Fundamentals of Industrial Oil Hydraulics and Pneumatics
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Lecture no 22A
Module no 06
Design Analysis of Gear Pumps - I

Welcome to today's lecture, this is on Design analysis of Gear pumps and this part 1.

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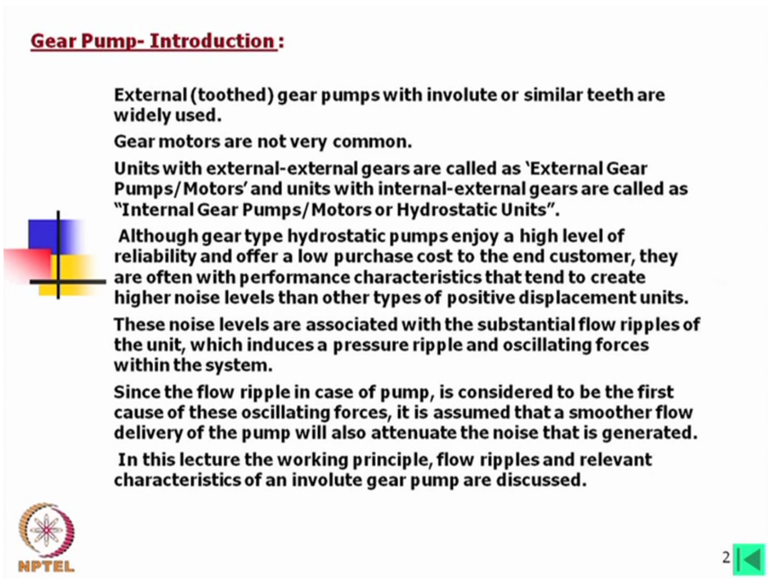

Now external toothed gear pumps with involute or similar teeth are widely used in fluid power applications. Gear motors are not very common, although there are some models of gear motors but this is not very common with involute teeth. Gear motors with different teeth such as orbit motor is very popular. However the involute teeth gear motor is not that popular because it is not much beneficial to use such units for motor. Now units with external-external gears are called as external gear pumps and motors and units with internal-internal gears are called as internal gear pump or motors. Or the together either it might be pumps and motors because they basically features will be same, they will look alike, only there is minor change in valves, they are usually called hydrostatic units.

So you may call external gear hydraulic units hydrostatic units or internal gear hydrostatic units. Although gear types hydrostatic pump enjoy a high level of reliability and offer a low purchase cost to the end customer, they are often with performance characteristics that tend to create higher noise levels than other types of positive displacement units. These noise levels are associated with the substantial flow ripple of the unit which induces a pressure ripple and oscillating force within the system. Since the flow ripple in case of pump is considered to be the first cause of these Oscillation forces, it is assumed that a smoother flow delivery pump will also attorney at the noise that is generated. In this lecture the working principle, flow ripple and relevant characteristics of an involute gear pump are discussed.

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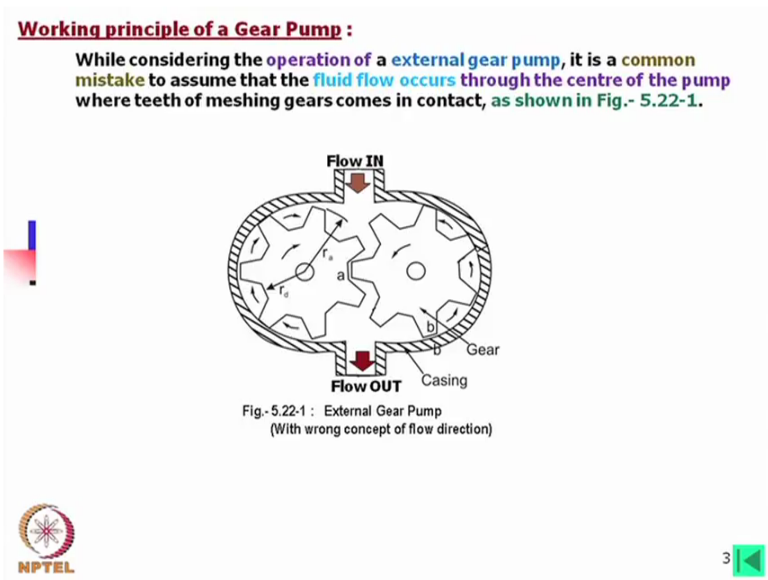

Now 1st of all, what is the working principle of gear pump, while considering the operation of external gear pump it is common mistake, it is common mistake to assume that the fluid flow occurs through the centre of the pumps where teeth of machine gears comes in contact. Normally, if a gear pump this (())(3:52) which seen then many people may think this as the gears are rotating in this direction so this must be flow in and this must be flow out, but actually it is not, it is otherwise.

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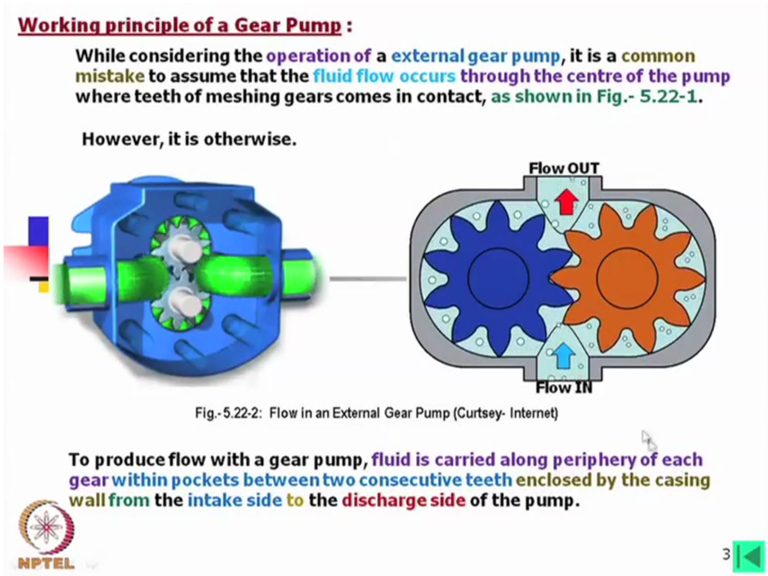

If you look into this picture that is in that case you can see that it is rotating in the clockwise direction, let us assume the blue one is the driving one, among these 2 one of them is driving one, in this case let us consider this blue one is the driving one then what we observe the oil is coming in from this side and then it is being trapped and it is going out, okay. And if you look into this view, this oil is coming in like this and oil is going out like this, so this should be this is the direction or the flow so, in fact again what we think that okay direction of flow we have corrected.

Now how it is the oil is being pumped out and we should remember that input side pump is usually atmospheric pressure whether in the output side pressure is high, this is the working pressure. And as earlier we have learned, it might be in the range of from 10-15 mega pascals in case of gear pump, not may not be as high as in case of piston pump that is cylindrical piston pump when it can attain up to 30-40 mega pascal but 10 to 15 mega Pascal are very common. Then again how it is being pumped because we need a lot of pressure to generate, now again looking into this figure 1 may be confused with this fluid pump, bucket fluid pump.

You might have seen in the faddy field, usually you will find a wheel with bucket fitted on the periphery that is used to take fetch water from the pond or water source to the ground. Usually ground level is slightly higher than the water level and in that case what is done the it is as if scooping the water and then it is put into the ground. Now that is something like bucket elevator, there the water is taken into the cavity of the bucket captured into the bucket and that is taken out, if we think of pressure, pressure is almost same. Now in this case it is not that, it is not that the fluid is being taken by this pocket and it is going simply out. There is the suction and compression at some by these gears, now to run that we must find out that where is the suction and compression chamber in that.

So to produce a flow we should say with a gear pump, fluid is carried along the periphery of each gear within pockets between 2 consecutive teeth enclosed by the casing wall from the intake side to the discharge side of the pump. Now in that case if we ask what is the how many chambers are there, we can say that within 1 regulation there are how many pockets, this is exactly the double of the number of gear teeth. So one may consider that is the number of chambers in a gear pump, so number of chambers are high as we have learned earlier that with the increase in number of chambers, actually the ripple reduces.

Then the 1st statement I have told that there will be high ripple that is due to the fact that here this compression and geometry of the volume discharge is such that we have very high peak of the ripple. So it is like that, within one revolution if there are say 10 teeth, so 20 chambers, so there will be but there will be actually 10 ripples because 2 pockets are given the flow together and you will find that ripple is not same as in case of the piston cylinder. So there is difference, we will show that curve later of this lecture.

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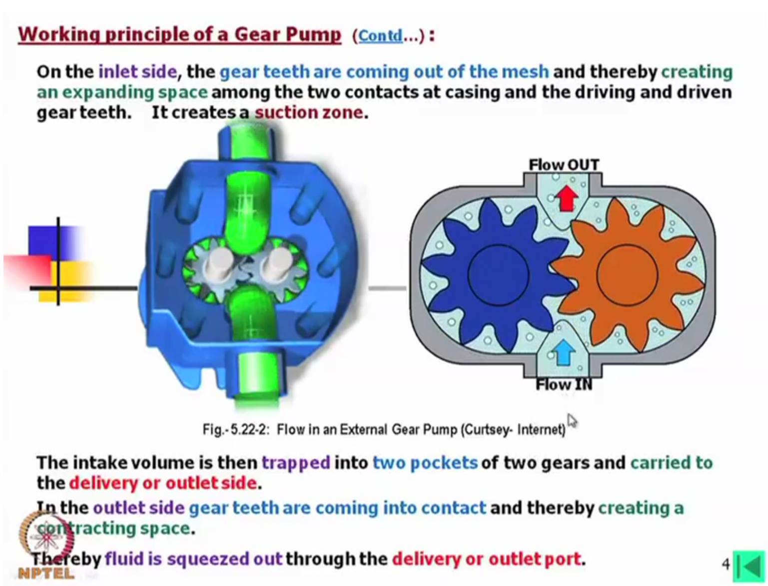

Now, on the inlet side the gear teeth are coming out of the mesh and thereby creating an expanding space among the 2 contacts at casing and the driving and driven gears. Say here, if we consider this contact point that is separating the high-pressure zone to low pressure zone and from this point to this point we will find when the gears are moving, this space is increasing so there is a suction, whereas in the other side this space is that area is decreasing that we will derive that how much area is decreasing or increasing that has to derive to find out the flow rate of such gear pump.

Now it creates a suction zone, the inlet side it creates a suction zone then and intake volume is then trapped into 2 pockets, here is one pocket, here is another pocket of 2 gears and carried to the delivery or outlet side. And in the outlet side, gear teeth are coming into contact and thereby creating a contracting space. Thereby fluid is squeezed out through the delivery or outlet port so this is the pumping action in case of the gear pump.

And in case of motor if we think of the motor then this this process is just reverse, what will be in that case the oil will be pumped in from this side okay, in case of motor and volume will expand so this this will be then high-pressure side and this will rotate in the same manner, where as the low-pressure oil will go out but it will transmit that this pressure will be converted into the force and torque. And that can be taken out either from both the gear or from a single gear, okay.

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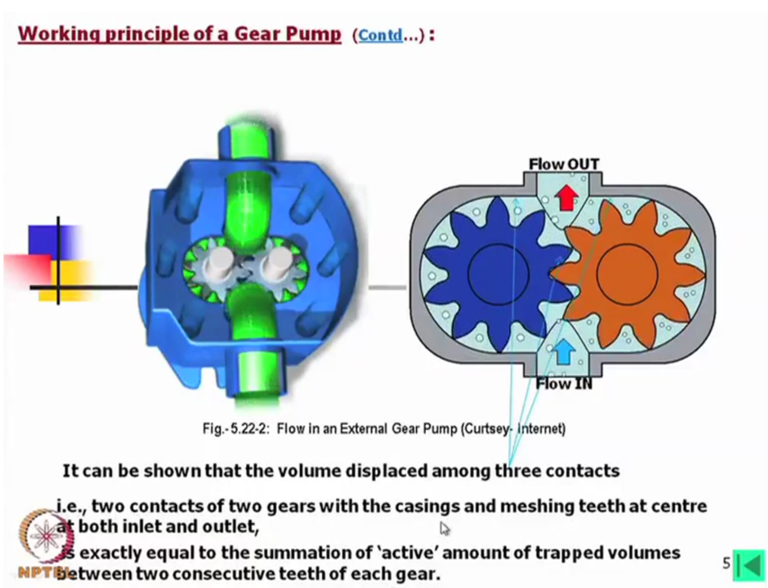

It can be shown that the volume displacement among 3 contacts, this is the 3 contacts, one contact is here and another contact is here and another contact is here, and then 2 contacts of 2 gears with casing and mesh meshing teeth at centers at both inlet and outlet. So one chamber means just consider the instantaneous the last teeth in contact with that this side and this side and the contact point here. So this is the compression chamber and similarly, the suction chamber usually this contact point here and the 1st contact point coming to the wall and the side the 1st contact coming to the wall okay.

Exactly equal to the summation of active amount of trapped volumes between 2 consecutive teeth of gear of each gear. This means that what is the total expansion or the compression, their volume, this volume will be exactly equal to these 2 pocket volumes but here is one confusing statement is there. If we just measure this area one pocket and here one pocket this will be same and if we multiply them then that would give the total volume but it is not that. It will be these 2 volumes – this entrapped volume which is actually inactive, if you just if you stop this gear, you will find there is one small volume the same oil is coming back to the suction side that we will have to subtract that we should remember.

So while we are calculating this we have to consider this part however, if we calculate the total displacement of this area during the suction and compression then automatically that will be deducted from the calculated area, we need not bother about that how much pocket that means if we give more under cuts to these gears that would not matter that root fillet, if we increase the route fillet that will not really matter this is one thing. Second thing which I am going to show now that one important fact is that actually each and every when it is passing through, there will be some oil leakage through the even through the contacts that we should take care of, in that way efficiency of this gear pump is lower than what is available with the cylindrical pump and Pistons.

Another part which I would like to describe here I would like to discuss that is, if we consider in case of pump, here the pressure is low maybe the atmospheric pressure, whereas here the pressure pressure is high-pressure. Now if we consider the pressure curve here, you can see this pressure is not increasing not increasing not increasing and then when it is being exposed, suddenly the pressure is increasing if it is in case of ideal ideal case. But what actually happening that there is a small amount of leakage so what we will find whatever pressure here may be some pressure is increasing and then suddenly it is being exposed to high pressure.

So that causes some sort of the fluid bore noise, to reduce that as well as to balance this I mean imbalance in the torque, what is done that this somewhere a pocket is made here and that is connected at the midpoint and then again another pocket is made here and then it is connected here then ultimately pressure distribution is something like this. This is high-pressure and gradually it is decreasing, that sort of pressure distribution will be better for the performance and that is done by such grooving. However, we shall not discuss anything about such grooving because for that we need some more complicated analysis, but anyway I will show you the flow between these 2 teeth.

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Now look at this figure, as you can see just how the flow is taking place. In that case so this is rotating in the clockwise and this is so this must be which side is the delivery side, this side or this side? So this side is the delivery side. As we look into this delivery side, not much thus flow phenomena is being looked in but what we find as if there is a flow is going taking place in the side say look at this and at that moment what you find that fluid is sudden fluid is going out that is a leakage, that leakage very important and in designing such gear pump, controlling such a leakage for correct estimating of such leakage such leakage for the analysis of gear pump is very important, okay.

Now of course within the scope of this lecture we cannot analyse such thing, only what we will analyze again this contact say this is look at this contact, this is the separating chamber, now this point is separating the chamber, now this point is separating the chamber, now this point is separating the chamber, so something is happening there, analyses of this will be very important.

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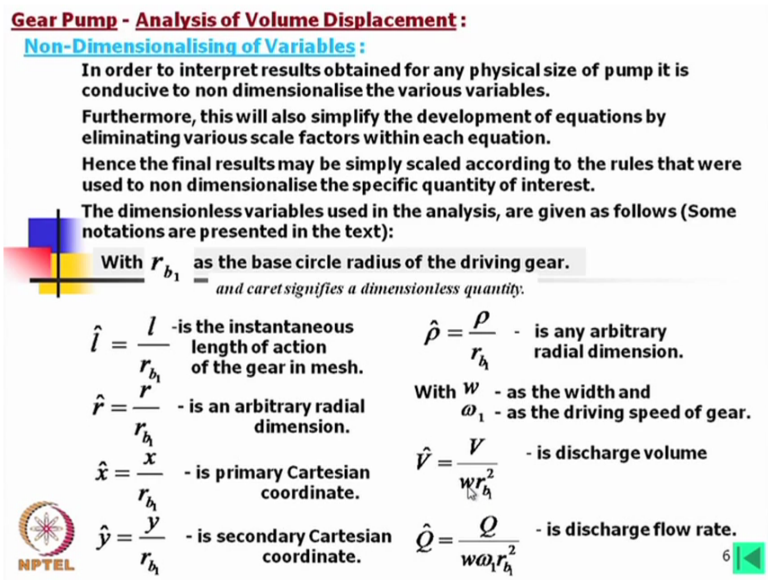

Now now we shall try to find out what is the volume displacement. What we would do, if we can calculate the variation of this area with shaft rotation order with respect to time then we can multiply the width of this gear, in case of piston pump what we do, we multiply with a constant area with the variable sprock length, that length is varying but the area remain constant. In case of this pump this gear pump what we find that area is varying and the width remain constant. So if we can analyze the variation of area with the shaft rotation, we can easily find out the volume displacement and then considering the time rate we can find out the flow rate theoretical flow rate.

Now before going into the analysis, we have non-dimensionalised the variables that is important that we should understand how we have non-dimensionalised okay. Now what is the benefit of that? You know that in order to interpret results obtained for any physical size of pump, it is conducive to non-dimensionalise the various variables. This means that if we know the shape geometric shape of the teeth number of teeth see parameter should be remain same remain constant. Then what we can do, irrespective of size we can go for analysis, then whatever result is available in non-dimensional form that we can simply multiply with the real size some size with respect to we have non-dimensionalise then we can get the actual size of this pump.

Say for example, we have taken a pump of the gear 10 teeth for the driving and 10 teeth for the driven, then maybe module is 5 say, then whatever flow we are getting if we reduce this module to 1 definitely flow rate will be less, it may be 5 times less or maybe something else that can be found out from the non-dimensional parameters. But once we designed this pump with a particular module particular teeth module, we can find out what will be the real output, input, et cetera, now how it is nondimensional that we shall examine now.

Before that further more this feel also simplify the development of equations by eliminating various scale factors within each equation. Hence the final results may be simply scaled according to the rules that we are used to non-dimensionalies the specific quantity of interest. The dimensionless variables used in the analysis are given as follows, see this. Now here for linear parameter we have considered the reference parameter as the base circle radius of driving gear,, perhaps you know that ones that gear say involute gears, we say that this is the module, this is a teeth number and this is the pressuring of course pressure angle should be there.

Then one dimension of that gear will be fixed that is normally we make a mistake that pitch circle is the fixed dimension but it is not. The fixed dimension will be the base circle that all which the involute is generated okay. Now if you introduce the correction, correction means it might the profile it shifted, in that case we may have different pitch circle radius. Even with uncorrected gears if they mesh they mesh with a enlarge expanded centre distance, we will have a new pitch circle radius but the base circle radius will remain constant that is why this parameter is taken as the reference parameter for non-dimensionalising.

Now, another information is there that we have used a caret to denote the dimensionless quantity. Say in that way now l, it is actually instantaneous length of action of the gear in mesh, I will explain this again with figure of the gears and non-dimensional means with caret or hat you can say, so l divided by r b1 is the non-dimensional parameter. Similarly r is a arbitrary radial dimension, this is also divided by the same base circle, in that way any in the Cartesian coordinate x-y coordinates are also non-dimensionalise and the Rho that radial dimensions it is not arbitrary, you can say this, this is from the contact point actually radius from the contact point. This is purely arbitrary and this original radius and this is the specific radius with respect to the point of contact okay.

Now this is the linear dimension, we have non-dimensional in this way but there are some other dimensions also for example, the volume, et cetera. In that case what we consider the width, we consider the real width of that machine or the width in the parameter we consider divide the volume by width and then also speed. Now this volume, volume is discharge volume this is divided by the width w that means what the non-dimensional that is per width unit width and then area is divided by simply by r b1 square so this volume is non-dimensionalise. Now we can non-dimensionalise this volume by this dividing by a volume, we (())(26:43) r b1 square. But again flow rate we have to introduce what? We have introduced that Omega 1 what is the angular speed of this.

Or in other words, if we carry out analysis with non-dimensional parameters then if we find some volume non-dimensional volume, simply we will multiply the actual real parameter of the gear pump which you have considered, we shall consider that width, we shall consider that r b1 that the circle radius. And in that in case of flow rate we will consider the actual speed, but here one thing I must mention that this gear 2 gears mostly you will find they are with same number of teeth that means if this side is 20, other side is also 20. But it is also possible that one is 20, other is 18 or it may be 16, this is also possible.

In that case if we non-dimensionalise all these diameter with one base circle, in that case base circle will be different then we have to take care while we are considering the some real parameters. But it is possible because knowing the number of teeth we know the base circle of other valve will be the simply the ratio of this gear teeth so that is not we will not discuss it out, this is in case of external tooth gears. In case of internal tooth gears as you know that numbers will be different because if the ringer is of say 20 teeth then internal-external tooth internal gears of that one has to be at least maybe 4, 5, 6 less than that. So in that case you can make the parameters non-dimensional considering any 1 base circle radius but why we are converting the real data you have to consider the other base circle also with proper proportion that is not difficult but one to take care of that okay.

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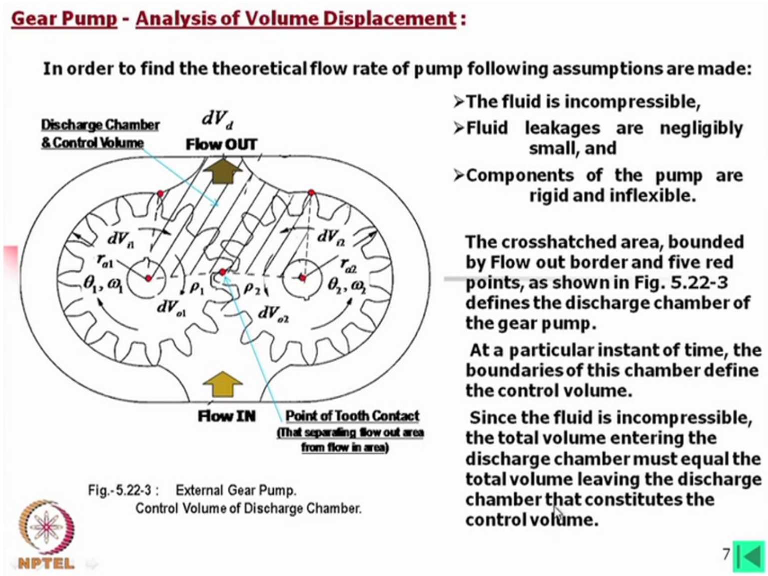

Now, so we will now go for this analysis, in that case as you see that these gears are rotating like this and this is flow in and this is flow out. Now here is what we have written, this is say look at these dimensions r a1, this is the teeth circle radius which is call also addendum circle radius so r a1 in that case we have considered r a2. This is gear one and as well we have considered this is the driving gear and this is gear 2, this is the driven gear. We have shown the same number of teeth here so these dimensions will be same but only while we are analysing this, we will consider that these are different. And then the Theta 1 is the angle of rotation of this one and Omega 1 is the speed of these gears and these 2 volumes I shall discuss during the analysis.

However, this Rho 1 is the distance between the contact point to the Centre and you should remember this Rho 1 and Rho 2 is varying as the gear rotates because this contact point is along the line of action and it is therefore when it is contact point moving from one side to other side and definitely this Rho 1 and Rho 2 are varying. Now we have made some other assumptions, these are the fluid is incompressible because to consider the change in area we have to if we consider the compressible then we have to consider that not only due to this area change but also due to the compression the volume may change, I mean the total quantity of the fluid may change, so if we consider incompressible then we need not bother about that.

Fluid leakage are negligibly small and neglected, also components of the pump are rigid and inflexible that we have to consider, although the in some analysis this pipe is considered that flexible that will complicate this analysis. Now the important thing is that to analyse this as I told that these 3 points will constitute the chamber, now when this point is moving in this direction due to this clockwise rotation of that then this point is moving, but what you find that after a small amount of rotation then this chamber will be connected because this path will be open, then immediately at that instant we have to consider this contact point and this point.

So this we have to be careful or we should take care while we are considering such area that there should not be change in sudden opening of this area width when this one pair is in contact. Maybe when the second pair is coming in contact then again the same phenomena is there, if within this contact zone if one teeth come I mean they open passage is opened then analysis will be complicated that also we have considered. That means when one contact one pair is contact is there then same teeth is here, only this point is moving on the valve okay. Now if we consider the hatched area so that area if we can calculate that area including gears that means the metal part and then fluid part.

If we consider the variation of this area with respect to this contact point that definitely will give us the variation of area during the rotation and from that we can find out the expansion and compression. At a particular instant of time, the boundaries of this chamber defines the control volume, we will consider this as the control volume. Since the fluid is incompressible, the total volume entering the discharge chamber must equal the total volume leaving the discharge chamber that constitutes the control volume.

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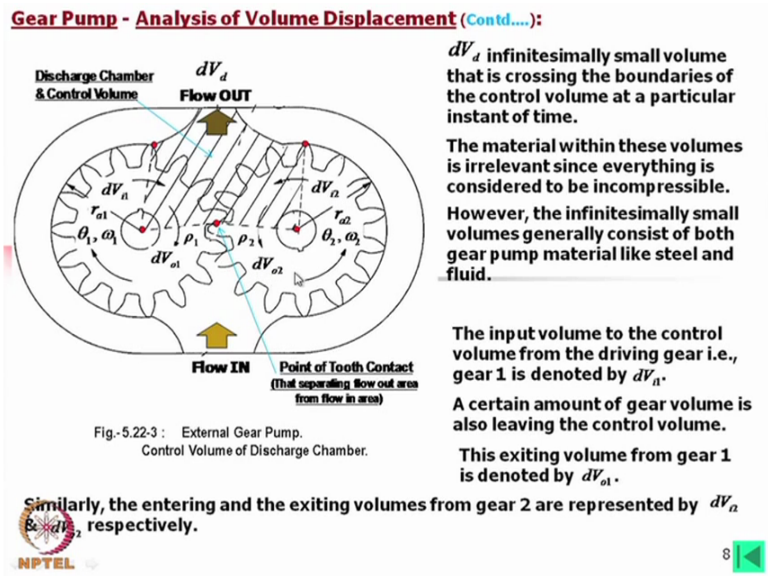

Then we consider dV d is the flow out and for the analysis we consider a very small amount of that is crossing the boundaries of the control volume at a particular instant of time. The material within these volumes is irrelevant since everything is considered to be incompressible okay. However, the infinite small volumes generally consists of both gear pump material like steel and fluid. In this case we have considered steel, might be something else also so but the thing is that one is that material solid material and another is the fluid material.

The input volume to the control volume from the driving gear that is gear one is denoted by dV i1, so when this is a very small amount d Theta rotating, what we find the dV i1 is going inside this control volume that means within this boundary what we have considered, this boundary itself is varying but we have to consider in terms of flow in and flow out, the volume in and volume out. A certain amount of gear volume is also leaving the control volume. This existing volume exiting volume from gear 1 is denoted by dV 01 whatever volume is going out okay. Similarly, the entering and exiting volumes from gear 2 are represented by dV i2 and dV o2 okay.

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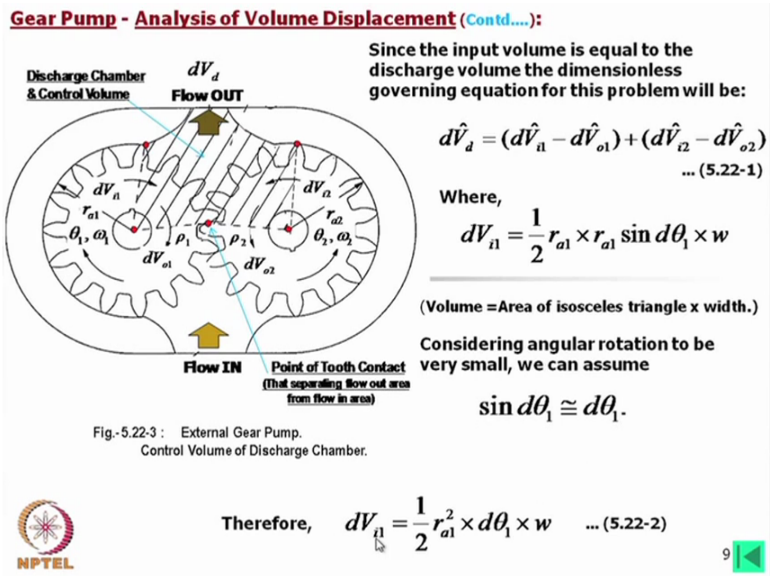

Since the input volume is equal to the discharge volume, the dimensionless governing equation for this problem will be; so dV d here we have considered non-dimensional parameters so input – output and here also input – output, so total this volume whatever this volume will be there, that must be going out. If there is no difference there will be no volume output, in fact in case of if you consider simply the bucket type of pump bucket fluid type of pump then this will be 0 both will be 0, there is no such expansion and compression in the control volume but in this case it will be we will find out that, where this input we can take if you look into this figure, half r ai into r ai sin d theta 1, so sine part of this one d theta 1 is more volume into w that will be the volume displacement.

If we consider this infinitely small amount of shaft rotation, so sine theta part will be something like this here, that part into this length which is r ai into half of that, we have considered the area of triangle and then multiply with the width of this gear w, this is simply volume area of Isosceles triangle and width. Considering angular rotation to be very small, now we can assume sin d theta 1 is equal to d theta 1 and in that case we can express the dV I what is equal to this much.

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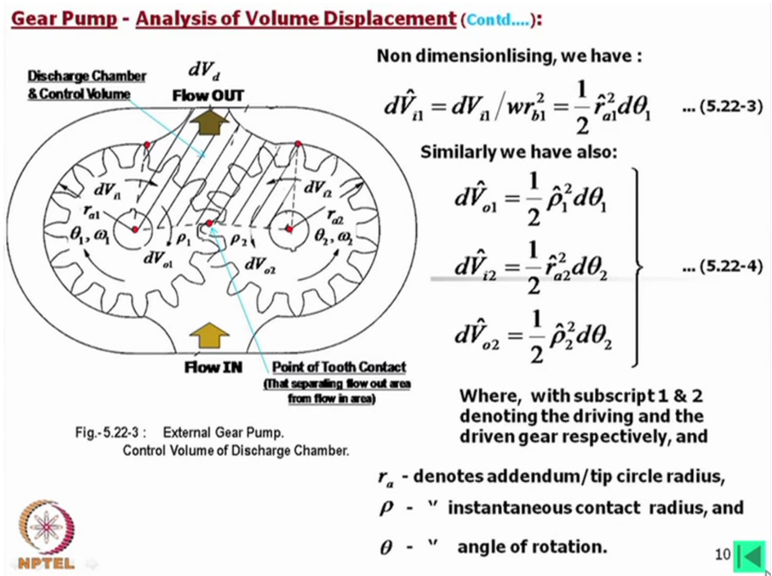

Now if we non-dimensionalise this volume, then we will get dV i1 is equal to half into r ai hat square into d Theta 1. Similarly we have also the relations found out in the same way, you can look into the equation these are same. In that case when we are calculating the volume out, we have considered this radius because this is the radius here, we have to consider a small amount of rotation and then this triangle formed by 2 arm is equal to Rho 1 in this case it were to go, so this will be the equation.

Now we can rewrite this equation, here subscript 1 and 2 denotes the gear 1 and 2, we need not mention already we have mentioned it and r ai is equal to the random tip circle, Rho is the instantaneous contact radius, Theta is the angle of rotation that we have already mentioned.

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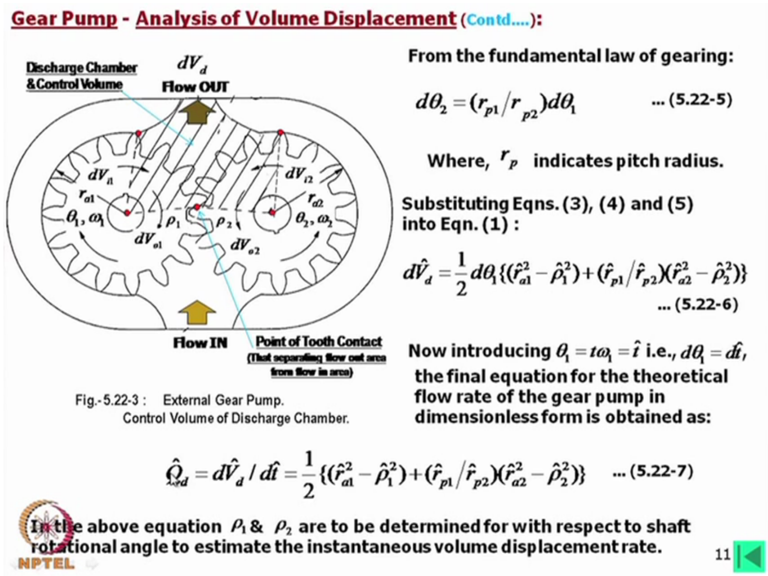

And then from the fundamental law of gearing now we have written these 4 equations, so there will be 4, substituting this into the main equation, we shall now from the fundamental law of gears we know again this relation is that d Theta 2 is equal to r pi by r p2 d theta 1 and where r p indicates the pitch radius, such equation 3, 4, 5 into equation 1 we get this relation, okay. Remember the 1st equation and then all the equations we have derived so far, we will arrive into these equations. Now, in this derivation they have considered the pitch circle but I prefer normally, one can prefer just directly the base circle, you can consider the base circle instead of the pitch circle.

But considering the pitch circle is that when on this this point will come on the axis on the line joining the 2 centers then this Rho 1, Rho 2 will be equal to the r pi and r p2, remember that fact so maybe this will be better to use at this moment but anyway this will give the same relationship. Now introducing Theta 1 is equal to tw 1, time into Omega 1 and that there at Theta 1 is equal to non-dimensional so we can consider that this itself is dimensional less time, the angle itself is the dimensional less time. This is a tricky thing but one can consider in this way because d Theta 1 will give you dt 1 rate of change of time if any there, so this this expression you have to initialize this.

Now we have that means non-dimensional time is equal to simply the angle of rotation. The final equation for the theoretical flow rate of the gear pump in dimensionless pump is obtained as;

now to find out the flow rate what we do, we do the we differentiate this volume with respect to the time and then we will find this final relation will be in this form, okay. Then now in this equation what we will find, at any instant say this is a fixed dimension, fixed parameter, this is also fixed parameter, once we know the Centre distance, these 2 are also fixed parameters. Only with time this Rho 1 and Rho 2 are varying so we have to calculate now these 2 parameters, once we can calculate we can easily calculate this one volume displacement.

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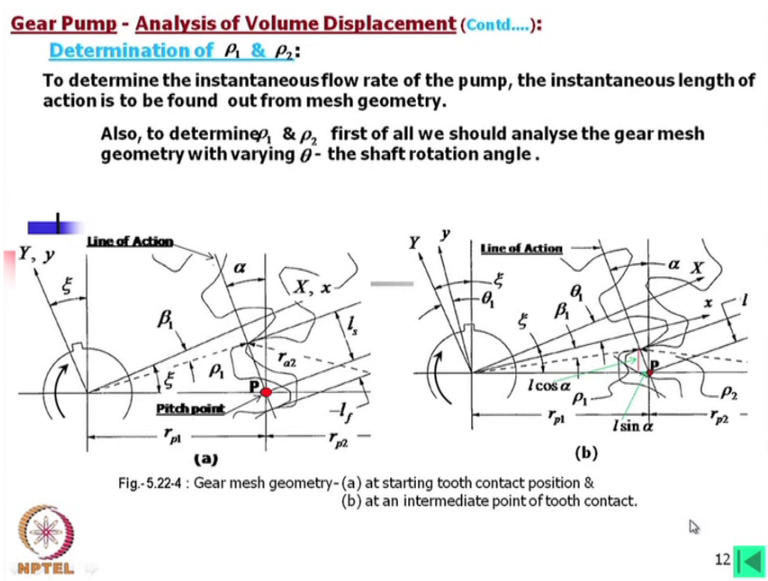

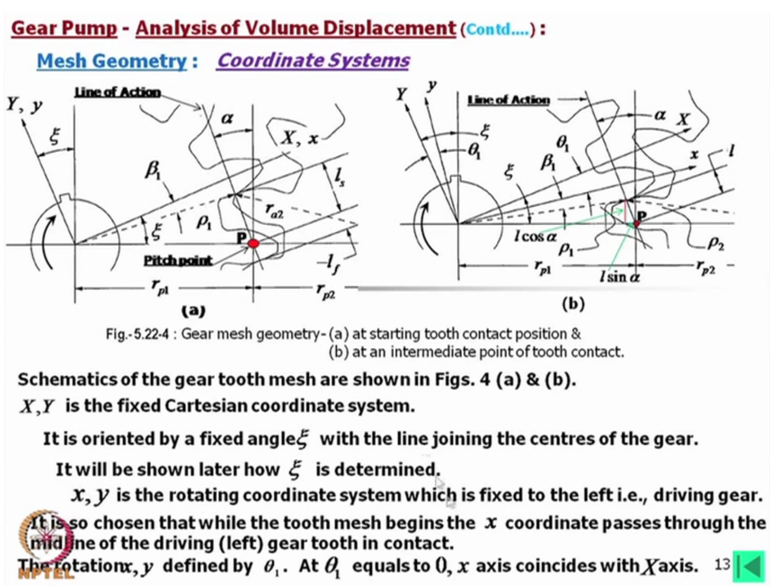

To determine the instantaneous flow rate of pump, the instantaneous length of action is to be found out from mesh geometry that means if you go back to this figure that we will find, we have to find out the instantaneous length of contact means from the pitch to the actual point of contact that lent we must know. Also, we have to find out rho 1 and rho 2 and then it will become very easy to estimate this flow rate. Now we consider this geometry of these gears, in in this gear say this is the driving gear, we have considered also the contact points. Now what we find that 1st of all we must consider the coordinate system to analyze the gears, in that case what we have considered that the midpoint of the teeth will be on this axis went Theta is equal to 0.

No not midpoint sorry I have made a mistake, this is a pitch point but we have considered that when one teeth is coming in contact 1st contact that is that Theta is equal to 0. Now we have shown that here the 1st point of contact and then we have also shown some intermediate point of contact okay. Now what else are shown here? That from the midpoint of teeth, so this is the angle Zeta and beta 1 is angle from that line to point of contact instantaneous point of contact, and then Theta 1 is the angle of rotation okay, these we have considered. And we have considered 2 axis system, which I am explaining in coordinate system and mesh geometry, X and Y, X and Y the fixed Cartesian coordinate system that is the fixed preference and this angle and it is oriented by Zeta angle with the line joining the Centre of gears okay.

That means what we have considered that when this 1st contact point is there, we have taken the axis through the midpoint of this one, which is oriented at this angle. And this coordinate system capital X and Y is a fixed coordinate and then we have considered the another X-Y coordinates. However, this will be determined later we will show that how it can be determined and x, y is in the rotating coordinate system which is fixed to the driving gears, this x and y that portion is fixed to the driving gear.

Now Theta is equal to 0, Theta 1 is equal to 0 when these 2 coordinates are coinciding that we shall we have to remember. It is so chosen that while the tooth mesh begins this X coordinate passes through the midline of the driving left gear tooth in contact okay, the rotation x, y defined by Theta 1. Theta 1 is the angle of rotation of the driving gear. At Theta 1 equals to 0, x axis coincides with the capital X axis that I have already explained.

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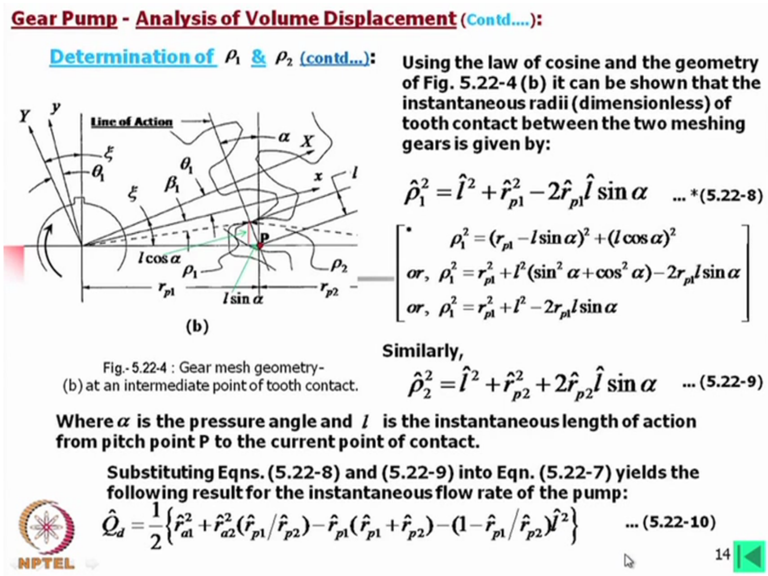

Now using the law of cosine and the geometry with reference to figure 4 b, which means that we have considered the intermediate point of contact, some point of contact, it can be shown that the instantaneous radii of tooth contact between 2 machine gears is given by this rho 1 is equal to l square + r p1 – 2r pi l sine alpha. Now this derivation is shown here, you can see this, you can write down, this square must be equal to r pi – l sin alpha square + l cos alpha square. Now if we simplify this, we will arrive into this form which we have non-dimensionalise and used in this equation, this is not difficult to find out. Similarly, root square will also will be expressed as like this, but remember the sign.

Where alpha is the pressure angle and l is the instantaneous length of action from pitch point P to the current point of contact that that means this is the pitch point and this current contact is defined by l. Now if we substitute 8 and 9 into equation 7, we will get this equation in this form, here comes this l square okay.

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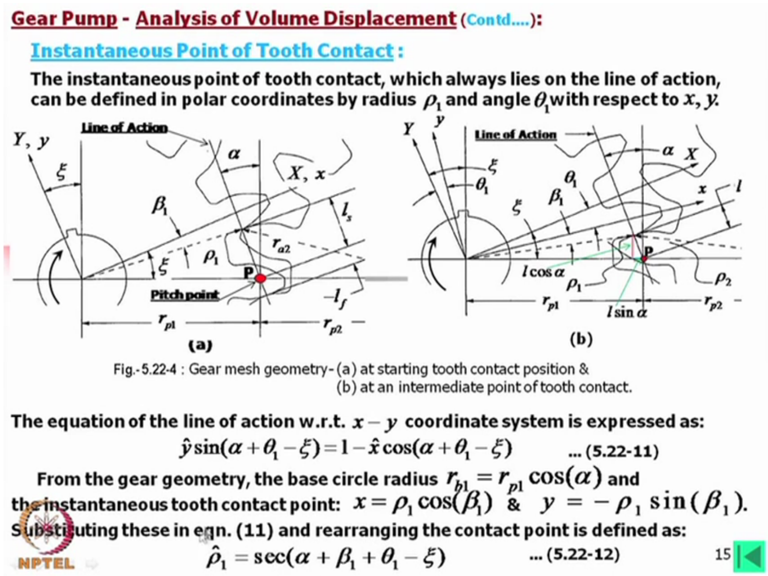

So the instantaneous point of tooth contact which always lie on the line of action can be defined on polar coordinates by radius Rho 1 and Theta 1 with respect to x and y. Now the equation of the line of action with respect to x, y coordinate system is expressed as y dash sin alpha – theta 1 the angle of rotation – this. If you just look into this geometry and you will arrive into this equation is equal to 1 – x dash cos alpha = Theta 1 – Zeta, this is the equation of this line of action. Then from the gear geometry, the base circle radius r b1 is equal to r p1 into cos alpha that is note. Now keep in mind that alpha in this case for alpha we have considered that is the working pressure angle, not the actual pressure angle or the standard pressure angle that you have to remember.

This means that, while you are going for this analysis you have to know that what centre distance they have made and the gear correction is there and from there we have to find out the actual working pressure angle that we can put. But in normally in case of these gear pumps we do not change the centre distance so even if the corrections are introduced there the Alpha remains constant that is it remains standard one. The instantaneous tooth contact point x is given by this, y is given by this, this is simple. Then we substitute this in equation 11 and rearranging the contact point is defined by this equation.

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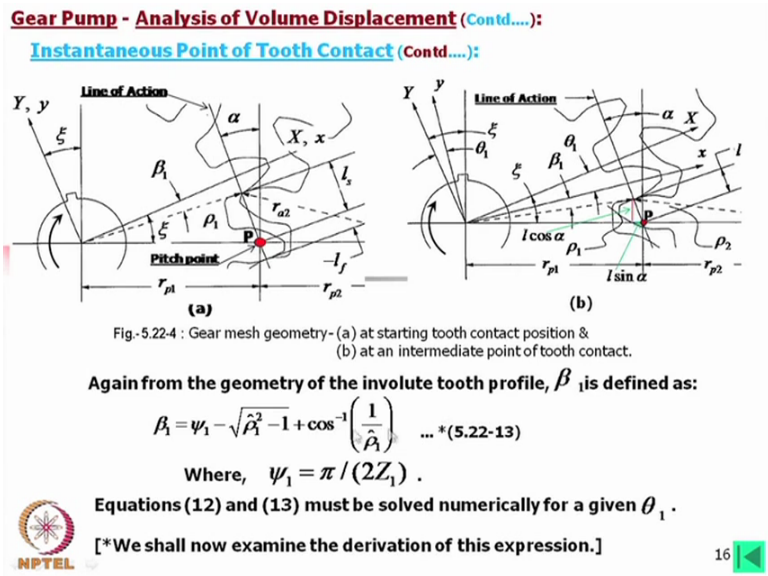

Now, again from the geometry of involute tooth profile, beta 1 is defined as beta 1 is this angle, this can be defined by this, how we will see in the next slide but we should look into this Si 1 angle is Pie divided by twice Z 1 okay, Pie divided by twice Z 1 means we have taken this angle, not this angle this, so I want here it is not shown in this figure but this is Pie by twice Z 1, Pie by Z 1 is equal to the 1 half the pitch and this is half of this stiff angle that is why we have to show this one this angle to be defined is given here. Equation 12 and 13 earlier equation must be solved numerically for the given Theta 1, okay these equations you can see that there is no solution, direct solution we have to solve it numerically. However, this is also analytical solution will be available, we will come into that but let us see this how this expression how this is derived.

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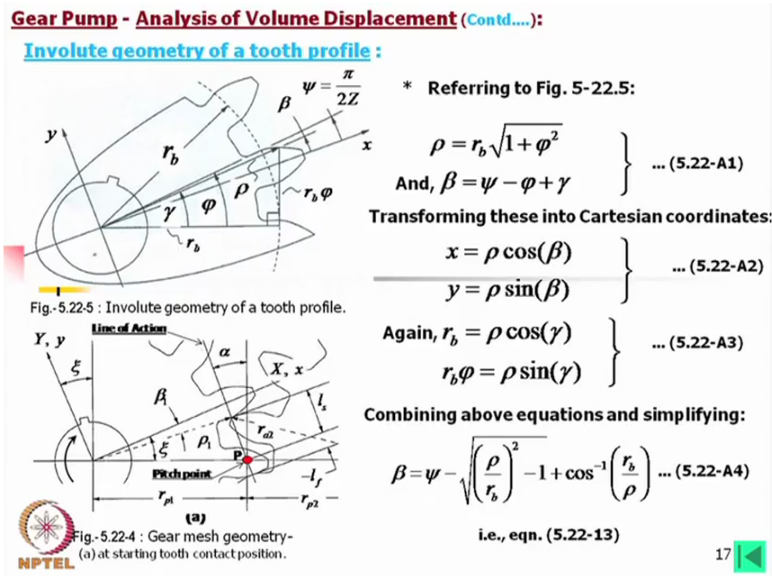

Now referring to this figure you can see this, now you can see this angle here this angle is given, so this angle is this much, this is okay. We have considered this angle that means this is half of the teeth, this angle is the half of the teeth angle okay. Now here these angles are defined and then we can write down this equation this Rho is equal to r b into 1 = this angle, this is the r b and this is the angle so we can define like this okay. Now next beta is equal to from the geometry you can find out this and then transforming these into Cartesian coordinates, we can write down these equations and also r b is equal to Rho cos gamma and this is equal to this is from the property of involumetry, we can find out this also. And then after substituting all these equations together, we will arrive to this equation which we have used as equation 13.

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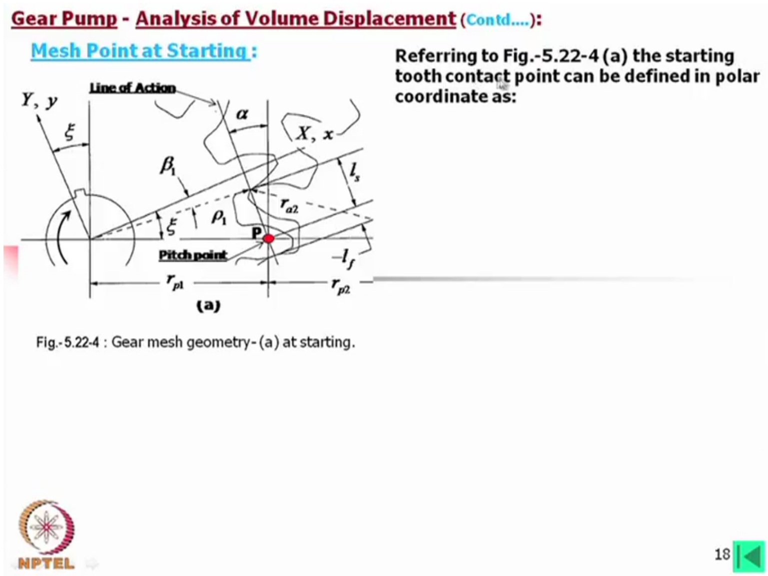

Now next referring to the starting tooth contact point through this figure, we will now find out the starting tooth contact point. Now this coordinate again is given by this one, if you look into this geometry we can have this equation. Now here, s indicates the starting point, this Rho 1 at starting that is denoted by Rho 1s, as well ls is the length of instantaneous contact point at starting.

And the starting length l s is expressed in this form from the geometry and then from equation 15 and 13 angle zeta angle Zi now can be derived with respect to this starting contact point parameters and this angle is equal to is expressed by this equation. So now we can derive what is this angle, if we know this angle of contact this, we can calculate this angle also, where this Rho 1s is equal to Rho 1 which already I have defined.

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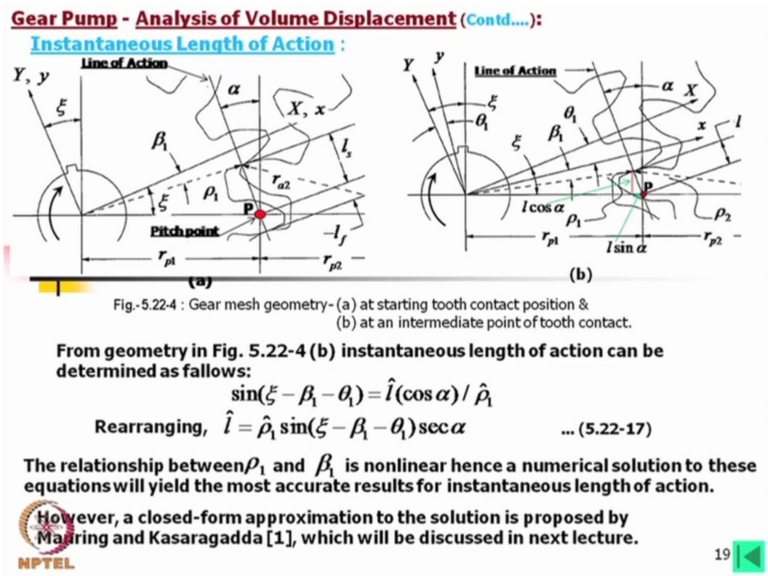

Now instantaneous length of actions, again looking into this figure what we find, sin of this angle is equal to given by, this is of course in the dimensionless form so this we can find also, this l length we can find out from this equation, which we have rearranged. And once we can calculate this, we can find out this also but still remember, these relations are non-linear hence numerical solution to this equation will give the most accurate results for instantaneous length of action okay.

However, a close form approximation to this solution is proposed by Maaring this you will find in the reference which will be discussed in the next lecture. Now I would like to say, these all such analysis were not available till date, although the gear pump maybe of being used from 1950s or so but this analysis only recently this analysis was done, which really showing how this area can be calculated, anyway we will discuss this in the next lecture.

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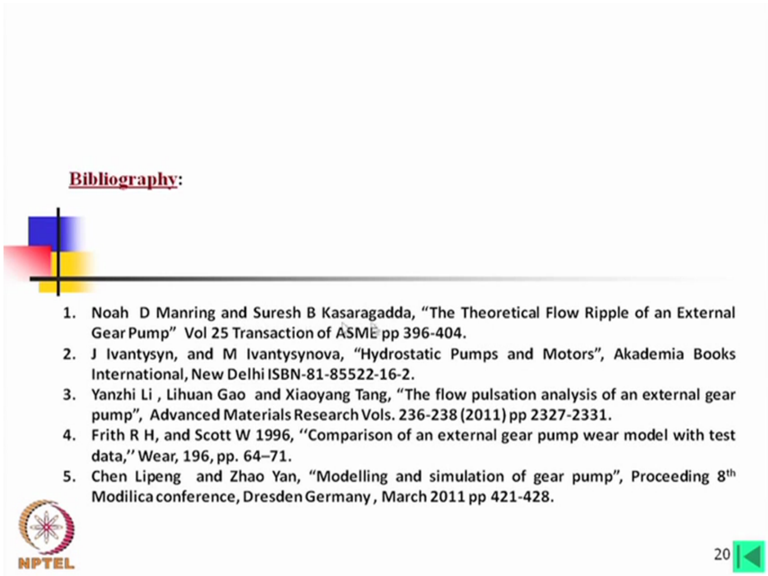

This paper is important if you have time then you can go through this paper, unfortunately this year is not given but I think it is in the 2004 or maybe 2005 only this work this research work was done. However, we will discuss also there is another method to find out such volume in the next lecture and what we will find that you can also go through this paper if you are interested, if you want to learn more about this. But if you read this and these 2 papers, you will mostly understand what I am trying to tell you and how to find out this volume displacement. Okay, this will be continued in the next lecture and I thank you very much for listening.