Fundamentals of Industrial Oil Hydraulics and Pneumatics
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Lecture no 21
Module no 06
Analysis of an Axial - Piston Swash Plate type Hydrostatic unit

Welcome to today's lecture, this is continuation of the analyses of an Axial Piston Swash plate type hydrostatic pump. Earlier we have learned the pressure ripples, their features also what the will be their input torque of the shaft and volume displacement, et cetera. In this lecture I shall discuss about the bulk configuration in a better way and thereby we also discuss about the pressure ripple and then one important aspect, which is called swash plate torque. This means that the torque required to hold the swash plate in the position or in case of fixed displacement what are what the torque experience by the swash plate are.

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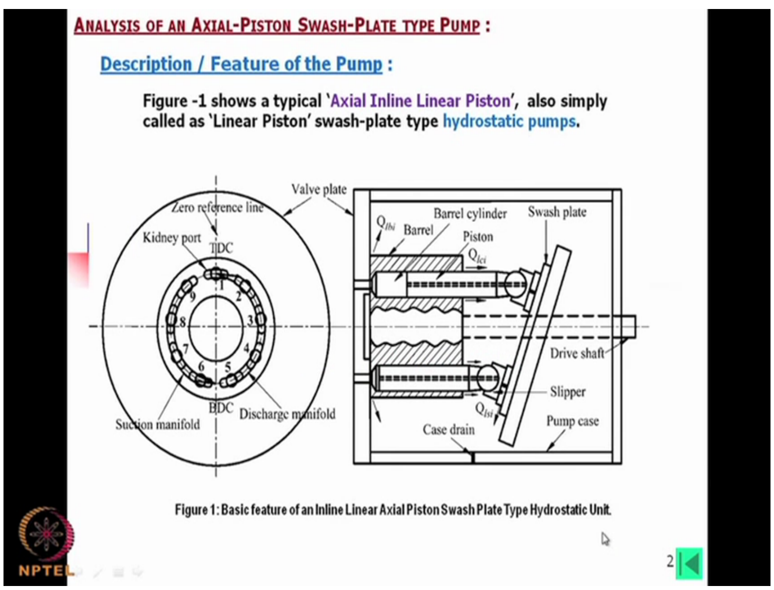

Now, this is the axial Piston pump which we are studying already I have described that this is rotated, the Pistons are laid in a barrel which is rotated by an input shaft and then this the end of such Pistons are with hydrostatic thrust bearing, which is called slipper pad. This is moving on the swash plate, which is inclined for fixed displacement, this inclination is fixed, in case of variable displacement this angle can be varied. Now as the pressure changes during the rotation, the suction and compression as well as in the valve, there are also transitions zones depending on that, torque on the input shaft as well as the swash plate shaft and they vary.

So while we are changing the angle particularly in case of variable displacement or when this is being used for a fixed I mean constant displacement or some constant pressure, constant torque, whatever may be the future, in that case we need to control that swash plate torque so for that it is essential to know what the torque, what is the torque acting on the swash plate at an instant.

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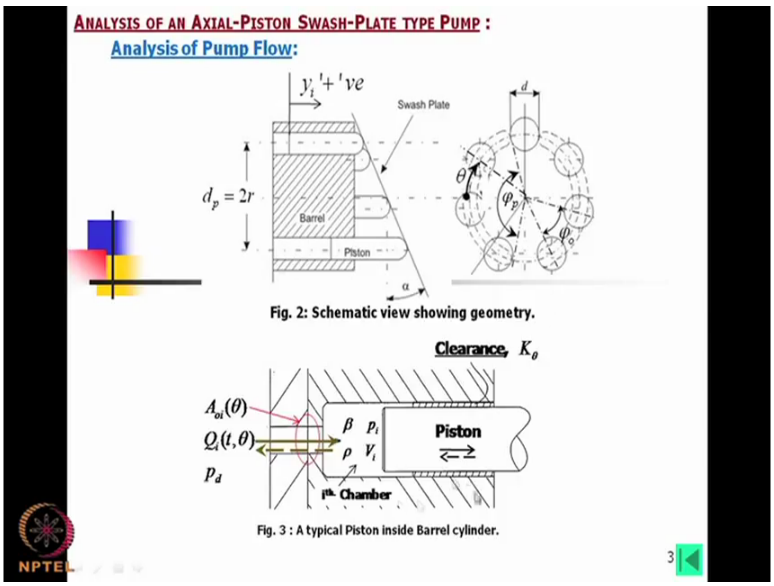

Now this is the geometric configuration which already I have discussed, between these 2 pistons which are equi-spaced, the angle is denoted by this one and this angle Si p is the valve angle total spade on the wall and the Theta is the angle of rotation of the barrel that is rotation of the shaft. If we look from this side then this theta begins from the left-hand axis the axis system is not shown here, which already I have described in earlier lecture. Now this is the typical leakage shown, I mean this typical figure for showing the leakage past the pistons so for that we need to consider the pressure, volume then bulk modulus, density and also we should consider the area of the orifice, flow, pressure, et cetera, et cetera.

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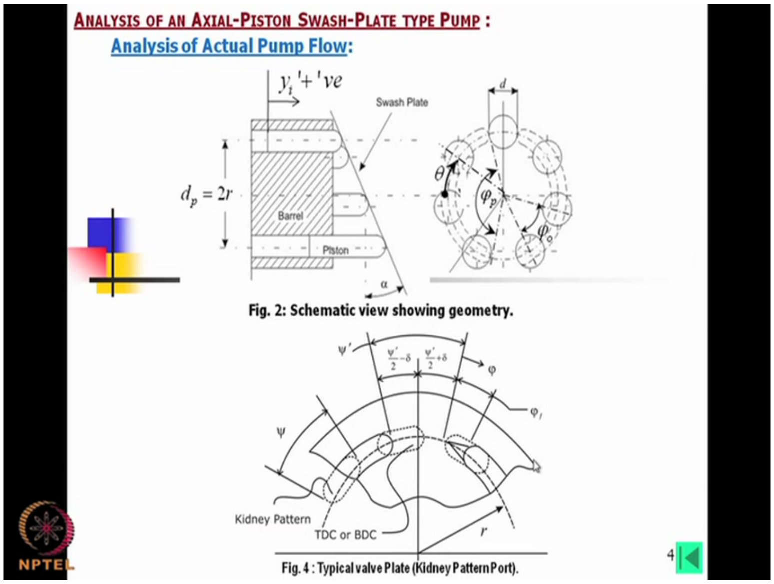

Now the valve which is called kidney port, so there is a valve plate which is not shown. On the kidney port I mean on the valve these are the kidney port if we consider from here to here and if we consider here to in this direction. One obviously one is for the suction, one is for compression or delivery, suction and delivery, input and output. Now this here although it is shown, apparently the cylindrical hole the other end of the barrel, but usually this end is with also another kidney port, this is kidney pattern because this is written kidney pattern port. Now the dimension or the angular spade between these 2 kidney ports shows that the length of this will exactly fit on the space.

This means that momentarily there will be no flow in or out to a chamber, to a Piston and next moment it will go inside this kidney port may be suction or maybe discharge. Now, looking into this end say it is rotating like this, then while it is entering whether it is discharge or suction, there is a groove is provided. Reason is that, if it is exposed to this main port directly then rate of change of area is very fast and the change in pressure in the in the fluid inside the Piston becomes very much transient, 1st thing. 2nd thing is that there is a there will be huge noise when it will if it is directly connected to this, if this group is not provided, therefore sometimes this groove is called silencing groove.

Here is the geometric zone what is the spade of this groove and this is the spade of kidney port on the barrel and this is the spade of this dead band it is it should have called dead band. Now this is again in some design these are not exactly equi-spaced that means in one-side it is less and other side it is more, this is also to reduce the dynamics because the barrel is moving, the port is then coming into contact, coming into this kidney port contact. The if it is made equal then due to the dynamics, actual time maintenance will not be equal, or in other words we can say that this for the optimum performance it is found that if one angle is less made less and other angle made higher, the performance is better, where this angle may be equal to this angle or this angle is slightly larger slightly larger than this angle.

If we make this is less than this angle then what will happen there will be direct connection from this charge to input that output to input and vice versa so immediate pressure drop will be there and it cannot be controlled, huge leakage will be there.

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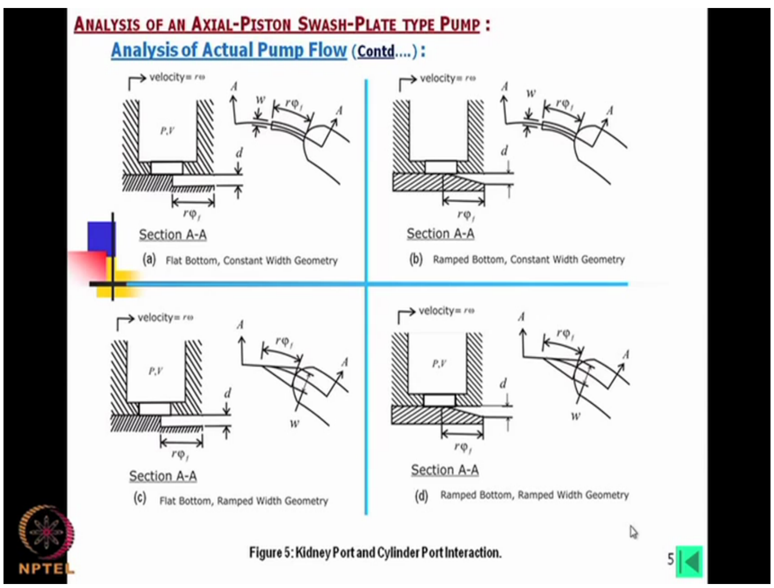

Now, if we think of the port that silencing groove, now this is the main port on the valve plate and this you can say this is a through whole through which the oil is going from one side and from other side it is going to the pipeline. Now, the silencing groove it may be of flat bottom and constant width. Now when this, this is the kidney port of the barrel when this move gradually on that, initially opening will be less, gradually it will increase and then finally it will increase to the whole port will be open to this main port, okay. Now it can be made simply like this which is a if we think of the manufacturing, it can be made by milling cutter in mill cutter like that.

Now next one is the ramped bottom constant width, so ramped bottom means this is an inclined bottom. If you look into this, how to calculate area, we will always consider this from this corner this perpendicular area that is the minimum area open to this port. In this case also directly we will instead of considering this area we will consider this area in perpendicular direction what is the opening. That means character whether we have to consider the flow is going like this, we have to consider this diagonal. In this case we can see this is more effective, so it is gradually increasing. Now in comparison to this we can have also it is like a pyramid okay, so no this is flat bottom that means this area is increasing, gradually this is increasing, this width is increasing however, depth remains constant.

Now as we understand machining of such thing is very-very difficult because here practical this is the vertex of a triangle and depth is like this. Probably, I cannot think of machining with ordinary tools but this is also explained that this can be used. But most common is that ramped bottom ramped width geometry that means this is inclined as well as the size is also increasing. I will show how to calculate the area, one of that I will take for an example and I show that how the orifice area can be calculated.

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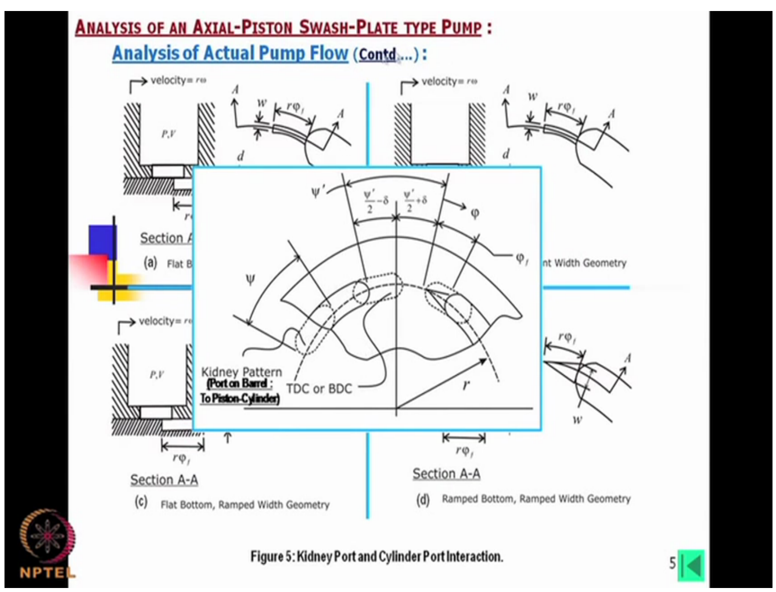

Now this is you can now see these are the 4 possible configurations and these are provided for the silencing groove.

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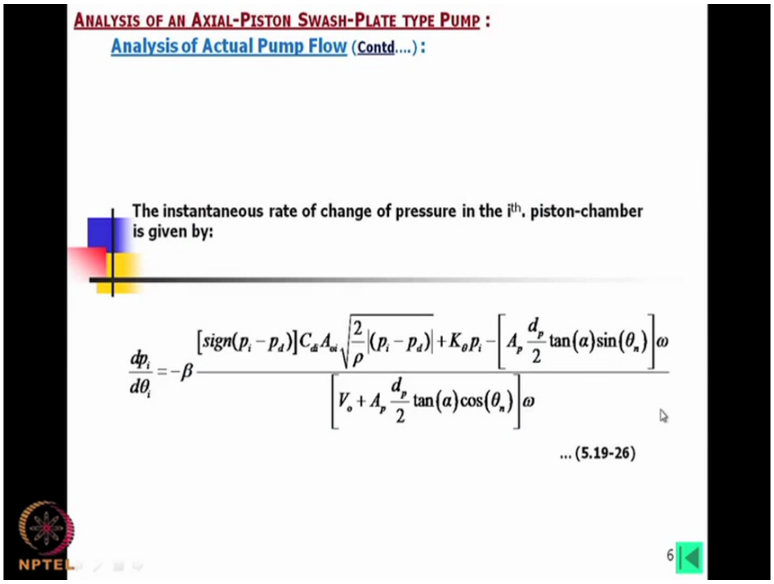

Now earlier, I have already discussed that the instantaneous rate of change of pressure in the ith Piston Chamber is given by this equation, I am not repeating this because earlier we have discussed. Here we consider the sign, depending on that direction of flow can be determined so this is the pressure drop and this is the leakage characteristics K Theta. So this of earlier lecture, if you remember in lecture 19 we have discussed this.

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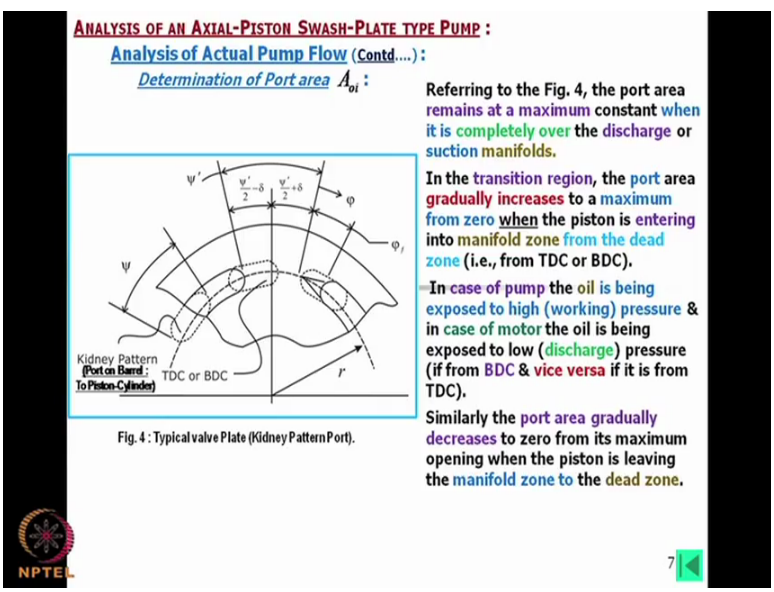

Now, again I am continuing with Actual pump, Actual pump means in this case we will consider the leakage and others to find out the pressure ripple. Now the port area remains at a maximum constant when it is completely over the discharge or suction manifolds that means when this port kidney port will come over this main port then it will be maximum. In the transition region the port area gradually increases to a maximum from 0 when the piston is entering into the manifold zone from the dead zone that is from the top dead centre or bottom dead centre. In case of palm, the oil is being exposed to high working pressure and in case of motor the oil is being exposed to low pressure if the BDC and vice versa if it is from top dead centre.

That means if we consider from it is bottom dead centre then that means in case of pump it is completely full with fluid. Now then next moment it will be connected to a port, in case of pump definitely it is a high pressure because it is going to discharge. In case of motor it will be the reverse so to understand this language you have to think in that way. Similarly, the port area gradually decreases to 0 from its maximum opening when the piston is leaving the manifold zone to the dead zone.

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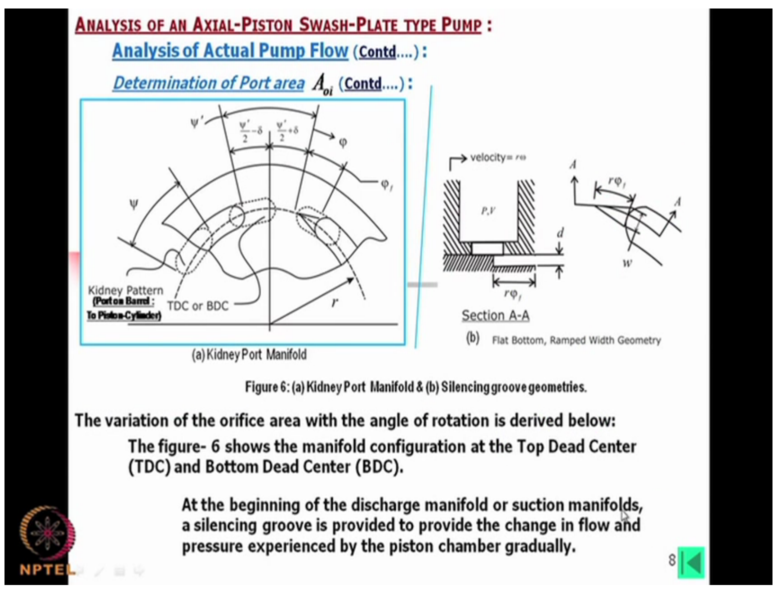

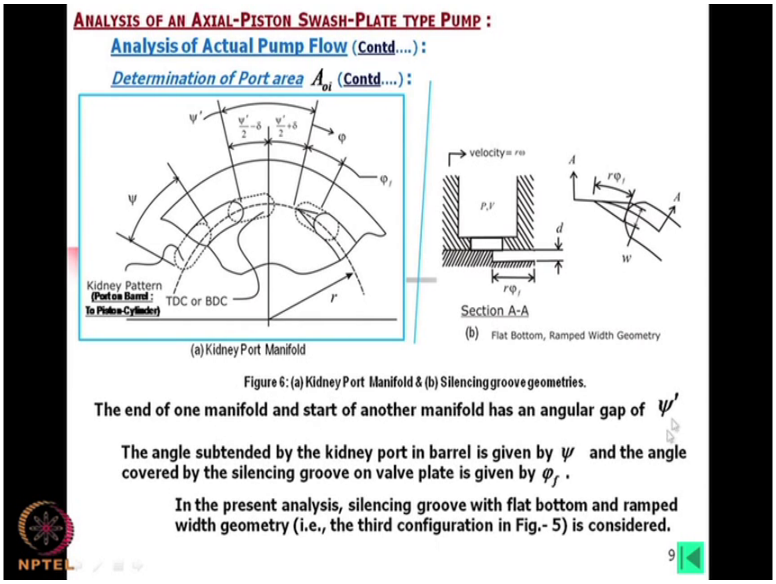

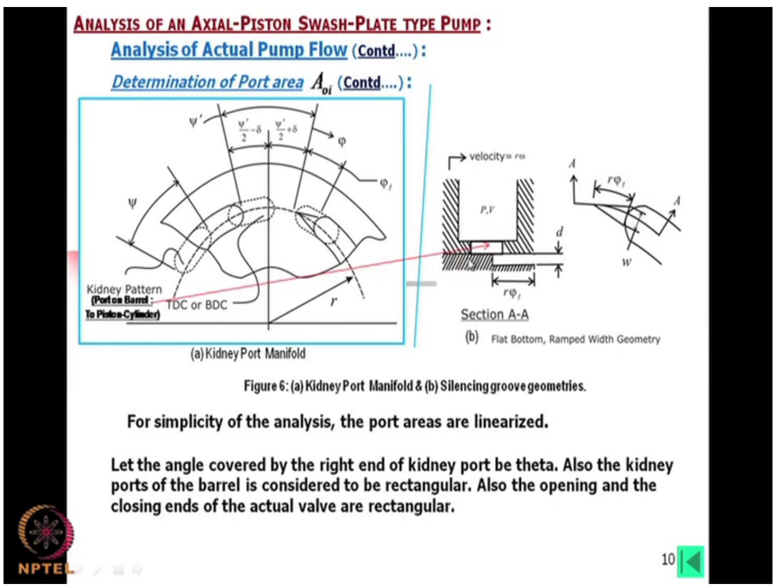

Now in earlier analyses I had shown that we will consider the flat bottom and ramped width geometry. This is just to show the variation of the area but this is not practical as I told that machining of this might be problem. Now at the beginning of the discharge manifold or suction manifolds, a silencing groove is provided for which this is the area already I have discussed. And then these are the angle shown here different angle in earlier lecture I have already discussed so I am not repeating here. Now we are considering this area I mean this kidney port pattern, this is that this one of this here.

Now area when we are calculating, we are considering this area. Let the angle covered by the right end of the kidney port be Theta okay. Also the kidney port of the barrel is considered to be rectangular, kidney port of the barrel is considered to be rectangular that means there is slightly curved but we will consider it is rectangular that means we will consider that this is a rectangle and this is a circle. This circle diameter will be width of this one and then length this from this center to this centre, we will consider this, actually it is curved.

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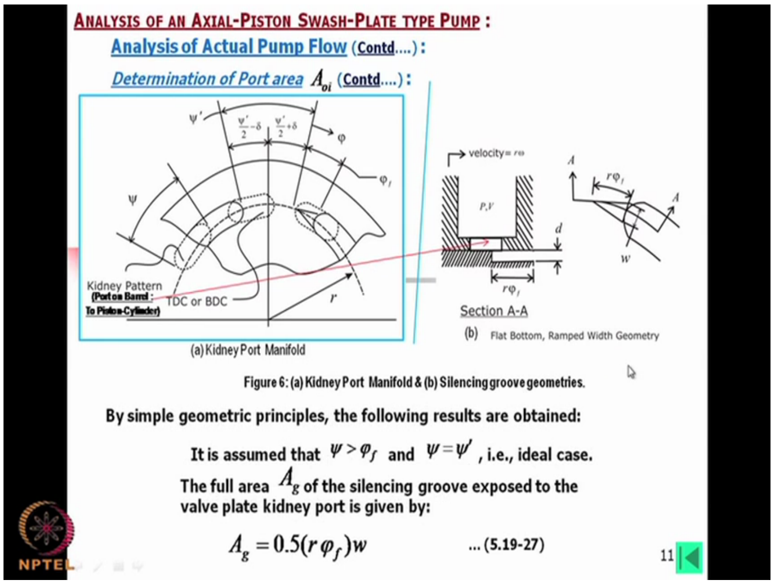

By simple geometric principles the following results are obtained; it is assumed that this angle is greater than this angle okay, we have assumed. And we have assumed also this angle is equal to this angle and the small variation for the spade of this right-hand side of the axis and left-hand side of the axis is not considered in this analysis at all. Now then what is the area we consider in this case as it is a flat bottom? In this case we consider whatever exposed of this area that becomes clearly this is the height and this is the base and half of that, did you understand my point? In case of the flat bottom and ramped width we simply consider whatever area is opened here not in this diagonal direction, directly we will consider this area is the orifice area who, so it is calculated like this which already I have described in earlier lecture.

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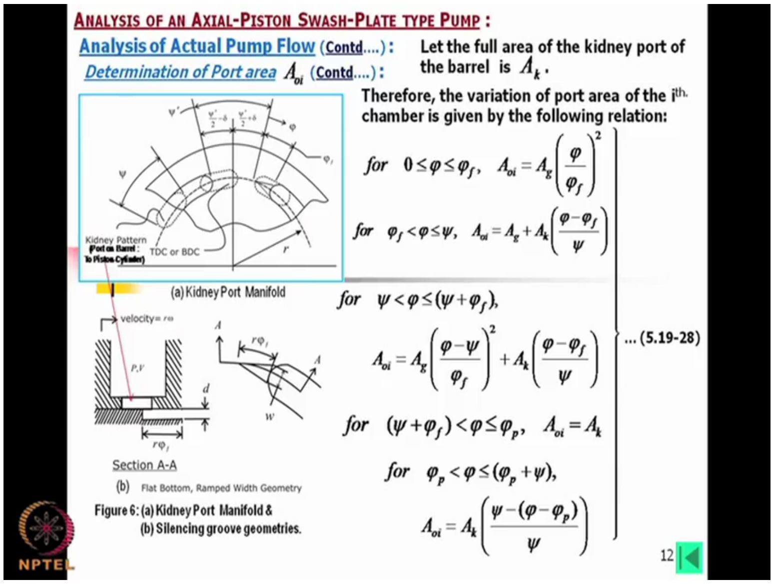

Now the important factor is that what is the total opening from the barrel to the port? That you can see that let the full area of the kidney port of the barrel is A k, therefore the variation of the port area of the ith chamber is given by the following relation. Now when this angle that is the lapping of this one, we have considered now angle from this line, this is the angle of rotation. Our actual angle of rotation we have considered Theta but while we are describing this area exposure, we have considered that this is the 0 position means the tip of this is here. Now this is increasing so when this tip is within this limit then area is given by this one.

This is A g is the total area of this angle, so it is given by this is total area of the silencing groove flat area. So then when it is after completing this it is going inside then for that we may consider this is the area, so this area is already exposed that means this triangle is already exposed. Then a part of this area is going inside this kidney port, the width here is same, this kidney port width and this kidney port width is same. So this if you carefully just observe that these are the simple geometric calculations, now when it is further moving further that means this angle this angle is less than this and no sorry this angle is greater than this and less than this then we find this area is completely different area = a part of that has come over that and then for the condition when it is fully on this port then this is the total area.

Yeah for this, this is the total area and this condition is that when it is just going out. Remember, that at other end there is no silencing port, if it is rotating in this direction, silencing port is always when it is entering other end there is no silencing port. So this angle is the total spade of this kidney port, so when it is crossing then the angle is given by this one. So my suggestion is that to calculate the angle and to calculate the area, we should go through these equations and we need a practice otherwise we will not be able to find out what are the areas. These are simple geometric lessons but it needs practice okay.

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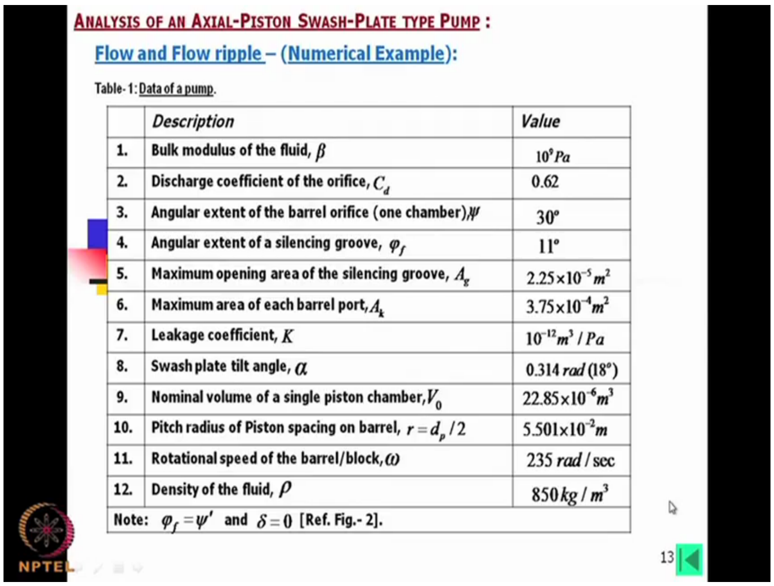

	Description	Value
1.	Bulk modulus of the fluid, β	$10^9 Pa$
2.	Discharge coefficient of the orifice, C_d	0.62
3.	Angular extent of the barrel orifice (one chamber) ψ	30°
4.	Angular extent of a silencing groove, φ_f	11°
5.	Maximum opening area of the silencing groove, A_g	$2.25\times10^{-5} m^2$
6.	Maximum area of each barrel port, A_k	$3.75\times10^{-4} m^2$
7.	Leakage coefficient, K	$10^{-12} m^3 / Pa$
8.	Swash plate tilt angle, α	0.314 *rad* (18°)
9.	Nominal volume of a single piston chamber, V_0	$22.85\times10^{-6} m^3$
10.	Pitch radius of Piston spacing on barrel, $r = d_p/2$	$5.501\times10^{-2} m$
11.	Rotational speed of the barrel/block, ω	235 *rad* / sec
12.	Density of the fluid, ρ	$850 kg/m^3$

Then the pump which we have considered here, this is I have already shown earlier but we consider the bulk modulus is 10 to the power 9 Pascal and coefficient of discharge is irrespective of the size of the orifice is 0.62, angular extent of the barrel orifice is 30 degree and angular extent of the silencing groove 11 degree, maximum opening area of the silencing groove is 2.25 into 10 to the power - 5 meter square and maximum area of the each barrel port is 3.75 into 10 to the power - 4 meter square, as you see this is smaller than this. This is the if you consider the top of this silencing groove, this is the area. Remember we will consider A g as this area for the orifice when we take this flat bottom type for the pyramid type that is ramped bottom and as well as the ramp opening for that we have to calculate such angle separately, I will show that.

Now leakage coefficient here we have considered K, I think it was given K Theta here, so we consider the leakage coefficient, again when I was discussing about the leakage coefficient I was told that depending on the length of the stroke, length of the opening, this actually it would change but if we find that stroke length is not very high because the swash plate angle for this pump is not very high, in that case we may take further analysis K is equal to constant where we have taken in this case.

Now swash plate angle alpha is maximum 18 degree and nominal volume of single piston chamber is 22.83 into 10 to the power - 6 meter cube very small, and r is equal to that is radius is 5.501 into 10 to the power - 2 meter that is 55 millimetre, d p is 110 millimeter, rotational speed of the barrel say nominal speed we have considered 235 gradient per second, density is 850 KG per meter cube and we have also considered this angle, 2 angles are same and the Delta that is the deviation of the both the side distribution, those are 0.

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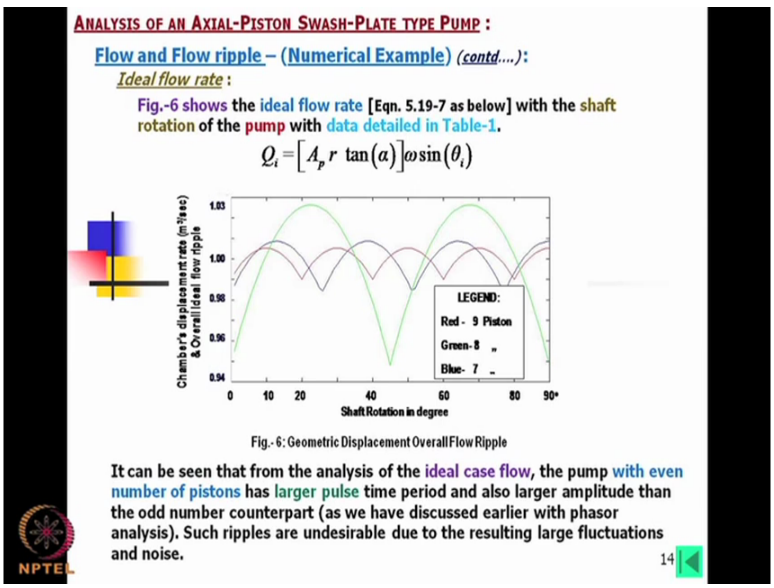

Now with these values 1st of all we have this curve plot that is following the equation, this is ideal the geometric displacement. Now what we have taken with the same diameter, we have taken number of Pistons is 9 pistons, we see the diameter of the piston is same but we have taken this radius d p in such a way that stroke length is varying, stroke length is different for 7 chambers, 8 chambers and 9 chambers so that total flow that is swept volume remains same for the same inclination okay. So that is why we see that if we take the average through this that will be same but if you think about the ripple, in case of 7 the blue is 7 chambers.

Now if we go for 9 then this ripple is reduced than 7, but in between that if you go for 8, ripple will be more. You can examine from this equation as well as from our the flow ripple analysis which we have done by the phasor analysis. It can be seen that from the analysis of the ideal case flow, the pump with even number of Pistons has larger pulse time period and also larger amplitude than the odd-number counterpart. As we have discussed earlier with phasor analysis, such ripples are undesirable due to the resulting large fluctuations and noise.

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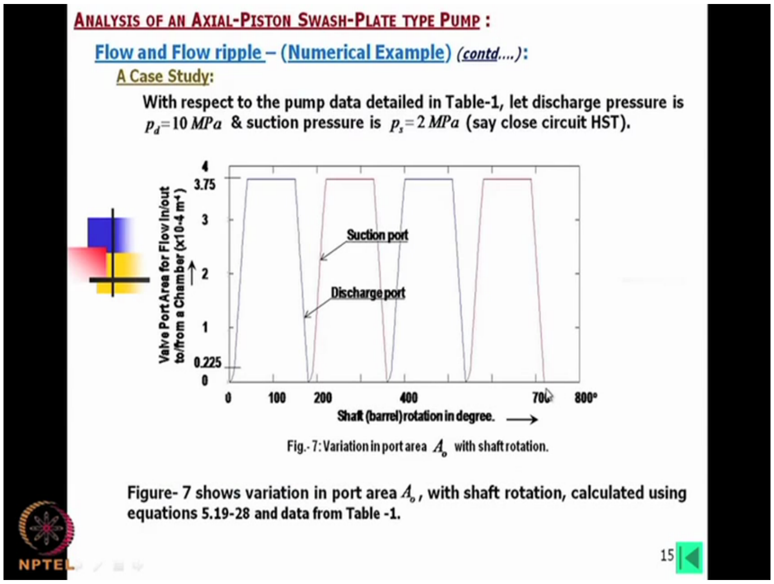

Now in this analysis what we have considered, we have considered the variation of area while we are calculating this flow, pressure, et cetera. Now, these are the data shown and what we find the valve plate area for the flow of in-out is varying in this way, say this this variation due to the you can say that silencing groove and other end there is no groove so it is a straightforward so this is a sample calculation we have just calculated what are the areas. And as you find for when the full kidney port is coming on the main ports, the full opening is there so this area you can compare with the area of the barrel kidney port okay, so these are the area and then we plotted the pressure.

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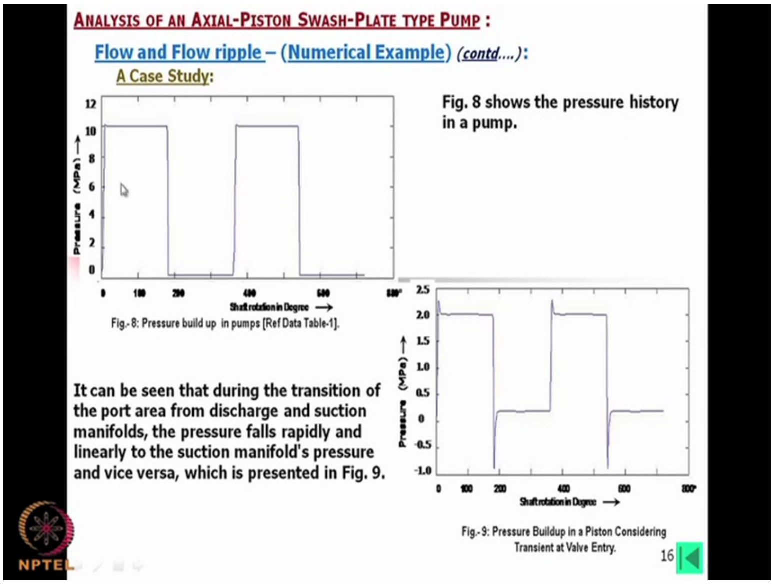

Now this is for the now this is this as at the valve entry and this is the exit. Now as you see that as this area just from the dead zone when this area has started opening momentarily, there is a pressure rise that means when the piston is coming on the, it is full and it is pressure in case of pump it is in the delivery side and it is in the dead zone, there will be momentary rise of this pressure, this depends on the area it is opening. Or in other words I would say that depending of different ports you will find that this pressure rise will be different however, if we provide that the Delta has some value, you will find there will be pressure fluctuation there. So then the question is that why we provide that, we will find that overall losses and overall performance will increase although there may have a spike.

But this actually I would say this calculation is done on the basis that apparently when it has reached the dead zone the initial opening of the orifice for a small angle, we have taken a small angle and area is too small but in reality this pressure rise may not be there, this will be truncated.

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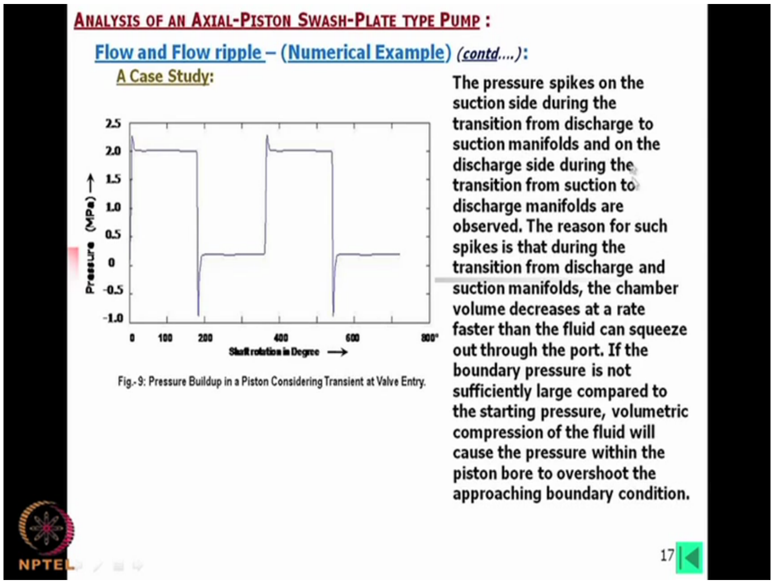

Now here I have described that oil this pressure rise, which I have already told.

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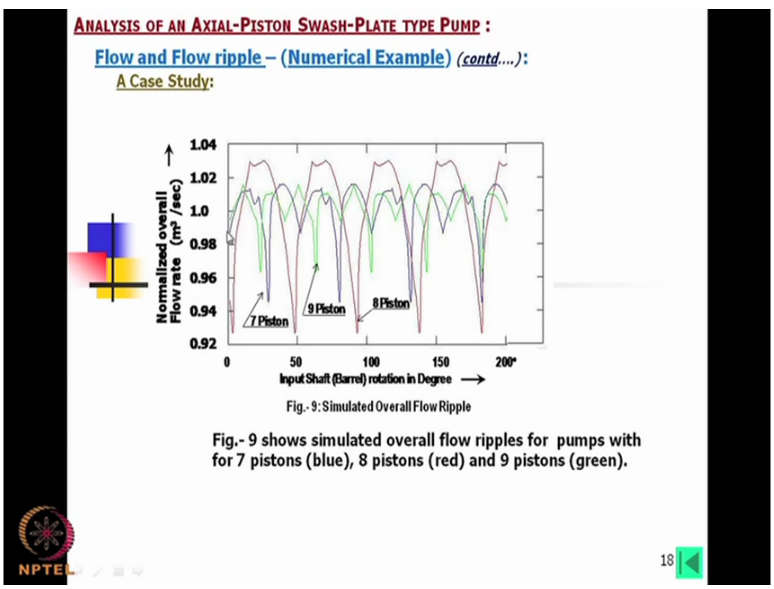

Now initially I have shown the what will be the flow rate but if we consider the leakage, if we consider the pressure fluctuation with all such considerations, your actual ripple may become like this. Experimentally it is also shown that it is not a smooth curve, there will be fluctuation like this. So this is of course not the experimental on considering this curve is drawn on the basis of considering the leakages, considering the pressure rise, considering the change in area when the port is opening and closing and this will be the theoretical curve, this may not match with experimental curve because while we are conducting the experiment, there are some more unknown factors which may not be considered in theoretical analysis.

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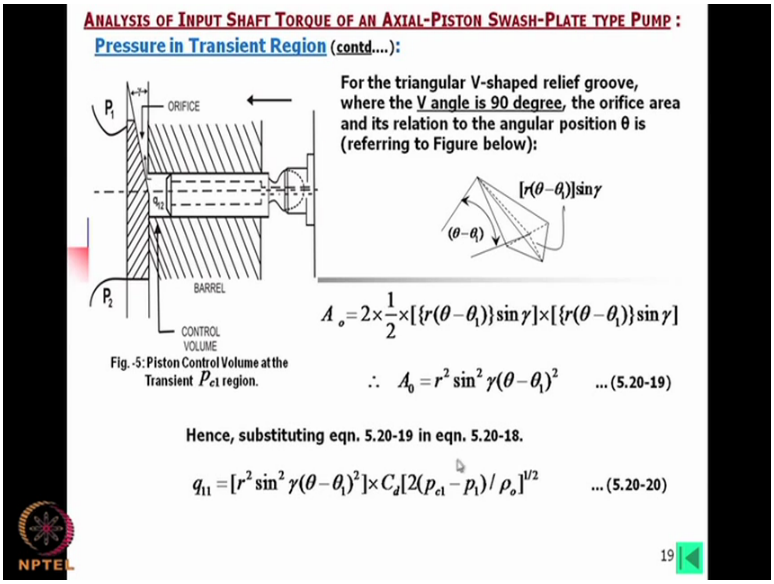

Now pressure in transient region, in this case to analyze the pressure what will be the pressure variation while a piston moving from 0 degree to 360 degree that we will analyze first, then that pressure is considered for the further analysis of the flow, pressure, et cetera. Now in this case we have considered this V angle is 90 degree that means if we consider this groove silencing groove this is the we have considered the V shape that means this is a ramp in this direction as well as ramp in this direction. Now, this angle is 90 degree that means this angle is considered 90 degree okay.

Now for that if you look into this geometry, then this altitude will become this angle, now what the orifice area and in relation to the angular position theta is given by this angle where in this case again we have considered the Theta angle with respect to the bulk rotation, so this angle is considered as a total angle Theta – Theta 1. So this means that from a reference point, the position of this tip is Theta 1 and our total rotation is Theta. So when we know this Theta and we know this position, we will use this angle, whereas this angle is the angle of inclination which is not shown okay, angle of inclination that means this angle, with respect to this surface this angle is gamma.

Now therefore this area can be calculated simply using this expression, so this is the height and as it is 90 degree, this is the base, half base if we consider this side so this is the area of one side and multiplied by the by 2 will keep the total area. Now look at this, in this case we are considering this area as the orifice, not the surface area. In case of this configuration of the silencing groove we have to calculate in this way. Now the area is in this case can easily be calculated, we do not consider the area of the valve barrel port okay. So now we substitute this into the earlier equation, this was the earlier equation what we used so to find out the flow q 11, here is the gamma angle is shown, this is the gamma angle that means this angle is the gamma.

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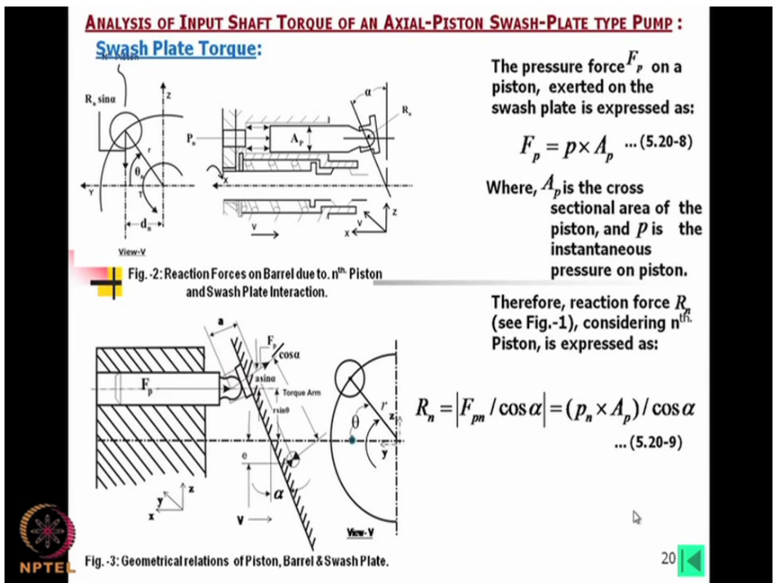

Now let us calculate the force, we will now consider 1st the torque, we now we consider here is the pressure so we consider a force A p along the piston, this is simply the pressure into the area of the piston okay, A p is the cross-section area of the piston. Now what will be this force reaction force offered by the swash plate? This will be definitely F pn divided by Cos Alpha, pn is the n is the nth piston so for the nth piston this will be the force. When we are calculating the force of a piston then we consider that particular position of the piston and the pressure, the instantaneous pressure at the piston.

So this is simply why this is we have simply divided by the Cos Alpha because force acting in this direction so definitely in this direction the force is opposite to this to generate I mean to offer that much resistance in this direction, the inclined plate has to offer this much force which is the which we have to find by dividing by Cos Alpha, this is from the simple geometry and force analysis.

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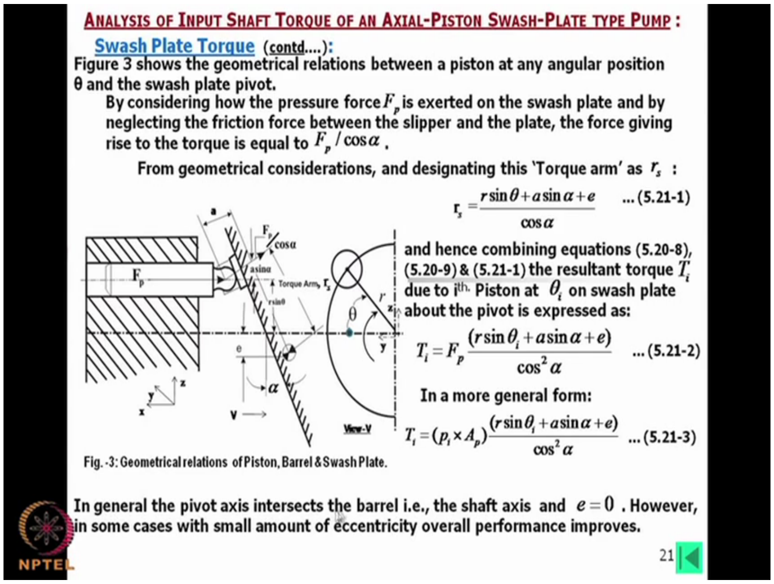

Now, so this already I have described so this much force is acting over there, then torque arm this is the torque arm that means we are calculating the talk about this point, so we have to consider this force and this arm length to find out the torque about this point okay. Now here one interesting point is that we have considered that this pivot point of this 1st plate not intersecting the axis of the barrel shaft or axis of the input shaft in case of pump, output shaft in case of hydraulic motor. Now this eccentricity in the if it is asked why this eccentricity is provided, again I would say which is very small eccentricity it has been found that pump performance or motor performance improve for some cases, not all cases.

Due to some range of the operation, this eccentricity is beneficial that is why here the eccentricity shown. While we are calculate in this arm we have considered the eccentricity but in our final calculation we have neglected that that means we have considered E = 0. So r s can easily be calculated, this is r sin theta this is r then sine component this is r so this is sine component of that then a this is the length of the… Actually I would say that here all those 4 it is on no it is right, we have to consider this length from this pivot from the centre of this ball joint, this is the slipper pad so this is ball joint from here to the pivot point, this length is A so we have to take this component also then this length e divided by cos Alpha is this arm.

We consider this length, we considered this length and then this length divided by cos of swash plate inclination gives us the torque arm. So we combine the earlier equations and these equations and we finally get this is the T i, in this case again we have considered ith ith chamber, not the nth, nth was for the force analysis, sorry this is a mistake we should have considered there also ith chamber, so this is only for one piston okay, one piston this will be the torque. Now similarly we have to consider all the pistons and here instead of this force, we directly we are considering the instantaneous pressure and the area of the piston so this is in more general form. Now in general the pivot axis intersects the barrel that is the shaft axis and e = 0, however, in some cases a small amount of eccentricity for overall performance.

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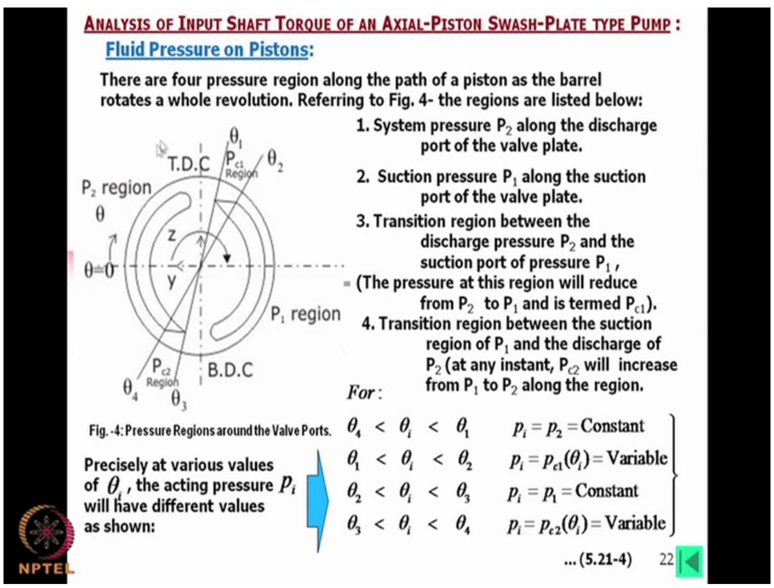

Now next is we have to consider the pressure distribution at different zones, system pressure P 2 along the discharge port of the valve plate, this is P 2 is the system pressure so this is the P 2 region so when it this is the delivery side, this is the delivery side that means 1st plate is inclined in this way, this is coming out of the screen, this is going inside there in so that while the piston is moving from this point to this point, it is gradually it is being compressed. So this is the high-pressure zone region, then P 1 along this suction port, this is the low-pressure (())(43:37) next the transition region between the discharge P 2 and the suction P 1 in case of pump, the pressure at region will reduce from P 2 to P1 and is termed as PC 1 so this is the PC 1 region, only this region.

The next phase we consider that this is the PC to region, where the pressure is gradually increasing. So in equation while we are calculating the pressure, each for each and every piston when they passing through this region we have to carefully consider that pressure that means while we are calculating for a particular piston, we have to consider what are the pressure there. Some pistons have P 1, some piston is having P2, some piston in having P C1, some piston is having P C2 so while we are calculating the swash plate torque, for all piston we have to calculate separately and there we have to some up them. So therefore precisely at various values of theta 1, the acting pressure P1 will have different values as shown like this.

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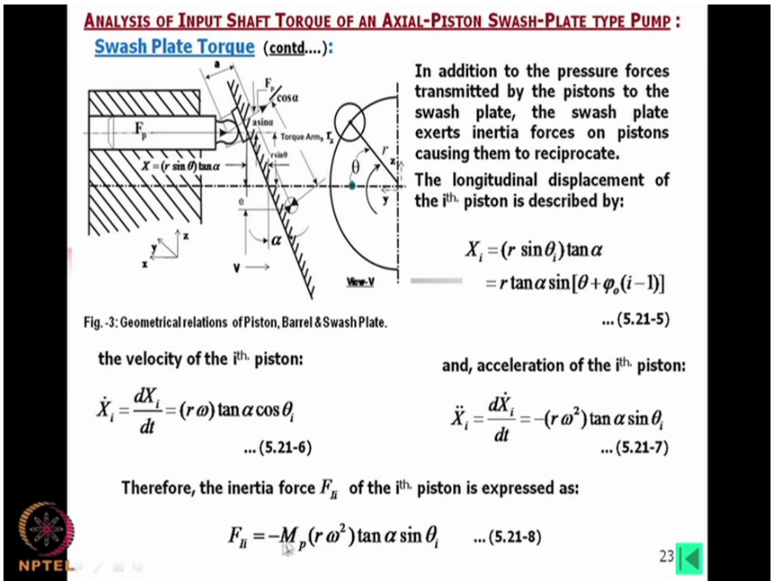

We can now we can divide into different zones and we can write it down what are the pressures over there, what I have described there this is written in the equation from A. Then in addition to the pressure forces transmitted by the pistons to the swash plate, the swash plate exerts inertia force on pistons causing them to reciprocate, do you understand my point? Say this pressure force are there as well as there is inertia force for the pistons so that is to be considered for the particularly for the transient region. While we are thinking the control of swash plate or by the what are the actuation system, we have to control the torque to keep the swash plate in a position or to move the swash plate from one position to other position so we consider this force.

First of all we calculate the longitudinal displacement of the ith piston then this you can calculate look at this, it is rotating like this so 1st of all we will consider, see this is the theta angle, we will consider this is r and then this is the sin theta then this is inclined so we consider the 10 components of that so this becomes we are calculating while we are calculating this 1st torque, we have to calculate this perpendicular height. This becomes 1st this value and then with the Tan Alpha we get the real this torque arm position from this pivot point. Look at this, we have here also neglected the e there is no eccentricity.

Now the velocity of the ith piston then if we differentiate with respect to time, it gives this value and as well the acceleration we find we can calculate further differentiating with respect to the time right. So therefore, the inertia force of the ith piston is expressed by this equation, this is the acceleration term and this is the mass of the piston okay.

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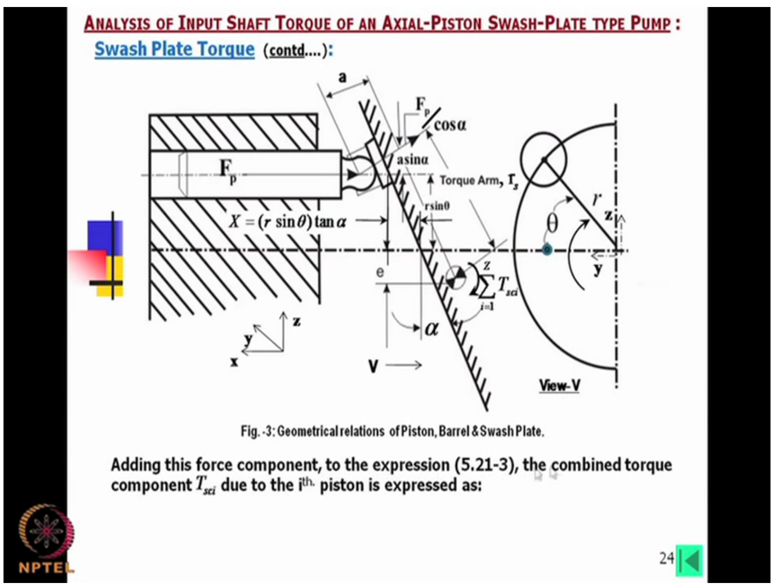

Now adding this force component to the expression the earlier expression, the combined torque that is swash plate control of for the ith piston is given by this expression okay, it is acting over here, this is only for ith piston okay.

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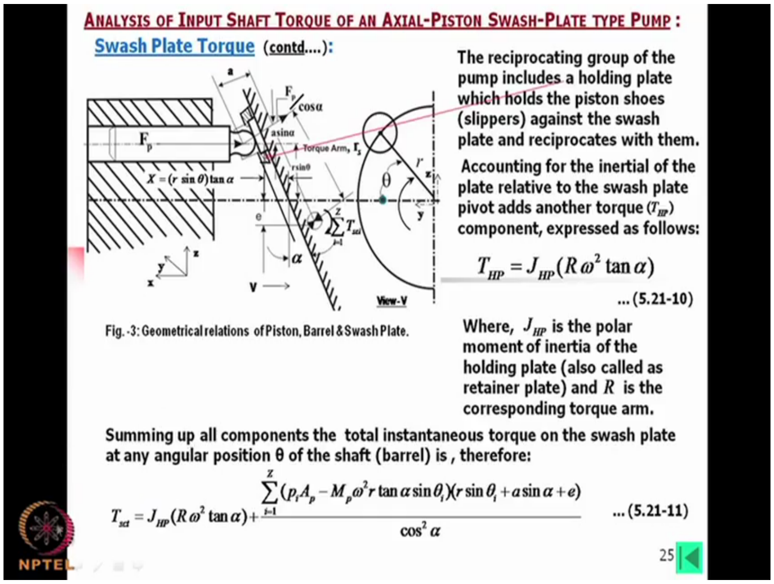

Now this reciprocating group of the pump includes a holding plate, which holds the piston shoes against the swash plate. Now actually here say if we consider the how these pistons are moving, forward motion is possible because this is raving on that but there is no physical connection between these 2, only this is a if the compression force is there then it will move. So while it is moving in the other directions, how this will be pulled out? Actually there is a plate which is sometimes called retainer plate, that retainer plate is held by a spring here against the barrel so this always keeps this slipper pad in touch, in contact with the swash plate. Now we must consider the dynamics or the inertia of this retainer plate also.

Now, while we are considering this accounting for the inertia of the plate related to the swash plate pivot and another torque T hp component expressed as follows. This is equal to that moment of inertia and R Omega square Tan Alpha, the derivation is not shown but as if we have considered mass of that and this is acting at a if we consider that this is the polar moment of inertia, this is radius of (())(50:31) and Omega square Tan Alpha will give us that torque. Now we add this torque with also the original torque what we have calculated. So now the submission of this that is the for each piston inertia = the pressure force torque due to this part = the torque due to the retainer plate. So this is precisely the equation for the torque of controlling or holding the swash plate in a position or to rotate the swash plate at an instant.

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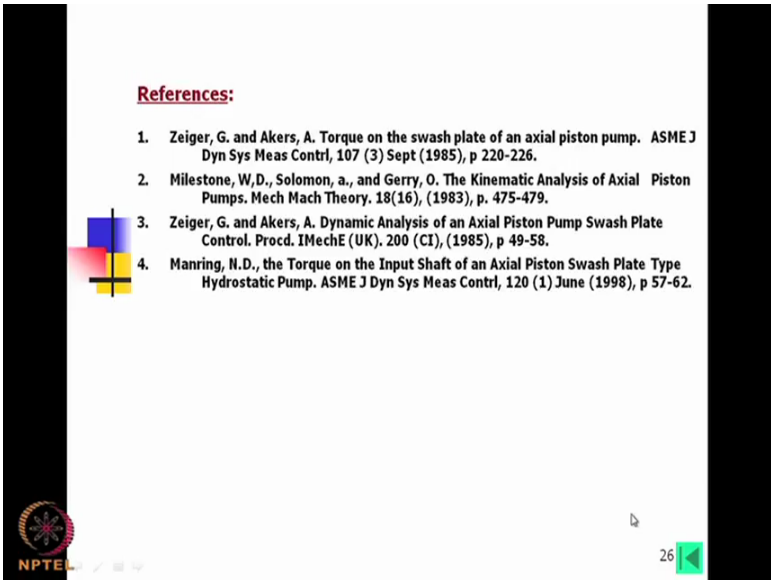

So these are the reference, I suggest that you should read this paper 1 particularly 1 and then 4. But if you would like to know the details about the dynamics, you should also follow these 2 papers. If you look into this, this was published may be in the same year but in other issues, this was published in ASME, this was another journal but 1st they found out this a nominal torque and then they did the (())(52:14) analysis, whereas to know the Kinematic details you should read this paper. Okay thank you.