Fundamentals of Industrial Oil Hydraulics and Pneumatics

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Lecture no 22B
Module no 6
Design Analysis of Gear Pumps - II

Welcome to today's lecture, this is on design analysis of gear pumps part 2, this is continuation of part 1.

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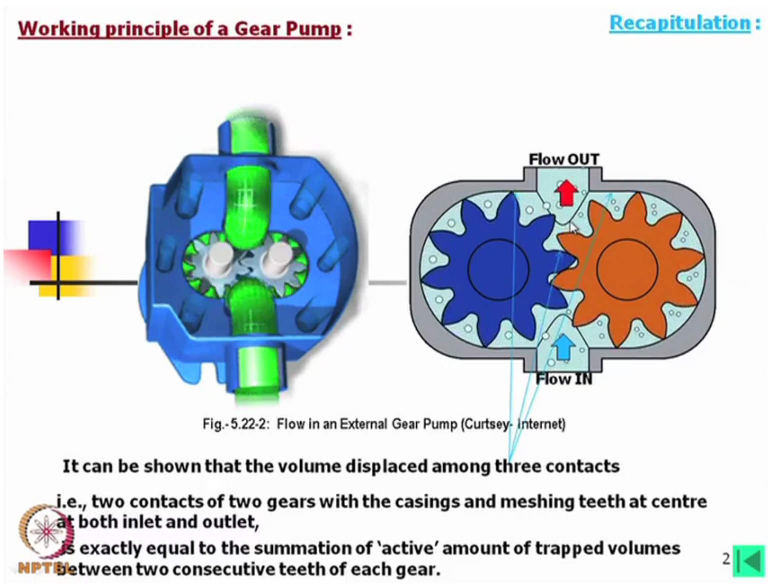

Now, last time what we have learned that in gear pumps external toothed gear pumps if we consider the blue one is the driving one then the flow is in here and then that is carried out to the other side of the pump and there must be the volume expansion and volume compression in this zone. Actually we would say, it can be shown that the volume displaced among 3 contacts; one contact with this valve, one contact with this valve and then contact between tooth, that space is will be in compressive phase and this will be in expansion mode here. That is 2 contacts of 2 gears and casing and meshing teeth at center have both inlet and outlet, okay.

And the volume displacement is exactly equal to the summation of active amount of trapped volumes, the active amount means if you look into this, some oil is going back to the other side so active amount is the what is the volume displacement happening here okay.

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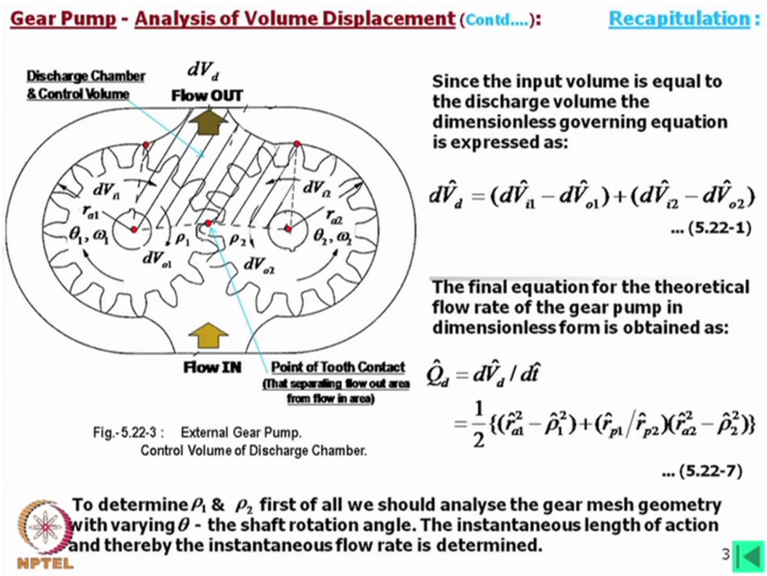

Now also we learn that, if we consider this is the external pump in that case what we have to consider, we have to consider a control volume which is constituted by the point here, the point at the mesh and the point at the other side of the valve. Now what is happening when this is rotating in the clockwise rotation then this is rotating in anti-clockwise direction, this point is going download whether these 2 points coming closer. We have to consider the contact point I mean when this just left this valve, from that point to the situation when this point again will leave this valve.

So in that way it can be shown that this area that is varying and this area varying in nature that gradually this area is being compressed. So that can be derived or that can be written in equation form by considering a small amount of going in here and going in here and going out from this zone here, now this is for d Theta 1 rotation of the drive gear so we can write down this equation in this form right?

And then the final equation for the theoretical flow rate of the gear pump in dimensionless form is obtained say Q d = half into r a1, r a1 which is the random or the teeth circle radius of this gear drive gear, Rho 1 is the instantaneous radius at the contact point, r p 1 by r p2 is equal to the ratio of this pitch circle radii of these 2 gears which is normally is 1, and Rho 2 is the contact distance from here. So if we would like to equate in this way, what we have to calculate each and every instant we have to calculate these Rho 1 and Rho 2 because other dimensions are fixed, so if these 2 are varying naturally this dimensionless flow is also varying.

We have to keep in mind, this derivation is actually derivation of this area into the thickness of the pump which is constant okay. And we have also learned in the dimensionless form what we did to make this flow in dimensionless form. We divided this into the r b square and the width as well as divided by the speed to get this flow rate dimensionless flow rate. Now to determine this Rho 1 and Rho 2 first of all we should analyze the gear meshed geometry with varying theta, theta is varying, the shaft rotation angle which is shaft rotation angle and the instantaneous length of action and thereby the instantaneous flow rate is further determined as…

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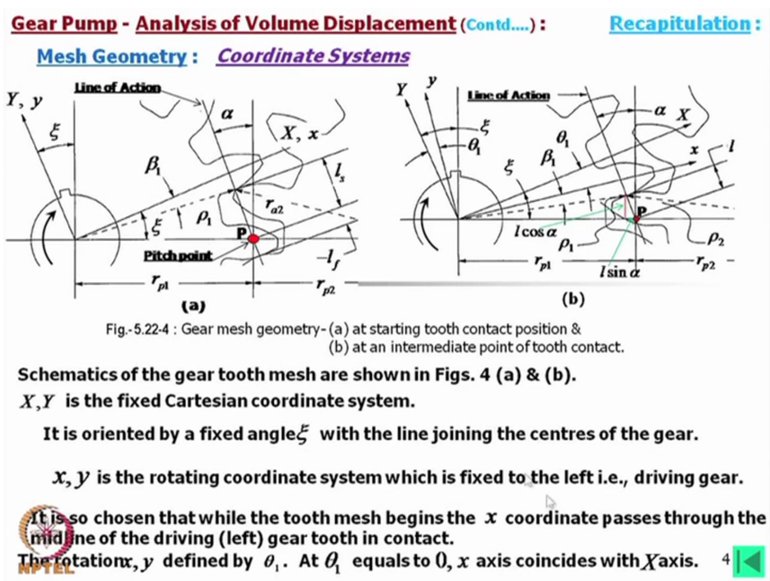

Before that I would say that while we are doing some geometric analysis, First of all we must know what are the axis systems we have considered? In this case if we consider capital X and Y, this is fixed to the reference frame that is this is a fixed axis which as you can see this is X and Y, capital X and capital Y. And again that we should say that this it is oriented by fix angle Zi with the line joining the centres of the gears okay, this is the Centre of line joining the centres of the gears, from here this at this angle. Now we must know what is this angle, then we have considered another axis system x, y which is fixed to this gear the driving gear.

Now it is like that that x, y small x, y axis is through the middle of this 1 teeth which with which just the contact had started. You see this is rotating in the clockwise direction so this is in the anticlockwise direction, the contact had just started here. At that position if we consider the half of this teeth then we get this line which is small x-axis. Now when Theta is equal to 0 then this x, y axis small x, y axis when side with capital X, Y axis. That means, by knowing this axis position with knowing the number of teeth, et cetera, then we can find out what will be the capital X, Y axis we have considered.

Okay, this is not difficult task, we can assign that and then what we do, we consider this geometry, we are trying to calculate Rho 1 and Rho 2 and then we consider this point is moving gradually and then we find out this length in this pressure angle and from there we find out this length Rho 1 and Rho 2, ultimately we find out the length of contact and ultimately we express the flow equations in terms of length of contact. It is written here how this x axis is considered when Theta 1 is equal to 0.

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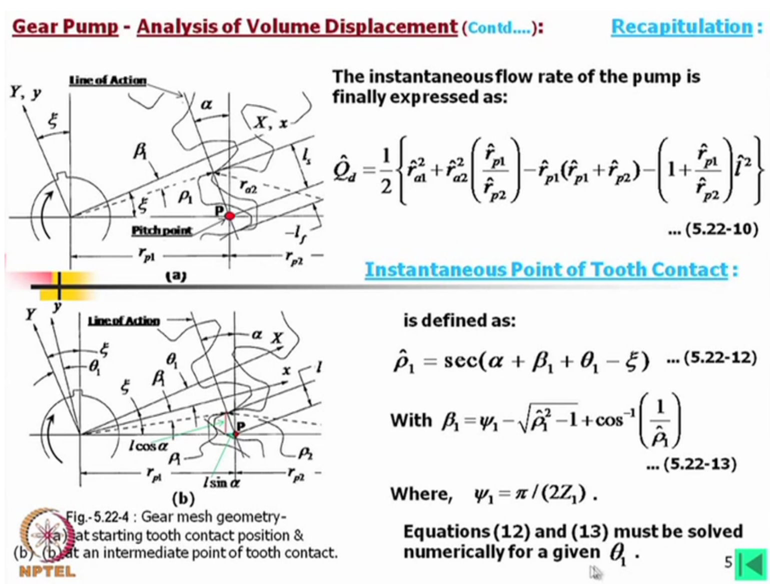

Now ultimately we can replace this Rho 1 and Rho 2 and the equation is expressed in terms of length of contact instantaneous length of contact, which is the contact from the pitch point to the instantaneous contact we have to consider this length okay. If we can calculate this length then this will be expressed easily because other terms are constant. Now, what is instantaneous point of tooth contact? That if we define that in terms of this contact radius and angle then contact radius is expressed in this form, you can check this geometry and you will find that this is the expression.

And then we consider this beta 1 angle which was last time shown, this can be derived this Beta 1 angle in this form and then finally with Si 1 = Pie by 2 Z -1 that is this angle is equal to nothing but half of this angle, half of this. This is pie by Z 1 is equal to tooth thickness basically, I mean 1 tooth angle and half of that okay. So possibly this angle Si 1 is defined here, no not defined but just consider this angle is calculated from the relation of the teeth number only. Then equation 12 and 13 must be solved numerically for a given Theta 1.

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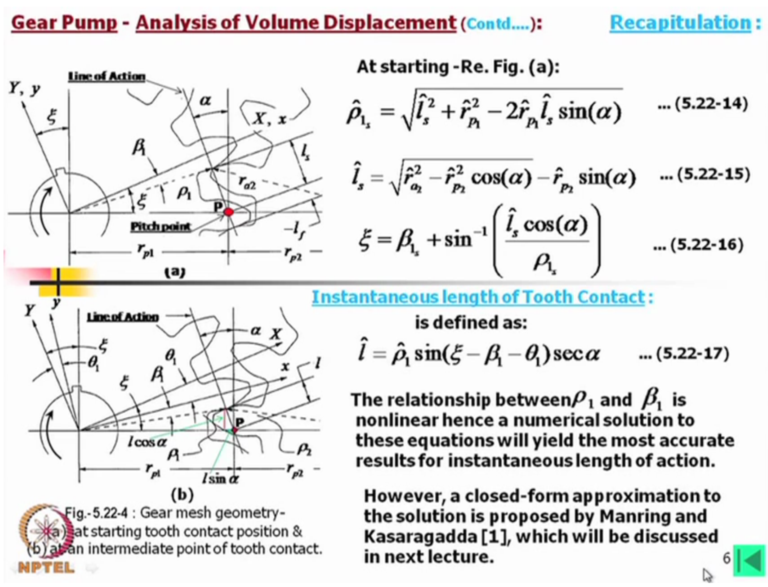

Now we shall consider at starting this equation, the Rho 1 at starting will be expressed in this form and then contact length from l s in dimensionless form is also expressed as like this, then we get this angle expressed by this expression where this is also at the starting and then the instantaneous length of tooth contact is defined as l is equal to Rho 1 sin of this angle and sec Alpha. Now the relationship between Rho 1 and beta 1 is nonlinear, hence the numerical solution to these equations will create the most accurate results for instantaneous length of action. We can solve this numerically but what I would like to mention, say it is Alpha, Alpha is the pressure angle, now we have to take care of this Alpha.

Here we will consider that might be with generated by a standard pressure angle, say 280 degree pressure. Then when they are meshing at if they mesh at their standard pitch circle radius then Alpha remains constant that is working pressure angle remains same as 20 degree. But if you change the Centre distance, definitely this working pressure angle will change. Now we should keep in mind sometimes the teeth number are taken less than the allowable teeth number (())(13:42) and in that case working pressure angle changes, therefore we should always keep in mind this pressure angle is equal to the working pressure angle, not the standard pressure angle.

While we are trying to find out the base circle radius of such case then we consider only the standard pressure angle there, so we should not confuse with the working pressure angle and the standard pressure angle, or such expressions we have to consider and the working pressure angle. However, a closed-form approximation decide told that we can we should go for the numerical analysis I mean numerical solutions to find out the flow but (())(14:42) Maanring and its co-author is Kasaragadda, they have proposed a closed form solution. So now to arrive into such closed form solution first of all we have to consider all contact points.

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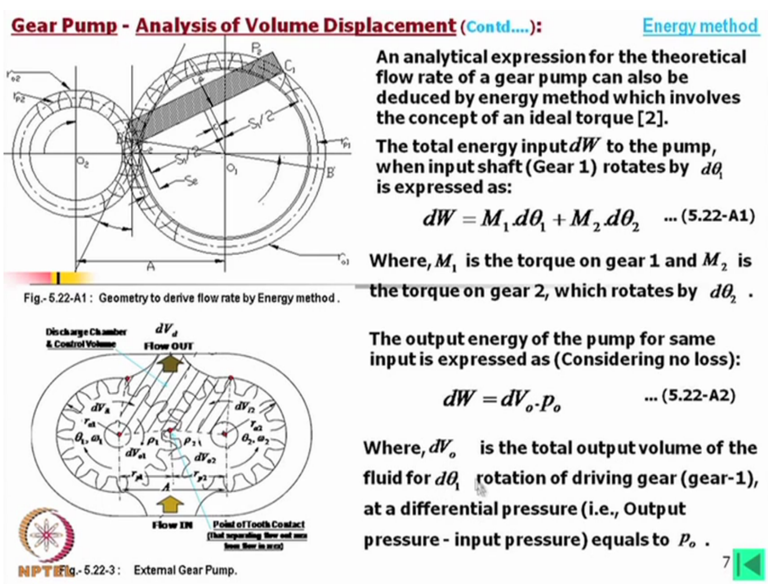

But before that I would like to mention in the method which we have discussed so far what we have done, we have considered a control volume and we have control volume, for that what we observed that basically the area is changing and the width remain constant. So if we can neglect the change of area variation of area with theta then we can find out the flow variation. However, there is another method in which what is done, we consider that those 3 points definitely but instead of considering this area what we consider, we consider a beam joining this instantaneous tooth contact point and the point where this tooth is just separating the expose zone to the trapped zone okay.

Now if I consider that as a beam say this point is point of contact, maybe here the casing is or might be here anyway, if you take these 2 points what we can consider this is we can consider a beam which infer load which is equal to the pressure. The load is equal to the pressure into maybe the the area that is uniform load. By that way we can find out that how much result in load acting on this line at the midpoint and we will find that from that midpoint we have a distance to the Centre of the gear so therefore, if we multiply that distance into this load we will find a torque, so that torque is acting on the gear. Similarly, if we consider the other gear also we will find another torque which is acting on the other gear.

Now this total torque is transmitted by the shaft, so in that way we can calculate what is the total power or total energy that definitely that is rotational angle into that talk, okay. If we consider the power the rate of change of I mean speed in that case we have to consider speed and the torque. Now this equation we are considering the energy, not the power energy, in that case what we consider that M 1 is the torque here instantaneous into the angle of rotation d Theta 1 and M 2 is on the other gear and multiplied by d Theta 2 okay, so this is the total energy.

Now, again this energy can be expressed in form of the rate of change of volume and the pressure. Now this pressure is the differential pressure that means output pressure – the input pressure, in this case we consider the input pressure is 0 so we have considered the output pressure here. Now, if we consider these 2 equations then we can easily calculate because the same pressure we are considering for these 2 so we can easily calculate rate of change of volume with the angle of rotation.

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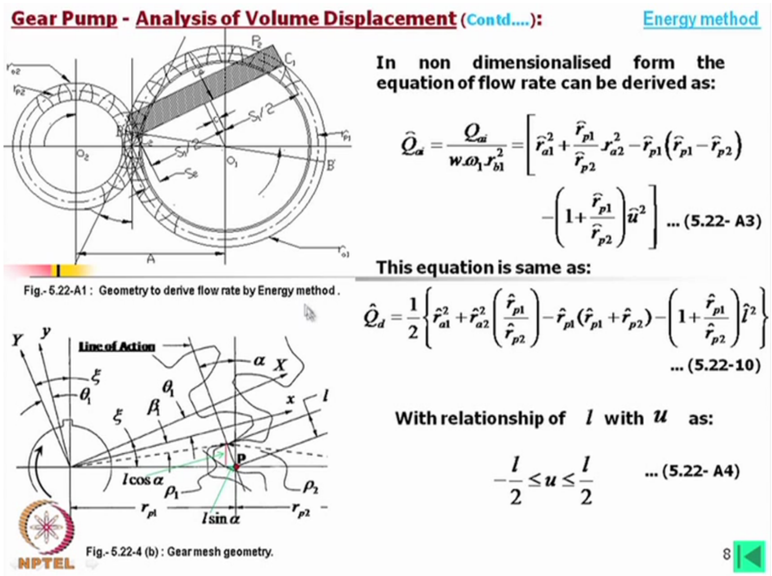

And from there again considering this change of this length that means change in torque which of course constant pressure because geometrically as the volume is being displaced here, there is a compression on this side expansion then definitely this length is also changing according to it. So we have to find this length instantaneous length to find out the instantaneous torque. Now again be have to consider the geometry, through this geometric analysis we arrived into these equations with this energy method, right. Then if we compare with this equation earlier which we have derived considering the control volume, these equation is more or less same, only what we find that here the half is not there and here the instantaneous square of the instantaneous length where this is some value of this u.

Now we can relate this u with l with this relation, while this was derived in fact this can be also derived within this form, but here the geometric analysis was different from this axis consideration so in that way the formula is coming like, but what is found that these are basically same. Or in other words, now if we would like to find out the numerical solutions for the flow then whether we use this equation or use this equation, we will arrive into same result. However, to for the closed form solution we consider this equation 10 and we are trying to find out whether this can be solved. Now closed form solution means you have to find out you have to relate this l with theta with approximation or with some solution.

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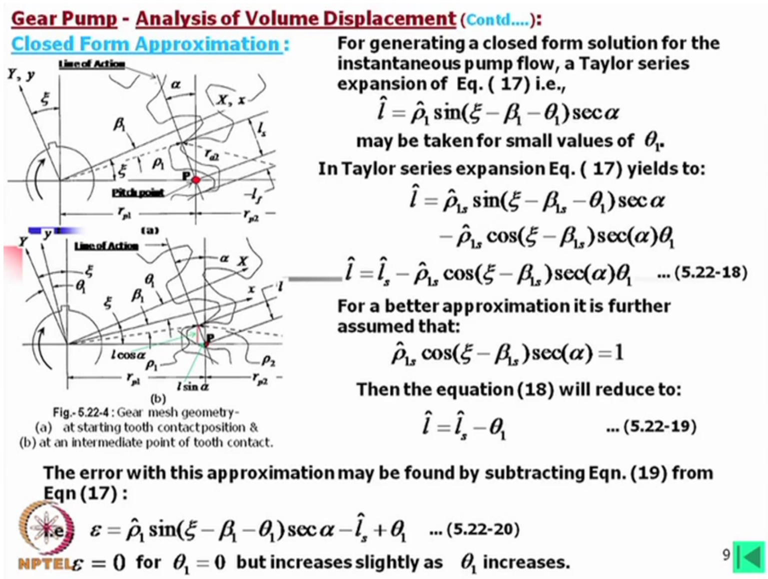

Now let us see that what the Maanring at all have done for generating a closed form solution for the instantaneous pump flow, a Taylor series expansion is done. In that case l we have expressed in this form for very small value of Theta, I mean we have to consider for this case for such equations, the Theta 1 varying very discreetly with small amount. Then in Taylor series expansion we can express this instantaneous length of contact is in this form where Rho 1s is the initial Rho 1 that is at the starting s means at the starting point. And as well because this angle is fixed, it is not varying where the beta 1 is varying, this beta 1 is varying as this gear rotates and theta 1 for which we are calculating this one it is small, we have considered.

And then this is the working pressure angle so we can express in this form obviously the other terms, there is Taylor series means there are other terms also which is negligibly small and we have neglected those, so this simply expressed in this form. Now l is then expressed l s – Rho 1s cos a Alpha beta 1s, then if you look into this l then l s – when it is just moved, we can calculate this part and l is expressed in this form. Then this of course could be considered but again further approximation is done that this Rho 1 s cos Zi – beta 1s into sec alpha is equal to 1.

Why this is considered? This from the geometry it can be proved this is very close to 1, we can say that this equation 18 is now reduced to this form and we may consider that there is some error which error can be found out by subtracting 19 from 18 and then it is found that at Theta 1 is equal to 0 you can see that it is 0 error is 0, but it will increase slightly with the increase in theta 1. But remember this Theta 1 we are considering, what should be the maximum Theta 1? Theta 1 is equal to 0 then what will be the maximum Theta 1? Just one pitch rotation, 1 pitch rotation means that 2 Pie or actually not even 1 pitch, one base pitch you can say and for that the total rotation is equal to 2 Pie by Z 2 pie by Z.

And Z is usually taken maybe 20 at the lowest maybe 10, not less than that, at least I have not seen, I I do not know whether it is 10, but usually you will find it is in the range of 20, so now you can find out 360 divided by 20 is 18 degree. And you can examine that for 18 degree when Theta 1 is 18 degree, if you calculate these angles, you will find that this 0 is not 0, slightly more but still it is it will be within acceptable limit. However, if we when this close solution if you plot the graph for flow ripple, then we can only examine how much difference will be there.

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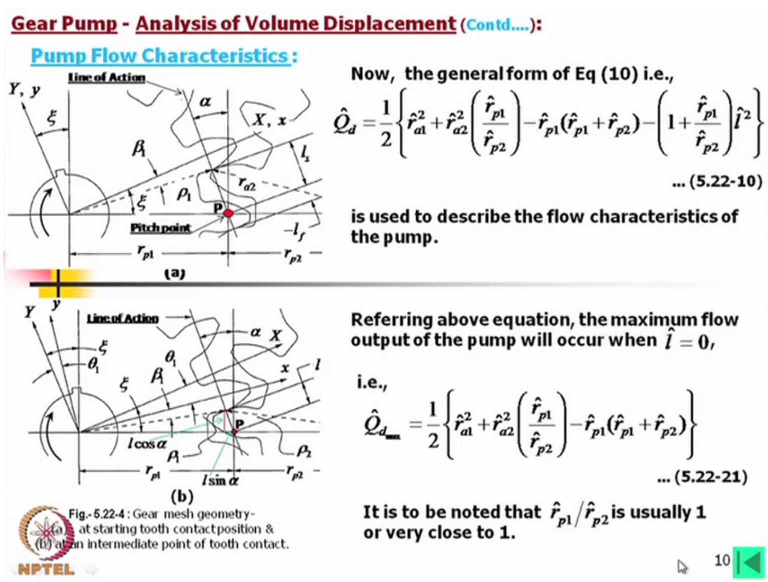

Now the again if we use this general form of equations, this l we have to express what we have derived and the maximum flow now we are trying to find out when the maximum flow will occur. If you examine this equation then when l is equal to 0 then there will be the maximum flow. Now this maximum flow simply if you put this 0, this is expressed by Q d Max and we find in equation 10 this simply this term is not there. Now again one interesting point I would like to mention that r p1 by r p2 usually you will find 1 that means they are of same size, but in some design they are not equal however, this number is not very big and what we will find this is close to 1, either it will be less than one or it will be at the 1 but it is very close to 1 that means this term is always close to 2.

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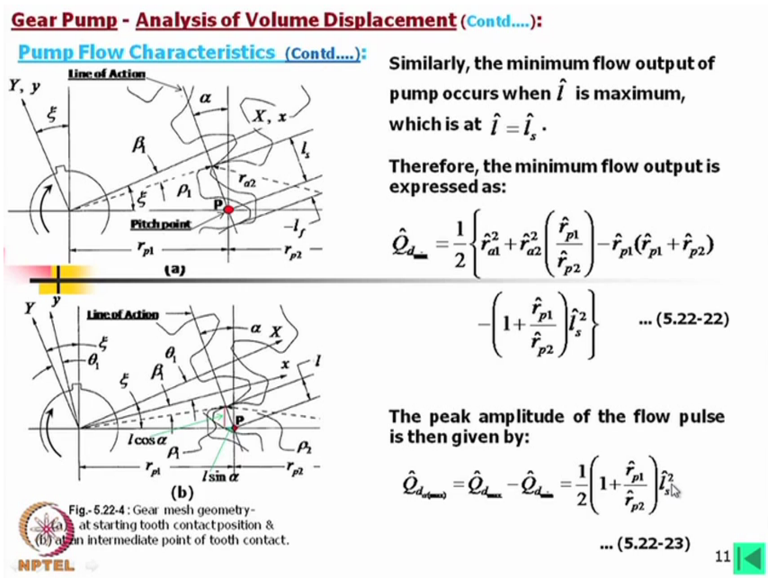

Similarly, the minimum flow output of pump occurs when l is maximum, that means when it is at the starting because we have in this geometry we have considered that from this starting point it is gradually approaching to this point and our total length is definitely at starting and then final length. But this length will gradually decrease and however, this length will be maximum when it is l s.

And substituting that what we find that this will be the minimum flow okay. With this negative sign with this as a maximum value, the total value will be minimum therefore, what we can find in ripple the maximum peak amplitude from lowest point to the highest point definitely Q d max by Q min which is again expressed by this 1. And this can be easily calculated because this l s will be fixed, geometrically we can find out l s, we do not have to go for any numerical solution in this case okay.

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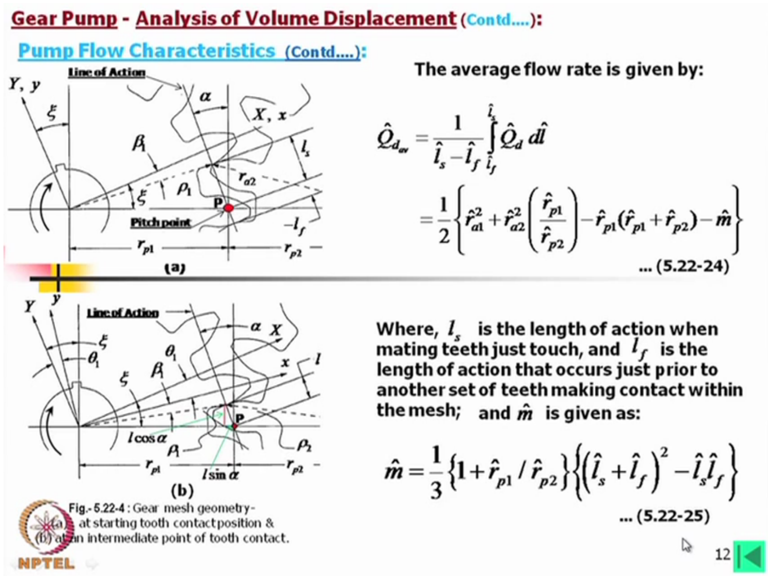

Now if we would like to find out the average flow then we have to equate all such flow from starting point to the endpoint that means we can make this integral. Now here just keep in mind that here – sign has been used because this – sign is in the opposite from the pitch point, actually this total length will be added that means we have to consider from the starting to the endpoint and which is which must be equal to what? This is equal to the base pitch, simply we can write here this is the base pitch length here okay.

Now then this integration becomes like this, where we have introduced a new term m which is expressed in this form. In that case again we have to calculate L S and l f, you can see this L S = l f that is actually total length, we are considering the actually total length. So this is while you are equating you have to little careful about this = – sign okay.

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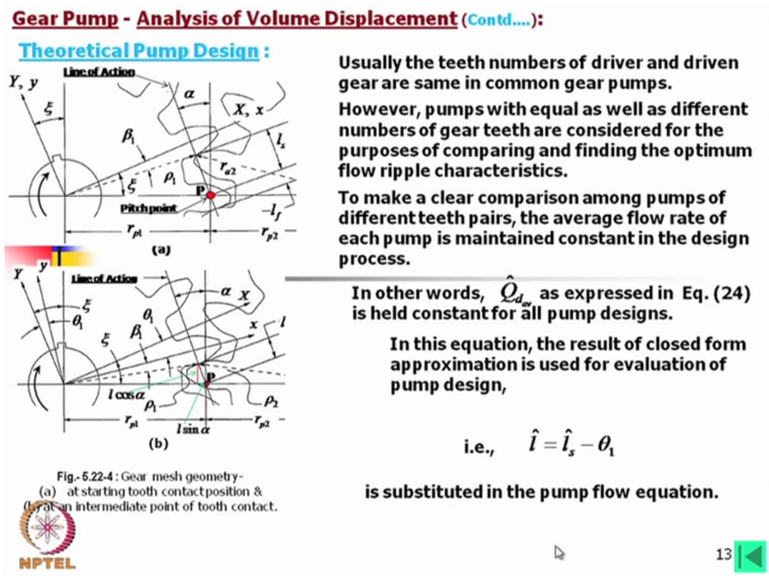

Now, usually the teeth numbers of drive driver and driven gear are same in common gear pumps right. However, pumps with equal as well as different numbers of gear teeth are considered for the purpose of comparing and finding the optimum flow ripple characteristics. Now this exercise is done by Maanring to find out whether is there any advantage to optimise the flow ripple by changing the gear ratio from 1. Now to make a clear comparison among pumps of different teeth pairs, the average flow rate of each pump is maintained constant in the design process. Now the advantage of dimensionless analysis is that we can we can find out the nondimensional flow and then we can multiply with actual dimensions to get the flow.

All in other words, depending on the flow rate what is done? Suppose we select the teeth number, we select then the speed of whatever it might be or Centre distance, working pressure angle, all will fix up so we can equate for non-dimensional form. Now then if you would like to have more flow from this pump, simply what you can do one is obviously you can flow rate if you want more flow rate, one is that you can go on increasing the speed, but otherwise for a fixed speed what we can do? We can simply change the module, we can simply change the size to have more volume output.

So this means that depending on the flow rate usually it is the speed, depending on the accuracy of this pump manufacturing accuracy of this pump there is a speed limitation. Say for example, in usually gear pump you may not find more than 2500 rpm, if we go for very high-speed then will be very high frequency noise will be there. Again of course that how it is the teeth are ground, all such things depends on what is the casing dimensions, how is the error in Centre distance, all such things will depend to have the flow. So then depending on that, depending on the purpose what you do, you finalise that what will be the speed.

Then if we want to increase the flow rate with that speed maximum flow rate what we do, we go for higher module, instead of 4 module we go for 5 module like this. Now in other words, Q d average as expressed in equation 24 is held constant for all the pump designs. So here I show you some results typical results of such analysis, in that case we have considered this is fixed. Now what we have considered, the result of close form approximation we have considered to find out the solution that is we have considered l is equal to l s – Theta 1, this expression from the approximation we have found.

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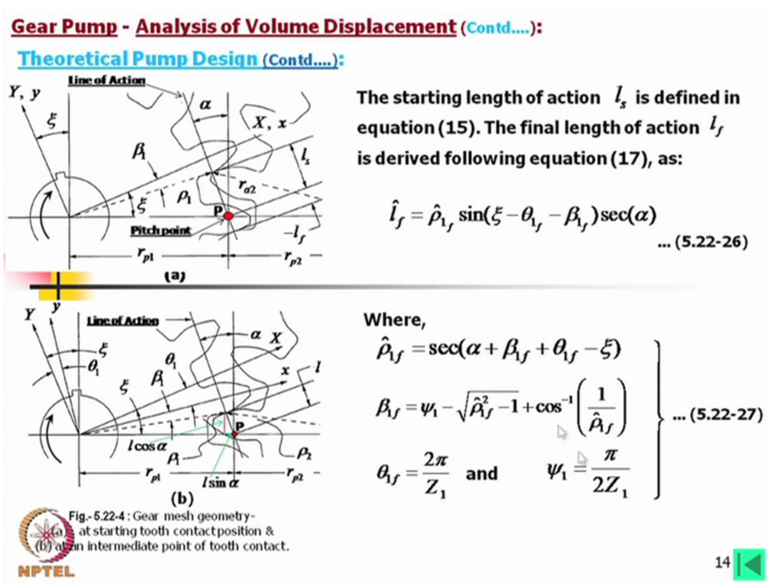

And then substituting this we get the starting length of action L S is defined in equation 15. The final length l f is derived as derived in equation 17 and then with this we get l f is equal to is expressed like this. Now this separately we have to calculate, now Theta 1 starting and Theta 1 final that you can say as I told that is simply 2 Pie divided by number of teeth of the driving gear which we are driving or gear 1 we would say.

So this calculation will not be difficult and we know this formula and from there we can calculate, we can calculate the final length of contact means this one where, this this expression for the final one is given by this one. And this is also as I told that we can find out this and then Theta 1 1f = 2 Pie by Z1 as I told, and Si 1 is Pie by 2 Z1 that is the half tooth angle okay.

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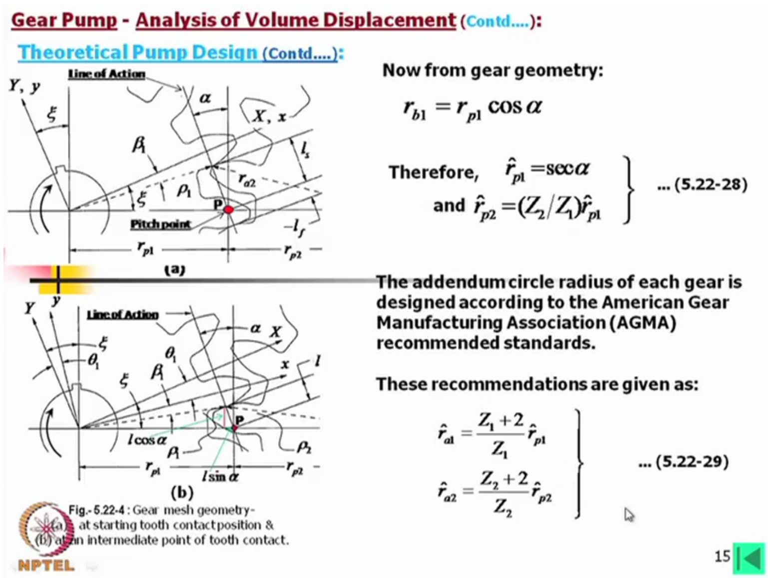

Again from the committee what we find, r b1 is equal to r p1 cos Alpha okay. Now here I would like to say that if we have considered this r p1 because r p1 is something fictitious, if you ask if you given one, if you ask to measure its pitch circle radius, Can you measure that? You cannot measure that, you do not know what is pitch circle radius? Pitch circle is the circle when 2 gears are meshing and if you divide the Centre distance by the gear ratio then only you will find the pitch point and then only you will find the r p1 okay. So if we derive in that way r p1 then we have to consider the working pressure angle but if we consider this as a standard then we should consider the standard pressure angle.

Anyway, this expression is that r p1 = sec alpha okay, with that r p2 is given by this one and the addendum circle radius of each gear is followed by the American Gear Manufacturing Association standard that is AGMA standard in which according to their recommendation for the gear pump, it is taken as the r a1 = Z 1 = 2 divided by Z 1 into r p1, which means the addendum factor is 2 by 2 is 1. Normally in standard gear we follow that, even for the gear pump the same standard is followed.

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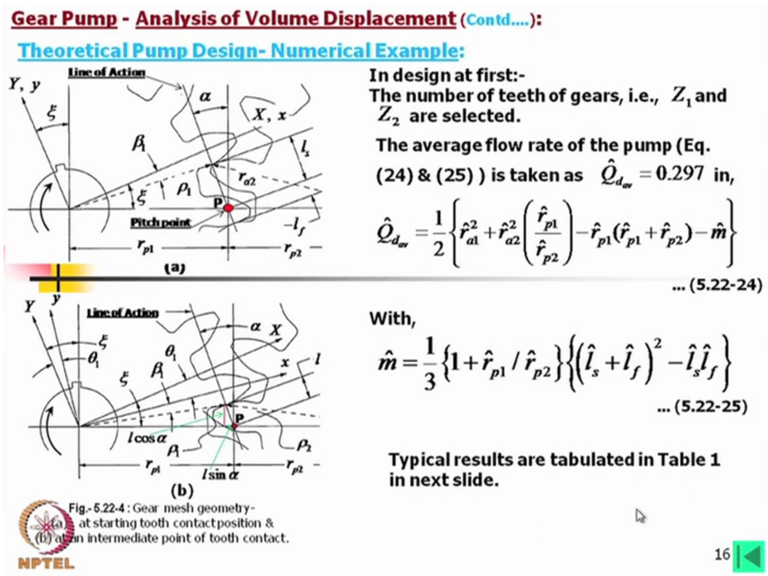

Now, in design at first the number of teeth of gears are selected that means in nondimensional analysis also we will be 1st tempted to take Z 1 = Z 2 and we can start with say teeth number is or for 20 degree pressure the 20 degree pressure angle, in that case we will probably take this Z1, Z2, very close to 17, which is the minimum teeth number for 20 degree pressure angle and probably we will compromise starting from 20 to decide maybe 16, not less than that, 16 teeth without gear correction is possible if we use the pinion cutter suitable pinion cutter, so in that case we may consider from 16 to 20 okay.

Let us consider, we have considered say 16, so 16 for Z 1 and Z 2 is also 16, now the average flow rate of pump in this case, in this calculation we are going to show some results Q d average is very close to 0.3 so it is taken 0.297, this is an arbitrary value, one can take 0.1 also. But we have to be careful that from some absurd value all the data will be absurd. So now we consider again this equation and m is expressed like this and in that way typical we get the some results which is shown in the next slide in 12.

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Gear Pump - Analysis of Volume Displacement (Contd....):

Theoretical Pump Design- Numerical Example (Contd....):

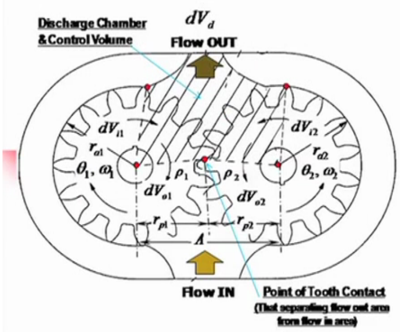

Fig.-5.22-3 : External Gear Pump.

Table 1 : Different Gear set with same average Flow rate $\hat{Q}_{d_m} = 0.297$

Z_1	Z_2	$\hat{r}_{p1}$	$\hat{r}_{p2}$	$A=\hat{r}_{p1}+\hat{r}_{p2}$
13	12	1.021	.942	1.96
13	13	1.022	1.022	2.04
13	14	1.023	1.102	2.13
13	15	1.025	1.182	2.21
13	**16**	1.025	1.262	2.29
14	8	1.031	.589	1.62
14	13	1.039	.965	2.00
14	14	1.041	1.041	2.08
14	**15**	1.042	1.116	2.16
14	16	1.042	1.191	1.76
15	12	1.061	.849	1.91
15	13	1.062	.921	1.98
15	**14**	1.063	.992	2.06
15	15	1.064	1.064	2.13
15	16	1.065	1.136	2.20
16	12	1.088	.817	1.90
16	**13**	1.089	.885	1.97
16	14	1.090	.954	2.04
16	15	1.091	1.023	2.11
16	16	1.091	1.091	2.18

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Now there may be question, for 12 number of teeth do you need a correction? In general we need correction but in case of gear pump the tooth load is not that high. For power transmission gear for reduction gearbox usually tooth loads are high if we use the steel and then for 12 teeth without any correction the root is weak, but in that case root is not that weak. Only problem is that we have to analyse the contact so that if the contact try to approach beyond involute then there will be some problem. Again if a cutter is designed properly that is the pinion cutter, we have considered in the state tooth or in the market production definitely home cutter then it is also possible that with 12 teeth uncorrected, the root fillet is not seriously undercut okay.

So therefore in this case because this is the dimensional analysis, what we can consider? Simply we can say that this is the module used for the flow, we will not consider the correction of gear teeth because with the correction of gear case then definitely this volume we have to while we analyse this contact point that will vary, we must introduce that correction part in that analysis, but here in this case we shall not consider or this calculation is made without considering that these are corrected here, this is uncorrected here. So 13 and 12 then in that series that Z 1 we have kept 13 up to this point and it is varied from 12 to 16 then gradually we have increased the teeth number and we have increased that driving teeth number up to 16, whereas driven from 12 to 16.

That means in this case only say this is one case where both the teeth are equal, similarly there is another case where both the teeth are equal and in case of this also 16 teeth there are 16-16 teeth okay. So 13-13 then 14-14, then 15-15 and then 16-16, these 2 are of equal teeth. If you only consider these 2 as we find that definitely Centre distance factor is changing.

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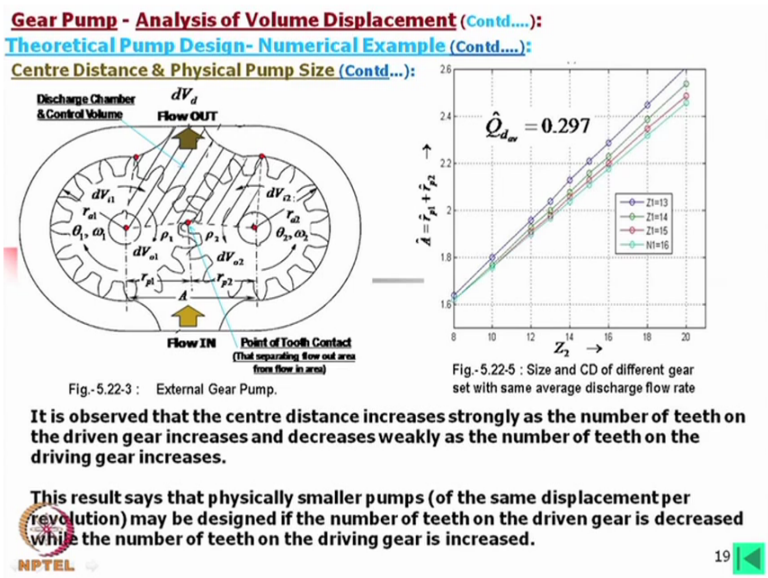

However, if we consider the whatever the flow ripple gears, we will I will show the graph but before that I would like to take I have expressed that r p1 and r p2 may not be the standard one, just mentioning the Centre distance we can mention what might be the volume because that will directly give us the nondimensional flow rate, we can compare with this nondimensional flow rate with the Centre distance.

Now a say this is the Centre distance and this Centre distance nondimensional Centre distance we have varied from approximately from 1.5 to 2.6 and then what we find? With different combination of teeth say if we consider the blueline then that is with 13 teeth in the, Z1 is the 13 and Z2 is varying from 8 to 20, we have a collected from 8 to 20, we have extended this graph but in the start was 13 something 13 was there a minimum teeth number. But what we find that this is the driving gear when it is 13 and this curve when driving gear is 14 and this is 15 and this is 16.

So from this chart we can have a conclusion that Centre distance increases strongly as the number of teeth on the driven gear increases and decreases weakly as the number of teeth on the driving gear increases okay. This means that you can judge this from this graph, what we find that if we increase the number of teeth of the driven gear then this increases rapidly. But on the other hand, if we when we increasing 13 to 16 then for a given number of driven gear this variation is not much okay whereas, if you change the driving gear 1 then there will be a lot of change. So from there we can have some assessment what pair we should select for the pump.

But again I would say that the normal practice is still equal, Z1 is equal to Z2 and there is really not much reason that this number 2 varied, if there is reduction of noise, flow ripple, et cetera that is nominal not much. This results say that physically smaller pumps of the same displacement or regulation may be designed if the number of teeth on the driven gear is decreased while the number of teeth on the driving gear is increased. So if we make this is small and this is big, the size of the gear pump will be smaller but I would say this is not much. Still for the sake of optimisation the size will be small. However, we are not considering what really will be to the noise and dynamics.

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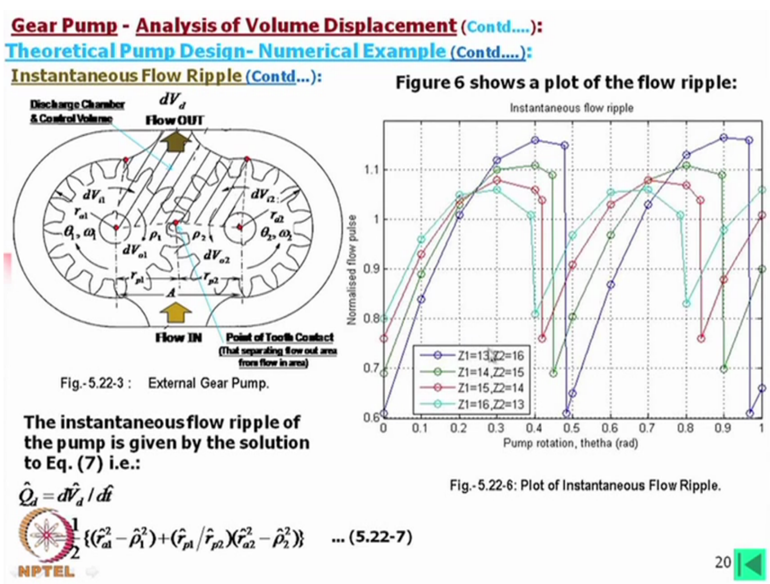

The instantaneous flow ripple of the pump is given by this solution of equation 7 that is this is this equation and then we find that this is the flow ripple for different cognition of teeth okay and this angle you can directly you can calculate by 2 Pie divided by the number of teeth of the drive gears is Z 1. So for example if we divide 2 Pie divided by no this is for half of the angle, so Pie by 13 how much it will be? 3 divided by 13, 3.14 divided by 13 will be approximately 0.25 or so.

No so this is 2 pie by 13, 1 full cycle yes 2 Pie by 13 and then 2 pie by 14 like this okay, so I think this one for 16 teeth in 16 teeth is 0.4, whereas for the 13 teeth it is 0.5. Anyway difficult question is not that difficult but you can see this ripple, the ripple is increasing with the definitely if the teeth number driving gear teeth number is small.

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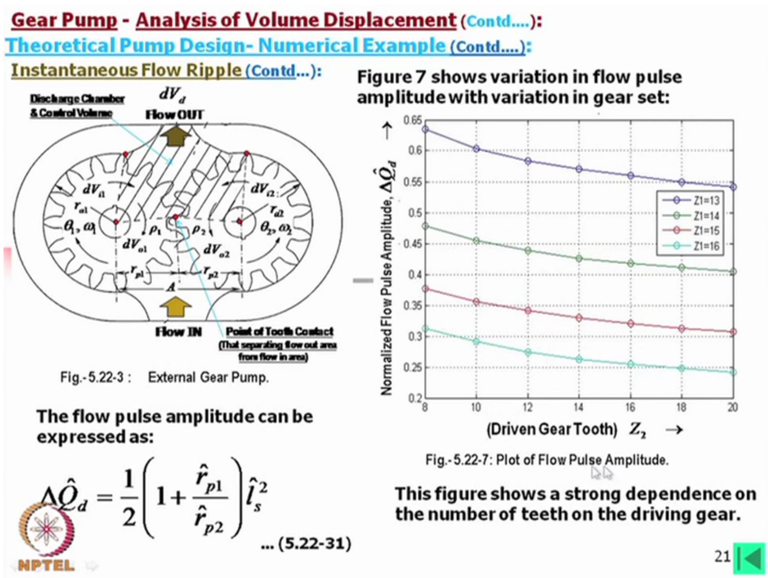

Now we should also consider the flow pulse amplitude which is expressed by this equation and then this we can if we plot this value with the driving increase in the number of teeth here and in this access this amplitude then we find for different teeth, Z1 is 16 here and Z1 is 13 and definitely amplitude will be higher for in case of small number of teeth. And what the observation that strong dependence on the number of teeth on the driving gear, otherwise this flat is not this curve is almost flat you can say with the increasing number of teeth of the driven gear.

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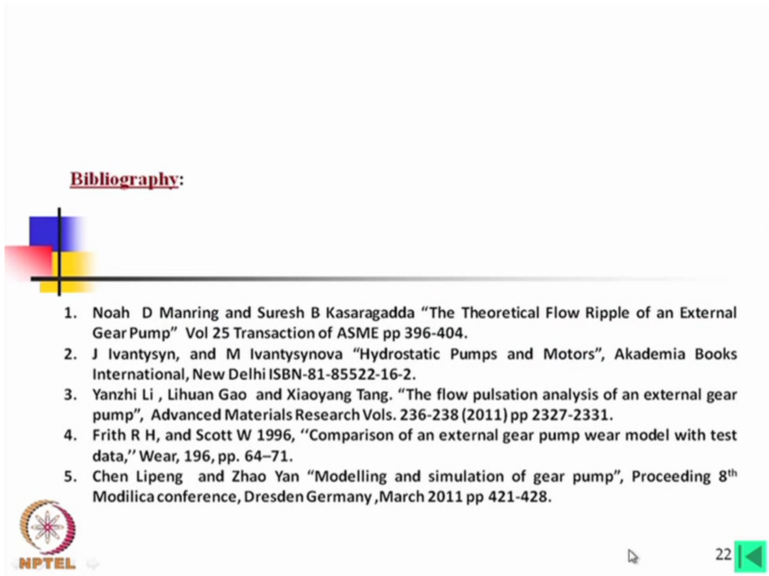

So we end here.