 50 Hertz, it is going to remove that 50 Hertz signal from this actual signal. So, this is where a notch filter is used. So, if I have such filters, I can also have an estimate of the frequency response of the signals, okay. Now, we will look into the, another theory behind frequency domain analysis. And that is what is known as this Fourier analysis.

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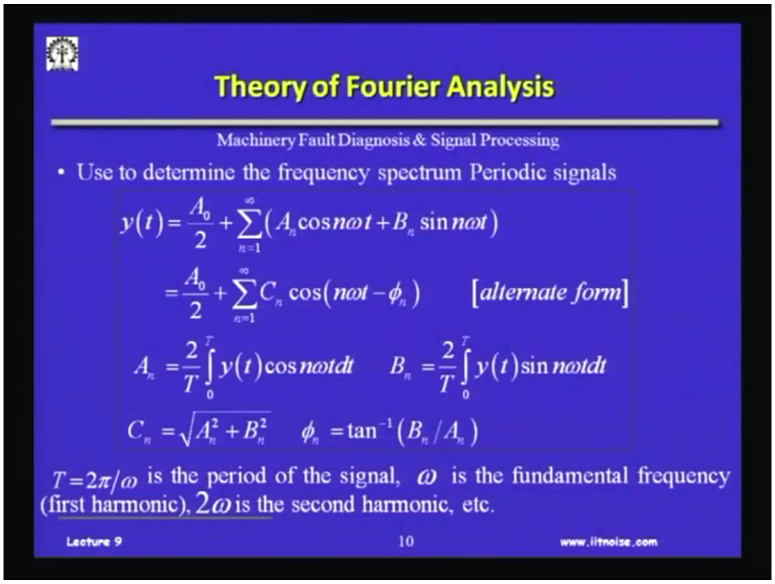

And I am sure this is a very, very important analysis which is used in engineering. In fact, mathematicians say, there are lots of 10 great inventions or discoveries in mathematics. Fourier series is one such analysis features. Fourier series has a lot of engineering applications: be it engineering, with mechanical, civil, electrical, be it in medicine.

Everywhere, Fourier analysis is useful in the sense that the most important thing is, anything, I will say, what this, you anything can be broken up into its frequency components. Be it signal, be it physical dimensions with the surface roughness. They can all be broken up into different sums of sines and cosines, okay. And that is a why this signal is, this is very, very important.

So, let us go back to, how to determine the frequency spectrum of periodic signals. Now, as you all know, what is the periodic signal, signals which repeat with time; and Fourier series says that the signal has to occur from plus infinite to minus infinite and periodic in nature.

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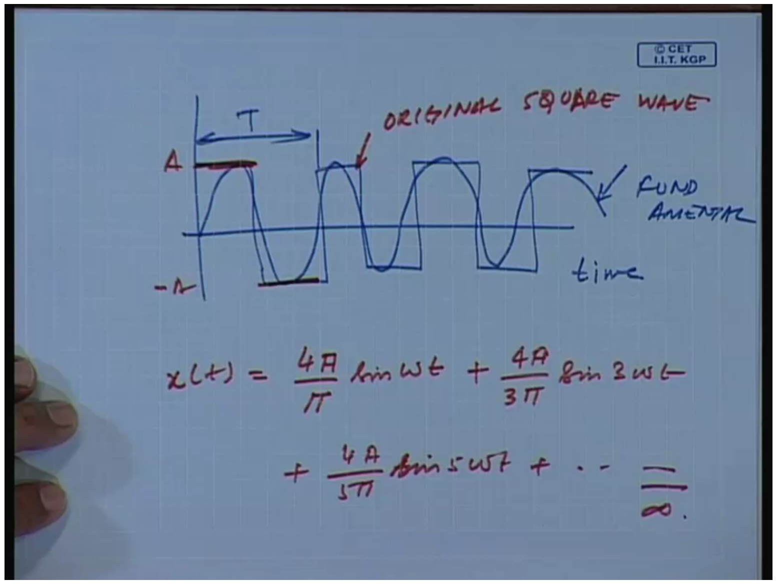

So, such a signal y(t) where, y is in, in the time domain can be broken up into components as A naught by 2 plus, n is equal from in cosine n Omega t plus B n sin n Omega t, where n goes from 1. Fine, 1 goes to finite. There is another way of representing this. But, we can find out the components An and Bn, by this integration, ok.

I will not go into details of this. But I will just tell you the fact that, yt to determine An and Bn. You will see from this equation yt has to be mathematically known to me. And then, we have a serious problem here. The real world mathematical signal which I have the real world machinery signal, which I measured, I do not know, it is yt. I cannot represent it in a mathematical expression.

If I was able to do it represent it mathematically, I could very easily find out it's An and Bn's by doing this integration. And that is a problem. We will see how we can overcome this. So, this capital T is given by 2 pi Omega is the period of the signal, where Omega is the fundamental frequency or known as the first harmonic, that the fundamental it is always known as the first harmonic.

And twice of that is the second harmonic and so on. And this you will find in any mathematics textbook, in the theory of Fourier series or second year Fourier series. And this is the very generic form of such an analysis and I will not go into the details of such an analysis here.

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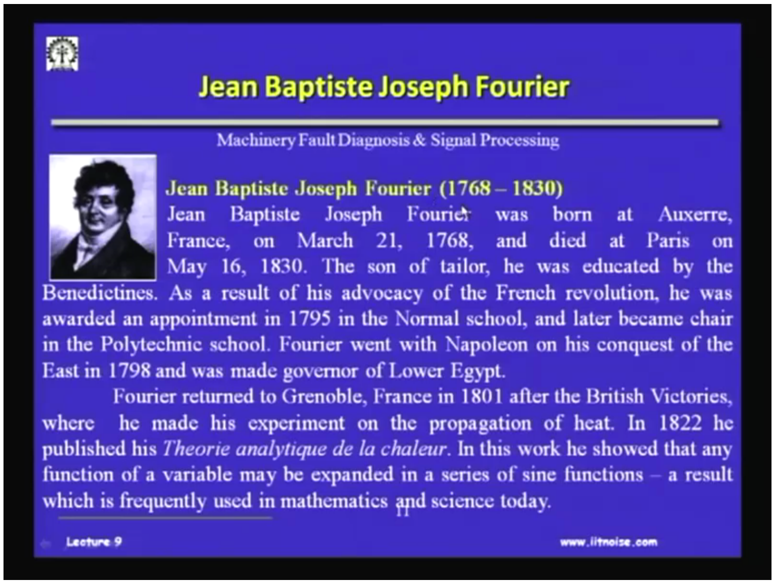

But, I must tell you because of this Fourier, you can all make a note of it; and because I always find it, we should know about the great mathematician who has given us this Fourier series to us. Jean Baptist Joseph Fourier, and you must have heard of a Fourier law of heat conduction. It is the same person, ok. And we will see.

In his work, he showed that any function of a variable may be expanded in a series of sine functions; A result which is frequently used in mathematics and science today and which we are going to use in this class.

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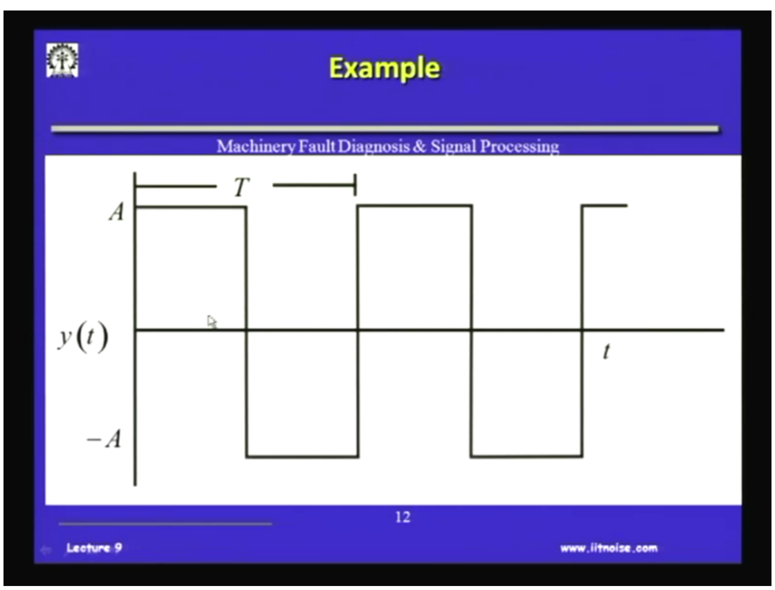

So, to begin with, if I have such an signal where in its time period is T, and amplitude is A, I can pretty well say, well, this signal has a frequency of 1 by T. So, if the signal is pure tone, I will get I would be thinking, yes very easily I can construct back the signal well if I was to construct, I will only get a sine wave back, is it not? If I just take the fundamental, but then, I am not getting my original signal.

My original, this is my original square wave, okay. But then, if I expand the Fourier series, there will be the other terms and other terms are very, very important.

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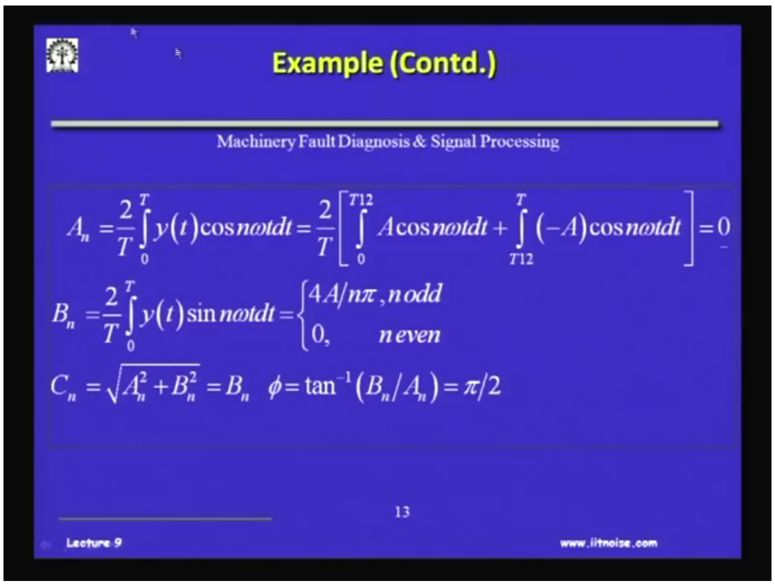

So, if I was to take the definition of this Fourier, apply this on this square wave, and I was to estimate, An say, An comes out to be 0 because the frequencies divided from 0 to T by 2. If you go T by 2, it is this there is a typo error. This is not 1; this is P by 2, okay. So, 0 to 2 by 2, this is sorry, excuse me, this is A and T by 2 to T, it is minus at the second half.

So, this one is A and this one is minus A. So, I have broken up the signal into two halves. And then, similarly if I do this, I will land up with in component of this form A n is 0 and Bn is equal to 4A by n pi. So, square wave can be broken up into different components.

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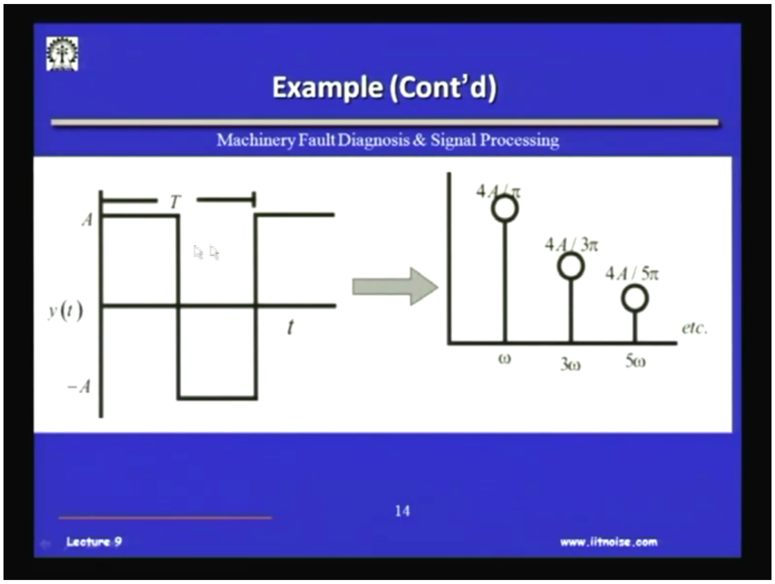

And thus, if I was to represent a square wave in the frequency domain, this is the square wave which we originally had. For T Fourier series expansion, by this expressions here, I will land up with such an expression, wherein this is, at the fundamental is at 4 a by PI, the second harmonic in this case, it is 300 mega because the second one does not exist. So, it has Omega 3, Omega Phi, Omega 7, Omega and so on.

So, A square wave xt can be broken up into 4A by PI sine Omega t + 4A by 3 PI sine 3 Omega t + 4A by 5 pi + sorry, sine 5 Omega t and so on. And if I take this term, still infinite, if I add them up together, I will get back my original square wave. And with confidence I can say, well, I have broken up in this Fourier, this square wave by Fourier series into its individual frequency components.

So, a square wave is just not a sine wave, but a summation of such sine waves .Omega, 3 Omega, 5 Omega, 7 Omega so on, and I am sure in your math classes, you must have done such exercises for triangular wave. In a first year circle, the teacher would have given you breakup of such a wave, periodic waveform into such Fourier components.

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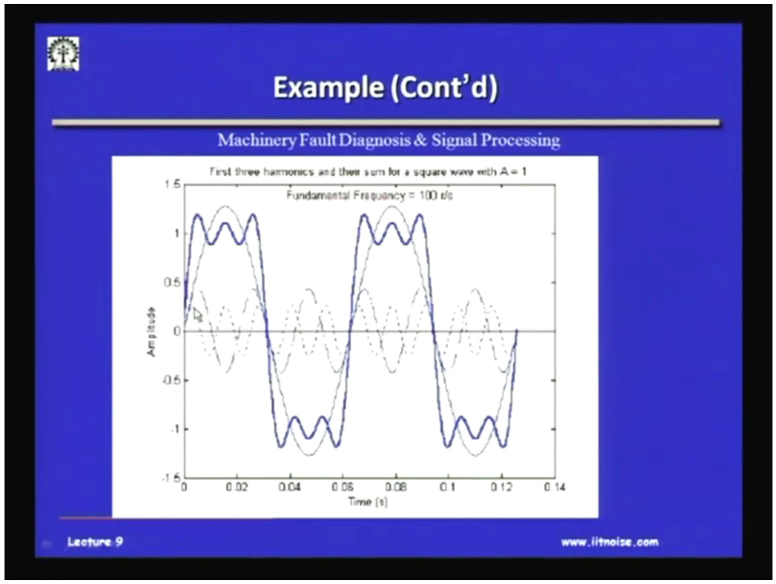

Now, you notice interesting thing here. I have now put all of them together. All the harmonics, individually they have been plotted and eventually, they have been added up. This is the sum of the first 3 harmonics with an amplitude of A = 1. And if you see this dark blue line, we have quite not got back the original square wave.

Had I added more terms, of course, we cannot go up to the infinite but a large number of terms, we can, you will see that you will get back your original square wave. What this means is, this square wave consists of frequencies of Omega 3, Omega 5, Omega 7 Omega etc and so if I get such a signal I will know this is its frequency content.

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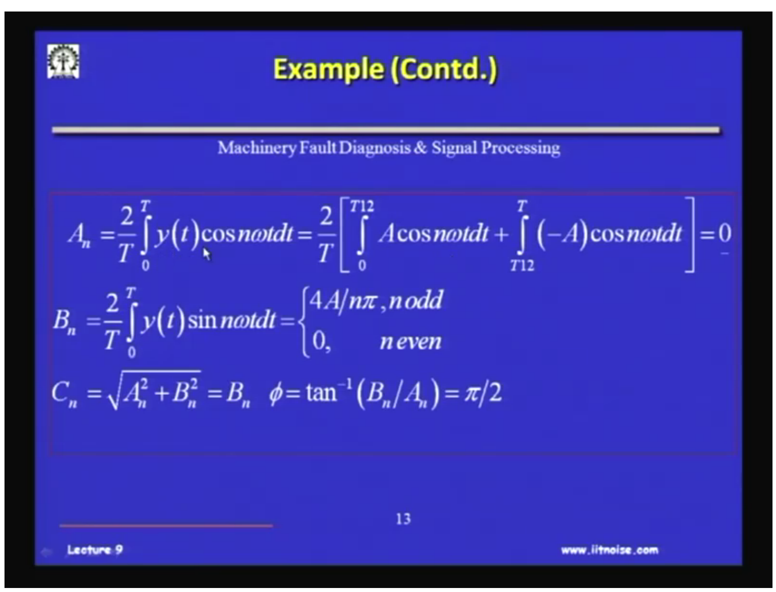

But the problem still rises; that I do not know, yt and that is, the something we have to live with it. So, we have to do what is known as a mathematical or a numerical estimation of the signal and try to go from it.

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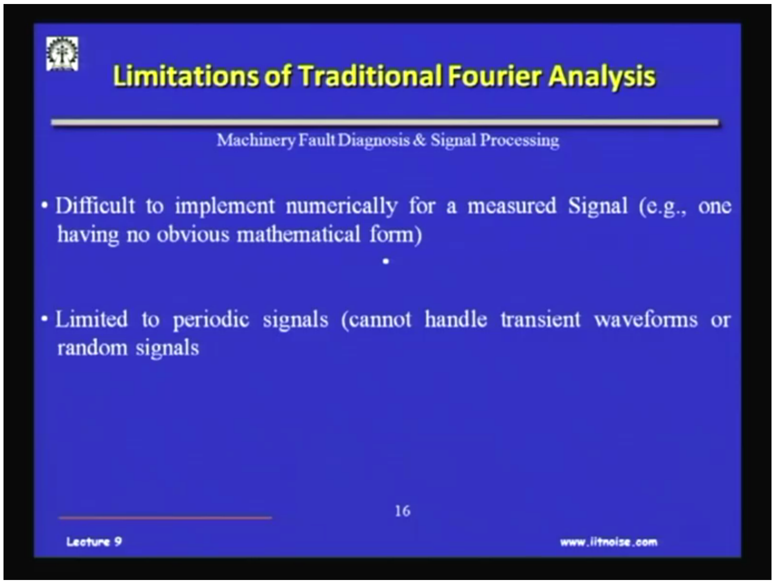

So, you will see the limitations of the traditional Fourier analysis. Difficult to implement numerically for a measured signal; because one obviously has, no obvious, mathematical form and which we shot; if I knew yt, I would have done this integration very easily got back. And another limitation of this Fourier analysis is, it is limited to periodic signals. It cannot handle transient waveforms or random signals.

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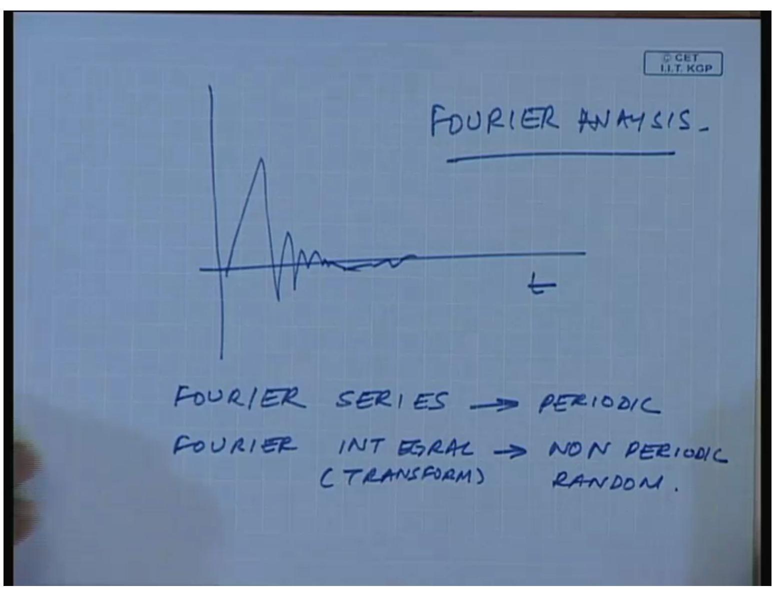

For example, a period, a signal, we just, occurs like this. I possibly cannot find out its periodic components because it is not periodic. And these are the limitations of Fourier series analysis okay. So, well, how do you do about, go about it?

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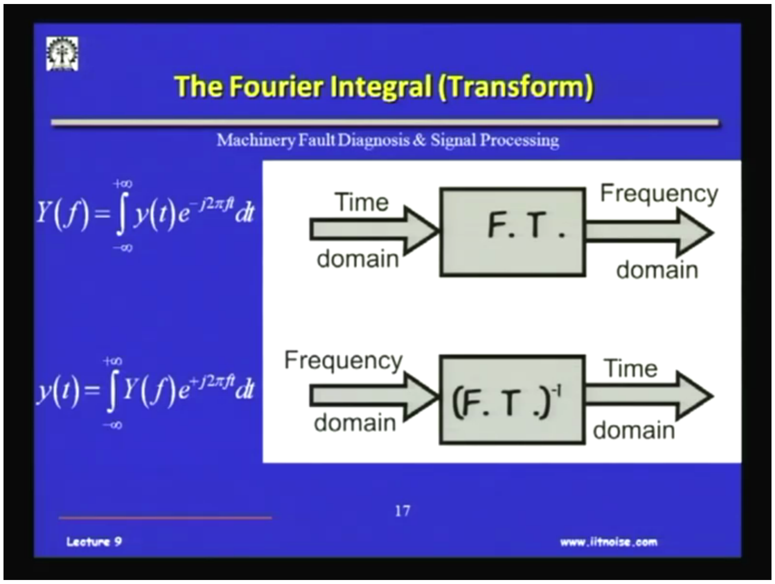

There are the Fourier series, there is an integral method to do it, wherein instead of summations, I do an integration from minus infinity to plus infinite. And in such a way, I can transform a signal from the time domain, to the frequency domain by doing this forward transform, okay. And we will see how this is done. And the vice versa is also true if I have a signal in the frequency domain, I can get back its individual time domain components.

And this is known as the inverse Fourier Transform okay. When we have it in the integration form, we call it as integral. Or in the digital form, it will be known as the Fourier transform, okay. Digital Fourier transform, okay. And this can be all done digitally, okay. But, in order to do this Fourier transform, I need to have representative of the signal, ok.

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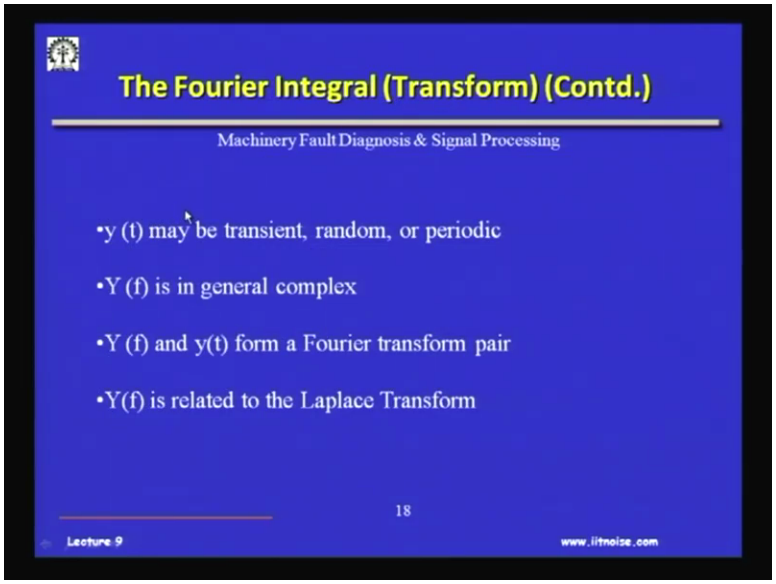

And the Fourier transform or integral, this yt could be anything; may be transient, could be random, could be periodic, okay. And the Fourier transform is a complex quantity, okay. And then, we always have a Fourier transform pair, wherein, the signal can be related from time to frequency or frequency to time. So, we have this Fourier series which is for periodic functions.

We have the Fourier integral transform, which can be for non periodic and the random signals. And then, if you look back here, this integration could be done numerically. Once I do numerically this integration, I need to have an estimate of the signal yt, at every numerical steps, okay.

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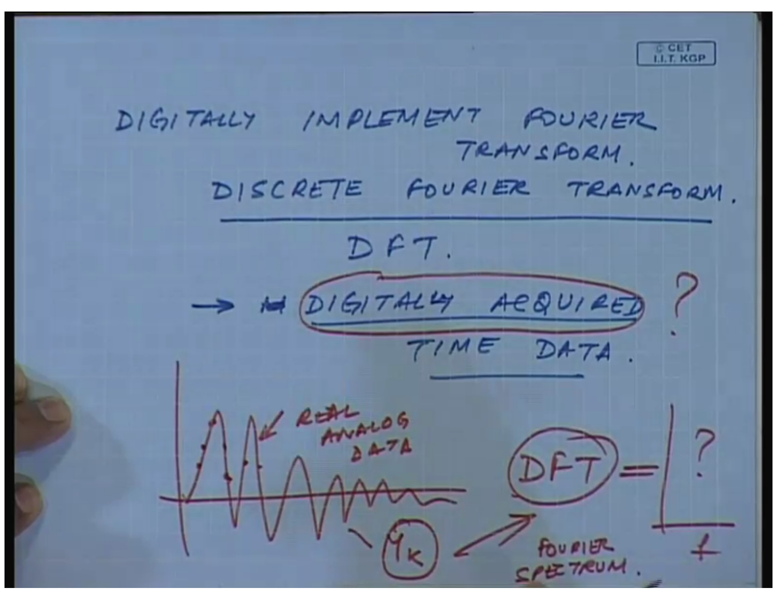

So, to digitally implement the Fourier transform, I can call it as digitally implement Fourier transform or sometimes it is known as Discrete Fourier transform or DFT. I need to have Digitally Acquired Time Data. Then, this integration can be done. I will. So, the problem is now, how do we get this digitally acquired? I have a signal which is occurring like this I want to do is DFT so that I can get the frequency spectrum.

But then question is, how do I do DFT? Well, there are many algorithms to do, you to do DFT. We will discuss that in the subsequent classes. But most important is I need to digitally acquire the time data. And that is known, what is known as the computation computer aided data acquisition. It has to be acquired by a computer converted from analog to digital sample data.

This is a real analog data. I have to digitally sample them, so that this digital data yk can be put in the DFT algorithm to get the Fourier spectrum. One such full spectrum is obtained by us, we can see, how this analysis can be done, okay.

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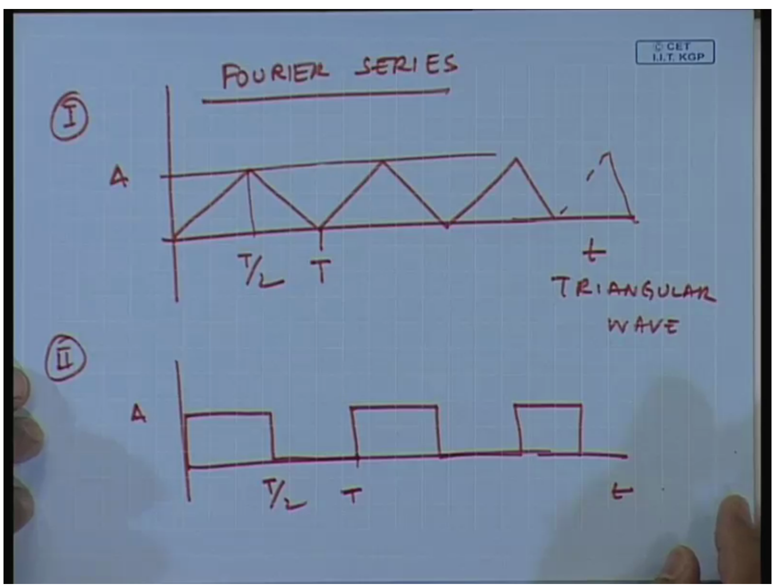

So, as a homework I could ask you to try out the Fourier series of, find out the Fourier series representation on these signals. You can take them as A, and this is T and T by 2. And this is going to infinite, this is a triangular wave. Another is, you could have T by 2, T, A, ok a positive sided square wave, where the negatives are chopped.

You can find out the Fourier series expansion of such waves. This could be homework, for all of you, to do, okay. And we will either put the results in the website or we will discuss that in the subsequent tutorial classes, this lecture, and okay. So, in the next class, this would conclude this lecture on the frequency domain signal analysis.

But we still do not know how to implement this with the real world signal. But the problem I pose to you is, we need to acquire this analog data into a digital form. Once this data is there, there are many algorithms which will be putting in, to find out the frequency estimate of the signal. And we will just discuss that in the subsequent classes, okay. Thank you.