

# Fundamentals of Industrial Oil Hydraulics and Pneumatics

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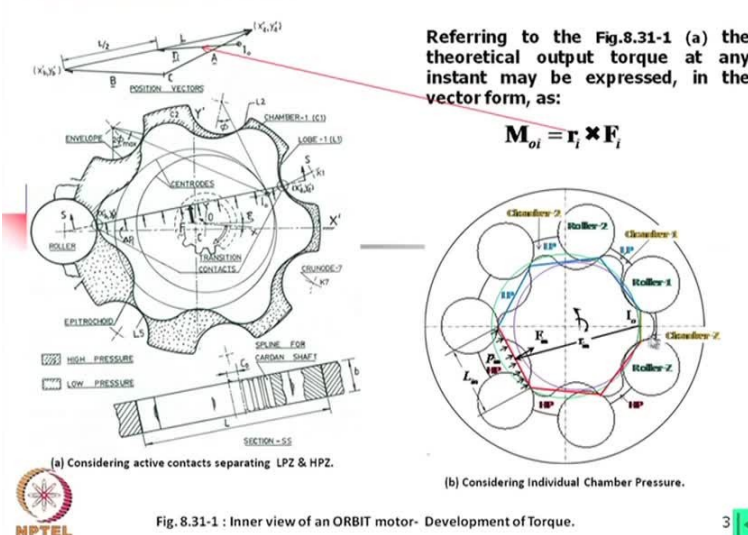
## Lecture 31

### Design Analysis of ORBIT Motor - III: Output Torque and Stress, Deformation, Gap at Contacts.

Welcome to today's lecture. This is continuation of design analysis of ORBIT motor part 3. Today we shall discuss about how to find out, how to formulate the output torque, stress, deformation, and gap at contacts.

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#### Output Torque – ORBIT Motor :



Now if we look into the figure, this is an inner view of ORBIT motor where we have drawn a line through the contacts which are separating high-pressure zone from low-pressure zone or in other words, the expansion mode to the compression mode. Now what we have done? We have joined, we have considered a line here and if we consider the width of this star and ring, then we may consider this is an beam where the pressure are uniformly distributed over this length joining 2 points.

Then we can consider the total force acting at the middle of this beam so equivalent force. Now the torque must be equal to this equivalent force  $F_i$  into the distance  $r_i$  which is from the instantaneous centre of rotations. Now here, one may confuse that we are considering the torque

about the star while we are considering such a distance. The reason is that what we have considered? This is the equivalent system we have considered.

In that equivalent system, what we find that as if there is a fixed drum in which this drum is rotating. Here of course, if we consider the drum, that is a friction drive which is not positive but let's consider this at a 2 gears. So definitely, to calculate the torque what we can find that as if this body is rotating about this pivot point by this force. So definitely to rotate this body about this point, whatever the torque that is in vector product,  $RI$  into  $FI$ , this is  $RI$  into this is  $FI$ , this is half of the distance.

So that torque must be the output through this body which is nothing but the star. So we can write down this is the torque. Now this is sometimes it is called energy method but whatever may be the name, in this way we can find out the theoretical torque output of this machine. This is the distance  $RI$  we should always remember. Now I have drawn another figure here. In this figure, again I have shown, here apparently slightly it has rotated by this angle.

In this case, as if it has not rotated. It is at this 0th angle. If we remember our earlier axis system, so this is the X axis and this is the Y axis, this is the reference axis which is coinciding with the ring gear. Now here we must know because we will refer it with chamber or roller et cetera. In this case we have considered this is roller 1, roller 2, roller 3, et cetera et cetera therefore this must be the zth roller. In this case, as we see in this figure, we have considered 7 roller and as well we find the lobes and the star are 6 okay?

Number of lobes, the star and 6. Now also we consider the chamber 1 which is in between 1 and 2 roller, chamber 2 which is between 2 and 3, et cetera et cetera. So this chamber at the 0th position, that must be chamber 7 or Zth chamber which is in this case 7th chamber okay? Now obviously the instantaneous point of rotation of equivalent drum is now situated on the x-axis. Now this whole body of the star is revolving around this axis in the anticlockwise direction whereas this body itself rotating in the clockwise direction.

So output will be clockwise looking from this side okay? Now in this case we have considered the high-pressure zone and low-pressure zone separated by these lines. And these 2 contacts will be considered active contacts. 1st of all these 2 contracts are active contacts. All contacts which

is joining the instantaneous centre of rotations and the crunode which is nothing but the centre of roller. Then these are the contacts and among these active contacts, there will be 7 active contacts.

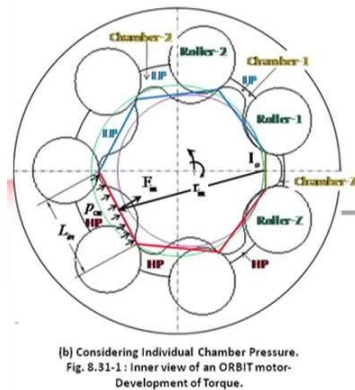
Among these active contacts, these 2 are you can say the critical contacts which are separating the chambers from high-pressure zone to low-pressure zone. Now instead of considering the axis I mean sorry the beam like this, we may also consider the all contacts points joined together and we get, in that case we will get 7 such line which obviously not equal in length. We may consider however that if we consider these are the critical joints which are separating high-pressure zone to low-pressure zone, we may consider in this side, there will be the pressure, each chamber is having equal pressure which is high-pressure okay?

Here of course, I have considered that apparently this is a neutral condition. We do not know what is the pressure history. It can be shown that even if we consider this is in high-pressure zone or if we consider this is high-pressure zone, the pressure, the sorry the torque calculation will show the same value, okay? Anyway, if we consider this is in high-pressure, these 3 are high-pressure, so this is experiencing high-pressure and again, on these lines, pressure distribution will be uniform, equal pressure but keep in mind while this is moving, these lengths are changing.

So these lengths may not be equal, are normally not equal. And on the other side this blue lines indicates these are separating I mean the chambers, adjacent chambers by these contacts and joining these lines, we consider the pressure is the outlet pressure. One may consider this is 0 and this is the differential pressure. This will also give the same torque value and this would give the same torque value. Now however, the equation for this will be different.

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**Output Torque – ORBIT Motor (Contd...):**



Referring to the Fig.8.31-1 (b), considering pressure in individual chambers, the theoretical output torque is expressed in more general form as:

$$\mathbf{M}_{ir} = \sum_{n=1}^Z (\mathbf{r}_{in} \times \mathbf{F}_{in}) \quad \dots (8.31-1)$$

Due to the inter chamber leakage flow through the capillary passage formed by the lobes adjacent to the active contact and at the flow distributor valve the pressure within a chamber may vary.

Neglecting such varied pressure distributions over  $L_{in}$ , but considering the average pressure of each chamber the force  $\mathbf{F}_{in}$  can be expressed as:

$$\mathbf{F}_{in} = |\mathbf{L}_{in}| \times \mathbf{p}_{cn} \quad \dots (8.31-2)$$



In that case, what we are considering say this is  $I$  is varying from 1 to  $Z$ , any chambers in that case, this is the length. So force will be at the middle of that. And then that multiplied by this distance. Say this is a typical one is shown here, only one chamber  $I$  have shown, the pressure is being distributed like this. Considering this equation will be better in a sense that actually in such machines we will find later that the gaps are generated here.

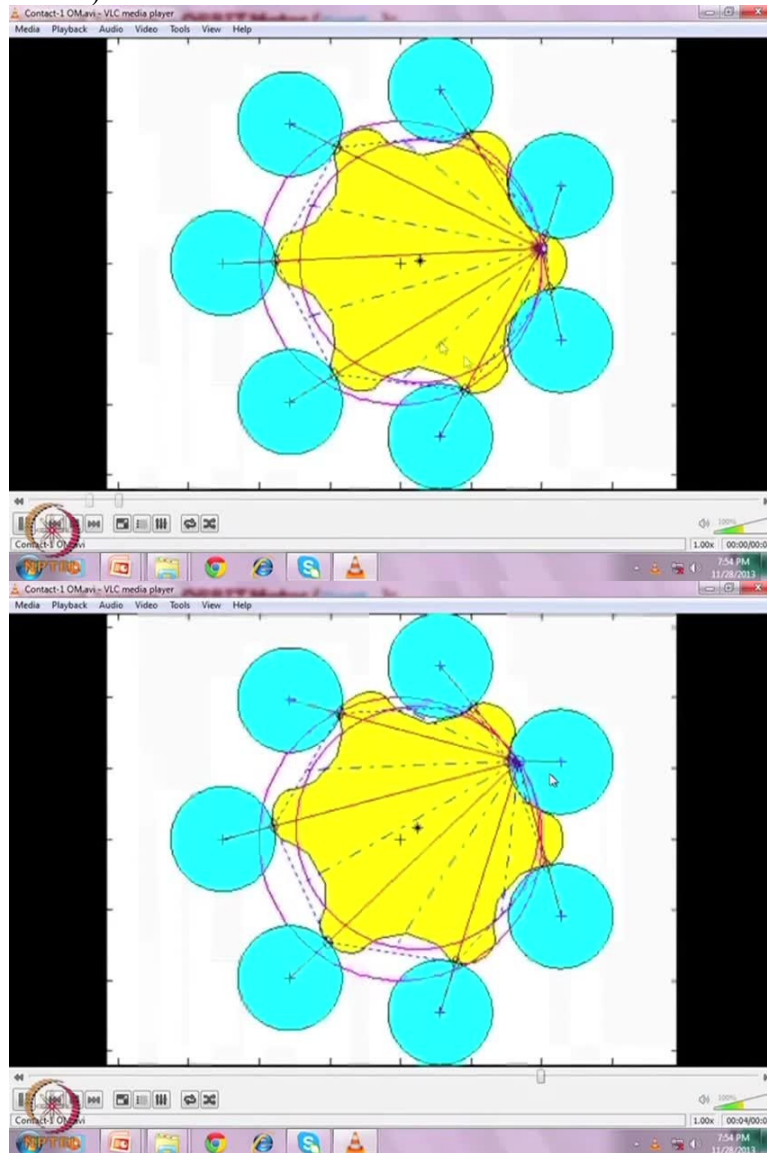
So case may be that pressure are not distributed over the whole length uniformly. It might be the pressure is non-uniform. In that case, if we can suppose this pressure distribution is nonuniform, then if we can consider such mapping, then we can accurately calculate the load and this distance, distance from that load to this instantaneous centre of rotation and this will give us the more accurate torque. That is the purpose of considering the pressure distribution in that way.

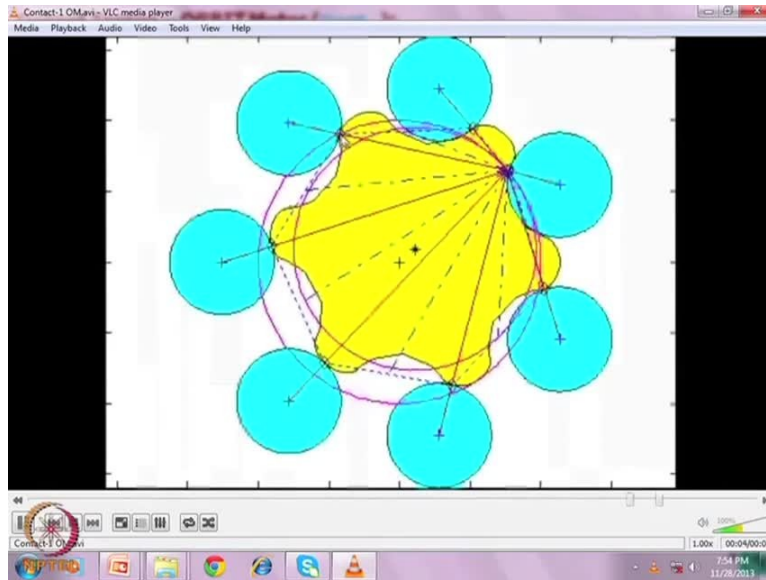
So here I have written that due to the inter chamber leakage flow through the capillary passage formed by the lobes adjacent to the active contact and at the flow distribution valve the pressure within the chamber may vary. Not only this, we should remember the distributor valve is very close to that. So due to that, there may have the pressure change due to leakage through that. So once the leakage through that all the paths are considered, we may have different pressure history instead of the ideal pressure distribution.

Now let us for the simplicity let us consider there is no variation of this leakage. Then to find out this length, we must know these contact points which can be calculated from the geometry for a

particular rotation for a given rotation of the output shaft and then we can find out that length and we may consider the pressure PCN that is in that chamber the pressure is PCN and the length is LIN and so this will be the force. We can calculate the force in this way. Now we can see the variation here.

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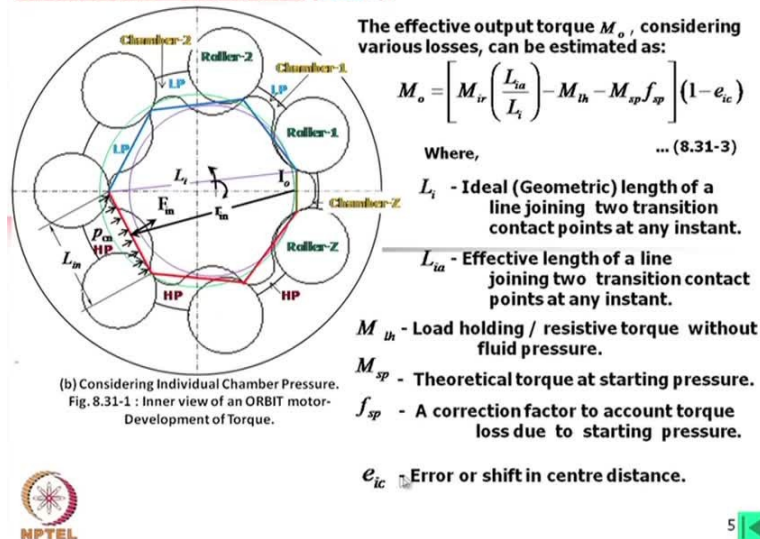


Look at this variation. You can see, these lengths are varying. Okay? Again we will see this. So we have just shown one chamber operation, rotation for one chamber. Again you can look this, how these distance are being changed and we have considered the this is the midpoint which can be, this is the main point. So this geometrically this we can calculate and easily we can find out the forces. We can define this distance and lengths in the vector form and we can do the vector multiplication okay? So in that way, torque can be calculated okay?

Now what we have calculated so far, that is the ideal torque output of such machine. Now in this machine, 1st of all if we consider the assembly of rotor and stator, we may consider that this is just exactly fit. This means, neither it is interference, nor any gaps at these contacts when these are assembled. In that case you should consider that there is no interference bit, nothing is there. Now due to the pressure, we have considered, we are considering now the pressure inside. Once there is a pressure means load and there will be deformations.

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### Output Torque – ORBIT Motor (Contd...):



This deformation means there will be gaps, the contact points will be deformed. In that case there are, we should consider the several losses. One is that at that contacts, when this is moving on that, earlier we have examined that this is having both, rolling as well as friction. So sliding. Rolling and sliding. So if we consider the combined friction there, definitely there will be one pressure loss and also, this what the length we are considering ideally, that may also not exactly because the contact has now deformed.

So length is changing. Due to that, there will be variations. Now let us consider that there is some interference fit to reduce the gaps, et cetera. In that case due to that interference fits, another additional torque will be required which will not be available at the output shaft. Apart from that, there may have some error also. Considering this, one can carefully express the output torque in this form. Now we will see that what are these parameters.

$L_i$  is the ideal geometric length of a line joining two transition contact points at any instant, this one. And then  $L_{ia}$ , effective length of line joining two transition contact points at any instant. What is effective length and what is ideal length? Ideal length what we are calculating initially from the contact points without considering any deformation at the contact points. Now if we consider there is a deformation, then contact points may shift. If that can be calculated, then we will consider that this is this will be definitely the deviation of the ideal torque. So that is there.

Now then what is MLH? It is called load holding. Resistive torque without fluid pressure. This can be calculated or experimentally can be founded out if there is some interference. Even if we consider the ideal fit, still there may have some friction of resistance to move this rotor. So that is why, we can find that initially, to start that motor, you need some pressure, just to rotate without any load. That you may consider the load holding torque.

That is that this means that we need a torque to start this motor without any load, that is load holding torque. On the other hand, this load holding torque is again you can may name that is a starting torque required. Now this obviously this increases depending on the fittings. If the fittings is too tight, in that case, this part will increase. That can be estimated also this is found normally from the experiment. Next, we consider the theoretical torque at starting pressure.

That means when it what we will see that we are giving a pressure but still this is not moving. So that pressure into the when pressure is working on this machine without moving, that is the load holding part. But once it starts moving, what will happen? Due to these forces present there, there will be a friction. So considering this friction coefficient or the correction factor we have given the name to account torque lost due to the starting pressure.

So there will be another loss. So that this means that actual available torque is this much. The ideal torque, then there is a variation in the this length. So that part, it will be slightly less than this actual torque. Then this load holding torque and then you know when it is starting, there is a friction, due to that there is another loss. So this is the total loss. Now this we have considered that there is no deviation in the eccentricity.


And if we consider the further deviation in the eccentricity, there will be further loss. So this amount and this much amount can be, can also be considered. So this is a proposed model to estimate the available torque okay? But we may this measuring such this error or shift in centre distance is very difficult. So this we may neglect. Or in other words even if possible, if we can measure or we can estimate that, then whatever additional loss, that can be attributed to this one. Okay?



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### Output Torque – ORBIT Motor (Contd...):

**Force Equilibrium :**



The instantaneous torque efficiency may now be expressed as:

$$\eta_t = \frac{M_o}{M_r} \quad \dots (8.31-4)$$

Again for force equilibrium:

$$\sum_{n=1}^Z \eta_t F_n + \sum_{n=1}^Z F_n = 0 \quad \dots (8.31-5)$$

Local deformation occurs at contacts initially due to interference fit (if any) and then due to fluid pressure.

Therefore, it offers the normal force  $F_n$  which is expressed as follows:

$$F_n = F_{fn} + F_{fpn} \quad \dots (8.31-6)$$

Where,

$F_{fn}$  &  $F_{fpn}$  - Are the reaction forces due to contact deformation for interference fit and fluid pressure respectively, at  $n^{th}$  lobe.

(b) Considering Individual Chamber Pressure.  
Fig. 8.31-1 : Inner view of an ORBIT motor- Development of Torque.

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Now the instantaneous torque efficiency now we can listen the ideal torque and this is the output torque, so this may be considered as a efficiency of such. Again for force equilibrium so this each chamber force and sufficiency must be equal to this ideal force at the contacts offered by this is the resistive force, this is offered by by the elastic deformation of the body. So we can equilibrium in this way. Now if we consider this is due to the ideal pressure, then this factor is 1 and then we will find, this force is equal to the pressure force okay?

But actually we find that whatever force is there, this is more than the pressure force. Now local deformation occurs at contacts initially due to interference fit if any and then due to fluid pressure. Therefore it offers the normal force  $F_n$  which is expressed as follows,  $F_n$  is equal to  $F_{fn}$  and  $F_{fpn}$ .  $F_{fn}$  and  $F_{fpn}$  are the reaction forces due to the contact deformations for interference fit, this is for interference fit and then fluid pressure respectively.

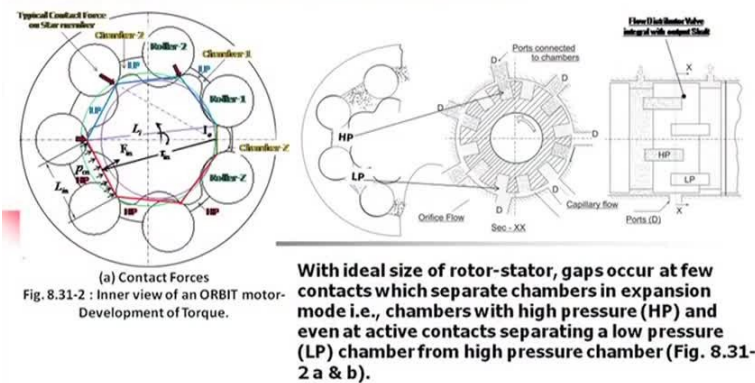
Say for example, there is no fluid pressure, let us consider. So we do not need or in other words, if we consider that there is no torque, output torque is required, so at 0 pressure ideally, just flow is given, it should be able to rotate but what we find that it requires some amount of torque because there is a interference fit. So this means that there is already some deform, contacts are deformed and due to that the force  $F_{fn}$  is there.

Now what we do, we add fluid pressure because the output torque is there. So therefore we should consider, at each contacts, there are 2 forces acting originally due to the interference and

then due to the pressure. If there is no interference, we can consider this part is 0 and normal force equal to due to the force due to the pressure.

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#### ORBIT Motor - Force Equilibrium & Contact Stresses :



However, depending on the pre-deformation due to the interference fit of star and ring the gap may not occur at all for a reasonably high operating pressure.

As the materials of star and ring are usually steel, it may be assumed that they have same Poisson's ratio ( $\nu$ ) and modulus of elasticity ( $E$ ).



Now with ideal size of rotor stator, gaps occur at few contacts which separate chambers in expansion mode. That is chambers with high-pressure and even at active contacts separating low-pressure chamber from high-pressure chamber. Now here I would like to mention that we are trying to find out the forces in each and every contact. Then we have considered that apparently, this, one we may consider that this line which is separating high-pressure to low-pressure, then this is a beam.

Now this beam is supported by how many supports are there? At least this will be the number of chambers divided by 2. Number of chambers plus 1 divided by 2, either this or minus 1 divided by 2. It has to be. This is for the odd number of chambers. We are considering odd number of chambers. So by that way, say for example, in case of the 7 chamber machines, this will be either 3 or 4.

Now in a beam, there are 4 supports or 3 supports. So this will be statically indeterminate problem okay? Now also if you look into this the support and their direction of force, to find out the direction of force, what we can do, we can join the instantaneous centre of rotation to the centre of the roller or centre of the crunodes of the envelop. So we are getting the direction of the force, direction of the support.

But if we look into the direction of the support, these are not perpendicular to the beam what we have considered. This is attacking angle. So the problem to find out this the loads not an easy task. We have to consider all the directions. Now again looking into the direction of the force, you may find that in some supports, some supports are not taking the load at all and as well you will also see that a gap is generated.

Now this gap say this is the high-pressure side, so definitely beam supports are in the low-pressure sides and these 2 are critical and we may find, at some instances, there a gap is generated. Once the gap is generated, you will find there will be the leakage. So estimating such leakage is essential to improve the performance of such motor. Also, what we the valve is very adjacent to this. So there also the possibility of side leakage like this here.

One is leakage through this contact, another is the side leakage from these valve ports. Considering all such things, we can have a redistributed pressure particularly where this capillary passage is there, we can have a distributed pressure over this chamber and that distributed pressure if we put on that, we find the different ports okay? However, depending on the pre-deformation due to the interference fit of star and ring, the gap may not occur at all for a reasonably high operating pressures.

What we can do? We can make this as an interference fit. This is basically such machines are called form closed. That means even if without any force, there will be all contacts, all 7 contacts in this case without any force and geometrically that contacts will be maintained. Now if we put the interference fit, interference fits means actually at some point we will describe. These rollers are oversized as well as the stars are oversized.

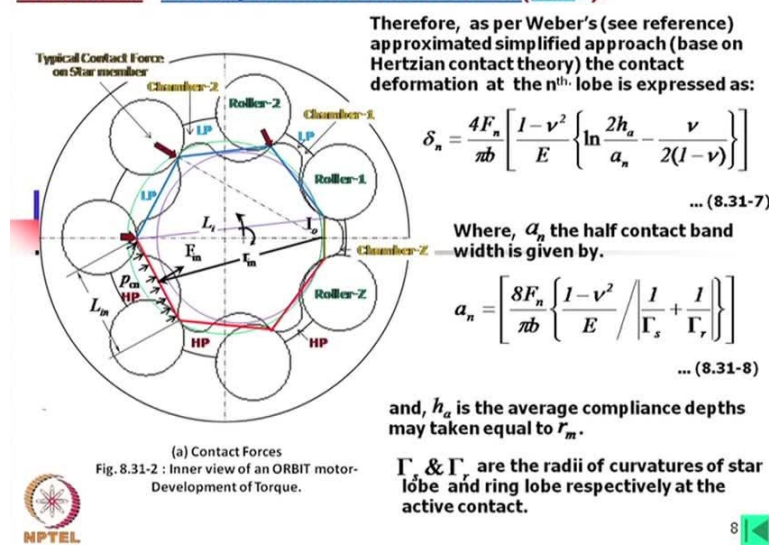
So in that case, what will happen, each and every contact should assembly is difficult but that is done with some technique, heating one element and cooling the other element, that assembly is possible. But once it is assembled, then there is whatever may be the small deformations will be there. Now with the increase in pressure, that deformation will gradually decrease and at one point with the higher pressure, definitely there will be gap but we can design such machines that up to the working pressure, there will be no gap at the contacts.

Only disadvantage of such machines will be there, at starting you need a starting torque, higher starting torque. But once it reaches at its working zone, you are not require if no leakage is there, efficiency will be higher although some initial torque were required. But you may find that overall efficiency is more or overall performance at least better than the ordinary, I mean without interference fit machines.

As the material of star and ring are usually steel, it may be assumed that they have same poisson ratio and modulus of elasticity.

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#### ORBIT Motor - Force Equilibrium & Contact Stresses (Contd...):



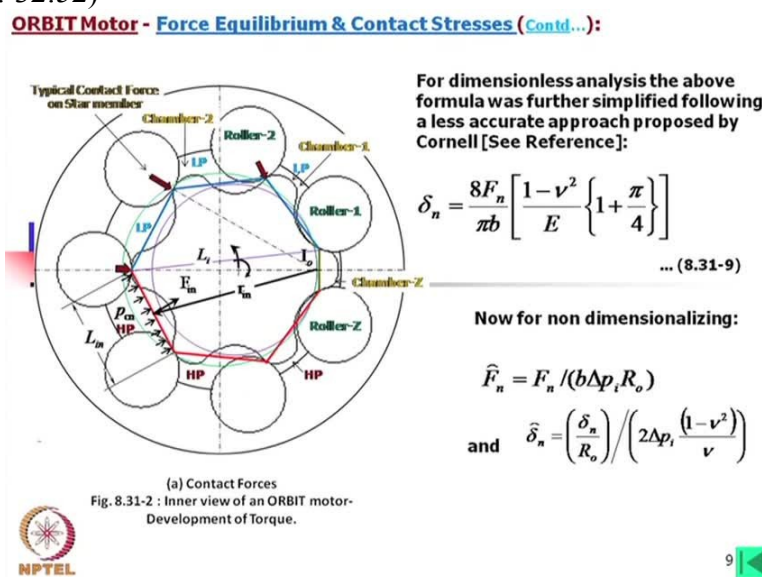
And then we can now we can consider the deformation at a contact is  $\Delta_n$ . This is along the normal directions. That means in this directions, this will be the deformation. So this expression will give us this deformation if the force are known. Here we do not know how much is the deformation, how much is the force. We are trying to find out the force at each and every contacts okay. Now  $a_n$  is the bandwidth half bandwidth which is given by this formula. This is from hetzian contact theory, so you can this perhaps known to you.

We have only written in this form. Here, this is the contact radius of the star and this is the contact radius of the ring. But you should remember, this plus sign means both are concave. Now if one of that but if you look into the star, this is having convex as well as concave. When this is a concave, then this there will be a minus sign okay? Minus plus sign. So you have to take care of that part. Now what is this one,  $h_a$ ?

HA is a depth upto which this deformation occurs. This is a complex things but this we have to assume for such machines what might be this depth. H is average compliance depth. In this case, we have considered this depth is up to the Centre of the roller. So this is equal to RM maximum but this is an arbitrary. However, if we consider this is half of that or double of that, you will find this overall contribution to this will be negligibly small.

So it is a reasonably good to assume this HA is equal to RM. So in all calculations we consider, this is equal to RM. This radius already I have explained to you, radius of curvature.

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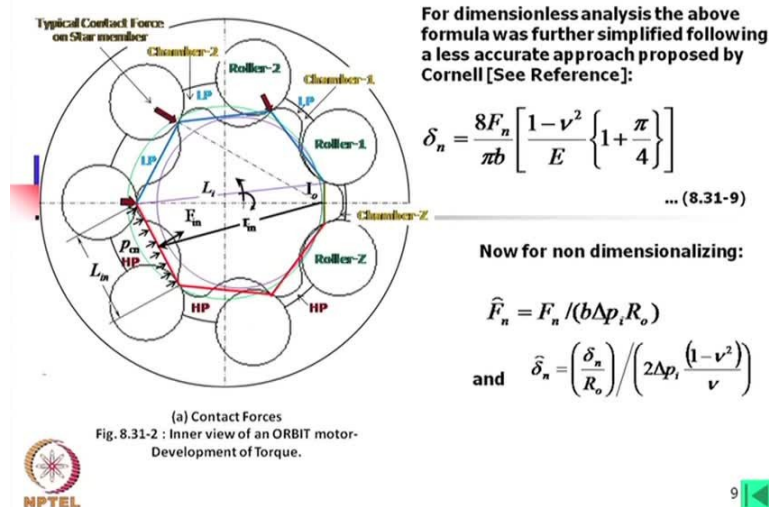
Now for the dimensionless analysis, this formula is further simplified which is given by this developed by Cornell and this can be written in this form. So this I have given this article as a reference, one can read to how this was derived. Once we reach here, then we can non-dimensionalise, it is like that. Now earlier, when we are non-dimensionalising a dimension, we are simply dividing that one by the radius of the base circle or the describing circle of the ring gear.

That is R0, we are considering this radius R0 of the bigger circle. Here it is a green circle. Whereas when we are trying to non-dimensionalise this force, what we do? we consider this force, normal force divided by B. That means this is coming per width and then divided by Del PI which is coming as a perunit pressure and then R0 is this radius, radius of the bigger circle. This, look into this unit. This is area and this is pressure. So this must be also force.

So this becomes a non-dimensionalised. That means once we calculate this deformations or force for a machine, considering this approach, then for actual pressure and actual width, we can find out what will be the actual deformation or actual force. And beauty of using such non-dimensional analysis is that you can analyse the machines for a particular geometry and then you can design, you can consider the pressure, actual width, flow rate, everything later to give the physical size of these machines keeping the geometric relation same. Now this is the deformations in the same way we can express in the nondimensional form. Okay?

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**ORBIT Motor - Force Equilibrium & Contact Stresses (Contd...):**



Now the contact deformation on star at the active contact nth lobe of ring can be expressed, this is actually this is in the vector form. So we can no, not yet, not in the vector form, we would say that this is, this what we are doing the actual deformations, total deformation from there, we are trying to find out what is the deformation in the normal directions. We have considered any deformations at any directions, then by this multiplication, we are finding out what is the magnitude of deformation along the normal line.

So for that, we need these operations. And then, also this actual area of the contact can be fined out. Because we have calculated the half bandwidth, so total bandwidth will be this much and B is the width of this star or ring. So this, you will get actual area okay? Then finally we can express this formula. The FN, nondimensional is equal to 0.44 into Delta N. Look at this, this is not for the both star and ring material as steel. We have considered that.

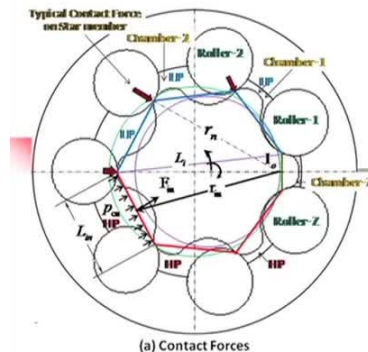


So, poisson's ratio is around 0.3. With that, we get these relations, okay? This is very simple relations. Any force in the normal direction is equal to 0.44 times of the deformations.

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#### ORBIT Motor - Force Equilibrium & Contact Stresses (Contd...):

##### Numerical Example (Contd...):



(a) Contact Forces  
Fig.8.31-2 : Inner view of an ORBIT motor-  
Development of Torque.



It is verified through the numerical results and discussed later.

And the load holding torque  $M_{lh}$ , i.e., resistive torque without fluid pressure, due to these forces may be expressed as:

$$M_{lh} = \sum_{n=1}^Z (\mu_c F_{ln} r_n) \quad \dots (8.31-14)$$

[  $\mu_c$  - Coefficient of combined rolling and sliding friction at contacts.]

This means that to rotate the shaft even there is no fluid pressure, the required torque is  $M_{lh}$ .

Speculatively for such geometric form closed machine, the summation of forces (eqn.13) may not become zero in all the angular position ( $\xi$ ) of output shaft and as the rotor (star) is virtually floating the summation will become minimum i.e., floating rotor will assume a comfortable position by deviating from its ideal geometric position.

So this analysis become easy. If the calculation of contact deformation, forces, etc, caused due to the interference fit or oversize star and ring, it is assumed that excess size is deformed in the direction of the normal to active contacts. That is in the direction of the line joining the instantaneous centre of rotation to the Centre of lobe or roller in ring. Now here I would like to mention, you see if we consider a simple cylinder and a hole in cylindrical body perfectly round and they are in interference fit, you may consider deformations over the contraction of the cylinder is uniform over the periphery, expansion of the cylinder is also deformed uniformly over the periphery.

In this case, we had to depend I mean such interference is taken care by few contacts only. So most likely, it is not uniformly distributed. Okay? And if you think, if you try to visualise, there is a possibility of that unstable conditions. That means, for such a interference fit, ultimately you will find this star will try to take a stable position. What is that stable position? You will find one of the chamber will be either in the outer dead centre or bottom dead centre.

Here you can say this is at the outer dead centre. So this is one stable conditions. Next next stable condition will be this one of these 2 chamber will be inner dead centre okay? So which is precisely can be found out by the pi divided by Z into Z minus 1. So every after that angle, this

will be in the stable conditions or in other words, you will find this machine, if you lift this machine, it will try to go to that position for its stability.

Now in other conditions, suppose if you hold that so after the other conditions what will be the situation in the machines? This machine, due to the pressure it is giving a torque. You will find apart from that, there is some unbalanced amount within this machines. That means to hold the shaft at that particular position, you need some unbalanced torque. Although the magnitude is may not be very high but this unbalanced torque is there. So in the force analysis, definitely such contact force is giving this unbalanced torque.

So while you are calculating automatically this should be should also be taken care of. So here I have shown that the equation using equation 9 and 10, one can try to find out the forces. But this we have to follow a particular technique that how we can find of these forces. Let  $F_{HN}$  be the reaction force due to the contact deformations for interference at  $n$ th lobe, then we can have that summation of all such forces due to the interference at no pressure must be equal to 0.

So we must satisfy that condition to find out this force and then we can consider this load holding torque must be equal to that that force into the coefficient of friction into this  $R_N$ . In that case, to calculate this  $R_N$ , we have to consider this context point to this distance because here, this is the normal force, so this force is in this transverse direction and that multiplied into  $R_N$  will give the load holding torque. This is the coefficient of combined rolling and sliding friction at contacts.

This means that to rotate the shaft, even there is no fluid pressure, the required torque is  $MLH$ . Speculatively for such geometric form closed machine, this summation of forces may not become 0 in all the angular position of shaft rotation and as the rotor is virtually floating, the summation will become minimum. Floating rotor will assume a comfortable position by deviating from its ideal geometric positions which I have explained.

It will actually will try to take a position for which one of the chamber is either at the top dead centre or the inner dead centre or outer dead centre or the inner dead, one is outer, one is inner, one is the top dead centre, one is bottom dead centre, whatever name you can give it to this. It is

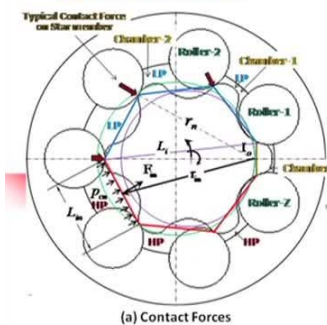


better to give this is outer dead centre and this is inner dead centre okay? We have also verified this in numerically and that can be shown.

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#### ORBIT Motor - Force Equilibrium & Contact Stresses (Contd...):

##### Model Calculations:



(a) Contact Forces  
Fig. 8.31-2 : Inner view of an ORBIT motor-  
Development of Torque.

In calculating the forces  $F_m$  part, first of all an arbitrary small deflection  $\Delta\hat{\delta}_n$  is considered (in the dimensionless numerical computation) in the opposite direction of the resultant applied force i.e.,  $\sum_{n=1}^Z F_n$  at  $I_o$  (See Figure).

Then its direction is varied in very small steps, in both positive and negative angular direction, to calculate  $\Delta\hat{\delta}_n$  and  $\Delta\hat{P}_n$ , till the direction of  $\sum_{n=1}^Z \Delta\hat{P}_n$  becomes opposite to

the direction of  $\sum_{n=1}^Z F_n$ .

Once the direction of  $\Delta\hat{\delta}_n$  is found out the magnitudes of deflection and force in each contact are calculated using the scale factor and incorporating the forces due to initial deformation.



For dimensional analysis following the Weber's formulae same approach is followed when a particular model of known dimensions is analyzed.

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Now in calculating these forces at PN part, that is the force due to pressure, 1st of all an arbitrary small deflection  $\Delta\delta_n$  is considered in the dimensional numerical computation, we consider any arbitrary value, small deformation in opposite directions of the resultant applied force. That means, suppose this is the direction of the force, we have considered these 2 points. Say shaft has rotated by a small angle for which these 2 are the critical contact points.

So we know these lines, we know this direction, we consider the transverse directions of this okay? Then we consider deformations are taking place in this directions. That means whatever deformation is there, deformation is taking in this direction which is perpendicular to this, 1st we consider. Let us give a very small because this is dimensionless quantity, let us take 0.01 is the deformations okay, numerical value.

Now what is there? Due to this deformation, we can find out the component and along the normal directions what will be the deformation at these points okay, amount of this deformation, then from there we can calculate what might be the contact forces there. So we can find out that a resultant force is there. Now we consider that resultant force, so we have considered only in these directions, now let us consider that these deformations, we are rotating.

Rotating means maybe only  $\pi$  angle about this centre. We are considering these deformations, we have rotated. For all such positions, we will find these forces. We consider which is the minimum force for which conditions for the same normal deformations. For which direction of that deformation, the summation of force is minimum, actually that is that would be the directions of this I mean deformations because there is no meaning that it will be the higher side. It will be always in the lower resistive part, lower sides.

So we find out 1st that positions okay? Once we find out that positions, we then that definitely not in the directions of the load, this is in the resultant in the some other directions. Now the component of that force, that summation of this resistive force due to the deformation is contributing towards the torque but the other component which is in the transverse directions of that force, that definitely generating an unbalanced torque of this motor.

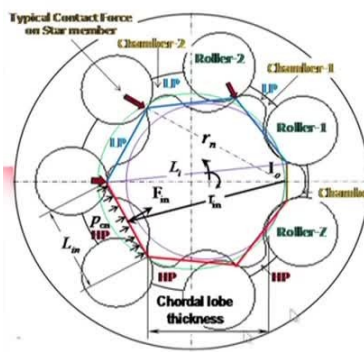
And you will in such calculation you will find, when one such chamber in bottom dead centre or at the top dead centre, this unbalanced force is 0. So after calculating such this force now if we would like to quantify, suppose we have a particular pressure operating at a particular pressure, we know other dimensions, then by multiplying this unit deformations for the given deflections, we can find out what will be the actual deformation, what will be the actual force, what will be the actual torque, what will be the actual unbalanced torque.

So this is the same what I have described. This is written here. And for dimensional analysis, the following members formula same approach is followed when a particular model of known dimension is analysed. Now what I have done, in a particular case I have taken later for the numerical calculations, one can find that non-dimensional analysis and the dimensional analysis using such formula and if you calculate that, there will be definitely some deviations.

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### ORBIT Motor - Force Equilibrium & Contact Stresses (Contd...):

#### Model Calculations (Contd...):



(a) Contact Forces  
Fig. 8.31-2 : Inner view of an ORBIT motor.



Table 8.30-1: Design data of few ORBIT motors.

SET NO.	BASIC PARAMETER				
	Z	$R_o$ (mm)	$\bar{A}_o$	$\bar{F}_n$	$M_n$ (Nm/10mm of b/MPa)
1	7	25.75	1.2567	0.369	+0.0405
2	7	20.50	1.5833	0.466	-0.0107
3	7	19.60	1.4946	0.324	+0.0078
4	7	16.18	1.8094	0.391	+0.0553

$\delta_c$  Over size in chordal lobe thickness of star.  
 $M_n$  Output Torque.

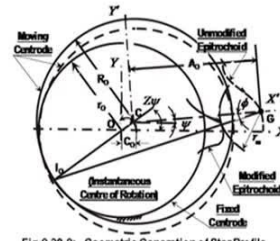


Fig-8.30-2: Geometric Generation of Star Profile

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Now we have calculated few such machines. Now in such machines, if you look say for example set 1, that is one machines. In that case, you can see this is the dimension, now this is you can find that plus some value, what does it mean? Actually this is a overall thickness, tooth thickness. This if you use a simple caliper, you will find that these 2, there will be some portion of the curvature which will be the tangent of which will be parallel.

So we measure this distance. Now for an ideal case whatever the distance, that we consider the ideal dimensions. Now we increase this dimension by this much only, this is a practical case we have taken, this much, then certainly this is an interference fit and what we find for such machine, we get this is the amount of torque. We can find out, we can have out of this machine. This is again, Newton meter per 10 millimetre of width per mega Pascal, this will be the torque.

So if you suppose you have 15 millimetre, so simply you multiply with 1.5 and pressure see another 15. So 1.5 into 15 into this, this will be the output torque of such machine, ideal output torque. If you consider 2nd machine in that case, this is slightly loose fit. Actually I would say, if you consider a machine without any interference, it should have some clearance. So this machine is without an interference whereas if you come to this machines, this is giving this is the torque. This can be calculated the formula we have proposed. Now here is just to show the dimensions, different dimensions and then you can see, these are the contact forces, directions are shown.

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**ORBIT Motor - Force Equilibrium & Contact Stresses (Contd...):**  
**Model Calculations (Contd....):**

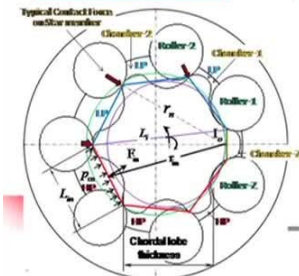


Fig. 8.31-2 : Inner view of an ORBIT motor.  
 Table 8.30-1: Design data of few ORBIT motors.

SET NO.	Z	BASIC PARAMETER				
		$R_o$ (mm)	$\bar{A}_o$	$\bar{F}_n$	$\delta_c$ (mm/10mm of bMPa)	$M_c$ (Nm/10mm of bMPa)
1	25.75	1.2567	0.369	+0.0405	11.59	
2	20.50	1.5833	0.466	-0.0107	9.10	
3	19.60	1.4546	0.324	+0.0078	8.71	
4	16.18	1.8094	0.391	+0.0553	7.2	

NPTEL

The deflection at the active contact without fluid pressure may occur due to various reasons.

For an example in a set, star and ring may be over cut i.e., the roller or lobe radius in star may be less than the ideal geometric size and the star is also cut with an oversize cutter, but the roller placed at the pitch circle with lower radius.

In that case deformation occurs at few contacts (without any fluid pressure) whereas gaps occur at the others contact, at any angular shaft position; due to inherent geometric property.

Therefore, for any contact position the geometric positions of the contact points on star and roller (or semi circular lobe in the ring) are calculated.

The projection of the distance between two points on the normal at the ideal contact point gives the deflection or gap at the contact point.

The forces are calculated.

The resultant torque required to rotate the star about the shaft axis is calculated using eqn. (14).

Then in model calculations, the deflection at active contact without fluid pressures may occur due to the various reasons. For an example, in a set, star and ring maybe over cut, the roller or lobe radius in star maybe less than the ideal geometric size and the star is also cut with an oversized cutter but the roller placed at the same pitch circle with a lower radius. This means that when because this is an interference machine, we can create such interference by several means. One is that keep all the rollers dimension, everything ideal, simply you increase the size of this.

So this is one way of interference. Another is that if this is intact, just you reduce the roller radius. Keep the roller size also same, only the pitch circle on which the roller is placed, just reduce that one. But you have to remember the way you have created this interference, in your geometric calculation, you have to consider that properly to find out the deformation due to this interference. So we should know that how it is is the interference is created. So this is described here.

The projection of distance between 2 points on the normal at the ideal contact point give the deflections or gap at the contact points. Okay? The forces are calculated and the resultant torque required to rotate the star about the shaft axis is calculated using equation 14 which we have described earlier.

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### ORBIT Motor - Force Equilibrium & Contact Stresses (Contd...): Model Calculations (Contd...):

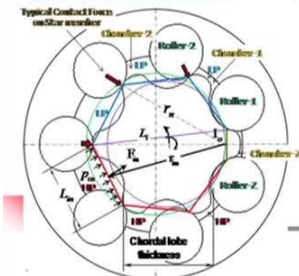


Fig. 8.31-2: Inner view of an ORBIT motor.  
Table 8.31-1: Design data of few ORBIT motors.

SET NO.	Z	BASIC PARAMETER				
		$R_o$ (mm)	$\bar{A}_o$	$\bar{F}_n$	$\delta_c$ (mm)	$M_c$ (Nm/10mm of b/MPa)
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3	7	19.60	1.4946	0.324	+0.0078	8.71
4	7	16.18	1.8094	0.391	-0.0553	7.2

NPTEL

Now the resultant deviation of the star (rotor) center from its ideal geometric position is assumed arbitrarily.

Then considering an infinitely small circular zone around the ideal point the actual center is shifted to verify the possible minimum unbalanced load condition.

The shift is made in such a way the rotation of the star about its own axis kept unaltered.

Clearly for any shaft position the unbalance i.e., excess forces can be separated and both the balanced and unbalanced frictional torques,  $M_{th}$  and  $M_{th(u)}$  respectively are calculated separately.

In these calculations geometries and kinematics of the rotor and star are to be considered.

$M_{th}$  would help in braking when inlet flow of motor is stopped with outlet is blocked.

Also the motor could hold a torque load even the motor is inactive.

Definitely the starting pressure would be more in such motor with interference fit star and ring.

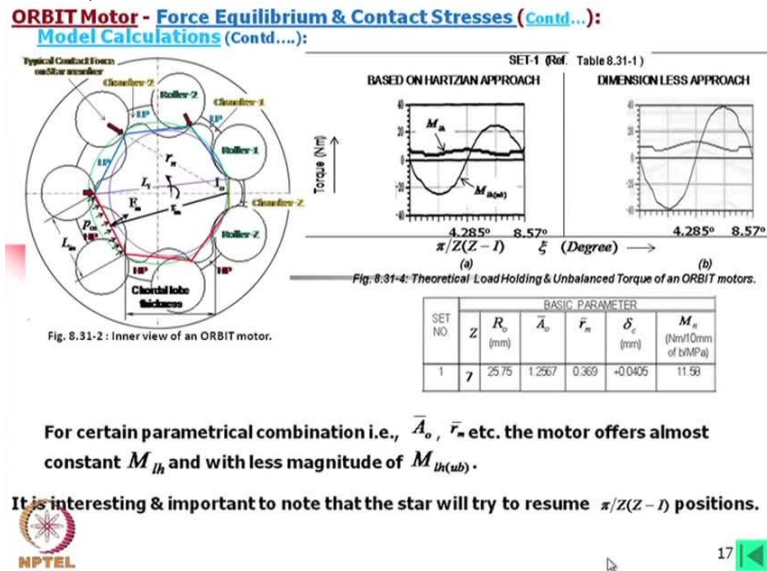
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Now the resultant deviation of star centre from its ideal geometric position is assumed arbitrarily. Then considering an infinitely small circular zone around the ideal point, the actual centre is shifted to verify the possible minimum unbalanced load conditions which I have described okay? And in that way, calculation is done and then we can find out at no pressure if there is any torque is required due to the interference fit we can have then there is one constant torque, load holding torque, average and another unbalanced torque.

In these calculations, geometric and kinematics of the rotor and star are to be considered. We have to consider as I told. This load holding torque would help in breaking when inlet flow of motor is stopped with outlet is blocked. Okay? So in some cases, this is required. In other cases, it is not required. So for the particular requirement, knowing the particular requirement, if we want to design machines, then we have to calculate or we have to follow such calculations.

So I have described here, definitely the starting pressure would be more in such motor with interference fit star and ring.

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Now I have shown the result also. In that result, what we find? We have calculated in both way and we have considering this geometry okay and in that case what we find, this is the load holding torque. You can see this is in Newton meter, very small out of torque but there is as well, unbalanced torque. This means, here you can see, this is the  $Z$  pi divided by  $Z$  into  $Z$  minus at that point, you can find that no unbalanced torque is there.

Whereas if you want to keep machine at that positions, you will find that some unbalanced torque is on this much will be required. And again, this is varying in negative and positive directions. For one directions, you will find that need to torque and other positions is automatically this is coming back to its positions. Now another interesting point is that this is the dimensional analysis and this is non-dimensional analysis.

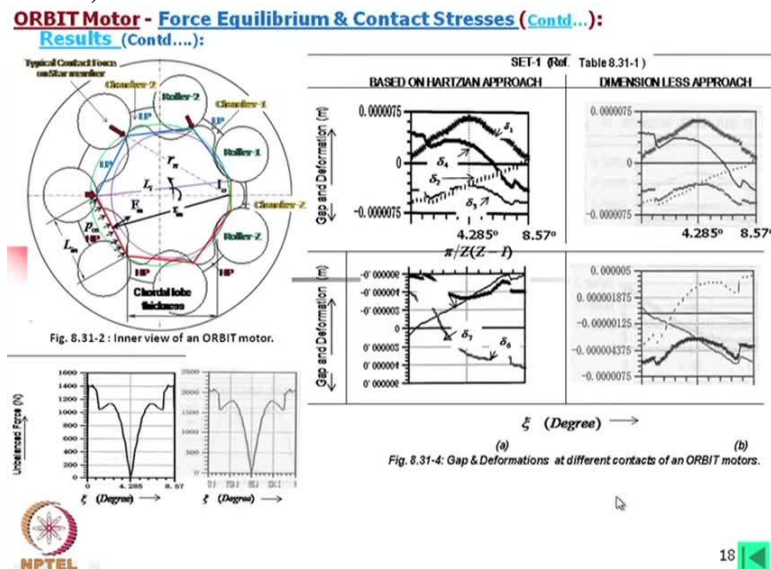
You will find there is a little difference. This amount is small. In that we will I would say little otherwise in comparison to this small, you will find the unbalanced force is coming more here. You see, it is also interesting to note that if we combine if we vary this dimension, it has been seen for some values, this unbalanced torque is also minimum as well as this is almost without much peak values. So those are actually better machines.

So this already we have described. That is we will try to assume always in such angles. Now another interesting point which I have mentioned that this nondimensional approach is more conservative, this means that if we calculate in this way, it will show always more stress value,



etc. so nondimensional approach in that way, we will be always safe side in the design. Not only this helps us because the beauty of using nondimensional analysis is that we can offer optimisation, we can optimise the machine, then we can finalise the actual dimensions. But in that process, it will be over designed but it no matter this will always give us a safe design okay?

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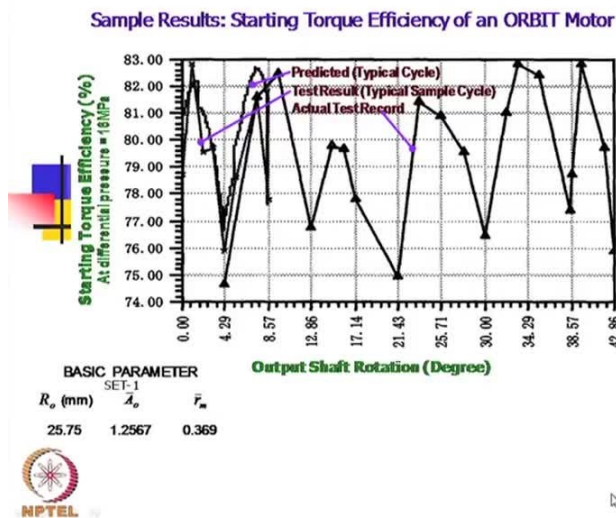


Now also if we consider the deformations at different contacts because we have to, while we are estimating such torque and et cetera, we have already calculated that deformations and as we find that there is also gap, say these contacts, this is a contact 1, that is a roller 1, this is roller 2, this is roller 4 is, this is roller 3 and for such angle of rotations and we find that sometimes this is from deformation to gap for different rollers. And if we look into this, this is the dimensional approach, this is the nondimensional approach.

But mind it, this nondimensional approach after that we have multiplied this factors to give to find out the actual dimensions, okay? Now this is for the other contacts. This is, there are differences but it is more or less similar. Now this is as you see, this is the unbalanced force, this is sometimes it is quite high. Of course, this is with respect to some pressure we have considered, this is not mentioned here but as you can see this, both the approach giving more or less same value but what we find is this with the viewers approach, this nondimensional approach, these forces is more but actual force may be less.

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**ORBIT Motor – Performance :**



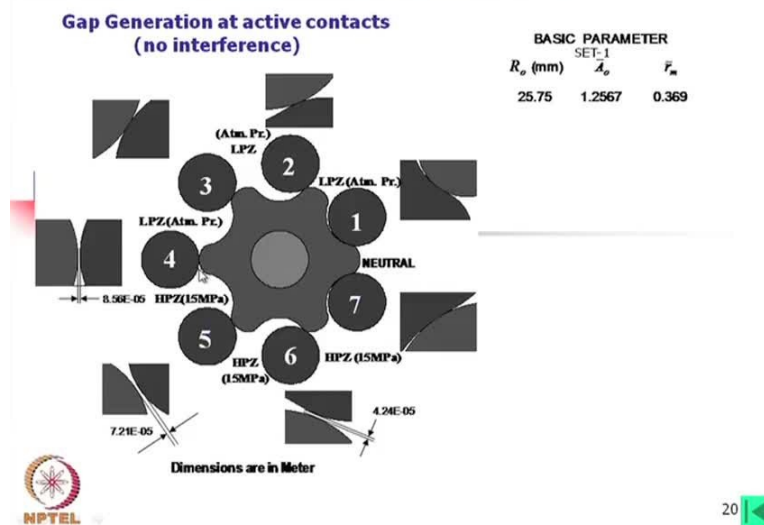
Now this is I would say that if experimentally it is carried out, then we have tried to find out that whether this approach is correct or not. Now as you can see, this is the predicted, typical cycle, this is the torque cycle it is giving. The test result as you see, this is obviously it is very difficult to measure the actual test. It is at random it is coming like this. So test result, typical sample, this is the sample and the actual test record is this one.

This, only one sample we have matching with this. And here, actual is like this. Obviously, the theoretical curve will not match with that but it is giving more or less similar trend and it can be fairly, accurately we can estimate what will be the torque, et cetera of such machines.



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**ORBIT Motor – Deformation & Gap at active Contacts (Contd...):**



Now this is using the FEM technique and this is for this is the value and this pressure has increased you can see, this 15 mega Pascal and this is the physical size. This will give us the physical size and at 15 mega Pascal, we may find that this or the gaps are generated. And this, as you see this is a very critical, at that moment, this gap is being generated. This gap of course, there is no harm because both are in high-pressure zone.

So there will be no leakage but this amount of gap is generated there. Only dangerous part is there but it is again not always. Momentarily, it will be there. Now this also can be realised from this machine.

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Now this is obviously not an orbit motor but this is an gerotor machines. In that gerotor machines, this is recently I visited technical University of Roslof. They are conducting such experiments using the steel rotors and stator but this is a very thick some sort of glass they are using for the cover and there with the photography they have found this. Of course, in this figure, you can see only the machines, I will show you another one where you can realise the you can see suddenly you are looking at this bubble, what is happening if you look into this?

When this is changing its phase from high-pressure to low pressure zone, these bubbles is definitely where the leakage is occurring. That means the gap is occurring. So theoretically, what I have tried to prove, practically also this is happening. This is a good example for inter chamber leakages. You can see this once again. This, looking into bubbles means this at the critical contacts, there is gap. Critical contacts means when it is separating high-pressure zone to low pressure zone and this is momentarily.

Theoretically, it is matching with my theoretical calculations whereas it is the those who have conducted such experiments, they have not yet calculated such things. They are only experimentally observing.

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**References :**

•Maiti, R. Active Contact Problems in Epitrochoid Generated 'Floating Axis' Orbital Rotary Piston Machines. *ASME Journal of Engineering for Industry*, 1993, 115 (3), 337-340.

•Colbourne, J. R. Reduction of the Contact stresses in Internal Gear Pumps. *ASME Journal of Engineering For Industry*, 1976, Nov, 1296-1300.

•Maiti, R. and Nagao, M. Prediction of Starting Torque Characteristics of Epitrochoid Generated Orbital Rotary Piston Hydraulic Motors, *JSME International Journal*, 1999, C 42(2), 416-426.

•Cornell, R. W. Compliance and Stress Sensitivity of Spur Gear Teeth. *ASME Journal of Mechanical Design*, 1981, 103(2), 447-459.



So these are the reference. If you can go through such paper, you can find out say for example, this paper where I will find that Webber approach and the Hertzian approach how it is simplified and nondimensionalised, here this prediction of the starting torque, et cetera is shown and this is the you can say this this is the 1st paper who calculated stresses for the gerotor pump mainly but he proposed this formulation part initially. However, I personally also calculated such cases in more rigorous way and in dimensionless form which I have described. So thank you very much for listening.