Fundamentals of Industrial Oil Hydraulics and Pneumatics
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Module02 Lecture04
Incompressible Fluid Flow Related to Fluid Drive

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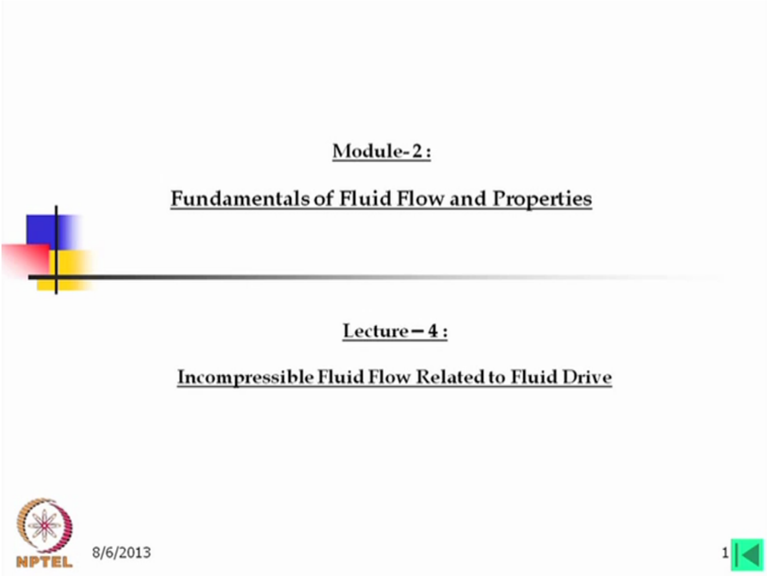

Welcome to lecture 4 incompressible fluid flow related to fluid drive. This is under module 2 fundamentals of fluid flow and properties.

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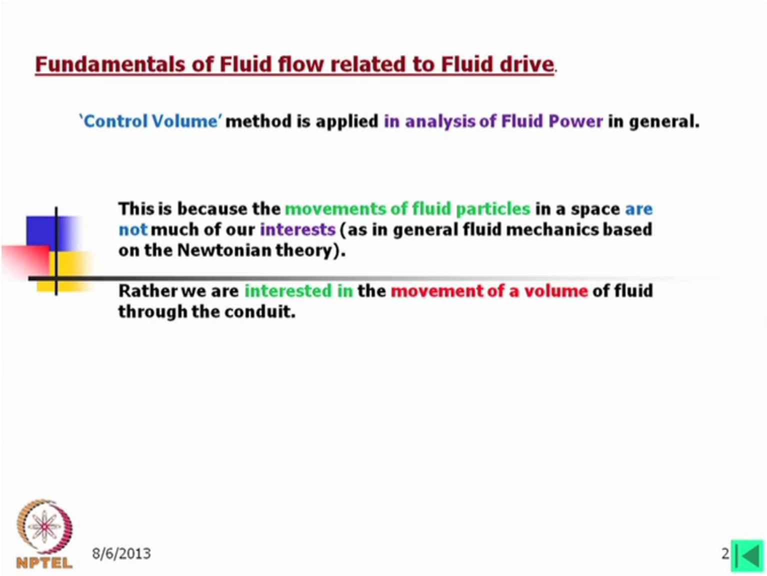

Control volume method is applied in analysis of fluid power in general. This is because the moments of fluid particles in a space are not much of our interests as in general fluid mechanics based on Newtonian theory. Rather we are interested in the moment of volume of fluid through the conduit.

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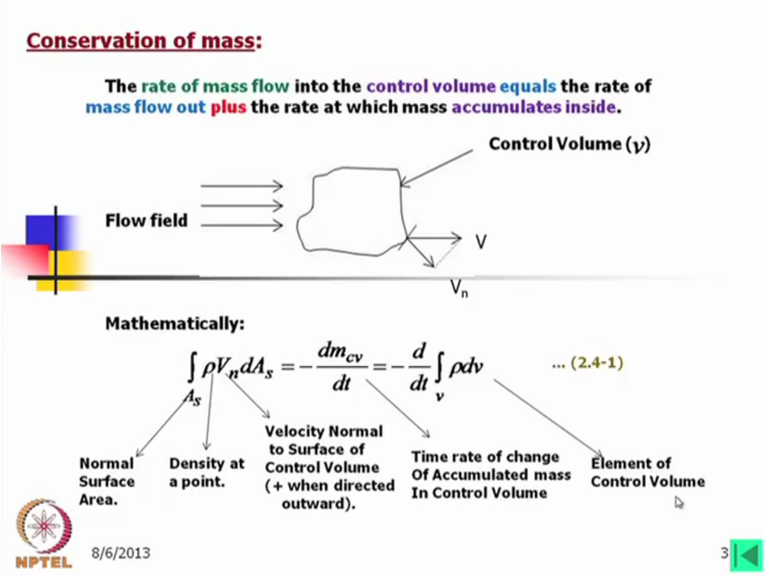

Now let us consider conservation of mass. The rate of mass flow into the control volume equals the rate of mass flow out plus the rate at which mass accumulates inside. Now let us consider a control volume, fluid is flowing inside this volume and then we consider a velocity in normal direction at a point on the volume which is Vn and obviously which is having a straight component V. Now mathematically we can write Rho Vn dAs is equal to minus dmcv by dt is equal to minus d by dt integration of Rho dv. Now in this case, As is the normal surface area, Rho is the density which we have already discussed, Vn is the normal velocity and this normal velocity, it is importantly is positive when directed outwards that means the direction shown is positive. That must be equal to time rate of change of accumulated mass in control volume, mcv is the accumulated mass. Now this means that the minus sign is given here, the reason is that if the mass is accumulated inside then Vn will be in the opposite directions. If it is going out then it will be minus that must be equal to the elemental mass of control volume we should consider the rate of change of elementary mass of control volume.

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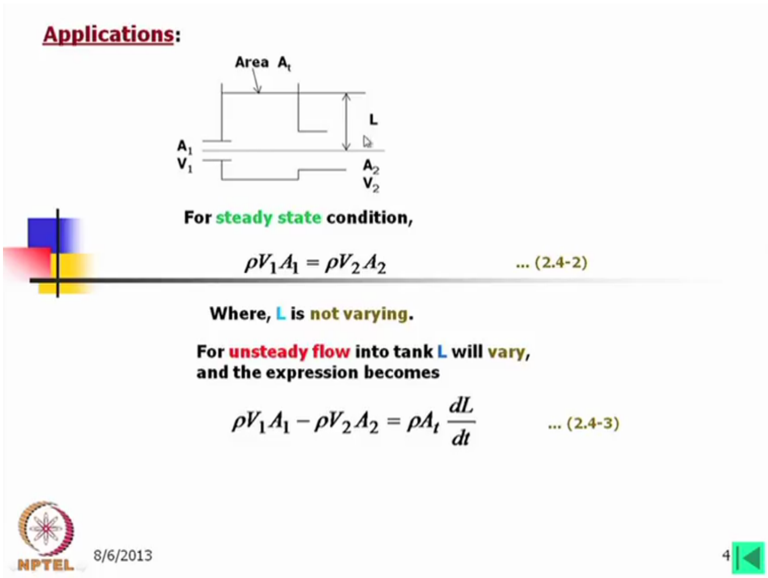

Now application of this conservation of mass or momentum theory is that. Let us consider a simple conduit sorry, a vessel with a conduit. Now flow in to this control volume through the left side where the area is A1, V1 is the velocity and that area surface area of this vessel is At and height of a datum (())(5:18) line is L where, the area is A2 and velocity is V2. This means that along this line when the flow is going in velocity is V1, flow is going out velocity is V2, area of this outlet is A2, area of this inlet is A1 and At is the surface area of this vessel and L is the height from this datum line.

Now for steady state condition, we can write Rho V1A1 is equal to Rho V2A2. What does it mean the steady state conditions that there will be no change in this volume? Where, L is not varying. Now for unsteady flow into tank that is this vessel, L will vary, and the expression becomes RhoV1A1 minus RhoV2A2 is equal to Rho At dL by dt. This means dL by dt is will be positive if this height is increasing and this is automatically it will be negative till L is decreasing.

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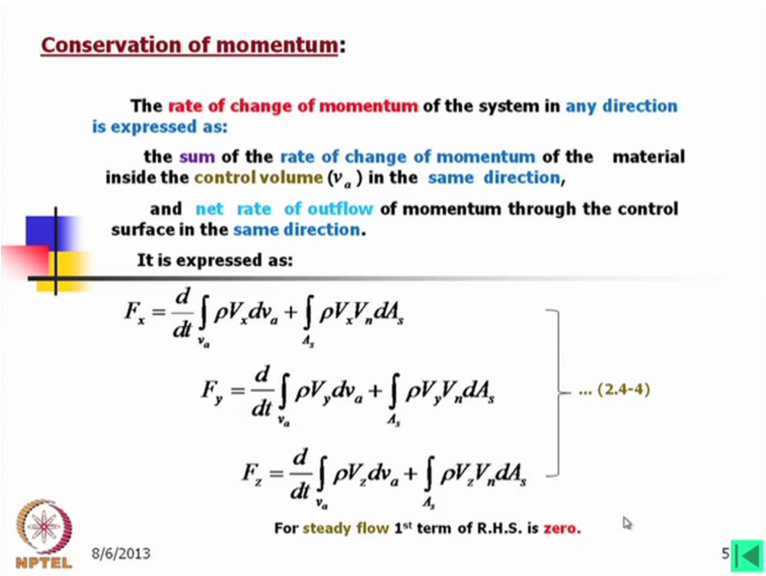

Now conservation of momentum. The rate of change of momentum of the system in any direction is expressed as the sum of the rate of change of momentum of the material inside the control volume, which we have considered as a va small va in the same direction and net rate of outflow of momentum through the control surface in the same direction. It is expressed as Fx is equal to d by dt integration of Rho vx dva over the volume va the control volume va This vx is the velocity in x directions plus integration of Rho vx Vn and dAs. It will be in the y directions as you look into these equations only we will considering here the expression is same, velocity in y direction and here also we consider the velocity in y directions. Similarly in z directions we consider a velocity vz here and also here. For steady flow first term of right hand side is zero. If it is a steady flow then there will be no change in this whereas, there may change in this surface area and accordingly this equations will be reduced.

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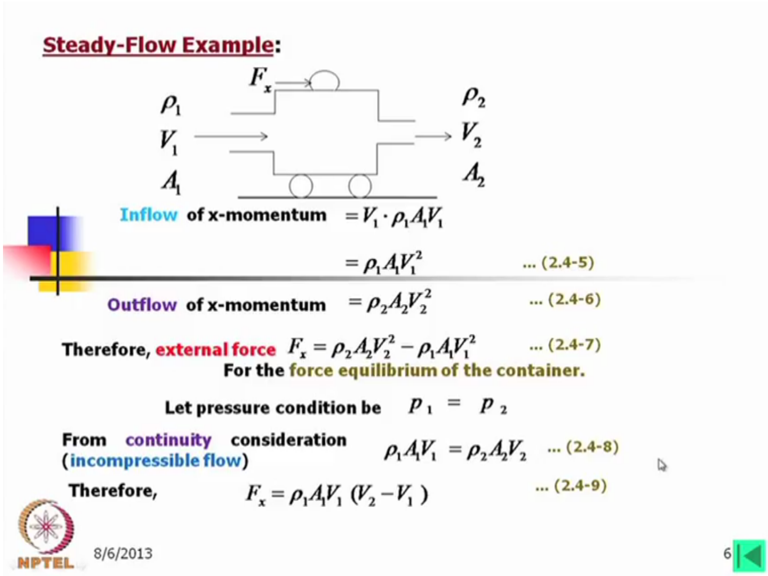

Now steady flow example, let us consider another tank. It is a closed. It is having a inlet and a outlet. Now let us consider at the inlet area is A1, oil is having density Rho1 and velocity V1. At outlet it is Rho2, velocity V2 and area is A2. Now Rho1 and Rho2 may be equal for incompressible fluid if there is no change in temperature. Now for this type of flow you will find that if we do not apply a load this will move, because this velocity obviously will be higher than this and a force will be generated here. Now to balance this for equilibrium we apply a load Fx and therefore, the relation we can write inflow of x-momentum is equal to V1Rho1A1V1 that is equal to Rho1A1V1 square. Is it clear? This is the volume flow rate and with the velocity. So we get this is the x-momentum.

Similarly, outflow we get it Rho2A2V2 square. Now equating, we get the external force Fx is equal to Rho2A2V2 square minus Rho1A1V1 square. This means that we have to apply this force for the equilibrium of this body. Let pressure condition be P1 is equal to P2. This means that here pressure and here pressure will be same. Then from continuity consideration incompressible flow Rho1A1V2 is equal to Rho2A2V2 and therefore, Fx will be Rho1A1V1V2 minus V1,s I think I have made mistake, it is not pressure conditions. The pressure condition remains same for which Rho1 and Rho2 remains same. Here I would like to mention the pressure remains same as well as temperature also remains same so that we get Rho1 is equal to Rho2.

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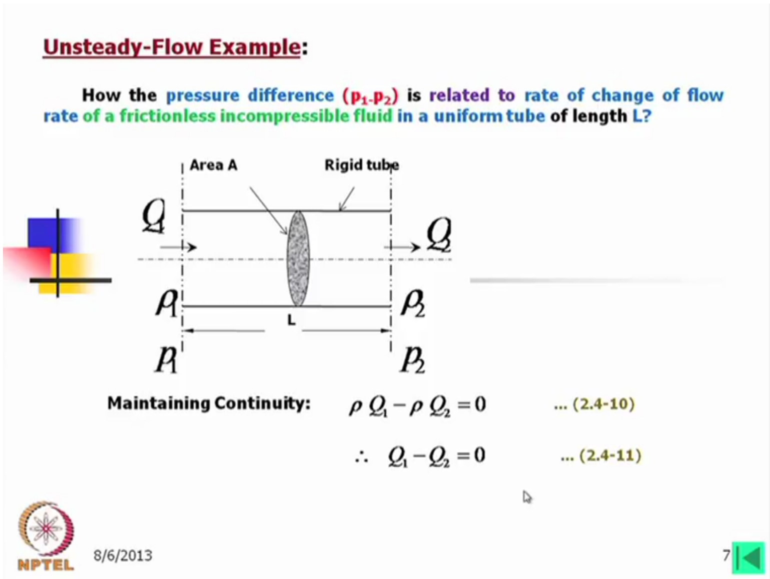

Now unsteady flow example. How the pressure difference P1 minus P2 is related to rate of change of flow rate of a frictionless incompressible fluid in a uniform tube of length L? We have considered a tube circular tube and we have taken a segment of length L. This is of uniform cross sectional area A and the tube is also rigid. Now flow in is Q1 and flow out is Q2 and we also for general tube we consider that density here was Rho1 and density this side is Rho2, pressure is here P1 and P2. We consider that pressure is changing. Maintaining continuity we can write RhoQ1 is equal to RhoQ2, again in this case we have consider Rho1 is equal to Rho2 that means density is not changing.

In most of the fluid power analysis for small conduit if length is not very large and if there is not too much change in pressure and the temperature, then we may consider Rho1 is equal to Rho2. This simplicity is required to avoid the complicacy in the calculations and in most of the cases; the error will be negligible small. Therefore we can write Q1 minus Q2 is equal to 0. This is for incompressible fluid.

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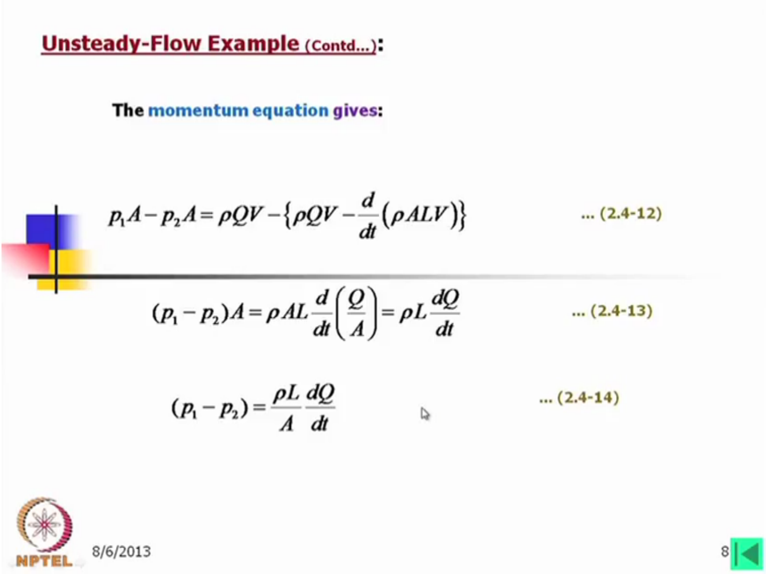

Now the momentum equation gives P1A minus P2A is equal to Rho QV minus RhoQV minus d by dt Rho ALV, A into L is the volume, V is the velocity and Rho is the density that is the mass. So rate of change of mass minus the mass in. Now this gives clearly P1 minus P2 into A that is the force is equal to Rho AL d by dt Q by A, if we equate this we will arrive here and this gives clearly RhoL dQ by dt as A is constant. This A will cancel out. This means that the force net force available due to this Rho is mass density into length into time rate of change of flow. This Q is flow rate. So this is time rate of change of flow rate. So final equations is coming P1 minus P2 RhoLA dQ by dt.

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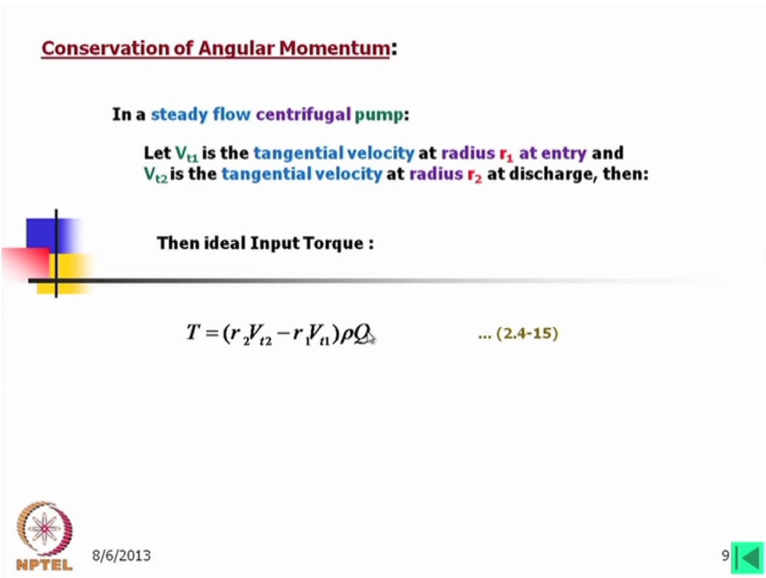

Conservation of angular momentum. In a steady flow centrifugal pump, let Vt1 is the tangential velocity at radius r1 at entry. If you think of the centrifugal pump, then you will find that entry at the toward the centre and outlet is towards the outer periphery. Now in fact, it works on hydrokinetic energy. In that case, when the impeller inside is rotating with the volume of fluid. Initially there is air, then this fluid mass gets momentum and it is discharge outside in terms it creates a suction pressure at the inlet. Anyway after let us consider it is pumping an incompressible fluid. In that case, let Vt1 is the tangential velocity at radius r1 at entry. This means that if I consider a circle at entry then tangential velocity of the mass of the fluid again a let us consider a control volume. Now same thing when it is being discharged, the tangential velocity is Vt2 at radius r2. Then we can write the ideal input torque is equal to T is equal to r2Vt2 minus r1Vt1 into Rho again density into the flow rate.

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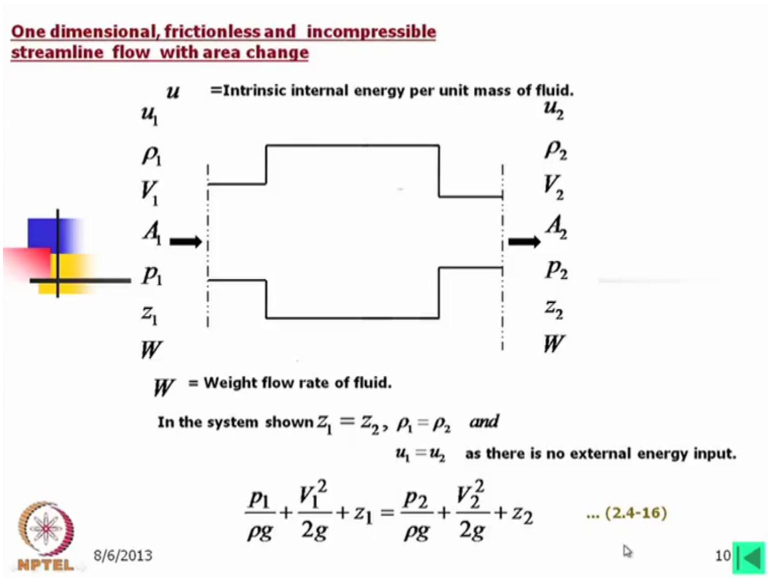

Now let us consider one dimensional frictionless and incompressible streamline flow area change. Now in this case, again we have consider a conduit, which is having inlet area is A1 outlet area is A2. In this closed vessel or closed tank, let us consider at the inlet the u1 is the intrinsic internal energy per unit mass of fluid. Now this is a little difficult to have the consent, because inside the fluid the internal intrinsic internal energy may change while it is flowing from one side to the other due to addition of some external energy or going out the energy from the inside of the conduit or the control volume.

Now this one again having a density Rho1, velocity V1, pressure P1, z1 is the height of this fluid from a datum line and W is the weight flow rate of fluid, okay. In outlet side also we consider u2, Rho2, V2, A2, P2, z2 and W. Clearly, this in this case, what we have consider in this system the z1 is equal to z2, because this is a horizontally placed we have neglect that part, Rho1 is also is equal to Rho2, because there is the change in pressure not changing the density or whatever change is there that is negligibly small and also u1 and u2 that means these is no change in intrinsic internal energy. Now then simply we can develop these equations and we can neglect also z1 and z2 as they are equal of these equations. Now these equations is very well known equations (())(22:22).

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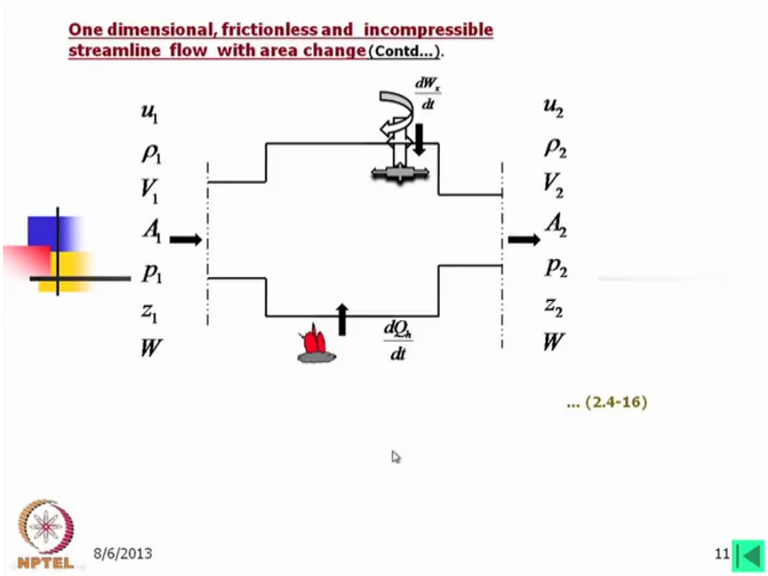

And with this equation, we will see next that if we now add some energy, then what will be the changes in these equations. Now here as I have told, this is a frictionless that means there is no friction between the fluid and the conduit, we have neglected that part. That is there always there in a conduit there will be friction, but we have neglected that part and again this is we are what we are developing these equation that is for the incompressible streamline no turbulent flow also, then equation will be different, only thing there will be change in area.

Now what we have done? We have added some external work. So that is symbolized by this say let us consider that (())(23:30) a fan or impeller is being driven and it is energy being added there. Now also we are adding some heat and there is so this is the heat flow inside. Now why we are considering the such things? The in fluid power usually this external work will be there on the fluid and this heat is automatically generated it due to the temperature rise due to change in pressure, due to the change in internal energy that we can consider in this form as well in some cases, heat also added from the outside.

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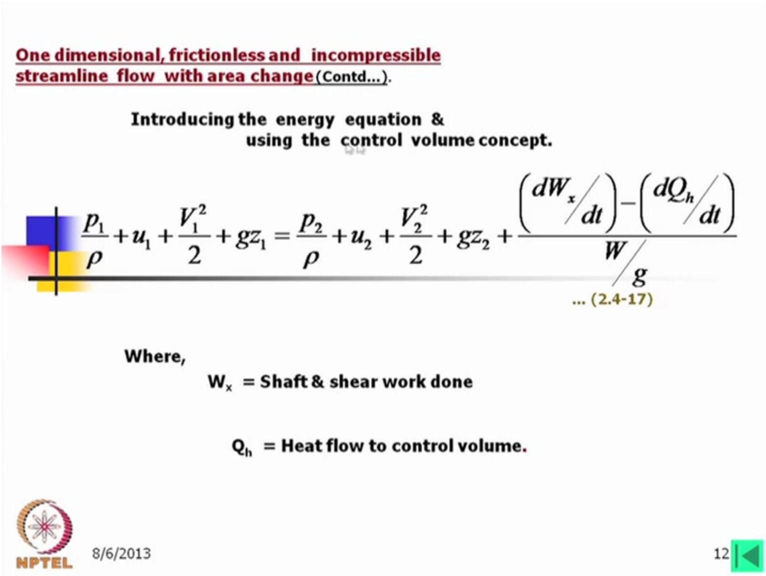

So what will be the equation? Now we introduce the energy equation and using the control volume concept. Then what we find that the same equation which we have consider earlier. We consider this u1 and u2, we consider that there will be change in you see this intrinsic energy as we are working on that, we are also adding the heat and we have we have also kept then this height term here and then the energy added is writurn in this form. This is the work added to this minus dQh by dt divided by W by g. Now Ws is the shaft or shear work done and Qh the heat flow to control volume. The here the question is that if we why it is negative. The heat flow in actually reduces to the this energy.

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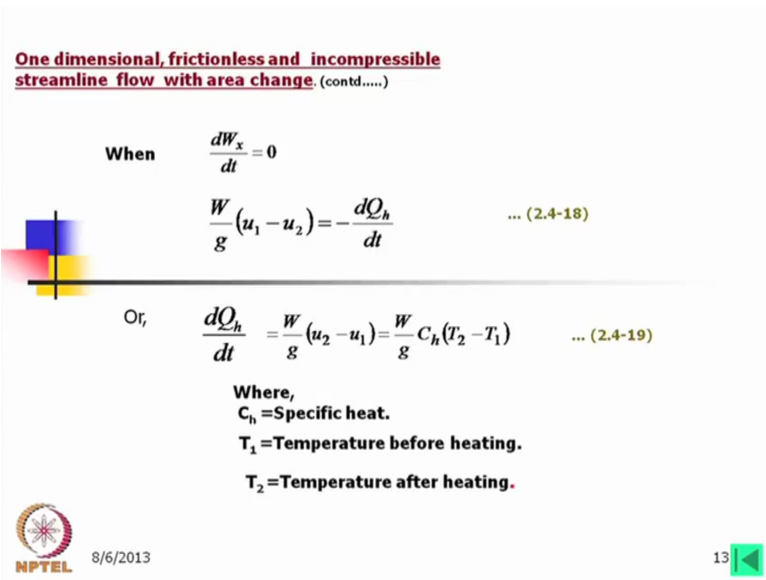

Now if we do not add any external work to the system then dx by dt is equal to 0. Therefore, the total equation will be reduced in this form. Obviously we have consider z1 is equal to z2 and we can write dQh by dt is equal to W by g u2 minus u1 is equal to W by g Ch T2 minus T1. Whereas Ch is the specific heat, T1 is equal to temperature before heating and T2 is the temperature after heating.

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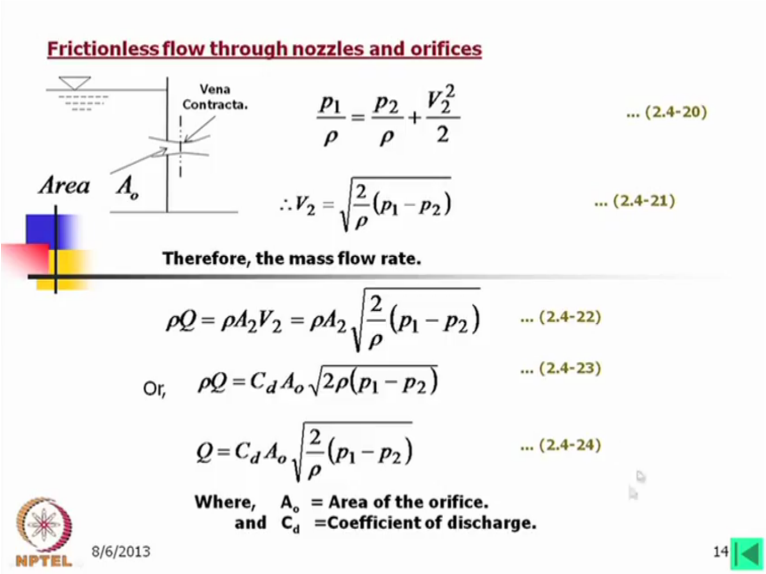

Now we consider frictionless flow through nozzles and orifices. Now here we have consider a again a vessels of large reservoir and where we are having a small hole, which is an orifice. Now this is known theory to you, you will find that oil will flow out and you will find that the stream will have a little less area than the orifice area at a certain distance and then again this area will increase. Now the minimum area as you know, it is called vena contracta. Now for this one we can write the equation in this form. Here, the velocity at the entry we may consider 0, we are considering this area a little away in the left side from the orifice and then we are considering the velocity V2 at the vena contracta. Then this equation can be writturn in this form that means velocity here we can write 2 by rho P1 minus P2.

Therefore, the mass flow rate can be writurn in the form of Rho into Q is equal to Rho A2 into V2, because A2 is the area here is equal to Rho A2 square root of 2 by Rho P1 minus P2. Now the A2 is the area at the vena contracta, which is not known. It is not it is neither we can measure the area there. Now to take care of that what we do that RhoQ into Cd into A0 square root of twice Rho P1 minus P2. Where, Cd is the coefficient of discharge and A0 is the area of the orifice. Now again you know that coefficient of discharge is equal to the area contraction coefficient and another coefficient velocity. So combining this Cd we use for fluid flow.

Now in case of fluid power with incompressible flow for a suitable orifice normally this Cd value to remain more or less constant. Of course that depends on the orifice area. In most of the calculation in fluid power particularly in oil hydraulics, this Cd value may be taken as 0.62 later when you will come to valve flow I will show you that how Cd can be taken more or less constant for fluid power analysis. Also a care is taken to make the orifice, there are various type of orifice in fluid power components starting from a circular hole to the variable area, but the care is taken so that Cd more or less given constant. The final equation here the 2.4.24 is Q is equal to Cd A0 square root of 2 by Rho P1 minus P2. This we should remember, because very often we need to use this equation. Other equations which I have shown normally when we analyze the inside flow in a valve then it will be repair (())(32:02), but normal cases this equation will be more useful.

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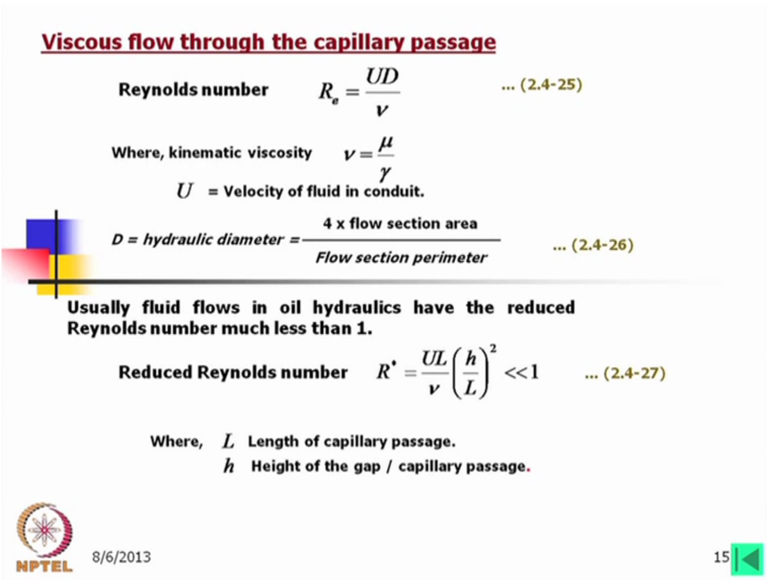

Now viscous flow through the capillary passage. Now here one important factor we consider which is Reynolds number. A Raynold as you know is the scientist, he developed these equations where, Re is equal to UD by Rho. Now this is the this is kinematic viscosity already we know that this is dynamic viscosity by density. Here of course we used in other form, normally we use Rho. Now U is equal to the velocity of fluid in conduit. Now normally this Reynolds number becomes very high, where D if we consider D is equal to hydraulic diameter that is equal to 4 into flow section area, whatever may be the area we consider 4 into flow section area divided by flow section perimeter.

Let us consider it is a circular one, then what we will do 4 into pi r square divided by 2pi r, 4 into pi r square divided by 2pi r. So this becomes 2pi r that is equal to pi d. This hydraulic diameter. Now in case of fluid power we consider a reduced Reynolds number, which is expressed as R star is equal to UL by sorry, this is not Rho Nu into h by L whole square. Normally you will find this is much much 1 and where, the L is the length of capillary passage and h is equal to height of the gap or capillary. This is for an example even in a pipe we can consider L is the length and h simply the inside diameter of the conduit, but this is more useful where the flow between two parallel plates or some capillary passage.

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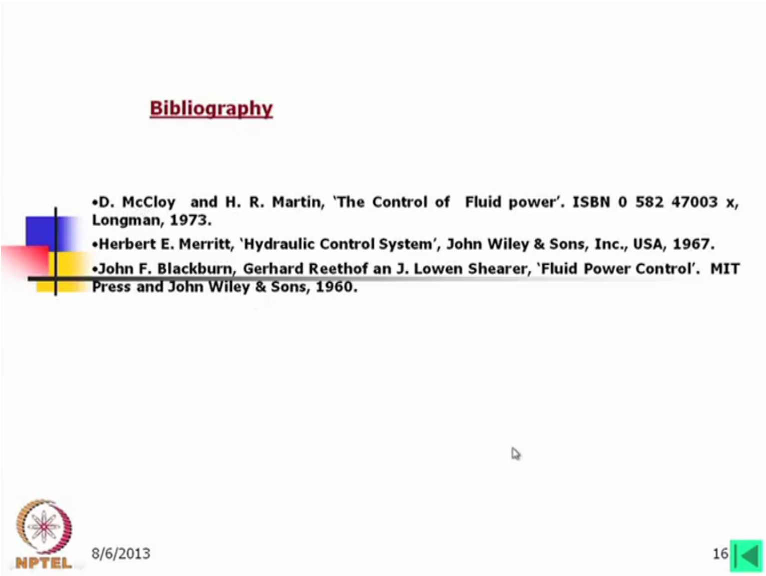

Now with this I finish this lecture and this note is prepared based on these two books the control of fluid power by Martin and McCloy and Blackburn and Reethof and Shearer by fluid power control. Also, I have followed the another book hydraulic control system which is by Merritt, but most of the equations that I have followed from these two books. Thank you.