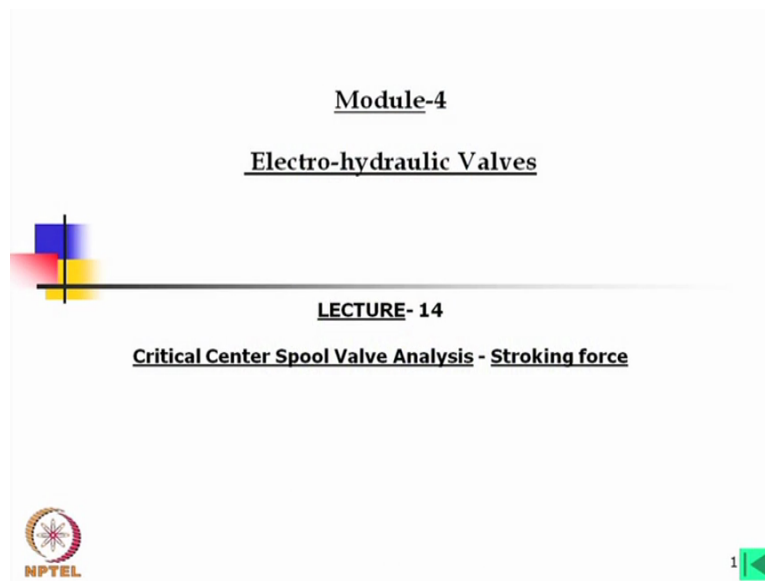


Fundamentals of Industrial Oil Hydraulics and Pneumatics
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Module 4
Lecture 14
Critical Center Spool Valve Analysis-Stroking Force

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We are continuing with module 4 Electro-hydraulic Valves and today's topic is Critical Center Spool Valve Analysis which we started in last lecture and today's portion is stroking force.


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Stroking forces

Force required to drive a spool at an instant is the instantaneous stroking force.

It has usually steady state, transient and dynamic components.

Not only at the vicinity of control point but also during spool motion from a position to other position the flow force (steady state part) dominates.



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Force required to drive a spool at an instant is the instantaneous stroking force. It has usually steady state, transient and dynamic components. Not only at the vicinity of control point but also during spool motion from a position to other position the flow force which is and steady state part of flow force dominates.

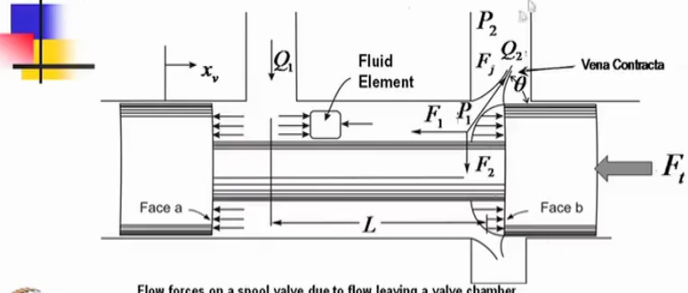
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Flow forces & static pressure forces


Flow forces on spool valves

It is named as flow induced forces, Bernoulli forces or hydraulic reaction forces.

The force induced as a result of flows through the valve orifices and valve body passages.



Flow forces on a spool valve due to flow leaving a valve chamber.



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Now we shall study first the what are the flow forces and static pressure forces. Now in this figure what I have shown a portion of spool valve with two lands and two ports. Now apparently when the right side port is closed still the left side port will remain open. Let us consider the null position is that the right side port is just closed and from there we have given a stroke x_v in right ward directions.

Now here I have assigned a force which is controlling this spool but it is not really the this force will have the direction in the shown direction, it might be opposite also why it is like that? While we are controlling the spool position it might be when it is opening we are giving the force in one direction but if it is overriding or over open then might be force is also opposite directions and particularly in a particular spool position the force may act in both the direction to keep the spool in that position.

Now first of all as I have told we will look into the flow force from the term you can understand the at the fluid is flowing definitely it will have some component acting on the spool forced component which we will try to open or close the spool and direction of motion or in the opposite directions. Now let us consider this opening this of course what it is shown here apparently it is a big but it will be very small opening.

Now if you have some idea about the orifice flow which I have discussed a little bit usually this will be in an angle this is called jet angle and if we think of the area obviously this area is rectangular if in case of full opening that means we have to consider the πd , d is the spool diameter length diameter. So it is like an umbrella flow is like an umbrella it is going outside with an angle θ which is jet angle.

Now again if we consider this area we will have an another area which is less than this which is called Vena Contracta. So we have to in fact we have to calculate the Vena Contracta area and then only we can find how much flow is there, pressure drop, etc. Now the flow is going out like this it is coming in now here it is full open we do not have any angle, we do not have any component of this flow in this direction.

However, in this case as the flow is going out it will definitely have a force inside this chamber. Now this force the fluid is being thrown in these directions, so the force is acting in these directions F_j now which will have definitely two components with respect to the axis of this spool and in the lateral direction also, okay. Now here is a good question that this force the oil is going out what will be the direction of the force, what is the component of the force inside this spool.

This is the for if the force is like that definitely the reaction force is acting over like this acting in this direction sorry acting in this direction. It is something like that a runner when he is starting if he there is a stopper on which he puts his legs and what is the reaction force on that it is in the backward direction the direction is moving. Similarly this force is having the

direction opposite to the motion of the flow that means flow is going like this and this force will act like this, interestingly you will find that this is helping in controlling this spool valve.

Now with this introduction I would say this is sometimes called flow induced force even it is called Bernoulli forces in the name of scientist Bernoulli and also it is called hydraulic reaction forces. Now in this portion what we are going to study that is the steady state part that means we are not any fluid transient here, we are considering the steady state flow is going out a jet is going out in these directions and we are studying what will be the forces there inside the spool.

Now look at this another thing is that we have taken a length L, how do we measure this length? This length is that middle of this orifice to middle of the orifice opened, so here if it is a x v so this must be this distance must be x v by 2 and this is the L. So if we know the geometry of this spool we will be able to calculate this L also. Now pressure P 1 we can measure here, this is inside and P 2 is the outside or we can say we should say that this section is section 2 at here as if section 1 and we can consider the pressure here to here is more or less same although there will be a flow but there is very little restrictions so P 1 here and pressure here also will be P 1, okay. Now the force induced as a result of flow through the valve orifices and valve body passages, which I have already explained.

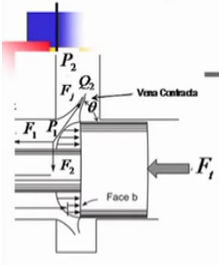
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Flow forces on spool valves (Contd...)

Referring to Figure, the steady-state flow forces (jet), F_j can be expressed as,

$$F_j = \rho V \frac{Q_2^2}{A_2 V} = \frac{\rho Q_2^2}{A_2} = \frac{\rho Q_2^2}{C_c A_0} \quad \dots 4.14-1$$

Mass Accn.



where Q = volumetric flow rate through orifice m^3 / sec
 V = control volume m^3 / sec
 $A_0 = w x_0$ = orifice area m^2
 C_c = Contraction coefficient, (dimensionless)
 C_v = Velocity coefficient, (dimensionless)
 $C_d = C_v C_c$ = Coefficient of discharge, (dimensionless)
 ρ = mass density kg / m^3
 w = Area gradient of orifice ($= \pi \times \text{spool land dia at orifice}$) m^2 / m
 A_v = Area of valve land, m^2
 θ = Jet angle, degree.

NPTel

Now referring to this figure the steady state flow force that is jet, F_j can be expressed as, this is ρ into V the V is the velocity of the fluid and then Q^2 is the flow rate divided by

A_2 is the area here into V is the velocity these are the standard equation which we have not derived but this is well known equation and this we have to accept at the present moment.

If we further equate this one it will ultimately will be in this ρQ^2 square by $C_c A_0$ where C_c is the coefficient of this is a component of coefficient of discharge, so this is a coefficient of velocity and velocity part and there is another component which is the velocity of sorry the coefficient for the area contraction, okay.

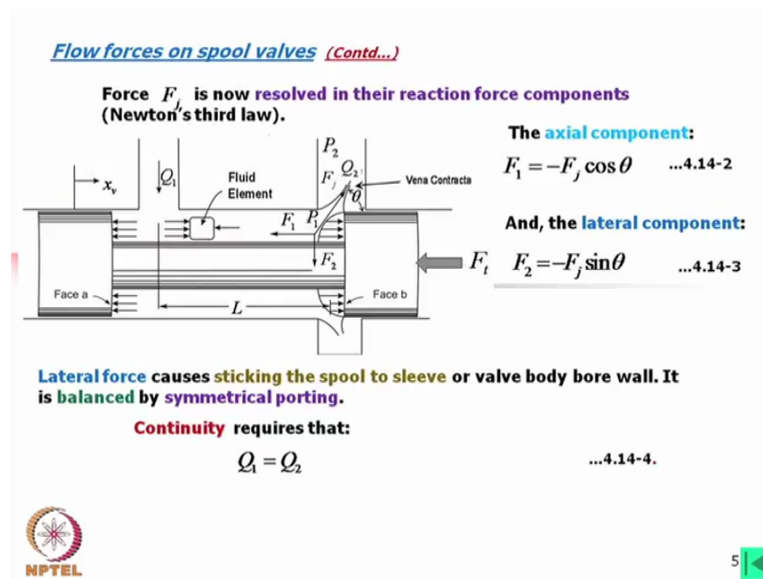
Now if we look further into this then Q here Q^2 is the flow through the section 2 in general Q will be volumetric flow rate through orifice, okay this is. I have also shown a set of units this is for SI units V is the control volume sorry this is not the velocity control volume it is also expressed in meter cube per second that means a volume flowing through this control volume, okay.

A_0 is the orifice area, now here w is the area gradient that is πd and x_0 is the motion general this is in general terms, whereas actually we have given the x_v is the motion, this is general area. C_c is the this is called better name is contraction coefficient and C_v is the velocity coefficient, okay whereas C_d is equal to $C_v C_c$ coefficient of discharge. This means C_c is the area contraction coefficient and C_v is the velocity coefficient, okay.

Then ρ is the mass density this is Kg per meter cube and w is the area gradient I have already explained it might be if it is not full then instead of π we will use some other actual angle of opening we should use there an angle of opening divided by 2 into the diameter or angle of opening into the radius of the spool land diameter, right. A_v is the area of valve land, this is in meter square and θ is jet angle, this is in degree.

Now if we closely observe this definitely this is the mass rate it is volume into the mass density so this will be the mass of the fluid flowing and if I look into this acceleration part. So mass into accelerations that gives us the jet force.

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Now force F_j we will now resolve into two components one is the direction of the axis F_1 and another is F_2 , the axial component F_1 can be written as $F_j \cos \theta$, why it is minus it is if we resolve this force that will have the component in this directions but this is we are considering what will be reaction due to the jet force so the minus sign is coming over there. So clearly this will be $F_j \cos \theta$.

Similarly the lateral component F_2 will be given by minus $F_j \sin \theta$. Now interestingly lateral force if we consider one lateral force then this will definitely will try to stick this spool to the sleeve it is called sleeve on which this is put usually there will be valve body and then there will be a sleeve, so it will touch the sleeve in case the sleeve is not there, directly it will put a pressure on the bore wall of the valve body.

However, it will be balanced if you look into this it is like an umbrella as I told the fluid like an umbrella and in that case it is everywhere the F_2 is acting towards the centre of this spool centre line of the spool. So it will automatically cancel and also this means that at the position here itself it is being cancelled. Now however, if there is some imbalance that is taken care by the symmetric porting and (15:59) and design we have to carefully design so that this F_2 is cancelled.

But in some cases due to say manufacturing error and other thing some small amount of forces may disturb this spool motion then that will be the additional force which we have calculate separately. Now we shall now consider the continuity equation which gives Q_1 is equal to Q_2 , whatever flow is coming in the same flow amount has to go out through the

other port it is completely we considering here assumed that it is completely incompressible fluid, okay.

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Flow forces on spool valves (Contd...)

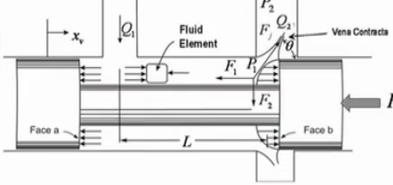
Therefore, using orifice equation:

$$Q_1 = Q_2 = C_d A_0 \left[\frac{2}{\rho} (P_1 - P_2) \right]^{1/2} = C_c C_v A_0 \left[\frac{2}{\rho} (P_1 - P_2) \right]^{1/2} \quad \dots 4.14-5$$

Substituting in eqn. 4.14-1:

$$F_1 = \frac{-\rho (C_c C_v A_0)^2 \left[\frac{2}{\rho} (P_1 - P_2) \right]}{C_c A_0} \cos \theta$$

Therefore, F_1 along the shown direction is finally:

$$F_1 = 2 C_d C_v A_0 (P_1 - P_2) \cos \theta \quad \dots 4.14-6$$


IMPORTANT: This force will try to close the port.

NPTel

Now what we can write so we now use orifice equation and we can write Q_1 is equal to Q_2 because this is same, this is coefficient of discharge into A_0 , A_0 again I am considering the area for the orifice area has open, then 2 by $\rho P_1 - P_2$, P_1 is the pressure inside, P_2 is the outside of this orifice this if you remember the figure. Now this is again we are resolving into C_c and C_v , the contraction coefficient and velocity coefficient, okay.

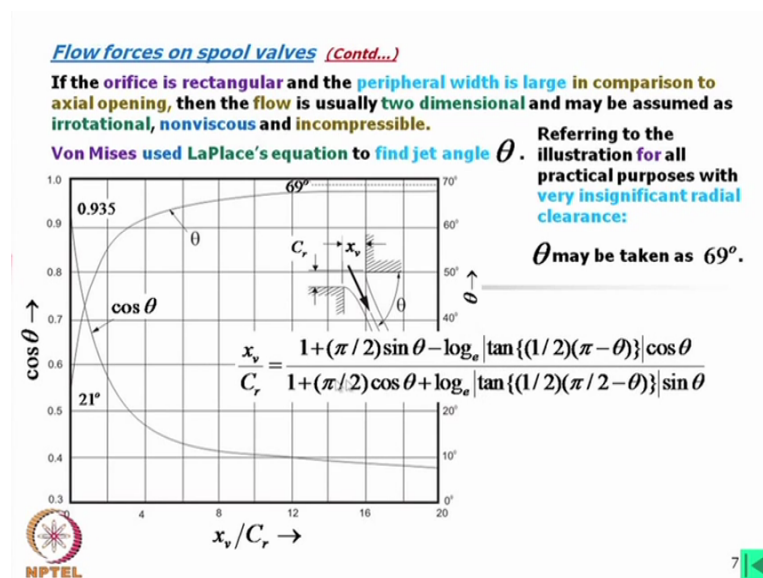
Now this we substitute in equation 4.14 this first equation which we consider for the F_j , then from there we can calculate that F_1 is equal to $C_c C_v A_0$ whole square 2 by $\rho P_1 - P_2$ divided by $C_c A_0$ minus ρ , this derivation I have not shown but if you substitute there automatically you will arrive into this, okay. Now therefore F_1 along the shown direction is finally if we further equate this one then we will have $2 C_d (C_c) C_v A_0$ minus $P_1 - P_2$ into $\cos \theta$ here of course this is minus sign will be there for the direction we have taken, okay.

So for our for the clarity again we can see this figure this $P_1 - P_2$, P_1 will be higher than P_2 and we can write down this equation if we know this jet angle. Now this jet angle is not known rarely but the valve orifices are designed in such a way that from the standard experimental data we can assume what might be this angle we will see that. Now one important observations the force F_1 we are interested in force F_1 because F_2 is being cancelled.

So this force will try to close the port why it is like that if you look into this if there is a inside a force is acting like this then while this oil is going out in general we may be in the impression that as the oil is going out probably it will try to open the port further, okay but it is really not like that. The force component if we mathematically what we have analysed that will in fact we try to close the port this force will act in this directions and it will try to close this force.

This means that while we are opening a port we are giving say suppose we are opening the port in the rightward directions. So we have to ultimately we have to give force from left to right but due to the flow force the force is being opposed that means if we leave that force it will be automatically closed that means it is helping in controlling, okay.

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If the orifice is rectangular and the peripheral width is large in comparison to axial opening, that is x_v , then the flow is usually two dimensional, okay and may be assumed as irrotational, nonviscous and incompressible which already we have assumed. Now what is two dimensional in this case this means that whatever the flow is going out that is having the motion in the along the axis and along the lateral directions there but if we come inside or any other places the fluid is not having any tangential motion, there is no rotational motion and at steady state we are considering the steady state force at steady state this is there will be no transient that means the flow is laminar no turbulent flow is there, if there is a turbulent flow due to that there will be forces will be different we are not considering that at the present moment.

Now the Von Mises, Von Mises is a scientist he used Laplace equations to find jet angle and he plotted a very interesting curve this was experimentally verified and then he proposed that how we can find out this jet angle for any positions. Now here in this direction what we find this is x_v by C_r what is x_v , x_v is the spool displacement and C_r is the radial clearance that means this is a geometric dimensions which we can measure which is known value and x_v is also known because we are giving this motion to the spool displacement to the spool.

Now this is in this directions we have put the $\cos \theta$ and this is the curve for the $\cos \theta$ and this θ angle for this rectangular type port rectangular type spool valve. So what we find normally if this is x_v by C_r is in this range it is 20, C_r how much I told that clearance is about 25 micron, this is 25 micron. So if usually this spool opening for practical purposes when we open it so it will be 20 times than this clearance has to be, while we are opening at that stage definitely it is less but for practical workable zone it will be 20 or more but for that what we find that θ is almost stable and very close to 70 degree and which may be for all practical purposes it can be taken as 69 degree.

However, if we take 68 degree, if we take 70 degree the effect on that this equations will be not very much, it will very close to that but in practise we all consider this angle is 69 degree you can see this what is x_v here what is C_r here and for that if we consider now this is it is 69 degree then we consider the insignificant radial clearance that means what we have 25 micron is very small and we will consider the 69 which I have explained already.

And here also he proposed this x_v by C_r relations with this some empirical relations which also in case if you do not have this graph from where from this practical value you can calculate also θ of course it is not an easy task to calculate this θ but it is possible still it is possible we can calculate this θ .

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Flow forces on spool valves (Contd...)

At small orifice openings i.e. at the beginning of opening θ is close to 21°

The steady state force then expressed as:

$$F_1 = 2 C_d C_v w (\Delta P) \sqrt{x_v^2 + C_r^2} \cos \theta \quad \dots 4.14-7$$

However, experimental results show higher values than estimated values and stiffer curves near the null point, by using above formulae.



A small orifice opening at the beginning of valve opening we should say valve opening theta is closed to 21 degree which I have shown if you observe while the valve is opening jet angle is almost 21 degree this is of course for the rectangular port. Now then for that this F_1 can be expressed like this, this is again not we are not considering the direction but the magnitude can be taken like this, where $\cos \theta$ can substitute 21 degree.

However, experimental results show higher values than estimated values and stiffer curves near the null point, that means what we have considered this curve this will be more stiff and may be this value will give some higher values but this null position is only at the beginning and they are usually we are putting their input but we are not controlling at that position, if we have to control of course that is also will be taken care of.

So this force whatever there this may not have much I mean ideal value from this but still we can use this and this will be overcome at the beginning there will be higher forces but this we can overcome (sorry this is there is some defect).

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Transient flow forces:
Transient flow force F_3 due to fluid mass is expressed as:

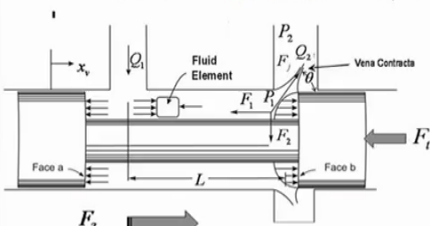
$$F_3 = Ma = \rho L A_v \frac{d(\theta_1 / A_v)}{dt} = \frac{\rho L d\theta_1}{dt} \quad \dots 4.14-8$$

Where, L is damping length & A_v is the cross sectional fluid area.

Now consider eqn. 4.14-5

$$Q_1 = Q_2 = C_c C_v A_0 \left[\frac{2}{\rho} (P_1 - P_2) \right]^{1/2}$$

Obtaining dQ_1/dt with $A_o = w x_v$:

$$F_3 = LC_d w \sqrt{(2/\rho)(P_1 - P_2)} \frac{dx_v}{dt} + \frac{LC_d}{\sqrt{(2\rho)(P_1 - P_2)}} \frac{d(P_1 - P_2)}{dt} \quad \dots 4.14-9$$


NPTEL

Now transient flow force we are now coming to that transient flow force, what is so far we have considered. Let us consider the laminar flow and due to this laminar flow there is a steady state flow force we have considered but there will be transient flow force also that means while the spool is moving the oil is moving inside and due to this there will be also the another transient force, another force which is transient force which is expressed by F_3 and this F_3 is equal to mass here the fluid mass into acceleration this is the general form I have written and that mass and acceleration can be written can be expressed in this form.

Here that is the mass density and then this is the length the what length we have considered that is the fluid inside this spool chamber and A_v is the spool land area and then this will be not theta this will be Q , this will be Q , dQ_1/dt by A_v , A_v is the spool area so flow rate divided by area means it is giving directly the velocity divided by dt so this is given the acceleration part.

So please note that it is not theta 1, this will be Q_1 so mass into accelerations, okay. Now F_3 say fluid is flowing like this so we can say this F_3 this transient flow is acting in this directions. Now where this L length is called damping length, this has great role on the valve performance because if we make it small then this length will be small and this force will be small, okay.

If we make this is large then this damping length will be higher and this force will be also higher. A_v is the cross sectional fluid area we have consider but you can say as well this is the land area because to consider the volume of the fluid we have written like this but

originally we have taken A_v is the area of this land and this is ring area we should consider this ring area, okay A_v is the ring area.

So now we consider again this Q_1 is equal to that continuity which gives these equations and then what we are doing we are obtaining $\frac{dQ}{dt}$ and $\frac{dV}{dt}$, I have already told that this will be $\frac{dQ_1}{dt}$, $\frac{dV}{dt}$ not θ_1 . So with A_0 is equal to w into x_v in reality we have given this displacement so our area will be orifice opening area will be w into x_v . Now this becomes if we remember these equations then this will be we are differentiating like this.

So L is there L is there and Rho is there but you see under root it is coming 2 by $Rho^{P_1 - P_2}$ $\frac{dx_v}{dt}$, okay. A_0 we have put is equal to $w x_v$, so while we are differentiating in this equation first part is the $\frac{dx_v}{dt}$ by $\frac{dV}{dt}$, okay. Now for the second part we do it $\frac{d}{dt}$ by 2 by $Rho^{P_1 - P_2}$ to the power half, so that considering this as a function first of all whole thing minus half minus 1 so this will be minus half so it will come into denominator and this half will come over here 1 by 2 , okay so 1 by 2 and then this will come in the denominator and then $\frac{d}{dt}$ of 2 by $Rho^{P_1 - P_2}$, so this is a constant so this will be 2 by Rho here so 2 by half I mean 2 into half that means 2 by 2 will cancel and then outside there will be a Rho that Rho if you bring into inside then that will be root Rho square which will cancel with 1 by Rho 1 by Rho will cancel here so ultimately it will become 2 Rho you see we initially we can consider here 2 by $Rho^{P_1 - P_2}$.

But while we are differentiating this then 2 by Rho is coming over here so that Rho if you bring inside it will become 2 Rho , just you can go through this exercise and one 2 here and for differentiation half here that is that two are being cancelled. So this portion is correct and then we can write that this will become $\frac{d}{dt} P_1 - P_2$ by dt , in fact in the book this expression is not correctly written so I am explaining here, this is from the Merrit book if you follow that it is not correctly written there.

So you should be careful about these equations but anyway if you agree with this from with this equation and differentiating this one we are getting F_3 is equal to like this.

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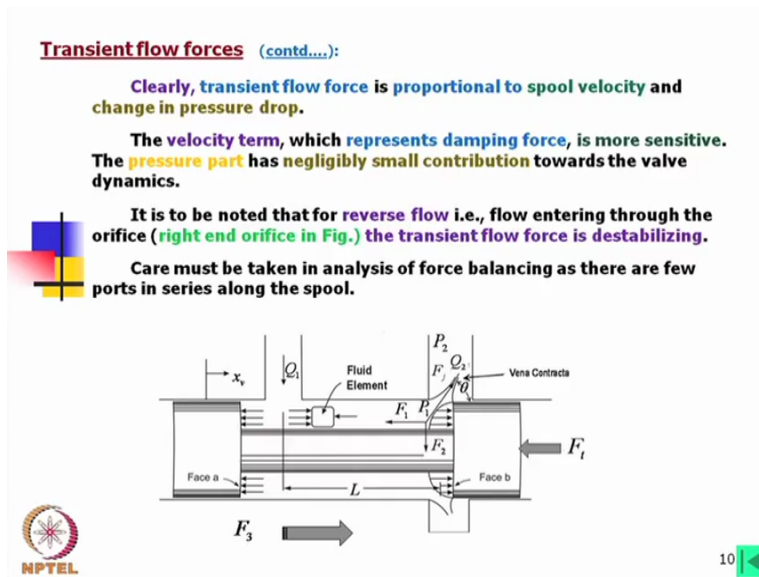
Transient flow forces (contd....):

Clearly, transient flow force is proportional to spool velocity and change in pressure drop.

The velocity term, which represents damping force, is more sensitive. The pressure part has negligibly small contribution towards the valve dynamics.

It is to be noted that for reverse flow i.e., flow entering through the orifice (right end orifice in Fig.) the transient flow force is destabilizing.

Care must be taken in analysis of force balancing as there are few ports in series along the spool.



Now again we are still continuing with transient flow. Clearly transient flow force is proportional to spool velocity and the change in pressure drop what we have written there, spool velocity means the total volume and depends on the what velocity the spool is moving the same velocity the fluid is also moving. The velocity term which represents the damping force the velocity term in that equation represent the damping force is more sensitive but the pressure part has negligibly small contribution towards the valve dynamics.

The equation what we have derived earlier there we will find that one velocity term is there which is dominant than the pressure part, okay. It is to be noted that for reverse flow that is flow entering through the orifice the right orifice the transient flow force is destabilizing that means if the flow comes from the other side than this transient force will disturb this spool in such a way it may become uncontrollable, you can just visualize this thinking over the when the flow is coming inside it.

Care must be taken in analysis of force balancing as there are few ports in series along the spool. So we are here considering one input that is again not which is a as if this opening this remains always open, so there is no jet angles and other things straight away the oil is coming, this is the left hand and we have consider only one port in the right hand side and then we are analysing the force.

But while you are taken a real spool and we find there are few ports for each and every port we have to analyse how much force is there and with correct direction, correct dimensions we have to add or subtract the forces to get the actual forces acting on the spool.

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Stroking force:

Referring to the Fig. let x_v is positive for orifice 1 & 3 the total force opposing the spool motion is expressed as:

$$F_R = 2C_d C_v (\cos \theta) w x_v (P_s - P_1) - L_1 \rho \frac{dQ_1}{dt} + 2C_d C_v (\cos \theta) w x_v P_2 + L_2 \rho \frac{dQ_2}{dt} \quad \dots 4.14-10$$

For symmetrical and matched valve,

$$F_R = 2C_d C_v (\cos \theta) w (P_s - P_1) x_v + (L_2 - L_1) C_d w \sqrt{\rho(P_s - P_1)} \frac{dx_v}{dt} \quad \dots 4.14-11$$

The pressure derivative part $\frac{dP_L}{dt}$ terms are negligibly small and not considered.

NPTEL

Now what the flow forces is having two components one is that steady state and transient, but while we are calculating the stroking force we have to consider also that spool is having mass so there will be force due to the movement of the spool when it is particularly accelerating when it is accelerating not in the if it is moving in the constant speed we do not have force but when it is accelerating then we will have the accelerating force.

Now let x_v is the positive the what are the positive directions are given for orifice 1 and 3, okay. The total force opposing the spool motion is expressed as we have we are considering the critical centre valve, so again we have come back to the original figure which we in the I described in the last lecture then we are considering the flow through orifice 1 and 3. Then if we carefully write down this equations then the actual equation it will be something like this, you can see this what we consider as L in this case it is L_1 and L_3 , L_1 and L_3 or may be L_4 which is again equivalent to L_2 so that is why it is written here L_2 , okay.

So if you now carefully write down these equations there then it will come like this. So for one it is plus and other it is minus because of the direction of flow, so you should carefully look into this equation consider the earlier equations what we have derived then this you take the damping length and with this damping length you look into the direction of the flow and if you write this equation for the two spool sorry two ports we are showing for one port now this is for two ports.

In this case for port 1 this part is minus, so this is the transient part and where this is a plus that means one opposing the other. So flow is going inside in this case flow is going inside so

Then for symmetrical and matched valves we can finally we get this equation is coming like this, this is the F_R , F_R means we are considering the total force due to the transient flow and due to the steady state flow. Now here the pressure derivative part dP/dt is negligibly small and consider because when we are using this derivations this part will also come which we have already neglected and here we have mentioned this we are not considering this part this is (40:58) because with the time this change in pressure is negligibly small although there will be but in comprising to this force what we have calculated you if even consider that is very small we need not consider this.

Stroking force (Contd...):

Now the **equation of motion of the spool** is as follows:

$$F_j = M_s \frac{d^2 x_v}{dt^2} + B_f \frac{dx_v}{dt} + K_f x_v \quad \dots 4.14-12$$

Where, M_s = Mass of the spool.

And, comparing with eqn. 4.14-11:

The damping coefficient B_f is expressed as:

$$B_f = (L_2 - L_1) C_d w \sqrt{\rho(P_s - P_1)} \quad \dots 4.14-13$$

and the spring coefficient (spring rate) K_f is expressed as:

$$K_f = 2C_d C_v (\cos \theta) w (P_s - P_1) \quad \dots 4.14-14$$

The diagram illustrates a fluid element of length L within a valve spool. The spool is displaced by a distance x_v to the right. The fluid element is bounded by Face a on the left and Face b on the right. Pressures P_1 and P_2 are indicated at the ends of the fluid element. Forces F_1 and F_2 are shown acting on the fluid element. A fluid element of length L is shown between two faces, Face a and Face b. The spool is displaced by x_v . Pressures P_1 and P_2 are shown at the ends of the fluid element. Forces F_1 and F_2 act on the fluid element. A fluid element of length L is shown between two faces, Face a and Face b. The spool is displaced by x_v . Pressures P_1 and P_2 are shown at the ends of the fluid element. Forces F_1 and F_2 act on the fluid element.

Now what we are considering the equation of motion of the spool now here this is I guess this will not be F_j because F_j we have considered for jet angle, this is the force due to the force due to the spool motion. M_s is the mass of the spool, so mass into acceleration due to this spool movement so that force plus there will be $\frac{dx}{dt}$ so this is the damping force fluid damping and here there is the here is the spring force, please note it we have considered the spring force that is K_f into x_v but this spring we are considering the fluid spring not the mechanical spring which is also there in the spool valve normally it is there we are not

considering the spool mechanical part here so far but it can be considered, it can be combined together.

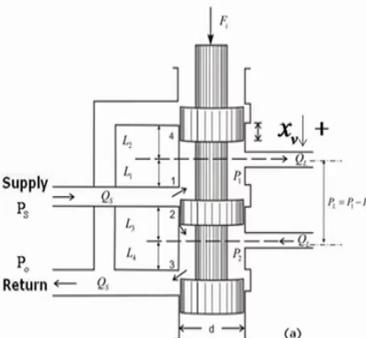
So this is you can say damping force damping part, this is the mass acceleration part and this is the spring force. So you can looking into this now what we will do this is the $M \ddot{x}$, what we will do we will compare with equations 4.14-11 that is the what equation we have developed this force considering fluid etc. Now with this if we compare then we will find that damping coefficient B_f is this part, remember these equations I am not writing these equations but in that equation you will find this is this into $\frac{dx}{dt}$ that part is there.

So we can say that damping coefficient is nothing but these equations here what is the L_2 and L_1 which we call damping lengths and C_d is coefficient of discharge and this is the area gradient, this is the mass density and then this is the pressure difference P_s we are considering the system pressure here because P_1 is the inside pressure there and similarly if we compare with the other part that where the x_v part is there, so spring coefficient will be like this this is the spring rate, okay.

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Stroking force (Contd...):

Analyses show that:



The steady state flow force acts as centering spring on the valve.

The transient flow force acts as viscous damping.

Both quantities are non linear because of changes in P_L .

If, $L_2 < L_1$ the transient flow force is negative and may cause valve instability.

Therefore, in design it is maintained that $L_2 \geq L_1$.

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Now this analysis shows that this steady state flow force acts as centering spring on the valve, we are not considering a the mechanical spring, the fluid spring that steady state part it is apparently trying to centre the valve because it is opposing the opening, okay. The transient flow force acts as viscous damping. So definitely L_1 , L_2 has great role about the spool motion both quantities are nonlinear because of the changes in load pressure, these two coefficient that is B_f , K_f as well as the force will be nonlinear, in fact these valves are servo

valves are highly nonlinear only what we do at the vicinity of the operating point we will linearize and then we are trying to apply the control there, okay.

If L 2 look at this figure, L 2 is this portion is less than L 1 the transient flow force is negative and may cause valve instability, that means in designing if you carefully look into the equations once again you will find the if L 2 is taken less than L 1 transient flow force is negative. So one thing is that we can take the L 2 and L 1 is equal to L 2 equal to L 1 for that it will become 0, there will be no transient force in that case equating total equation but it is again not good because there should have some damping too much damping is not desired again if there is no damping it will be also a problem, so somewhere we have to compromise usually L 2 is taken slightly more than L 1.

Therefore, in design it is maintained that L 2 is either equal to or greater than L 1. In normal practice we take L 2 is more than L 1, (L 1 is equal to L 1) sorry L 2 is equal to L 1 can be taken but we have to be more careful about the control of the spool force.

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Stroking force (Contd...):

Now the "damping ratio" δ_s may be defined as:

$$\delta_s = \frac{B_f}{2\sqrt{K_f M_s}} \quad \dots 4.14-15$$

Substituting the expression of B_f and K_f we get:



$$\delta_s = \frac{(L_2 - L_1) C_d w \sqrt{\rho(P_s - P_L)}}{2\sqrt{2 C_d C_v w (\cos \theta) (P_s - P_L) \times M_s}} \quad \dots 4.14-16$$

$$= \frac{(L_2 - L_1)}{2\sqrt{2 \cos \theta}} \sqrt{\frac{C_d \rho w}{C_v M_s}}$$

Finally,

$$\delta_s = (L_2 - L_1) \left\{ \frac{1}{2} \frac{C_d \sqrt{\rho}}{\sqrt{2 C_v \cos \theta}} \right\} \sqrt{\frac{w}{M_s}} \quad \dots 4.14-17$$

It is independent of pressure $[(P_s - P_L)]$.

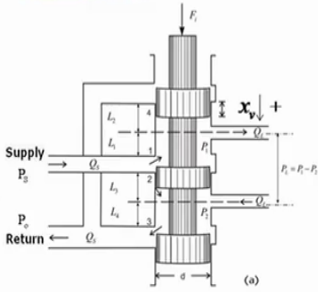
Now the damping ratio δ_s may be defined as B_f by $K_f M_s$, this is also standard it is called damping ratio. Now this damping ratio we can substitute these two in this equation and we can find that damping ratio is expressed by L_2 minus L_1 $C_d w$ under root ρP_s minus P_L and in the denominator 2 root over $2 C_d C_v w \cos \theta P_s$ minus P_L into M_s , okay that also can be calculated because all data will be known something we can measure and rest of the part we can calculate.

So if we this will be reduced to this form and finally δs is equal to L_2 minus L_1 half C_d root under root ρ divided by twice C_v C_d into $\cos \theta$ and then whole into w by M_s . Now interestingly what we find that this is independent of pressure whatever may be the pressure this damping coefficient, damping ratio it will not be affected on the pressure value.

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Practical design :

In a realistic design:



$L_s \approx 6d$...4.14-18

Where, d is spool land diameter.

And the damping length

$L_1 + L_2 \approx \frac{L}{3} = 2d$...4.14-19


Also, the spool rod diameter may be taken as,

$d_r \approx \frac{d}{2}$...4.14-20

The spool volume is also approximated as,

$$\frac{\pi}{4} d^2 L - 2 \left(\frac{\pi}{4} d^2 - \frac{\pi}{4} d_r^2 \right) (L_1 + L_2) = \left(\frac{1}{2} \right) \frac{\pi}{4} d^2 L = \frac{3\pi}{4} d^3$$
 ...4.14-21

Therefore spool mass, $M_s = \left(\frac{3\pi}{4} d^3 \right) \rho_s$...4.14-22



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Now we shall consider a practical design, so let us consider this is a valve and we are trying to find out what might be the force. Now in earlier lecture as I have told that L , L is the damping length it can be taken as $6d$ what is the d is the spool land diameter, okay. And the damping therefore the damping length L_1 plus L_2 (it is coming like this) this will be L_1 plus L_2 approximately equal to this sign is somehow it is not matching may be something wrong here L by 3 is equal to $2d$ you may consider L_1 plus L_2 is approximately taken as L by 3, which is equal to $2d$.

L is the total length, do not confuse with this L with the first L we consider, there we consider this L for the damping length and in this case L is the total length, some total length of this port but however, the important part is that L_1 plus L_2 is approximately L by 3 is equal to $2d$ or in other words we may consider L_1 plus L_2 is equal to $2d$. Also, this spool rod diameter may be taken as half of the length diameter, in practice it is taken like this.

So if we consider this then for we can consider the mass of the spool of course we are not considering the extra portion outside the active lands we are not considering. So if you now right down this, this is a very simple one and then ultimately you will find that the volume of

this spool for the length L which you have consider it is approximately $\frac{3\pi}{4}$ into d cube, d is the diameter of the land.

Therefore, mass of this spool can be taken as this value that is spool volume into ρ_s which is the density of the material spool material say subscript s means it is for spool material. Now here again I would like to mention this is a solid spool but in many case you will find the spool is hallow also that means there is a bore hole particularly tandem valve if you consider tandem type in that case there might have a hole through this spool, in that case we have to consider that part carefully but this is in general for a solid spool.

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Practical design (Contd...):
Significance of the transient flow force coefficient B_f , by computing the damping ratio.
 Substituting the expression of spool mass M_s in equation 4.14-17, the relation of damping ratio can be rewritten as:



$$\delta_s = (L_2 - L_1) \left\{ \frac{1}{2} \frac{C_d \sqrt{\rho}}{\sqrt{2C_v C_d \cos \theta}} \right\} \sqrt{\frac{w}{\left(\frac{3\pi}{4} d^3\right) \rho_s}}$$

$$= (L_2 - L_1) \left\{ \frac{1}{2} \frac{C_d \sqrt{\rho}}{\sqrt{2C_v C_d \cos \theta}} \right\} \frac{2}{d \sqrt{3\rho_s}} \sqrt{\frac{w}{\pi d}}$$

$$= \frac{(L_2 - L_1)}{2d} \left\{ \frac{C_d \sqrt{\rho}}{\sqrt{2C_v C_d \cos \theta}} \frac{2}{\sqrt{3\rho_s}} \right\} \sqrt{\frac{w}{\pi d}}$$

Considering damping length (eqn. 4.14-19) finally:

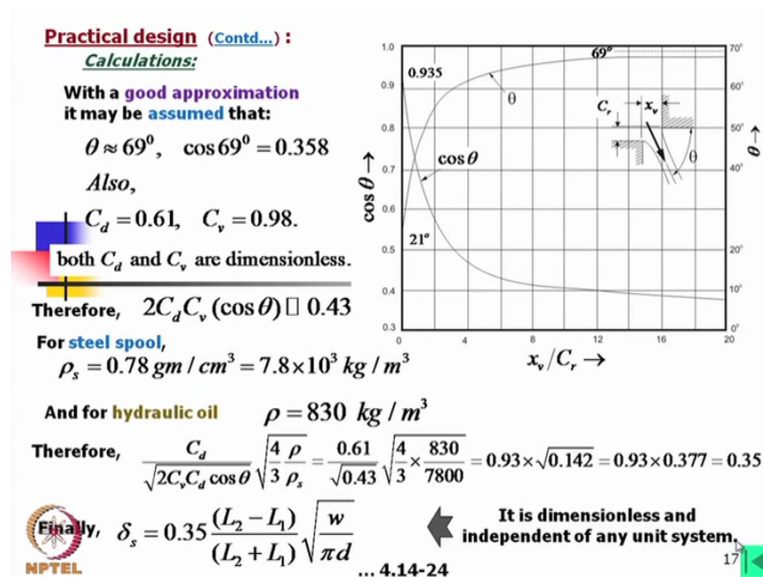
$$\delta_s = \frac{(L_2 - L_1)}{(L_2 + L_1)} \left\{ \frac{C_d}{\sqrt{2C_v C_d \cos \theta}} \sqrt{\frac{4}{3} \frac{\rho}{\rho_s}} \right\} \sqrt{\frac{w}{\pi d}} \quad \dots 4.14-23.$$

Significance of transient flow force coefficient B_f that is the damping coefficient by computing the damping ratio substituting the expression of spool mass M_s in equation 17, the relation of damping ratio can be rewritten as look into this we have now put this mass and we have rewritten this damping ratio which we can equate to this value, okay.

Then finally we get δ_s is equal to $L_2 - L_1$ divided by $L_2 + L_1$, you can see this L_2 is always greater than L_1 , if it is 0 the whole part will become 0, in that case this is a very we would say that ideal critical valve but that is in fact a difficult to control and then here we get the ratio of mass density of the fluid and the spool fluid and the spool and here the area gradient divided by πd . Now for full port opening this will become (0)(53:58) we will check that one.

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Now let us a sample calculation is like that we are again considering this curve and from there for a good approximation we consider theta is equal to 69 degree, so cos 69 degree will become 0.358, okay it will become 0.358. Also, C d we take 0.61, in some cases it is taken 62 and 0.6 very close to that and C v is taken 0.98, what is C v? Coefficient volume coefficient that is taken 0.98 that means may be very close to 1 and C d and C v are dimensionless both C d and C v are dimensionless.

And therefore, $2C_d C_v \cos \theta$ is approximately equal to 0.43, this i approximately becomes 0.43, okay and for steel spool we can consider 0.78 gram per centimetre cube, I think this is again wrong it will be 7.8 grams per centimetre cube Rho of steel 7.8 grams per centimetre cube which is but this is correct 7.8 into 10 to the power 3 kg per meter cube, this value is correct this is something wrong so this is correct. So in SI units we consider this value, okay.

And for hydraulic oil I told you that from 850 to 830 you can consider in this case we have consider 830 kg. And therefore, this part if you calculate this, this part become 0.35, okay and therefore, damping ratio can be rewritten in this form, okay. Still we have maintaining this part because of the reasons that w means totally full peripheral opening may not be equal to πd if it is a partial. It is dimensionless and independent of any unit system, okay.

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Practical design (Contd...):
Calculations of Stroking Force:

For a rectangular port, which is very common in spool valve, $w = \pi d$.

Therefore, $\sqrt{\frac{w}{\pi d}}$ is unity.


In such condition if damping length is 50% greater than the other (say $L_2 = 1.5L_1$), the damping ratio δ_s becomes 0.07.

On the other hand if $L_1 = 1.5L_2$ then δ_s becomes negative with same value.

It would create same effect but with negative value i.e., magnitude of δ_s can never be greater than $0.35\sqrt{w/\pi d}$. However, a realistic value is around $0.1\sqrt{w/\pi d}$. In practice $L_2 \cong L_1$ [but with positive damping]

Both viscous damping coefficient and spring rate maximum at null point, at $P_L = 0$.

$K_{f0} = 0.43wP_s$



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So for a rectangular port, which is very common in spool valve w is equal to πd the full open. Therefore, w by πd is unity and then in such condition if damping length is 50 percent greater than the other that means L_2 we have consider 1.5 times of L_1 , the damping ratio becomes 0.07 damping ratio becomes 0.07. Suppose if I take L_2 is equal to L_1 in that case it will become 0, because of this L_2 minus L_1 component but if I take 1.5 times greater than L_2 it is coming 0.07 which is reasonably a good value.

On the other hand if L_1 is equal to 1.5 L_2 this is just reverse then δ_s becomes negative with small value. It would create some effect but with negative value but with negative value, that is magnitude of δ_s can never be greater than 0.35 with w by πd . However, a realistic value is around 0.1 root w by πd we can keep this value, we can use this value. In practice L_2 is more or less equal to L_1 but with positive damping mind it that means L_2 will be slightly greater than L_1 so we can use this data for designing a real valve.

Both viscous damping coefficient and spring rate maximum at null point, at P_L is equal to 0. So with this value we can have K_{f0} is 0.43 w into P_s that is this is the coefficient at the null point that can be taken like this.

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Practical design :
Calculations of Stroking Force (Contd...) :

Mass of the spool.

$$M_s = (1.84 \times 10^4) d^3 \text{ kg}$$

Where, d is in meter.

Example: Let $d=12.5$ mm and pressure 7MPa

$$K_{fo} = 0.43\pi 12.5 \times 10^{-3} \times 7 \times 10^6$$
$$= 118.14 \text{ KN / m}$$


So if we would like to calculate stroking force then again this mass of the spool can be taken as like this if you know this diameter we have taken these relations, you see these relations this this equation only we can use if we maintain those relations because this ultimately we are expressing in terms of spool diameter. So the ratio is that L_1 plus L_2 is equal to L by 3 like this.

So now we take a realistic value it is 12.5 millimetre spool say about 30, 40, 50 (litre per second) no litre per minute for that type of flow a 12.5 spool diameter is good enough and let us consider this is a 7 Mega Pascal is thousand PSI not very high pressure we have consider. So for that K_{fo} is 118.14 kilo newton per meter, okay and then what will be the force? The for a stroke length 0.5 millimetre at null point spool force required approximately this distance in meter into the K_{fo} , that is x_v into this is like a spring constant, okay. So that will be the force we required.

So in that way this is in sample calculation I have shown how to calculate the force. Say what we find here that realistic dimension we have taken about half an inch or 12.5 millimetre and for that type of spool not at the transient normally at the when it is steady state condition 60 newton which is around 6 kg force is required this is not even a neither it is small, neither it is very big but usually there will be an electric drive so 60 newton force we have to generate so that electric drive it might be solenoid operated or may be the tort motor electrical tort motor may be there, okay. So with this knowledge today I stop here we will continue further on the this analysis in the next lecture also, thank you.