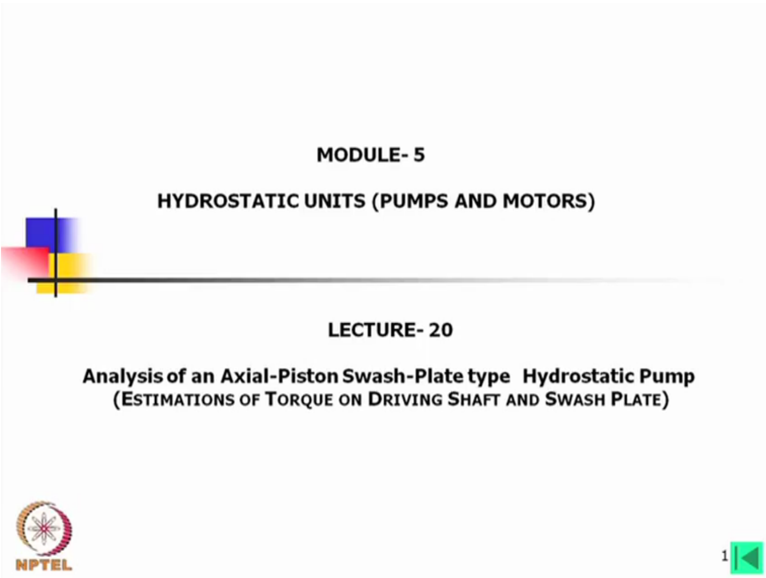
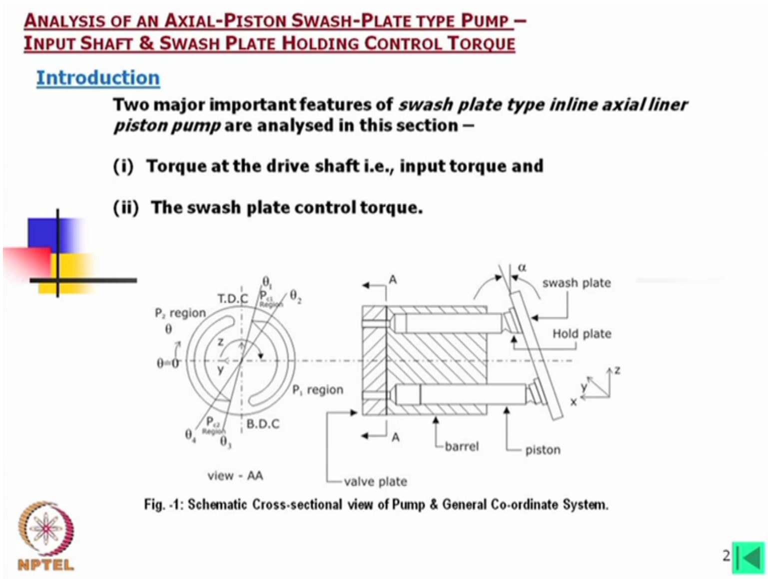
Fundamentals of Industrial Oil Hydraulics and Pneumatics
By Professor R. Maiti
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur
Module 5
Lecture 20
Analysis of an Axial Piston Swash Plate type Hydrostatic Pump (Estimation of Torque on Drive Shaft & Swash Plate)

(Refer Slide Time: 0:22)



So welcome to this lecture 20 Analysis of an Axial Piston Swash Plate type Hydrostatic Pump and in this lecture we will discuss about the torque on the driving shaft and swash plate.

(Refer Slide Time: 0:40)



The major important features of swash plate type inline axial linear piston pump are analysed in this section and we consider the torque at driving shaft that is the input torque to drive the pump and the swash plate control torque. Now as we are rotating the barrel and it is there is a suction and compression of oil definitely we need to supply some torque depending on the pressure.

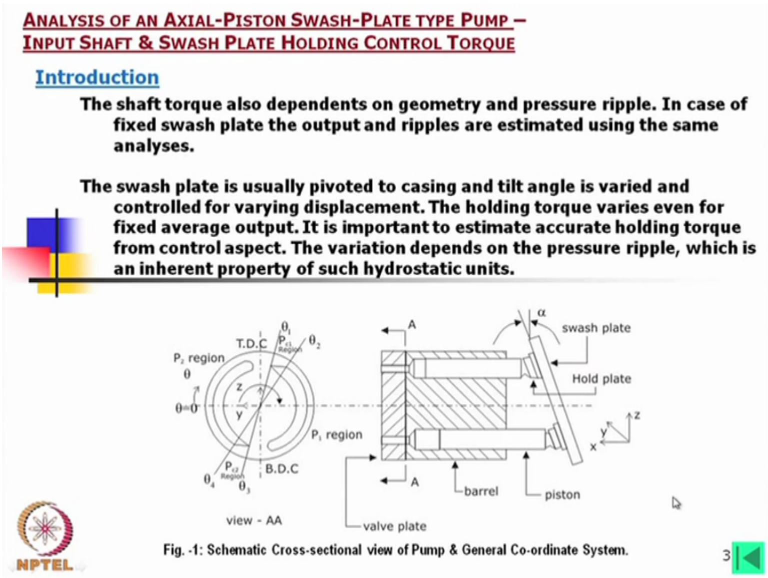
So this is clear this is not difficult to understand the driving the shaft which is driving the barrel this is having a torque which we can estimate knowing the forces acting on the pistons and swash plate. But if we say about swash plate control torque normally we will think that what it is. Now this swash plate it will be pivoted usually the pivot point is on the axis of the shaft that means the pivoting axis and the shaft axis they are perpendicular to each other but intersecting.

However, it might be also nonintersecting we will come to that later. Now the plate is tilted swash plate is tilted at an angle so definitely on the plate due to the forces due to the pressure on the pistons there will be force so definitely the swash plate will try to become in the vertical position, we need to apply a torque there to hold the pistons in this positions. For a fixed displacement pump the swash plate it is a just a plate inclined and this is fixed to the housing in that case this torque is experienced by the housing, we need not bother about this torque control of this torque only thing while we are assessing the performance we have to consider the fluctuation of this reaction torque.

However, in case of variable displacement pump where we are varying the swash plate angle definitely this torque coming on this varying mechanism and there will be also the variation I mean the fluctuation in this torque. If we rigidly hold by some mechanical means then there will be it will be like an fixed displacement pump that means we have to consider only with the variation in the torque but there will be no position variation of this swash plate.

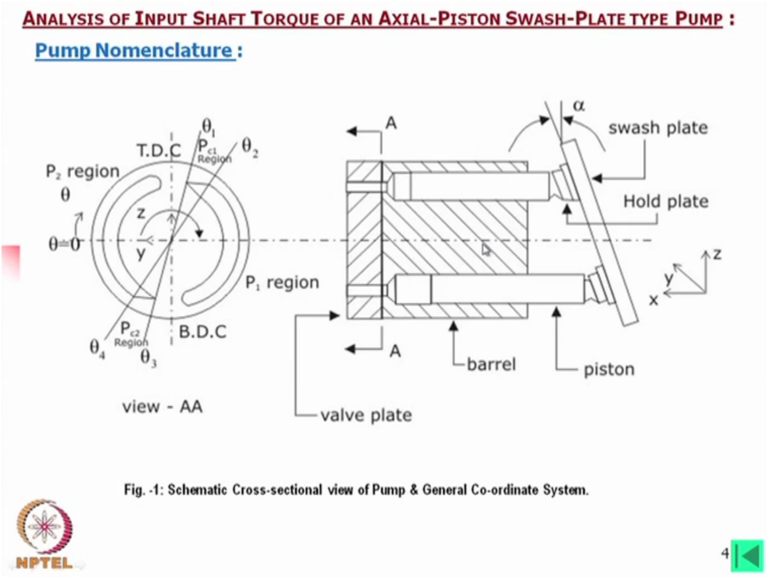
However, suppose we are controlling by the by an actuator hydraulic actuator then definitely on that on that actuator there will be vibration due to the variation of this torque. So estimating the swash plate torque is very important to control the swash plate positions of this pump of such pump of may be also motor.

(Refer Slide Time: 4:40)



The shaft torque also depends on geometry and pressure ripple. In case of fixed swash plate the output ripples are estimated using the same analysis, okay. The swash plate is usually pivoted to casing which I have mentioned and tilt angle is varied and controlled for varying displacement. The holding torque varies even for the fixed average output, it is important to estimate accurate holding torque from control aspect. The variation depends on pressure ripple which is an inherent property of such hydrostatic units. I have already discussed about this pressure ripple here we will show discussion little more.

(Refer Slide Time: 5:44)



Now looking into this pump I think next which can go into the next slides and here let me explain the components and the valve angles. Now this is already described this is barrel, this is piston and this is the here it is not very clear but this is the slipper pad and there is another holding plate which is holding all these pistons together simply a plate which is again placed between this valve plate and the swash plate by a spring.

In assembly say this swash plate will be pivoted and then there will be shaft which is passing through this swash plate usually the swash plate is having swash plate is annular ring type in between that the gap is that to pass the shaft and the swash plate is pivoted on the housing at two ends there is a not through shaft, you can imagine a ring sort of things, plate and at two ends the two small shaft or axials are connected which are pivoted in the housing.

Now in case of fixed displacement you can fix that one, in case of variable displacement one of the shaft is extended and there is another actuator is fixed so that this can be rotated. In some cases you will find away from this pivot point there is another actuation system on the swash plate which is actuating swash plate. So in housing the swash plate is put, then the drive shaft is put, then barrel with pistons from the valve plate side put inside.

Now if you simply put inside the spring which is inside the barrel fixed a by many means one of that you can put a circle you will find this end of the barrel is coming out of the housing. So if you like to cover plate including this valve plate then you will you have to put a pressure on that to bring this surface touching the barrel surface and the main housing after that you can bolt it.

Now this valve plate in some cases this is the integral part of the cover plate, in some cases this is another separate piece which can be put inside a cover but anyway you have to put a thrust. So therefore when it is in this compression mode you will find that this is always touching. In this case also when it is moving like this apparently the piston is to be pulled you will find that there is a holding plate and there is a half circular bearing sort of things which is pulling the piston out otherwise there will be detachment, okay.

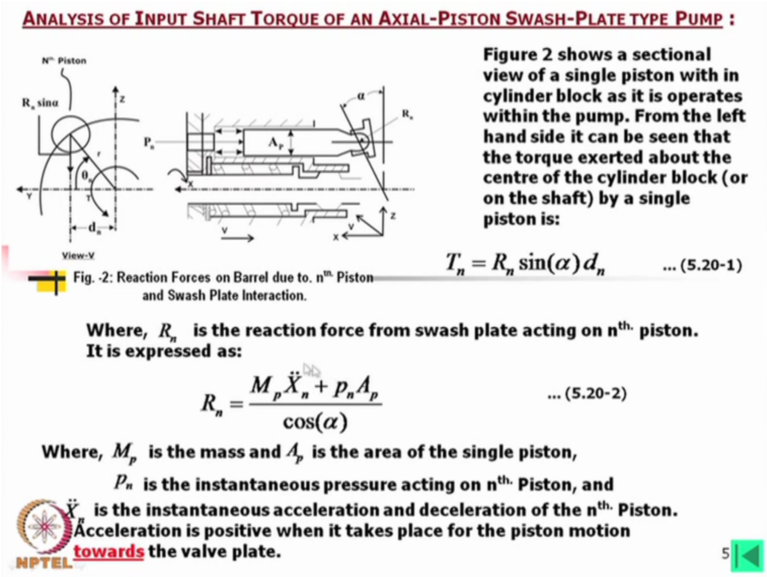
Now coming into this valve plate I have already explained what are the kidney ports and the silencing groove but we can now consider this different angles say theta is equal to 0 here, say and piston is moving the piston at that position will number 1. Suppose there is a 7 pistons it is not written that this is number 1, number 2, number 3 like this but while we are analysing we are considering one piston is one this axis that is number 1, then 2, 3, 4, 5 depending on the direction of rotations.

Now as such the angle for analysis we should divide into angles in 3, 4 zone. One is that we can have this from this point to this point the same analysis but we may consider from this point to upto this point when this kidney port end is coming to this point upto that we can analyse in one phase. Next phase gradually this area is increasing so we will consider the theta 1 and theta 2 is the silencing groove which is called transition zone that means theta 1 to theta 2 is called transition zone and then it is the P c1 one pressure that is in the suction pressure region, this is the compression zone, this is the suction pressure region.

So after this transition in suction zone then there is almost constant pressure upto theta 3 because when the leading edge of the kidney port of the barrel reaches at this point only then this phase ends so that is why we have considered the angle upto this point. And after that again another transition zone say that transition zone lasts upto the angle theta 4 and then again we are coming to the 0 position, so we can divide into few zones and we can carry our analysis.

Now here what we find this is P 1 region the suction pressure, this is P 2 region is the delivery pressure but this transition period this pressure is neither P 1 nor P 2 it is something in between depending on the as there is a trapped volume which is entering into other phase through variable orifices. So this pressure we have to consider or we must calculate this pressure build up to calculate accurately what is the torque required both in case of the shaft and also the swash plate torque.

(Refer Slide Time: 12:40)



Now this figure shows a sectional view of a single piston within cylinder block that is barrel as it is operates within the pump. From the left hand side it can be seen that the torque exerted about the centre of the cylinder block or on the shaft by a single piston is T n is equal to R n sin alpha d n, what is R n? R n is the reaction force acting. Say look at this this is there is the swash plate it is not drawn here, it is in this angle so this pad is also tilted in this angle, what is there a force is being experienced by the piston depending on the oil pressure here.

So this is definitely giving a thrust on the swash plate and swash plate in return giving a reaction force there. So this reaction force and their component in the T n sin alpha d n is the torque arm it is not given here. So we will consider this force oh I see this we are considering the shaft force you see to calculate the shaft force what we have to consider the force acting in this directions and that into the pitch circle radius, so why we have consider R n sin alpha into d n it should be d n by 2 I think, okay we will see that R n sin alpha I think not R n it will be d n by 2, d n is the pitch circle no d n is the torque arm itself, d n is the torque arm itself and R n sin alpha is the force in this directions, okay.

So I made a mistake because I was thinking that d n was the pitch circle diameter which we have used here but in this nomenclature it is considered that d n is the torque arm itself, we are considering this is as if in this plane but suppose this is here then definitely this component will be the R n sin alpha force and from here to this distance is d n so this is the torque for a single piston.

Now R n already I have mentioned that reaction force and this is instead of ith piston we have consider as the nth piston here and now we have written that R n is equal to P n into A p the P n is the pressure here at the nth chamber piston which is having the area this is not the diameter actually this is the area. So P n into A p is the pressure force acting on the pistons and we have consider the dynamic force that is with M p is the mass of the pistons and X n is the acceleration and that force this is acting in this directions.

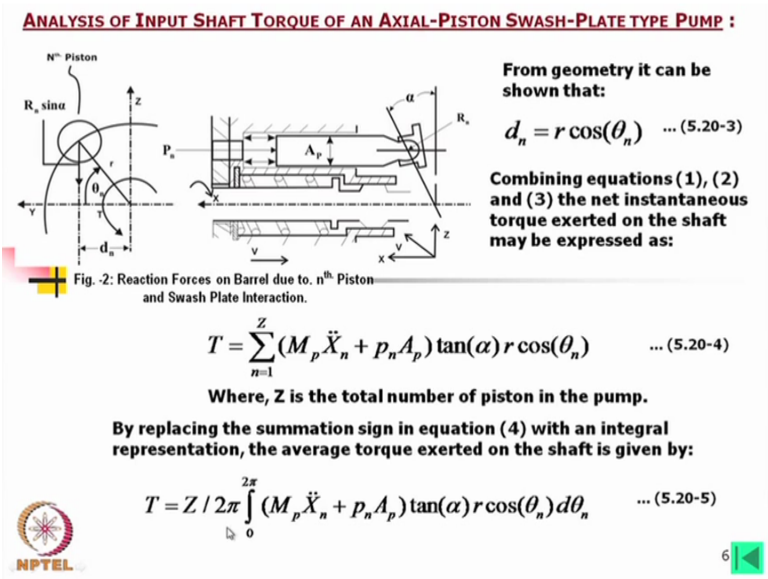
So this force will be divided by cos alpha take care of that many of us make a mistake particular while we are writing examinations we take this part correctly but we multiply cos phi but look at that this force is acting over here so to resist this force or if I think a plate in the alpha directions on that plate the thrust will be this component we have to divide this value by cos alpha, unless we give that much of thrust we cannot produce the force in this directions.

So definitely one important factor you can say so this this force is having another component in this directions which are acting at this point and as there is a force which is most undesired because this will here this barrel rapidly as well as or may not be rapidly but still this reaction force is harmful for motion of the piston and the barrel, motion of the piston inside this hole in the barrel.

So that is why we do not go for very high angle of alpha, usually 20 degrees maximum normally may be 20, 21 degree for inline piston pump this also I have discussed earlier and that is why for very high pressure we go for bend axis where we can go this alpha angle upto 45 degree but this type of piston this type of pump we should not go for more than 20 degree, okay.

So here again that x direction this acceleration is positive when it is moving in this directions positive y directions. Here we have assigned the coordinate system, okay so this is positive y for positive y X n will be positive no this is x directions so x will be positive when it is moving towards the valve plate, we must take care of that otherwise the sign will change you can feel it if we have to generate acceleration in this directions with plate inclined in this directions and the pressure force is acting in this directions then this two will be these two value there value will be sum up, just take care of that do not make a mistake there.

(Refer Slide Time: 19:58)



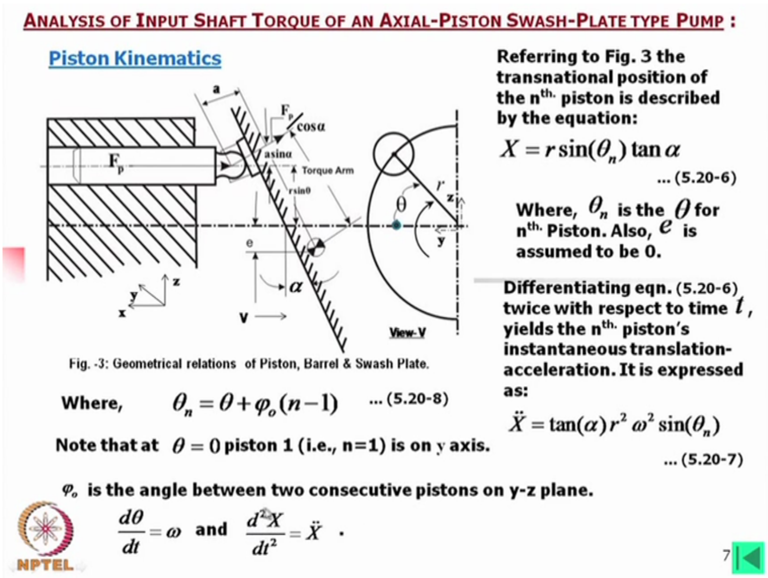
Now from geometry it can be shown that this distance is r cos theta n, okay this is r and cos theta n is the at an angle of nth piston so easily we can calculate what will be the torque arm. Combining equations 1, 2 and 3 the net instantaneous torque exerted on the shaft may be expressed as this is we have consider all the pistons which are giving the force. Now look at this in this case we have consider n is equal to 1 to Z and we have taken this summation of this.

Now one thing is there that P n is the pressure now we can consider the this pressure is the differential pressure that means for example that it is pumping well at 10 Mega Pascal, whereas in the suction side say pressure is 1 Mega Pascal say then we can separately calculate this for 1 Mega Pascal and 10 Mega Pascals or else what we can do we can take the pressure difference is equal to 9 Mega Pascal so when the particular chamber in the suction side this pressure is 0.

So this part will not come automatically this will become 0. However, this part will be there but this is again depending on the acceleration and decelerations sometimes it will help and sometimes it will I mean this will reduce the torque and sometimes it will increase the torque so this will be this will automatically take care of but I would like to mention here except for the transient except for the dynamic performance we can again neglect this part.

So for a pump or a motor if you would like to calculate the nominal torque at the steady state conditions you can neglect this part and this you can put 0 in the suction side and on the compression side for the pump this is whatever the system pressure. So in that case simply the this equation will become A p into the system pressure into this value and n is equal to 1 to Z dash which is in the compression phase in case of pump this will become very simple. By replacing the summation sign in equation 4 again we can use the integral form that we have used earlier also and average torque we can multiply this by Z and divide it by 2 Pi.

(Refer Slide Time: 23:18)



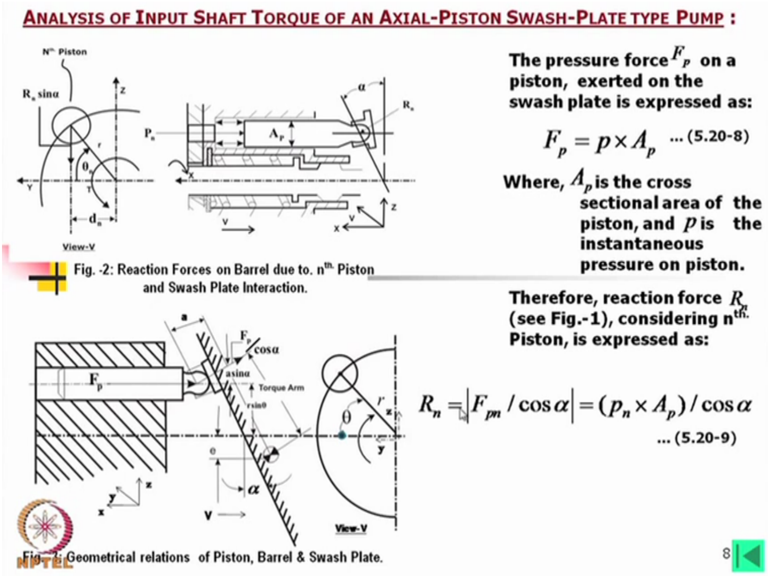
Now referring to figure 3 the translational position of nth piston is described by the equation which we have done also earlier we can write down this equation X at an instant is equal to r sin theta into tan alpha and then so theta n is theta for nth pistons also e is assumed to be 0 here look at this figure in this figure what we have shown that as if this pivot point is not intersecting this axis that means pivot of the pivot point of the swash plate is not on the axis of the drive shaft.

Now this as I told that this eccentric pivoting of the swash plate gives better performance of the pump because in the valve plate if I consider the all are ideal and slight eccentric position we give a better pressure ripple better pressure ripple for small eccentricity so this is sometimes provided. However, that is only for a particular direction other directions this will be reverse for that valve plate so we have to take care of that.

Secondly this also gives a while it is changing the swash plate angle that gives a more dynamics in comparison to if the pivot point is on the circle. So this is in special case this type of design is there is given some value but in all this analysis what I am showing we have consider e is equal to 0, okay. Now if we differentiate this one with time twice then we get this acceleration in this form, omega again we are considering this is a constant otherwise another term will come here that means if we would like to find out the transient of the pump when the pump shaft is acceleration that will be different from this one.

This we are considering the pump speed is constant steady state but we are considering the other pressure ripple within this steady state. Now theta n earlier we have defined also this we can find out in this way and theta is equal to 0 when piston 1 is in this axis. Now d theta by dt is equal to omega this we have already and so and d square X by dt square is equal to X dot which I think which is automatically understood.

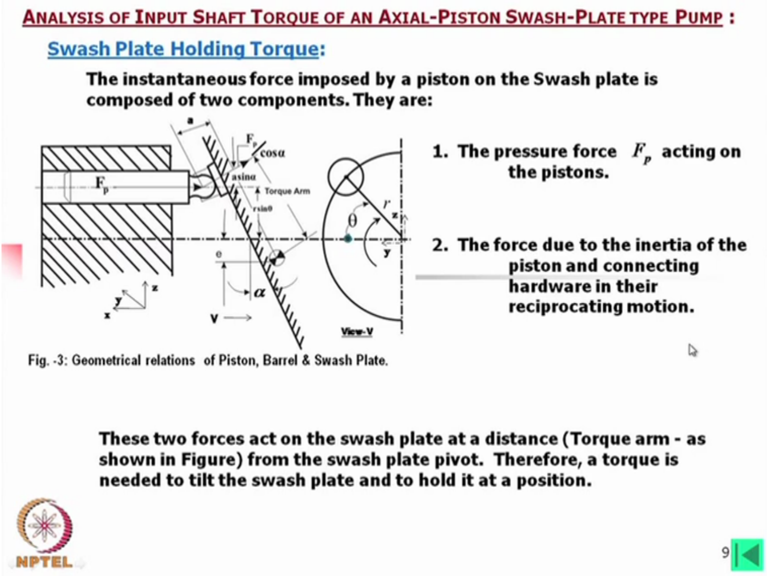
(Refer Slide Time: 26:40)



Now what we would consider we will simply calculate what is the F p what is this force F p is the force acting over there on the pistons here here it is shown this is the F p, F p is nothing but the p into A p. Now again here we have kept the p general but in case of nth piston we have to (push) put p n into A p because for a pump A p will given constant. So the force can easily be calculated from these equations.

Now R n this force now becomes at nth piston F pn whatever the force over there this force will vary depending on the pressure divided by this swash plate cos of the swash plate angle. So we can write down this equation in this form R n is equal to p n into A p divided by cos alpha.

(Refer Slide Time: 27:50)

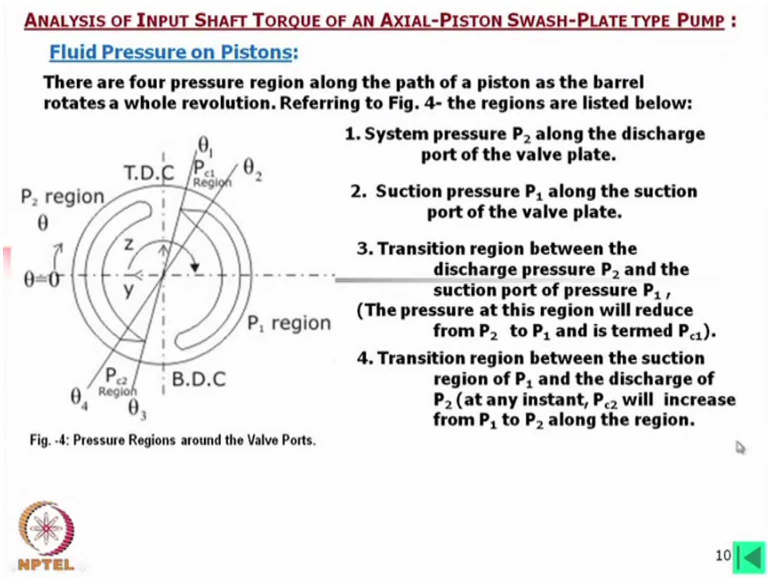


Now the instantaneous force imposed by a piston on the swash plate is composed of two components again. The pressure force F p acting on the pistons and the force due to the inertia of the piston and connecting hardware in their reciprocating motions which we have already shown. Now we are calculating trying to calculate the swash plate holding torque. Now look at this figure you see this is the force acting on the pistons.

Now for the equilibrium diagram what we can do we can remove the swash plate, we can put F p here force and R n in this direction, okay. So that we should say this equilibrium position obviously if we put R n we have to put a force over at this point also then it will be in the equilibrium and there is ofcourse there will become movement this will try to be rotated but that movement will also be balanced, this being hold held. So if we get all these three forces now which is creating a torque on this shaft that is this sin component on this but if we consider the what is torque to hold this one we simply can consider this force R n force that R n force into this torque come will give us the torque on the swash plate.

Now this R n force it is again varying depending on the piston position so that we can calculate and we can easily how much torque is coming on this shaft sorry this swash plate pivot and if we had to help this by a controller there we can also estimate how much force will be required on this controller.

(Refer Slide Time: 30:01)

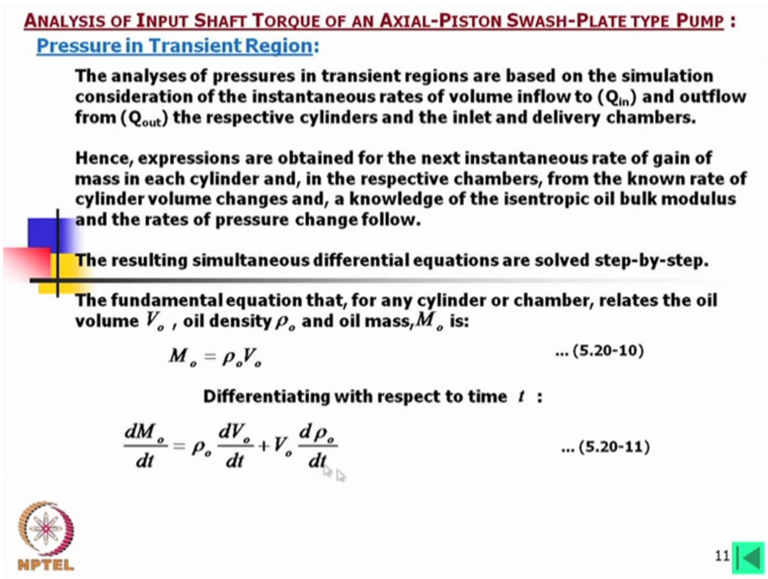


Now there the four pressure region which I have already explained along the path of pistons as the barrel rotates a whole revolution. Now referring this figure the system pressure P 2 along the discharge port of the valve plate suction pressure P 1 along the suction port of the valve plate, transition region between the discharge pressure P 2 and the suction port of pressure P 1 the pressure at this region will reduce from P 2 to P 1 and is termed P c1 P c1 just remember this term because this we are using later in the equations.

And transition region between the suction region of P 1 and the discharge of P 2 at any instant P c2 will increase from P 1 to P 2 along the region but I would say that we have apparently written that P c1 and P c2 both will be in between P 1 and P 2 but in some cases you will find that this P c2 will increase above the maximum of these two and also in another conditions this pressure may decrease below the minimum one say suction side P c1 may below the suction pressure.

Usually in positive displacement pump this is not below the nominal pressure there P 1 but in some cases it may come if it is below that obviously there is a suction head is being created but this is again the possibility is that there is a separation of the oil from the piston it is not touching the head of the pistons so this is not desired but anyway we can calculate this.

(Refer Slide Time: 32:20)

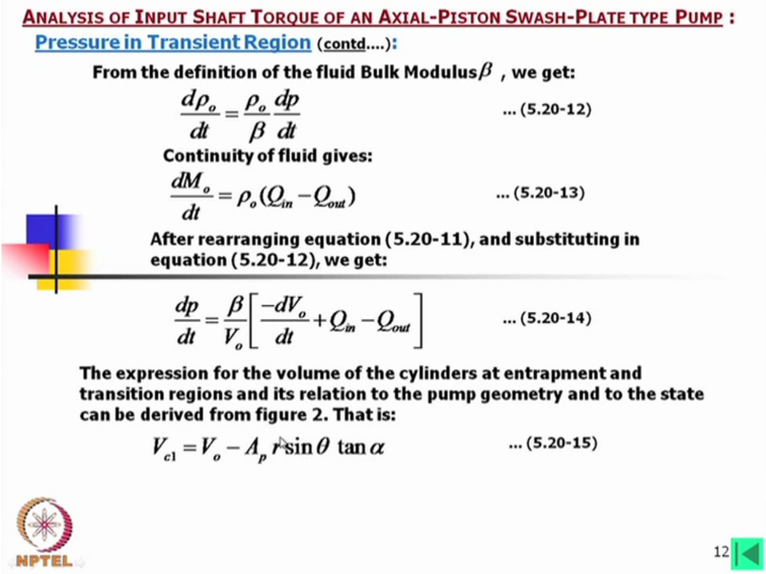


The analysis of pressure in transient regions are based on the simulation considering the instantaneous rates of volume inflow Q 1 and outflow from the respective cylinders and the yielded and delivery chambers. Hence, expressions are obtained for the next instantaneous rate of gain of mass in each cylinder and with the respective chambers form the known rate of cylinder volume changes and a knowledge of isentropic oil bulk modulus and the rates of pressure change follow.

Now this already we have shown in various form but here again I will show that how we can calculate this we should consider the bulk modulus also. The resulting simulation differential equations are solved step-by-step. Now first of all we will consider fundamental equation that for any cylinder of chamber, relates the oil volume V 0, oil density Rho 0 Rho 0 not 0 o we should say this is for the oil and the oil mass M o then we can write the here we are considering the mass of the oil note that not mass of the piston so that is why I have used mass of the oil is M o and o is the density of the oil and V o is the volume of the oil within a chamber.

Differentiating with respect to time t we will arrive into this form, okay this is not difficult to understand now this part sometimes it is 0 but still we will consider this also exists.

(Refer Slide Time: 34:30)

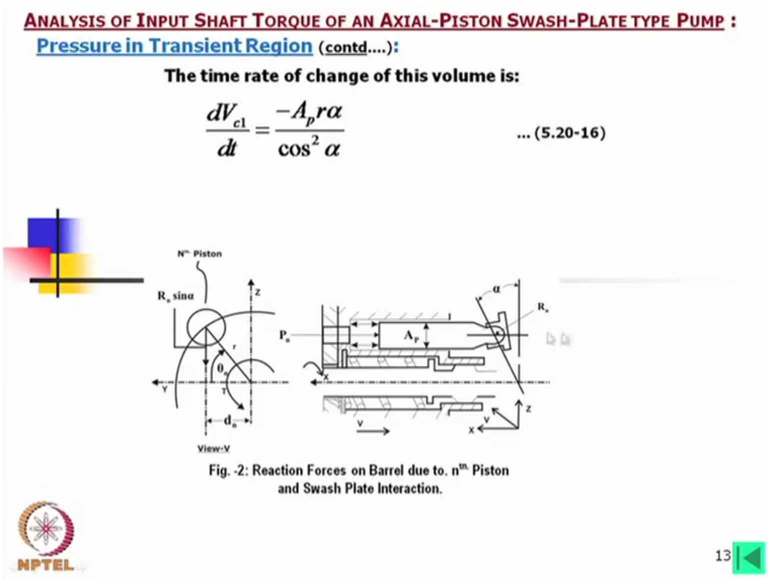


From the definition of fluid bulk modulus this can be beta can be expressed again in this form, okay this we have in earlier lecture I have shown. Then from the continuity of fluid flow we would write these equations remember this we have consider at an instant that means when we are considering these equations definitely for that time that pressure and temperature are not varying or there effect is negligibly small.

Now what we can do combining these equations we will arrive in this equation this is not difficult you can if you take a paper to write it you will be able to understand that will be arrive in this equations. Now the expression of the volume of a cylinders at entrapment and transition regions and its relation to the pump geometry and to the state can be derived from figure 2 that is V c1 V 0 A p r sin theta minus tan alpha that means this volume when this piston is moving this volume is varying so this (V 0) is the V o is the total volume when the piston is fully extended whatever volume inside that means the volume covered by the stroke as well as the ideal volume or interrupt volume inside.

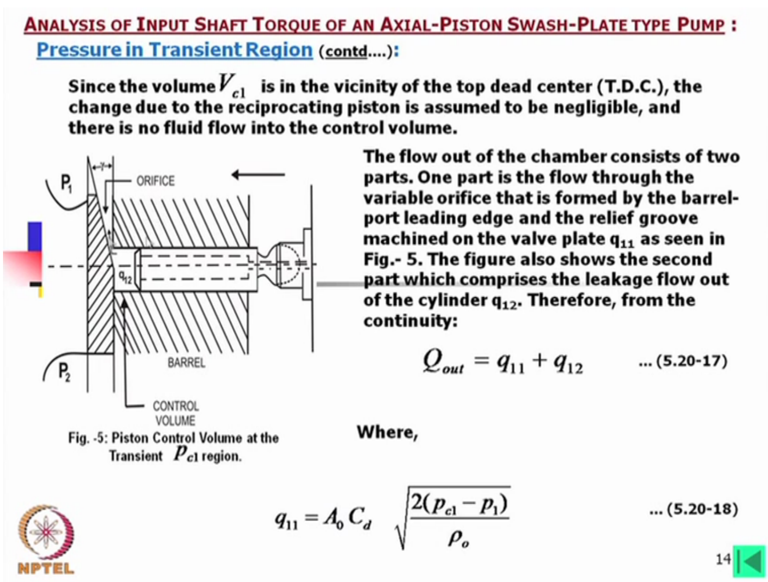
So at any instant one while we are calculating this oil we neglect sorry we subtract the portion it has moved inside so that will give the instantaneous volume of the oil.

(Refer Slide Time: 36:36)



Now we again differentiate this with time but here in this equation if you find that this is simply written in this form of this angle, how? Now what will be this change is this volume is definitely this area into the fluctuations or alpha is this angle this I would say this distance roughly this distance. So we with approximation we can write down this equation this derivation I am not showing you can go through this exercise and we will find that this V c1 dt can be expressed in this term, A p is the area r alpha is the this distance.

(Refer Slide Time: 37:48)



Since the volume V c1 is the vicinity of the top dead center remember what is top dead center and bottom dead center, V c1 is in the top dead center that means which one fully open ya no this is top dead center means fully closed one this smallest volume, V c1 is the vicinity of

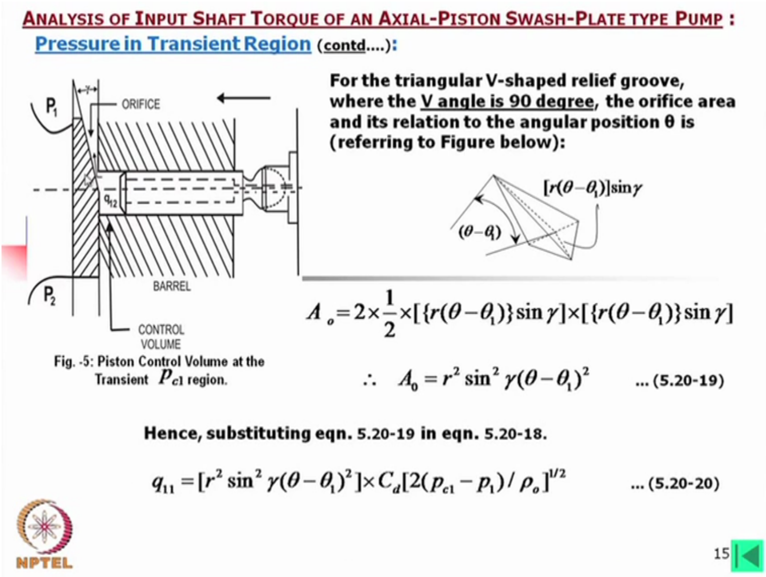
T.D.C, the change due to the reciprocating piston is assumed to be negligibly and there is no fluid flow into the control volume.

I would like to say this is we are doing it considering that the barrel port is exactly equal to this space between these two ports but if there is variation that means suppose this dead bend is more than this port definitely V c1 will be compressed. In that case you will find that there is a pressure (())(39:06) but the problem is that suppose these are critical that means both are equal still there is a leakage due to this leakage you will find that pressure may not rise upto that much or if when we try to rise then there will be leakage and there will be pressure drop one hand and the other hand if this is critical then you will find that there will be loss, there will be too much leakage from one side to another side so there we must balance this.

But for the analysis if we consider they are ideally equal then we neglect the compressibility of the V c1, only we consider this how much volume is interrupt but there they are being compressed inside that part we will neglect. Now the flow out of the chamber consists of two parts one part is the flow through the variable orifice that is formed by the barrel port leading edge and the relief groove machined on the valve plate so that is given by q ii small q 11 or 11 we should say small q 11 as seen in figure 5, this q 11 is not shown it will be here and the q 2 the figure also shown the second part which comprises the leakage flow out of the cylinder.

Now here one leakage is here, another leakage is here. So that is q 12 and this is q 11 unfortunately this is not mentioned here. So these two leakage we must consider. Now again to find out such leakages we have consider this orifice equations where we must estimate the P c1 and P 1 is the suction pressure and C d is coefficient A 0 is we should calculate the area very carefully for that situation.

(Refer Slide Time: 41:34)



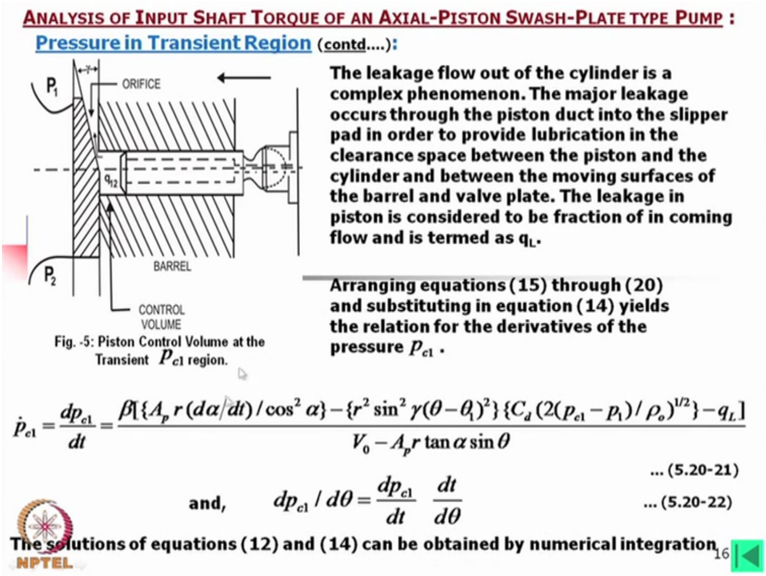
For the triangular V-shaped relief groove, where the V angle is 90 degree, I am showing this figure now which is 90 degree, this angle that means this angle is consider 90 degree this angle is 90 degree, okay. The orifice area and its relation to the angular position theta is given by we consider let us see this angle say this angle must be theta minus theta 1, theta 1 remember this is not for the piston one but for the region on the valve plate we have shown.

So theta is the position of this piston any pistons and we subtract theta 1 from theta but remember this theta, theta 1 within these zones suppose if we consider the theta is much the higher position then definitely it is not in this position but in that position if I mean if theta minus theta 1 is lying on this zone then definitely this will be the r theta minus theta 1, r is the radius if you remember this circle radius into sin gamma, gamma is this angle will be this height, okay.

Now the total area say one part area is because this is 90 degree, so this must be 45 degrees so this equal to this that means say r theta minus theta 1 sin gamma square into half is one area into 2 is the total area. So this is the area A 0 in that way this area is calculated, I was earlier mentioned that this is not the surface area say this is the barrel is on this plane on this plane of this triangle, okay. So definitely oil from the barrel it is going out or going in through this passage, so orifice area means we have to calculate this when it is on the silencing groove and there only in that zone that P c1 or P c2 will arise, okay.

So we are considering this area which is given by this equation. Now hence, substituting this into the earlier equations we get this is our the flow through q 11 that means this flow is q 11, q 2 is this.

(Refer Slide Time: 44:42)



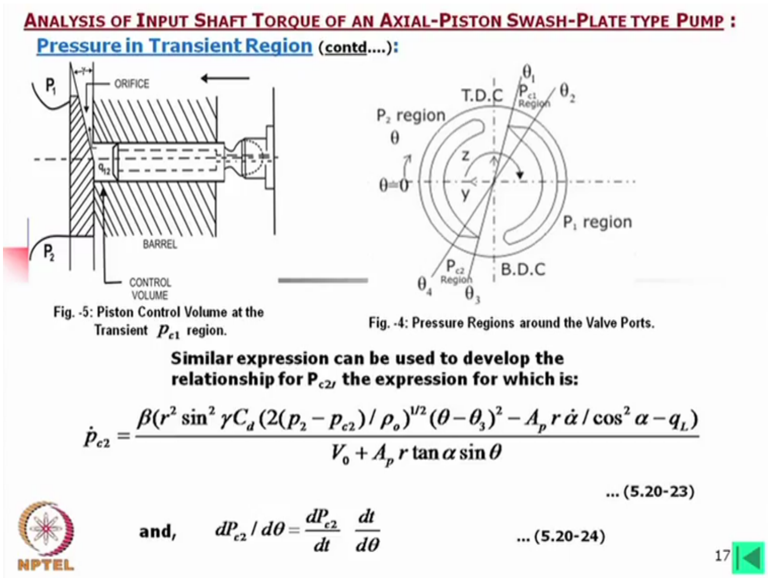
The leakage flow out of the cylinder is a complex phenomenon. The major leakage occurs through the piston duct into the slipper pad in order to provide lubrication in the clearance space between the piston and the cylinder and between the moving surface of the barrel and valve plate, the leakage in piston is considered to be fraction of incoming flow and is termed as q L, okay we must now consider the total leakage q L.

Now here what is mentioned say actually leakage is taking like this say oil is coming over here through this orifice major leakage is occurring at this required pad but this is required to lubricate this lubricate between this slipper pad and this swash plate. However, there is also a leakage through this path and there might have some other say through this region. Now this quantity is given by q L, now that is a pump characteristics. If we would like to measure separately for different region obviously we can do it but it is very much difficult.

So this q L normally which is a pump characteristics either it is given by the manufacturer or we may experimentally it is found out simply this is measured and depending on the pressure and the velocity we can have some empirical relations to find out q L. Usually there is a constant which we can put we can multiply with q L and we can we can find out what will be the q L, total flow into such factors will give us the q L.

Now if we take this time derivative of the P c1 we will find this equation is coming in this form, okay. So this will give us the pressure fluctuation with time at that transition zone. And the P c1 by d theta is this is simple we know it, so equation 12 and 14 can be obtained by numerical integration the earlier equations we can now we can find out by integrating.

(Refer Slide Time: 47:41)



Similarly for the other side where the P c2 we can have this expression in this form, okay and then where the d P c2 by d theta is given by this equation. Now after that so we have arrived here then we can have detail pressure map for the whole region throughout this region again I repeat this is P c2, then P c1, then P c1 and then P c2. So with this pressure now we can calculate whatever the force is coming on the swash plate we also know this acceleration of the pistons, we can consider that part and we know detail of the pressure, we can consider that part and then we can calculate the actual force coming on that and from that the we can calculate the arm also so at an instant we can calculate the torque but I think that we can continue this in the next lecture because this is we have to another detail part we have to calculate, okay which cannot be covered in this lecture, okay thank you.