Fundamentals of Industrial Oil Hydraulics and Pneumatics
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Module 8
Lecture 30
Design Analysis of ORBIT Motor - II: Geometric Volume Displacement

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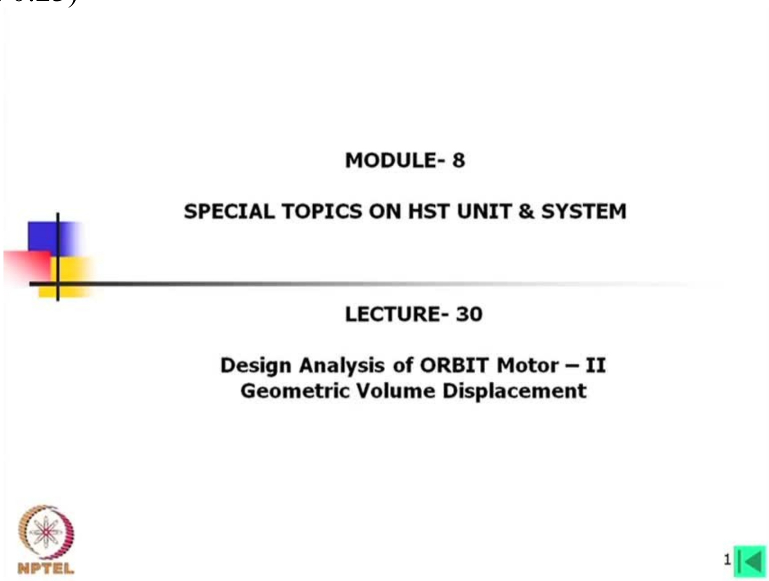

Welcome to today's lecture on design analysis of ORBIT motor-2 which is geometric volume displacement. In earlier lecture, the in part 1 I have discussed about the geometric design of the star and envelop that is ring of an ORBIT motor.

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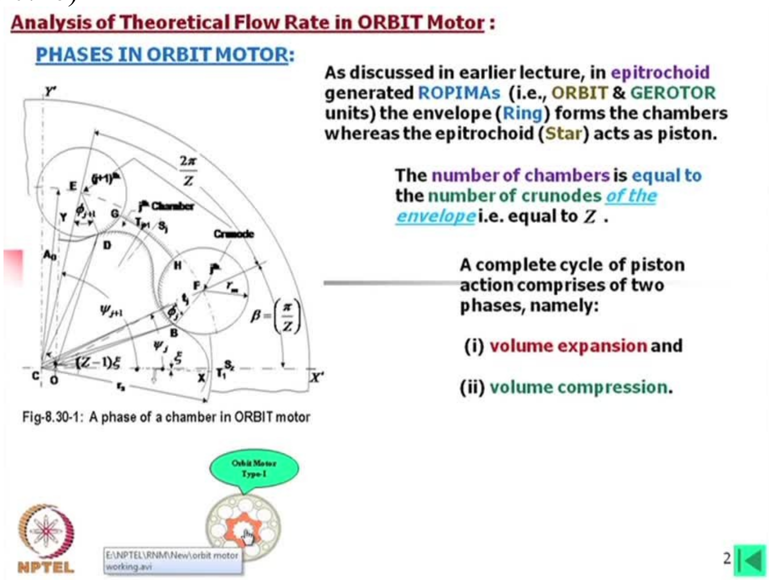

In this lecture, I shall show how to estimate the geometric volume of a chamber and then swept volume and how to find out the volume displacement rate with the shaft rotation. So as discussed earlier, the epitrochoid generated ROPIMAs that is rotary piston machines, orbit and gerotor units, the envelop ring forms the chambers whereas the epitrochoid star acts as an piston. If you look into this, so this is the chamber.

So one particular chamber is shown here which is formed by you can say that the outer member that is the envelop, that is ring is the cylinder and this is acting as a piston. Unlike the cylindrical piston machines, in this case the area varies and so variation of volume with the shaft rotation is equal to the variation of area multiplied by the thickness of the star and ring which is constant. It is also discussed, the number of chambers is equal to the number of crunodes of the envelop that is equal to Z.

So the number of chamber will be equal to the number of roller in this case or it might be sometimes integral part of the envelop whereas we know the lobe of the star will be 1 less than this. So here there will be 6 lobes if we take Z is equal to 7. Again, in earlier discussion what we have seen that the ripple is less in case of odd number of chambers. So while we shall discuss all the formulations, particularly we have to be careful formulations are like that, Z is always an odd number. That means, number of chambers are always odd. A complete cycle of piston action comprises of 2 phases, namely volume expansion and volume compression.

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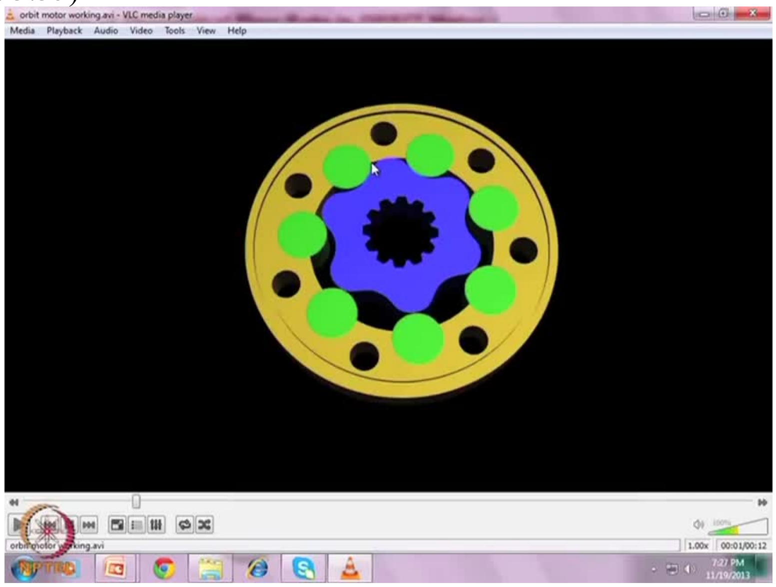

Let us look into this. So this is the star and this is the ring. This is modified epitrochoid what we have learned and this is basically enveloped and this modification is nothing but the constant difference inward shift of the original epitrochoid generated. So for which the envelope actually goes through this centrepoint envelop and then if we modify this envelop, what we find, this contact portion becomes circular arc.

If you notice this rotation, you will find that this contact is not up to the full arc. It is some angle, the maximum leaning angle you can say in both directions from the centre. So but the thing is that we can replace this we can either make it integral or we can replace this integral part by roller ok. Advantage of roller I have already discussed which is that we can replace this roller if this is worn out okay?

Now other part of the envelop is that this circle on which this rollers are kept, I mean on the outer body, this is slightly less than the circle which is called as big circle, slightly less than that. So this roller does not come out. And it is selected on the basis of at the topmost position of this roller. Okay? Now this we can call that at this position, the piston at its top dead centre okay. Now when it will rotate, you will find gradually this space is being increased, the black space is being increased.

That is the change in area. So if we can estimate the change in area, then if we multiply with the thickness of the star and this envelop which is equal nominally which is equal, we can estimate the volume displacement and that we will learn today. But one important thing here I would like to discuss that if we consider this is the bottom dead centre then at this position you will not find any other chambers in top dead centre.

Top dead centre will be which one will be the top dead centre? In next while it will start rotating you will find that this this one will become the it will reach at top dead centre this one, just a small rotation. Let us see. You can see this.

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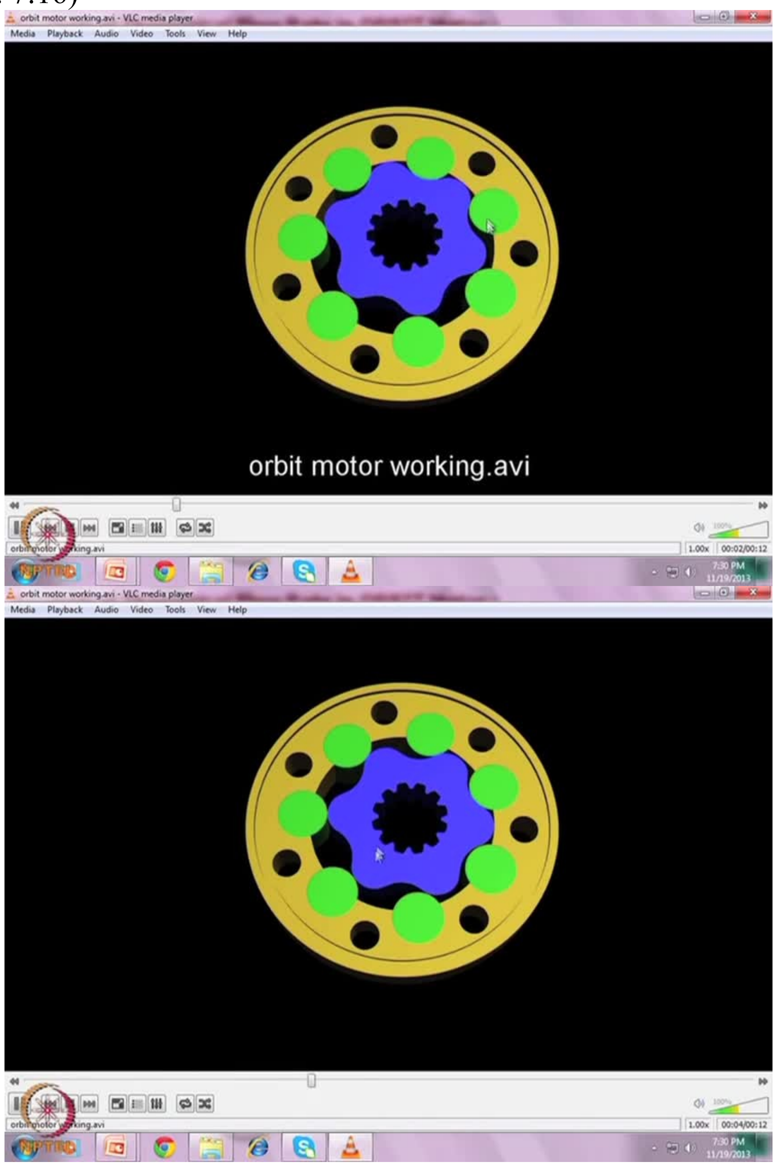

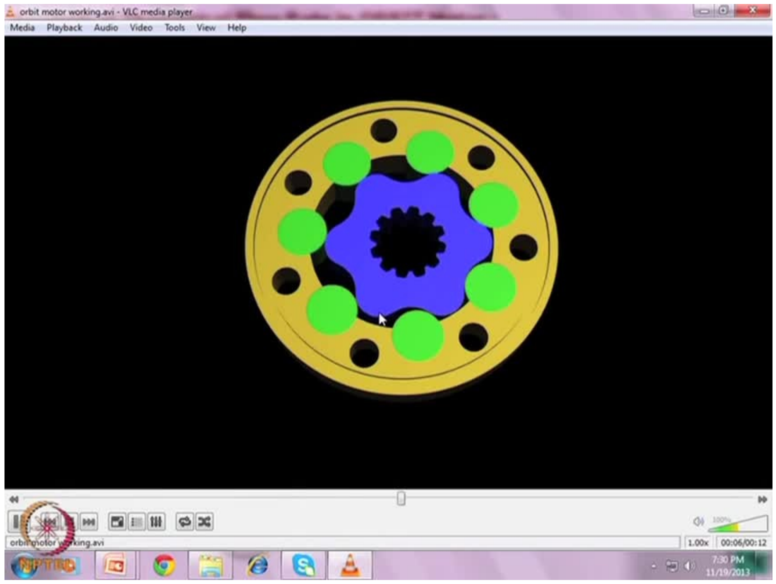

This already had bottom dead centre and then again say suppose this is the minimum, at this position you will find after a slight rotation, this is becoming a top dead centre. This angle can be easily calculated from the physics of such ORBIT motor, from the analysis of physics of such ORBIT motor.

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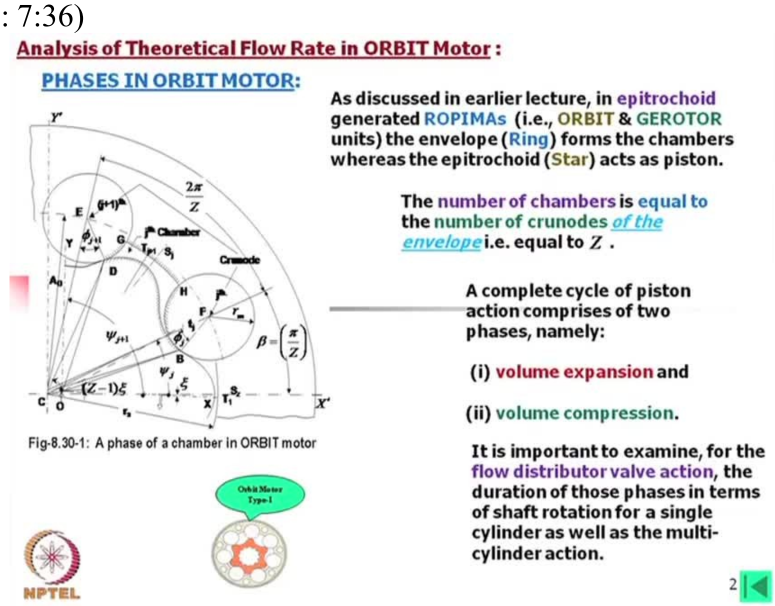

It is important to examine for the flow distributor valve action. If we analyse the flow distributor valve action, the duration of those phases in terms of shaft rotation for a single cylinder as well as the multi-cylinder action. So what we have to do, we need to analyse how long a chambers or what with respect to the shaft rotation, at what angle this bottom dead centre and top dead centre is occurring and according to that, we have to also look into this the distributor valve.

This would act accordingly. So this phase analysis is not shown it will not be shown in today's lecture but we will accept that when this is in expansion mode then one, the oil in channels are connected and when it is in the compression mode, the oil out channels will be connected okay?

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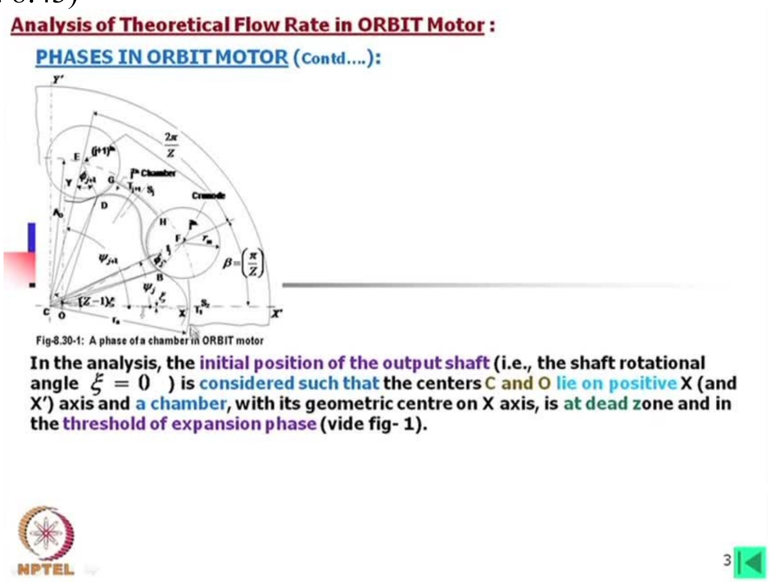

Now in the analysis, the initial position of the output shaft, that is the shaft rotation or angle theta is taken as 0. What we are considering with this axis when this placed is at topdead centre, that angle is 0. The centre C and O lie on the X X dot axis this is X dash and here is the X so it is lying on this axis. Both this axis of the star as well as the axis of X axis of the envelop, they coincides. Whereas Y axis is away at the Centre distance CO whereas this one is the this one Y axis through the centre of the envelop.

And that is one of its dead zone, we have called it the top dead centre and this is in the threshold of expansion phase, that means immediately after that if we rotate in the clockwise directions, here the expansion modes will start okay?

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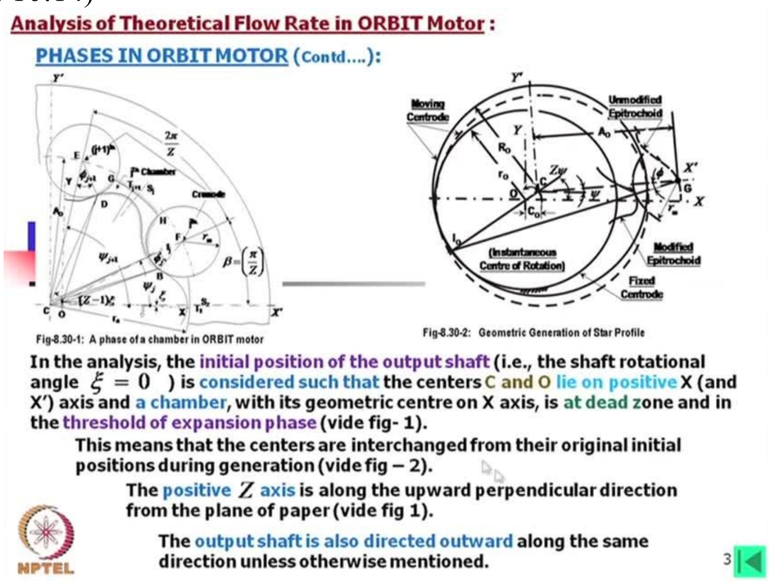

Now this means that the centres are interchanged from their original initial position during generation. So in the previous lecture I have explained that how this generation is done. In that generation technique, what we do initially that the Centre of the Star was here or the centrode fixed for star was here and the centre of the star sorry the ring gear or envelop was here. Whereas when we have started operation, we have just interchanged the Centre.

This means that while we are we shall consider the geometry for the envelope, we have to take care of these transformations okay? Now the positive Z axis is along this Z, do not confuse, this Z is not the number of the lobes but this is the Z axis. X axis, Y axis and Z axis is in the upward perpendicular direction of the plane of the paper, that is in this it is from the here will be the Z axis which is coinciding with the axis of the output shaft.

The output shaft is also directed outwards. So Z axis is coinciding with that which I have just told you.

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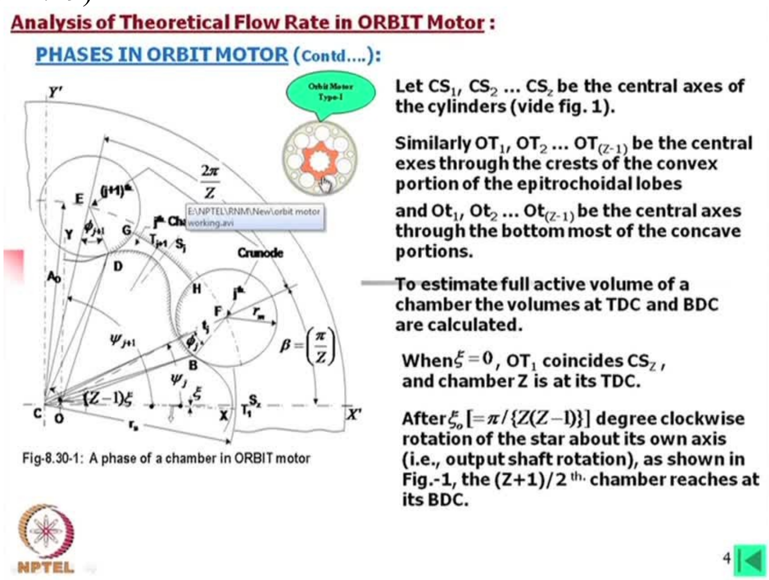

Now we have taken some axis on the star and ring. What are those? We have taken a CS1, CS2, CSZ with a central axis of the cylinders. This means on the envelop we are calling that envelop is acting as a cylinder. So on the envelop, from C the centre of the envelop to the Centre of this portion that is you can say the centre of the chamber, that we have designated as CS. Now according to the number of chambers, we have numbered the chambers also, we have given the S1, S2, S3, et cetera.

Now here I have mentioned as a jth chamber, any jth chamber but on the with respect to the figure which I have drawn, we should say this is chamber 1. This is chamber 1. In the phase analysis, I have shown that this is the chamber 1, next chamber is chamber 2 whereas the chamber on the x-axis, along the x-axis when the rotation is 0, in that case this chamber is chamber 7. So this is you say if you put this number, this is S7, this is S1 and C is the centre okay?

Similarly we have taken OT1, OT2 capital T, OTZ minus 1 with the central axis through the crest of the convex portion of the epitrochoidal lobes. So this is the convex portion the if you take the topmost point, the crest point, then we number is OT1, OT2, et cetera. But if you look into this case, we have taken as if the OT1 is here, it can be named otherwise this is the T into Z minus 1 because there will be 6 lobes, one lobe less than Z. so that is why number will be up to the Z minus 1.

Similarly we have also taken O small T1, O small T2, OT2 that is the that is through the lowest point of this profile okay? That is the concave portion we have taken this. Now these lines while we are analysing the physics, then which one is coinciding with which one, depending on that we are giving the number, we are assigning. This is easy to understand but in the present analysis, our main concern is to estimate this area at any instant.

You will find this area and you may find that these are not much useful here but while you are writing something, presenting something, you can say suppose OS1 has coincided with OT1 capital T1 or O capital T2 like this hmm? That is easy to understand. To estimate full active volume of a chamber the volumes at top dead centre and bottom dead centre are calculated. Suppose if we would like to find out what will be the volume expansion of a chamber, we calculate the volume at top dead centre and bottom dead centre.

Top dead centre means when the piston is at the top, that means this is the top dead centre. Whatever the volume at that condition, that is not the active volume, that is not the varying. That is a constant volume that will be always there. So we have to calculate separately this volume as well as we have to calculate when is the maximum volume at the bottom dead centre. And then if we substract this, we will get the volume of a chamber.

When the shaft rotation angle is equal to 0, OT1 coincides with CSZ, in that case CS7 if we take this Z is equal to 7 and chamber Z is at its top dead centre. Okay? Now after an angle that is theta 0 which is calculated as pi by Z into Z minus 1 we will arrive an another chamber which is given by Z minus Z plus 1 by 2th chamber reaches at its bottom dead centre. But I have doubt, I think this will be minus.

This will be true if we consider this is chamber 1 but we have considered this is chamber 1 so possibly this will be Z minus 1. So that we can examine.

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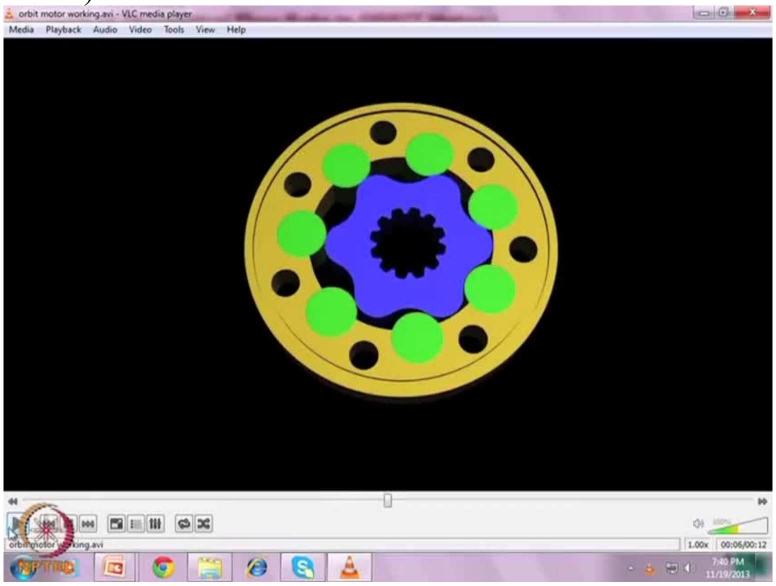

Let us see again. Say if we let us consider this is 1, no this is 7. So 1, 2, 3. So 3rd one is reaching there. So this will be Z minus 1 by 2. Not the Z plus 1, it will be Z minus 1. But this angle, say this much at its top dead centre when the shaft rotational angle is all 0 whereas this will be at its bottom dead centre, then the shaft rotation can be calculated as pi divided by number of lobes on the envelop into the number of lobes on the epitrochoid that is star, the ring number.

So suppose in this case we have taken 7 and this is 6, so pi divided by 42, it is around 4.3 degree or 4.27 degree for after which this angle will be this chamber will be at its bottom dead centre. There will be maximum volume. So what we can do? We put the shaft rotation angle 0, we consider this 7 chamber, we calculate the volume and next moment, after 4 degree of rotation, we consider the chamber 3 and we calculate the volume.

Then this volume minus this volume will give the volume of the one chamber one maximum volume displaced of a chambers okay? So this we have to do but we must calculate this area now.

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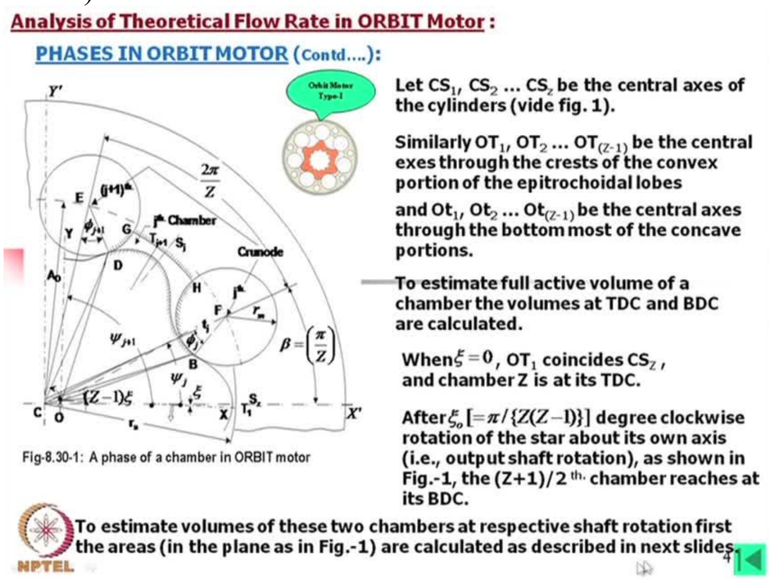

So this is what I have discussed. This is told here.

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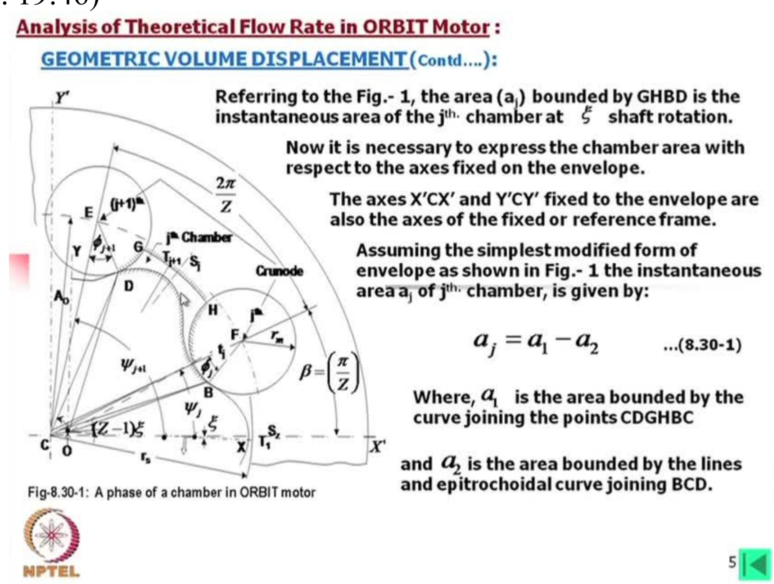

Now next slide is what we will see that referring to this figure, the area aj, aj means area of the jth chamber at instant when this has rotated by any which angle. It is bounded by, this area bounded by GHBD. Now it is necessary to express the chamber area with respect to the axis fixed on the envelop. We will find out this area considering this axis, the fixed axis fixed to the envelop or the ring.

So the axis XCX dash and YCY dash fixed on the envelop are also axis of the fixed reference plane. That means, this is fixed, so this you can say the axis of the fixed reference, so we will transform our coordinates with respect to this to estimate this area. Assume the simplest modified form of the envelop. What does it mean? We have assumed, this envelop is nothing but that there is a circular ring and this is on a ring with the inner circle of this one.

So this is the simplest form which we have considered. And the instantaneous area aj of jth chamber is given by aj is equal to a1 minus a2 where a1 is the area bounded by the curve joining the points CDGHBC. So we 1st we shall calculate CDGHB and again C, this area we shall calculate minus the area bounded by the lines and epitrochoidal curve joining BCD. So if we substract now this area from this area, we will get this instantaneous area of this.

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Analysis of Theoretical Flow Rate in ORBIT Motor :

GEOMETRIC VOLUME DISPLACEMENT (Contd....):

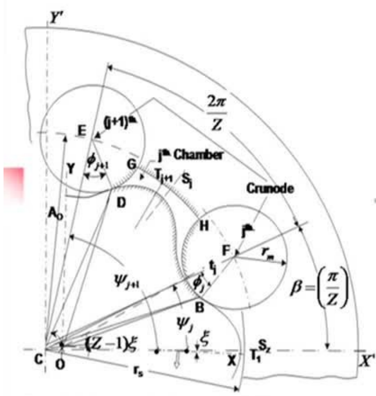

Fig-8.30-1: A phase of a chamber in ORBIT motor

Now from the geometry, the areas can be equated as follows (Ref. Fig-8.30-1) :

$$\Delta CDGHBC = \Delta CEGHFC - \Delta CEGDC + \Delta CFBC - \Delta HFBH \quad \text{...(8.30-2)}$$

Areas in RHS can be expressed as follows:

$$\Delta CEGHFC = \frac{\pi r_s^2}{Z} - r_s^2 \arccos\left(\frac{A_o^2 + r_s^2 - r_m^2}{2A_o r_s}\right) + r_m A_o \left\{1 - \left(\frac{A_o^2 + r_m^2 - r_s^2}{2A_o r_m}\right)^2\right\}^{0.5}$$

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Now how it is done? So geometrical, this is pure geometric analysis. Referring to this figure, what we find? The this is the area indicating CDGHBC is equal to area CEGH and then F, then C, 1st we consider this area, okay? So this is like that. We are considering this area okay? Now next we substract CEGDC. Now we substract this area and then we add CFBC, this area and then finally we substract HFBH, that area we exclude.

So ultimately we are getting this area. So this you can check yourself these equations and you will find that we are calculating ultimately this area okay? Now area in right-hand side can be expressed as follows. This is purely geometric analysis. 1st we should have considered this one.

This is, we have taken pi RS where RS is the radius of this inner circle. Hmm? Pi RS square by Z, that is the whole area minus RA square, this is we have taken one seventh of this area because this area will be one seventh and then we have taken RA square R cos you can you can just understand this how it is.

This is the cos inverse, R cos means cos inverse of this angle. So this is, this will give one pocket and then next we shall consider the another pocket, we will get this one. So this you just, you have to take this figure and this you have to understand. Here a0 square is this, this is the a0, this square, then RS square, we have taken this, this square minus RM square is this one. Cos inverse of that into RS square will give you this area okay? Now plus RM into a0 RM into a0 into 1 minus this will give this area. So we are going to get this area out of that.

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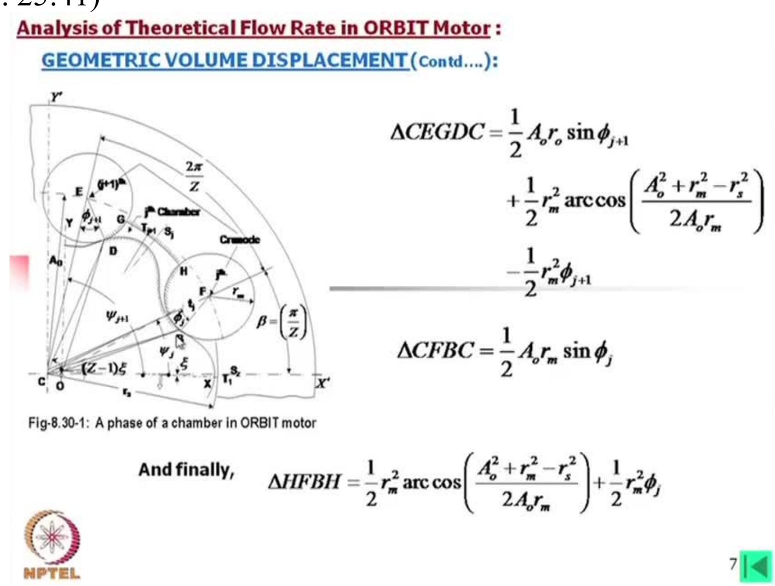

Next we consider the area CEDGC and again CEDG and C. That means we are getting this area. This is again if you can find out that 1st this is a triangle you have considered a0 into this is perhaps not R0, it will be RM this will be RM. This is RM. That means we are getting this perpendicular, A0 is this one. So we are getting the area of this triangle this one. Then plus half RM square into cos inverse of this angle, that we are getting perhaps this area okay? This area, we are getting this area. Minus ya this area.

Oh okay we are getting this area and then minus this, so we are ultimately getting this area here. Now next we will consider CFBC CFBC this is simply half into A0 RM by sine psi J. These angles are called leaning angle that can be calculated for rotation of the shaft which I have discussed in the last lecture. And finally HFBH is nothing but this angle so this area. This is RM square into cos of that plus half RM into psi J.

That means we have considered this area 1st and then we have considered this area.

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Analysis of Theoretical Flow Rate in ORBIT Motor :

GEOMETRIC VOLUME DISPLACEMENT (Contd....):

Substituting these values in equation (8.30-2) and then non-dimensionalizing area (CDGHBC) $\bar{a}_1$ is expressed as:

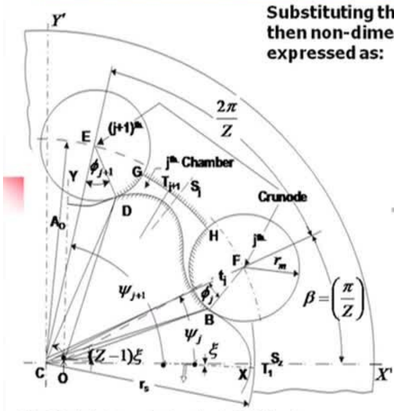

Fig-8.30-1: A phase of a chamber in ORBIT motor

$$\bar{a}_1 = \frac{\pi \bar{r}_s^2}{Z} + \frac{1}{2}\bar{A}_o \bar{r}_m \sin\phi_j - \frac{1}{2}\bar{r}_m^2 \phi_j - \frac{1}{2}\bar{A}_o \bar{r}_m \sin\phi_{j+1} + \frac{1}{2}\bar{r}_m \phi_{j+1} - \left[\bar{r}_s^2 \arccos\left(\frac{\bar{A}_o^2 + \bar{r}_s^2 - \bar{r}_m^2}{2\bar{A}_o \bar{r}_s}\right) - \bar{r}_m \bar{A}_o \left\{1 - \left(\frac{\bar{A}_o^2 + \bar{r}_m^2 - \bar{r}_s^2}{2\bar{A}_o \bar{r}_m}\right)^2\right\}^{0.5} + \bar{r}_m^2 \arccos\left(\frac{\bar{A}_o^2 + \bar{r}_m^2 - \bar{r}_s^2}{2\bar{A}_o \bar{r}_m}\right)\right] \quad ...(8.30\text{-}3)$$

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So now we will substitute this value in equations 8 30-2 and a non-dimensionaling now here is the question of non-dimensionalisation which I have shown an earlier case. What we find that area, any distance we have put a bar. The bar means it is the nondimensional value nondimensional parameters. So in that case what is done, each and every parameter, so if it is a length, that is divided by R0 what is capital R0? What is capital R0?

Capital R0 is the radius of the centroid, the bigger centroid, the centroid for the envelop. So we have divided by R0 to make nondimensional of any length, A0, RM, et cetera. So this means that while we are considering area, suppose we have calculated this one, what will be the actual area? Then A1 bar into R0 square, capital R0 square will be the area. That you should remember. Now in the nondimensional form, this can be expressed as that I am not reading it, so this when this equation is available, you can just sum up and nondimensional you will arrived into that. So now we have calculated only one area, A1.

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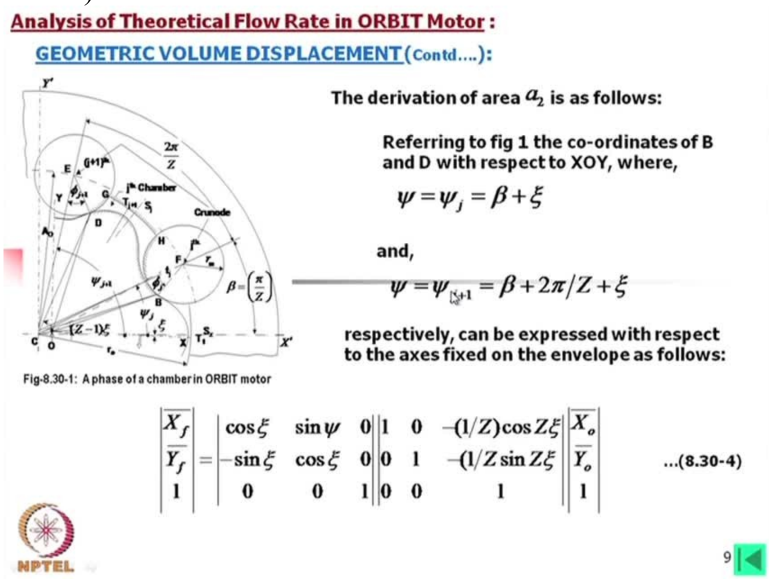

In the next page, what we do? We find out how the A2. For this A2, the coordinates of B and D with respect to XOY where this angle is with the reference to the original generation of profile, epitrochoids. You just look at this, this angle indicates the shaft rotation whereas this angle with respect to the original development of the profile. Now what you can find say this angle for degenerations, we take for the jth chamber, psi J is equal to beta.

Beta is in this case pi by Z. That means half of this angular spread between 2 rollers plus the angle of rotation okay? So beta plus angle of rotation we will consider for this one. Similarly, for the other one, this angle we will consider beta plus 2 pi by Z plus plus this angle the shaft rotations. So in the original formula of the profile we will put this value and this value to get the coordinates of these 2 contact points okay? So therefore these coordinates can simply be written in this form.

If you use this form, you will find these coordinates okay? So this is being transferred from the original coordinates what we have calculated with respect to the reference frame, okay? 1st of all, we have to calculate these 2 and then we will transform in this coordinates system. Next here have mentioned, this beta may be pi by Z or any other angle. Only we have to take care of the proper geometry.

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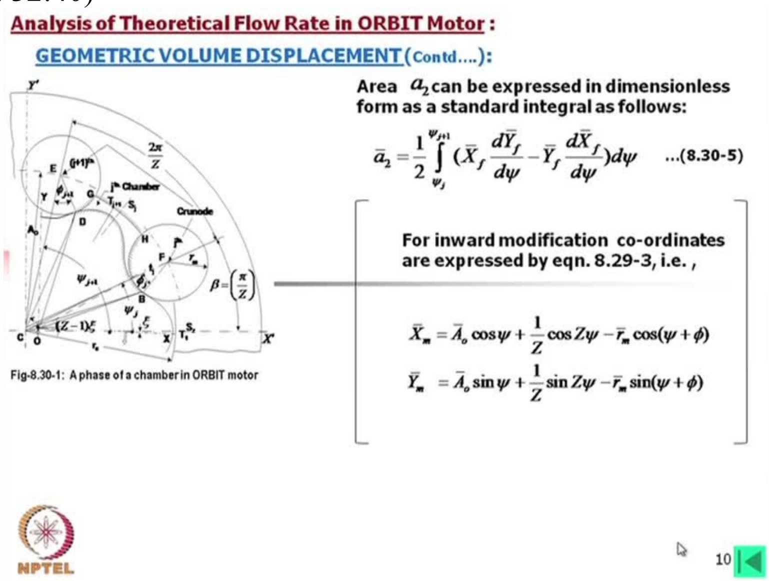

Now next we will express the A2 in the dimensional, nondimensional form. Now this is from epitrochoidal geometrically it can be shown that this integration will give us this area A2. If we would like to find out this area, then knowing these coordinates of these 2, we can differentiate like this. Now here, I have presented this formula, this is from the earlier lecture. So this is the formula for expressing the coordinates of the envelop at any this Psi angle. Okay.

Now in that case we have calculated this angle separately and we are adding to that. Remember, we have to calculate leaning angle while we are trying to calculate these coordinates of any contact point. That is you have to also take the formula in the earlier lecture to calculate this properly. Okay.

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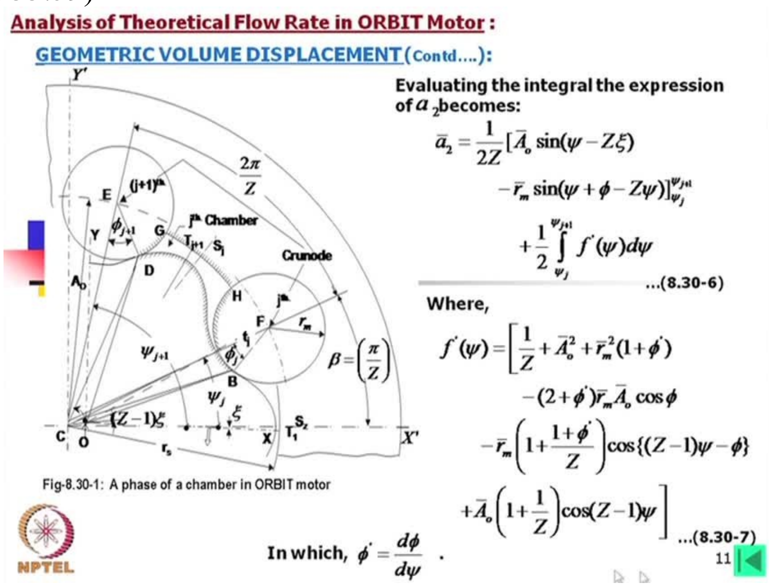

Now if we evaluate this integral, we will arrive into this formula but still one integration part will be there for which this can be expressed in this form. That now this integration is not presented in the any closed form, closed form solution is not given here, so we have to go for numerical integration to evaluate this, okay? But maybe a mathematician can he can find out something but otherwise we have to go for this numerical integration. This is that you can find out this also. This is not difficult to find out. This is the leaning angle. That means this angle.

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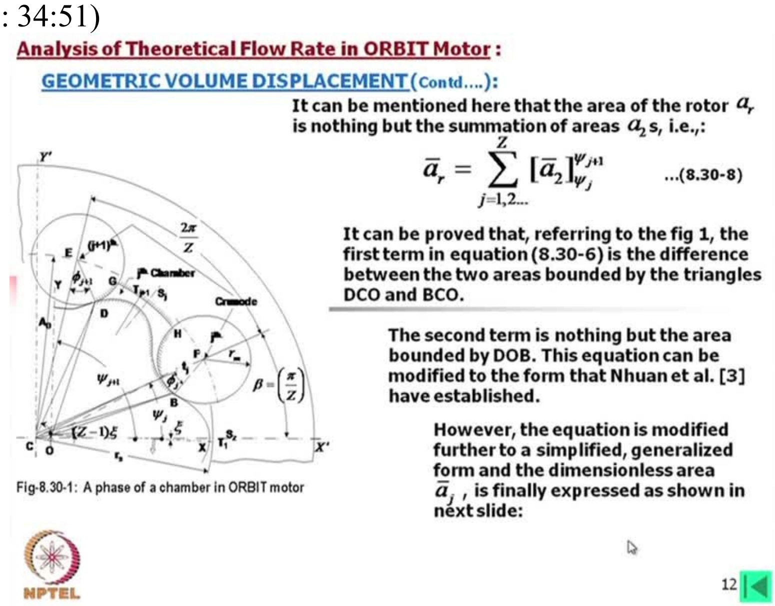

So finally okay before going into calculation these chambers, here one interesting note is given that now we have calculated in the as a A2, this area. Now at any instant, if we add all such areas together, that must give the area of the rotor which is important to find out the envelop I mean total area under the profile of the star which can be simply if we add all the area calculated like this at any instant, we can simply get this area of the star okay?

Now the 2nd term is nothing but the area bounded by DOB. This equation can be modified to the form, this one reference you will find that one reference, he developed the formula in other way. It was, to me it was more cumbersome but I have simplified this formula and to and we arrived into in this form which we will see in the next slide. Okay?

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Analysis of Theoretical Flow Rate in ORBIT Motor :

GEOMETRIC VOLUME DISPLACEMENT (Contd....):

$$\bar{a}_j = \bar{a}_1 - \bar{a}_2 = \frac{\pi}{Z}\left[\bar{r}_s^2 - (\frac{1}{Z} + \bar{A}_o^2 + \bar{r}_m^2)\right]$$
$$-\left[\bar{r}_s^2 \arccos\left(\frac{\bar{A}_o^2 + \bar{r}_s^2 - \bar{r}_m^2}{2\bar{A}_o\bar{r}_s}\right)\right.$$
$$-\bar{A}_o\bar{r}_m\left\{1 - \left(\frac{\bar{A}_o^2 + \bar{r}_m^2 - \bar{r}_s^2}{2\bar{A}_o\bar{r}_m}\right)^2\right\}^{0.5}$$
$$\left.+\bar{r}_m^2 \arccos\left(\frac{\bar{A}_o^2 + \bar{r}_m^2 - \bar{r}_s^2}{2\bar{A}_o\bar{r}_m}\right)\right]$$
$$-\frac{\bar{A}_o}{2Z}\left[\frac{(Z+1)}{(Z-1)}\sin(Z-1)\psi - \sin(Z\xi - \psi)\right]_{\psi_j}^{\psi_{j+1}}$$
$$+\bar{r}_m \int_{\psi_j}^{\psi_{j+1}} [\{1 + \bar{A}_o^2 + 2\bar{A}_o\cos(Z-1)\psi\}^{0.5}]d\psi \quad ...(8.30\text{-}9)$$

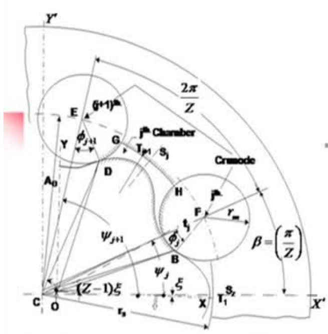

Fig-8.30-1: A phase of a chamber in ORBIT motor

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So what I have done? I have substituted all the findings, all the formulations in nondimensional form and ultimately we get this will be the area of a chamber at any instant. That means when the shaft is rotated by this angle ok, we will get this angle. Or so to say, I will consider if we consider any J and whether it is rotation is 0, 1 degree or whatever it might be, if we substitute it these values, we will get this area of particular chamber. If we consider the next chamber, we have to take all such values accordingly.

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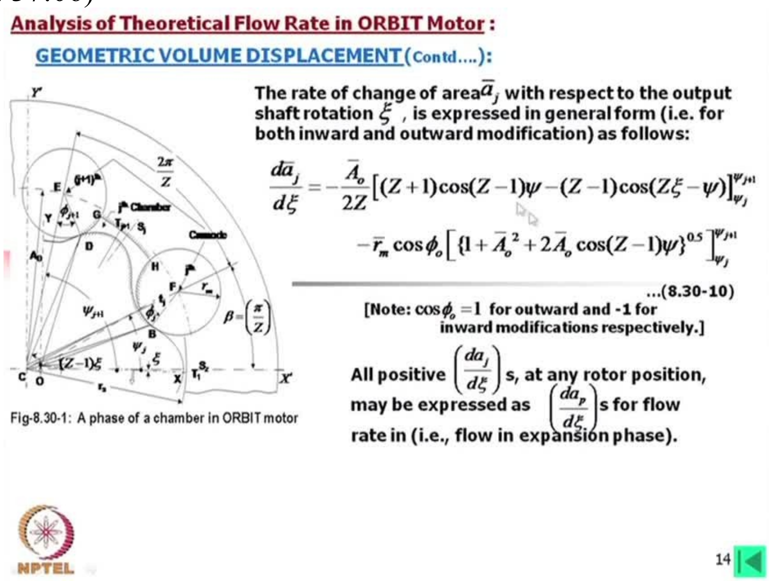

The rate of change of this area with respect to the output shaft rotation is expressed in general form for both inward and outward modifications as follows. Now what we have done? We have 1st calculated this area. Now here we have derived this as a rate of change of this area with respect to shaft rotation which now becomes in this form. If we look into this, this is basically you will find that this motion of this epitrochoidal area, this has no connection of the other geometric relations. It is obvious.

The fixed part is eliminated in this case. It clearly indicates that whatever shape we take for this envelop, keeping this active envelop portion intact, that means instead of taking this portion and this fixed portion as the circular even if we take a big hole or even if we make a curve like this, this equation will remain same. So because this is only the variation of the volume which is working volume.

Now if we get this one, then here I would like to mention one thing that this modification maybe inward and outward, this profile can be modified outward also. So in case of (())(39:05) this is modified in the outward direction like that is a small modification of course. So to accommodate these, whatever may be the modifications, in that case obviously it is inward directions. What we have done? We have used a equation cos phi 0. In fact this phi 0 we can put along this axis.

So when this angle is pi, that means this becomes minus 1 this is for outer modification. That means in this formula for the orbit model what we have considered, we will put minus 1, so this will become plus okay? Whereas if we modify the outward direction, we will put this is equal to 1. That means in that case, this angle is 0. And in inward modification, this phi 0 angle is pi. So it is coming minus 1.

That we have to remember. Now again, so this variation, this is obviously will valid for whether in compression mode and whether it is expansion mode okay but we have to identify very carefully as long as this is in say expansion mode, we will add with the other chamber which is in expansion mode because these machines, in orbit machines is also a DC machines, not alternating machines. All the flow is being mixed.

Suppose at an instant, we find the chamber 1, chamber 2 and chamber 3, this is in the positive value, so we will put this DAP that is in expansion mode. And whenever we will get say for the chambers, we will get this is minus we will put to that separately and we will that make that this is in the compression mode. But interestingly we will find that summation of these positive and negative will always constant.

That means flow, inflow, out will remain constant. And another interesting information I would like to give you here although it is not shown, but if you take one chamber to the corresponding chambers, you may find that the volume in and volume out is not same. Or in other words, when say one chamber is the expansion mode, let us consider in this case, or if I consider the fixed axis, in that case, this angle will be nothing but this 2 pi by Z.

That is the angle for any phase. If expansion and compression. Now this phase what we will find, 1, 2, 3, 4, 5, 6, 7 degree say such we we will make it the interestingly we will find that for a particular angle, the compression and expansion of a chamber is not equal. Whereas in case of ordinary piston machines, this is equal because whatever may be the rotations, whether it is suction or compression, you will find a single degree of rotation, this volume is constant.

In this case, these volumes are neither constant, nor it is matching with the expansion phase and compression phase. Whereas if we consider the overall, that is matching and it has to match. Otherwise this machine will become impossible. It will not rotate smoothly.

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Analysis of Theoretical Flow Rate in ORBIT Motor :

GEOMETRIC VOLUME DISPLACEMENT (Contd….):

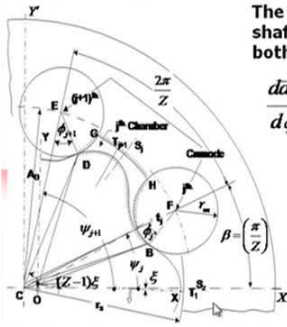

Fig-8.30-1: A phase of a chamber in ORBIT motor

The rate of change of area $\bar{a}_j$ with respect to the output shaft rotation ξ, is expressed in general form (i.e. for both inward and outward modification) as follows:

$$\frac{d\bar{a}_j}{d\xi} = -\frac{\bar{A}_o}{2Z}\left[(Z+1)\cos(Z-1)\psi - (Z-1)\cos(Z\xi-\psi)\right]_{\psi_j}^{\psi_{j+1}} - \bar{r}_m\cos\phi_o\left[\{1+\bar{A}_o^2+2\bar{A}_o\cos(Z-1)\psi\}^{0.5}\right]_{\psi_j}^{\psi_{j+1}} \quad ...(8.30\text{-}10)$$

[Note: $\cos\phi_o = 1$ for outward and -1 for inward modifications respectively.]

All positive $\left(\frac{da_j}{d\xi}\right)$s, at any rotor position, may be expressed as $\left(\frac{da_p}{d\xi}\right)$s for flow rate in (i.e., flow in expansion phase).

It is to be noted that $\left(\frac{da_j}{d\xi}\right)$ is independent of the shape of the inactive envelope. Hence, the area in equation (9), which is different for other approximated curves, is only useful to calculate the area (and volume) at an instant.

NPTEL 14

Anyway our purpose to find out the swept volume, so what we, it is to be noted that this rate of change of area is independent of the shape of the inactive envelop. This is an active envelope. Maybe say suppose this is the contact point up to this. So this portion is active and this circular arc and this circular arc, all inactive. So it really, this formula is independent of this area, whatever it might be.

Sorry whatever the area in equation 9 which is different for other approximated curves, is only useful to calculate the area and volume at an instant. Only thing, suppose if you would like to find out the trapped volume, unused trapped volume, then you have to consider the geometry.

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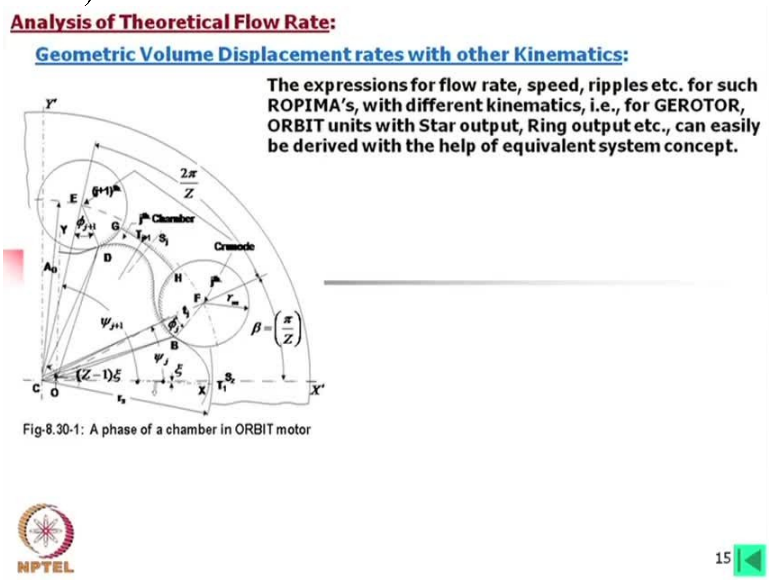

The expression of for flow rate, speed, pulse, et cetera, for such rotary piston machines with different kinematics, that is gerotor orbit units with star output, ring output, et cetera, can easily be derived with the help of equivalent system concept. What it is? Now orbit motor what we have seen the ring remains fixed and output is taken through the star. Star is having two rotations.

One rotation is the revolving actions around the central axis, that is the axis of the outer ring and it has also rotation about its own axis which is the output in case of orbit motor. And orbit, this version is only used as a motor because other pump version is not efficient or beneficial from the transmission point of view, volume displacement point of view. It it needs if we make an pump with orbit principle, it will need large torque for slow speed which is not available from the engine, so it is not beneficial. On the other hand, when we make it fixed axis, then which is called gerotor, also a special name is assigned, G roller when we use this type of roller.

When it is integral, usually call it is gerotor. Or you should call in general case it is gerotor unit that is fixed axis units. What is fixed axis unit? In that case, both rotates about their own axis but even they rotates about their own axis, you will find this compression, expansion, that variation of this area will occur, so that can be used as a pump as well as that can be used as a motor also.

Now what we have done, we are analysing all such area and everything with respect to the orbit motor. But it is possible that if we know the kinematics, so same formula also can be used for the other kinematics, that means gerotor units. How? Let us see.

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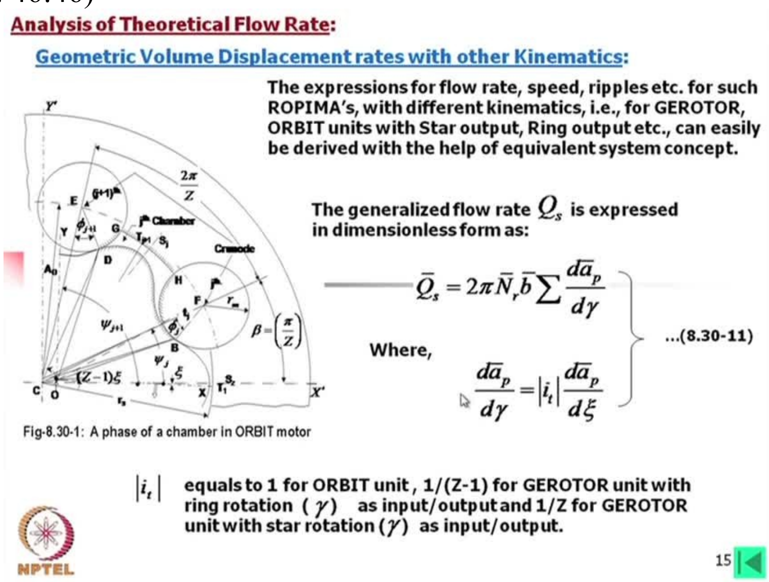

Now what we do? The general flow we can express that what we have derived that this is 2 Pi NR. NR is the output rotations, the B is the width, so DAP and D gamma, this gamma is general angle of rotations we have given. So this simply if you add them you will find out what will be the rotations and in in case of orbit motor, this will (())(47:19) theta and this is the output rotation of the shaft. Now this angle is nothing but transmission ratio into this angle.

This transmission ratio we have derived separately which is not given here. So directly if you would like to use this formula, you have to know this transmission ratio in terms of the gearing in between this star and ring which is tabulated form available one of the references. Okay? Now in case of obviously in case of orbit motor, we will put this as a 1. Orbit motor, this as a 1. So we can easily calculate this one or simply this will be replaced by this one in case of orbit motor.

Now here I have written is in case of one for orbit unit. So without following the table, you can still calculate this if you follow this one. For gerotor width, unit width, ring rotation when the ring is rotating, simply you put 1 by 1 by Z minus 1, so we will get this value and then this flow rate will be for that gerotor unit with ring rotations. For the star rotations, this will be 1 by Z, that we will find this the same expression. In case of orbit motor, this is one.

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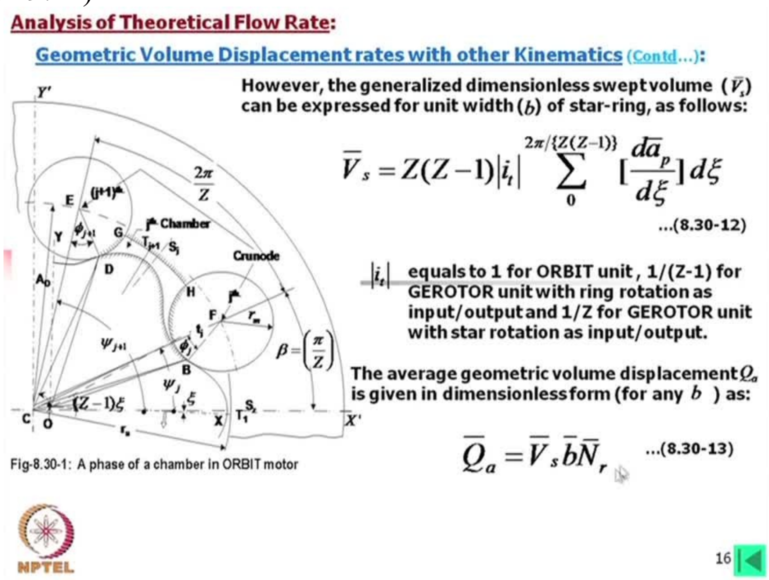

Fig-8.30-1: A phase of a chamber in ORBIT motor

However the generalised dimensionless swept volume VS can be expressed for unit width. Width is given by B of star ring as follows. What we do, in case of so this 1st we calculate this IT is 1, for say orbit motor and then we multiply with this. This is flow rate, volume in and we integrate up to this angle and then we simply find out, so this is I would say rather it is a complicated form to find out the swept volume.

But automatically from the flow rate, we should be able to get the swept volume. So this is shown here. But I will show you the simplest way of calculating the swept volume. Now this is again the same thing I have described here while you are using this formula, the average geometric volume displacement QA average, you see this if you estimate this one with respect to this, that we will find the flow fluctuation.

So what we should do I mean once we calculate this, we simply multiply with the width factor and because this is for the unit volume, unit width so we have to multiply here with B and then the rotational speed.

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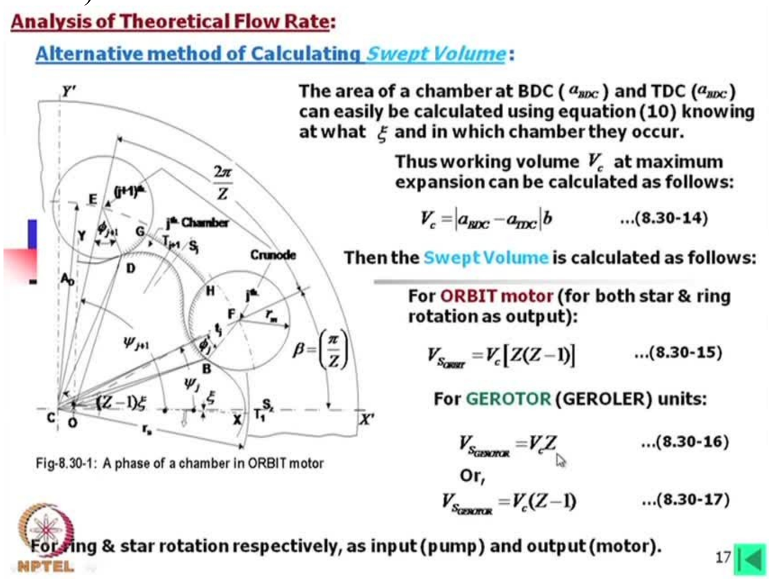

Now the alternative method of calculating swept volume, this is just to calculate the swept volume, what we can do? We can calculate the BDC and TDC at area. Area of the chambers at BDC and TDC and which I have earlier explained that if you for the geometry we have considered, if you consider this angle is equal to 0, then you can 1st calculate this area at chamber 7 which is the chamber at top dead centre.

And then after rotation of pi by Z into Z minus 1 angle, you can take the other one, that is Z minus 1 by 2, that is in this case this will be 3, 3rd chamber after this after that rotation of pi by Z into Z minus 1 and then you will get the bottom dead centre area. So and then, the working volume is nothing but the area, difference of these 2 areas into the width. Now the swept volume is calculated then as follows, for orbit motor, what we do?

Simply this volume of one chamber into this action in one revolution. That means how many such chamber actions will be there in case of orbit motor? Z into Z minus 1 times. So you simply you multiply this, you will get the swept volume orbit motor. Interestingly, in orbit motor, this is one respect to the other. That means, either star rotating with respect to the ring or ring rotating with respect to the star.

So in that case, there will be single formula to find out this swept volume whether star is rotating or ring is rotating. Whereas in case of gerotor unit, this will be either VC into Z or VC into Z minus 1 depending on ring and star rotation respectively because in that case suppose your star is output. Both are rotating, star is output. In that case, you will find that when the star on full rotation is done, actually 6 chambers are displacement is there, not the 7 chambers.

Whereas if the ring is rotating, then 7 I have mentioned 7, not 6. In that case, Z minus 1 and Z. So carefully if we lose, we can calculate the swept volume of the ring I mean gerotor depending on which one is the output.

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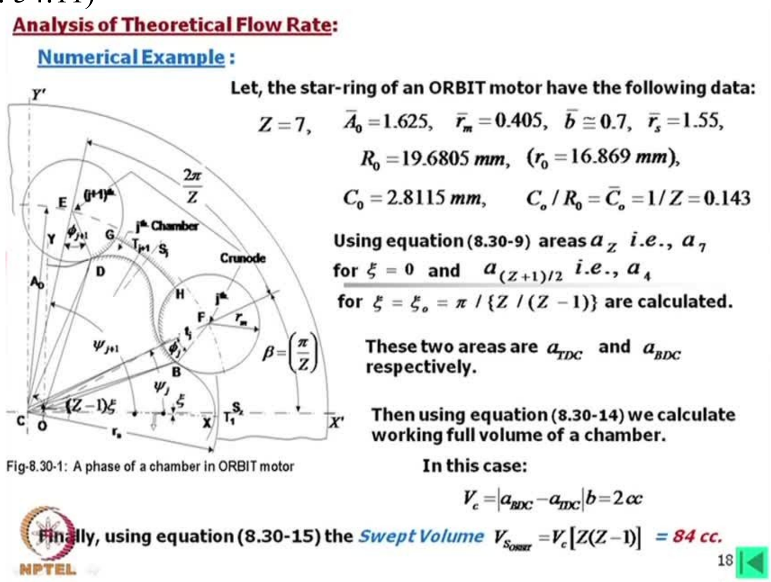

Let the star ring of an orbit motor have the following data. We will take an numerical example. Now in that case, we have taken Z is equal to 7, A0 bar is 1.625, RM bar is equal to 0.405, B is approximately 0.7. It is slightly not 0.7 some value is there. And RS, means this one is 1.55 whereas R0 is 19.6805, approximately 20. So you can imagine what is the actual dimensions okay?

Then this is R0 is the radius of the inner centroid, centroid for the epitrochoid which is 16.869 but this is not required because we have nondimensionalised with respect to this one. Interestingly, say we use A0, RM bar but we never use this C0 bar because C0 bar is nothing but 1 by Z. Because this you will find, this and a relation, R0 by capital R0 by small R0 is nothing but Z by Z minus 1. So and their difference will be 1Z, so this means that C0 bar is 1 by Z.

So in the formula you will find that not C0 bar is used, only 1 by Z is used. Okay? So with this data if we calculate this area, what we have done for the swept volume? AZ we have taken 1st

A7 we have calculated and then this is again I have made a mistake, this will be minus 1 by 2, the 3rd chambers and this is A3 and we will get this now, this area. These 2 areas are ATDC and ABDC respectively.

Therefore we will now use equation 14 to calculate the working full volume of a chamber which becomes this and you see this, I have not shown the calculations but to calculate this area, we have used this formula and ultimately we have found out that this full volume of a chamber, working volume will be around 2 cc, very close to that. So therefore the swept volume of an orbit motor of with this data, it will be 84 cc.

Per regulation, the flow required is 84 cc. So you can imagine that in case of the fixed axis, it will be either 7 times less or 6 times less than that for one revolutions. This is one, 2nd thing look at the size. Size is say A0 is 1.625. If you multiply with this 20, it will become 20 plus 12 30 or maybe 35mm here. And this might be another say 20, so it is 55 to 60 millimetre at the most. So diameter will be 120 millimetre which is less than 5 inch.

120 millimetre will be something like this. So this is the size of the motor whereas and thickness is only 14 millimetre. If you multiply with this, it is close to 14 millimetre but you see the so much volume is required for one rotation. Obviously, this is due to the gearing action inside. So in this way you can calculate the swept volume also you can calculate, you have to obviously you have to make the numerical integration to find out the volume displacement of such machines.

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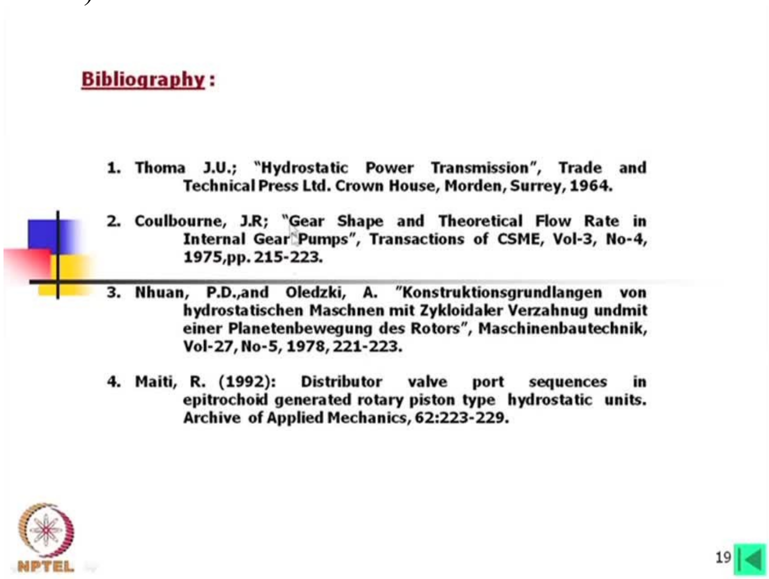

So these are the references. I would say that this paper, you will not be able to follow because this is in this is old as well as this is in German language. I think not in not even German language, it is something it is not exactly German language, I think these 2 people were from Austrian people. If you read this paper, you will have some idea about this fixed axis machine, not orbit motor, also Coulbourne, he derived such displacement formula for fixed axis and Thoma, he is I have used this, how to face is changed and other this, you can read this book also but if you would like to know that phase differences and volume change, et cetera, you can read this paper. Okay, thank you.