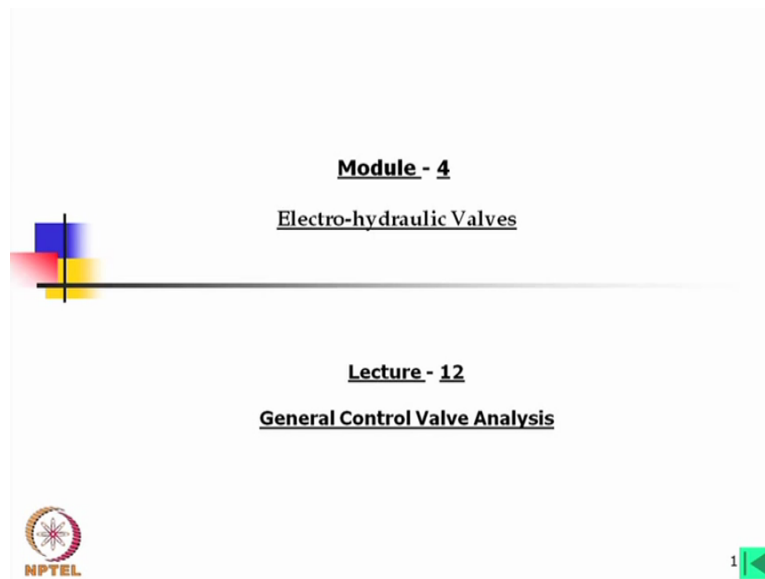


Fundamentals of Industrial Oil Hydraulics and Pneumatics
By Professor R. Maiti
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Module 12
Lecture 12
General Control Valve Analysis

Welcome to today's lecture.

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This is general control valve analysis, it is very important particularly with respect to Electro-hydraulic Valves. The difference with the ordinary valves and Electro-hydraulic valves or so to say servo control valves or proportional control valves the spool or may be poppet everything the ports they are manufactured with very care and they should have particular geometric configurations.

And this general analysis is essential for analysing even if an ordinary valve although we did not use such rigorous analysis but this analysis will be helpful for general valve design as well as the most accurate type say servo valve design.


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General Flow Equations: Four-way spool valve

Some general performance characteristics of spool type valves, useful for spool valve analyses in general, are presented in this section.

However, care must be taken in application of such general formulae for a valve analysis considering the valve's actual feature.

Same analogy is also applicable to other type valves like flapper nozzle valve etc.

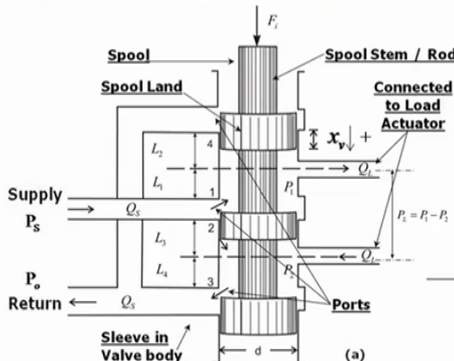


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Now first of all we shall develop general flow equations, we have consider four-way spool valve. Some general performance characteristics of spool type valves, useful for spool valve analysis in general, are presented in this section. However, care must be taken in application of such general formulae for a valve analysis considering the valve's actual feature. Same analogy is also applicable to other type valves like flapper nozzle valve etc.

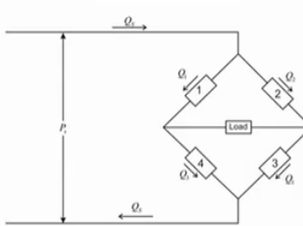
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General Flow Equations: Four-way spool valve (Contd...)



Four orifices of a three land-four way spool valve, as shown (a), are completely analogous to four arms of a Wheatstone bridge (b).

This analogy is often helpful in visualizing valve operations.



3

Now look at this figure what we see that this is Three-Land-Four-Way Spool Valve, if we look into the spool it is having it is put inside a sleeve which is which again inside the valve body say this portion other than this spool we can say this is a integral body you can say inside the spool can move, this you may consider as a sleeve inside the valve. Now here I

would like to mention that we can directly put this spool inside the valve body making a bore of the size and group for the valve.

But there are some problems one of that is that machining may be difficult as well this sleeve will worn out with the use of this valve and if we use a separate sleeve then we can replace that, whereas if we put this spool inside the valve body we cannot replace it. So due to various reasons we cannot use the spool which also can be accurately machine rather than if we machine the valve body for the bore.

Now in this spool this the larger diameter portion is called spool land, so we have three lands in this spool and look at this width, width may be different may be same already you have seen that we need different width for land for different purpose. Then the middle portion which is almost sometimes it is half of the diameter of the land is called spool stem or rod. Now these are the ports now these port configuration their width, land width, position of these ports with respect to other ports and the land width and distance between two lands all are very important and accurately manufacture for good performance of the valve.

Now we have already shown that in this body there is a supply of oil which is P_s is the supply pressure and oil is going out that is return with a P_0 the return pressure. On the other side of this sleeve what is there two holes which are being connected to the load actuator. Now if we look into this here, here actually I will show you another figure where we can understand that what may be the actual sleeve analysis and the grooves.

Now here if we look into this apparently this is the groove width which must be equal to this also may not be this drawing may not be very accurate but it will be equal and what we see from this point to this point they are having dimensions L_1 and L_2 and here L_3 and L_4 , later we will relate these distances for while we are we shall develop the equations.

Now these ports are also numbered as 1, 2, 3, 4 okay. Now this in the load side what we find that we have used a Q_L is a flow and here also the flow being written is Q_L , the same flow has to be there, this is incompressible fluid. However, apparently we can say that what is the supply flow that should be equal to Q that should be the load flow but they may not be equal due to the leakage that is essential for the valve analysis, we should consider this.

Now as such the pressure is concerned here what we find that P_L is called load pressure, P_L is called load pressure which is nothing but P_1 minus P_2 . Now what is P_1 ? P_1 is the supply pressure to the actuator and P_2 is the return pressure from the actuator, whereas P_2 is not

equal to P_0 . Similarly P_s is not equal to P_1 because if these valves are used for control of flow or pressure these orifices are such that there will be pressure drop there has to be pressure drop.

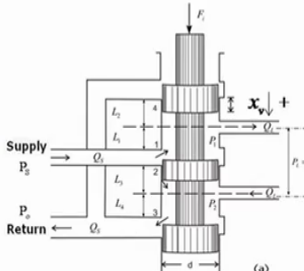
In fact just for your knowledge I would say if you look into a servo valve in many cases you will find servo valve operates functions at the half of the pressure of the system pressure. So suppose if you are using supply pressure 10 Mega Pascals you will find the valve actuator is being performed at about 5 Mega Pascal pressure. So so much pressure drop is there, so much power loss is there but there is no other way we have to accept that for the performance accurate performance, accurate control of the load motion.

Now four orifices of a Three-Land-Four-Way Spool Valve we have shown in this figure and this are completely analogous to four arms of a Wheatstone bridge you must have knowledge about the Wheatstone bridge which are used in electrical circuit analysis and this looks like this here what I have shown that Q_s is the system flow, whereas return also Q_s the system flow, then Q_1, Q_2, Q_3, Q_4 are the flow through flow through four valves sorry four valve ports Q_1, Q_2, Q_3, Q_4 this will be Q_4 sorry this will be Q_3 and this will be Q_4 there is some mistake is there and this is the load, load connection, okay and here the pressure difference P_s which we have already mentioned and the respective pressure P_1, P_2, P_3, P_4 's are there, okay. Now this analogy is often helpful in visualizing valve operations.

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General Flow Equations: Four-way spool valve (Contd...)

Flow through valve orifices are described by the orifice equations as follows:



$$Q_1 = C_{d1} A_1 \sqrt{\frac{2}{\rho} (P_s - P_1)} \quad \dots(4.12-4)$$

$$Q_2 = C_{d2} A_2 \sqrt{\frac{2}{\rho} (P_s - P_2)} \quad \dots(4.12-5)$$

$$Q_3 = C_{d3} A_3 \sqrt{\frac{2}{\rho} P_2} \quad \dots(4.12-6)$$

$$Q_4 = C_{d4} A_4 \sqrt{\frac{2}{\rho} P_1} \quad \dots(4.12-7)$$

In above four equations p is the system pressure. The drain pressure or outlet pressure p_o is considered to be negligibly small in comparison to other pressures.

Where, C_{d1}, C_{d2} etc. are respective coefficient of discharges through orifices A_1, A_2 etc.

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Now this here I have shown that valve displacement x_v and this is the positive direction we have considered the positive direction along the direction of the force being used force F_1

being used to move the spool. Now there is one confusion may be there usually neutral position is the position from where we consider the spool movement. Now if it is the overlap valve I have perhaps discussed some times that which is overlap if the land width is more than this port width then it will be overlap.

Now in case of overlap usually you will find the middle position of land is matching with the middle position of the port in that case for the first movement when the movement initiates for small width small movement there is no flow in the valve no flow. So calculating this x_v in the equations what we are using in the equation we have to take care of that overlapping portion.

However, in this case as if we have consider that this edge was matching with this edge and from there we have started counting the x_v the spool movement, okay. Let the spool is given a positive displacement along the direction from null or neutral position where x_v is equal to 0, what is null position? That neutral position when from which valve can move either right direction or the left directions that is called null positions. Now usually we prefer to use the critical central valve what the critical central valve means this means that width of the land and width of the port is equal but in fact the width of the land is slightly higher than the width of the port because there is radial clearance.

So even if it is closing the port there will be some leakage flow if the both the width are same to avoid that slightly overlap is used, okay. So we have to take care of that overlapping portion while we are ultimately calculating the x_v the spool movement or orifice opening inside the equations which we are going to develop. Neglecting the compressibility flow at steady state, now what is compressibility flow? I have described that when within a control volume within a trapped volume the oil is pressurized then there will be some motion of the fluid that is called compressibility fluid.

But compressibility flow but that can be neglected for general purpose calculations if we would like to more accurate calculation to find out the transient then sometimes this compressibility flow is also considered but in this case to understand the main analysis we have neglected that part. Now what I have shown in this equation that Q_L is called the load flow must be equal to Q_1 minus Q_4 .

Now if we look into this the here this is the oil is going in the load side and then this is entering here so some flow will go this side you may ask that why we have not consider the

other side actually this other side will be consider in other equations, okay. If there is any flow at this movements is very small it is in the order of 1 millimetre or even less in case of any valves.

So here this will be the much more important than this leakage, so we consider Q_1 minus Q_4 is very close to the Q_L neglecting this part, okay. Similarly Q_L the same load flow which is being return we can write the Q_3 minus Q_2 . Where Q_L is the flow through the load or simply load flow and for which P_L is the load pressure which is expressed as P_L is equal to P_1 minus P_2 , mind it this P_1 pressure will be inside here that is equal to the one side of the actuator.

Let us consider an linear actuator through one side of the piston this pressure is P_1 in other side of the piston pressure is P_2 . Now in this case again we use normally that symmetric valve sorry that actuator which means we have rod in at both ends of the piston normally for servo valves actuators this is the both direction the rod rodent is there so area is same in both the directions, okay normally it can be otherwise also but it is normally like that.

So flow through valve orifices are described by the orifice equation which we have already learned but in this case these are as follows. Now Q_1 is equal to $C_{d1} A_1 \sqrt{2 \rho (P_s - P_1)}$, look into this equation and the figure. So Q_1 is the flow through this orifice, okay that is equal to coefficient of discharge at this orifice into this area here the orifice area this area I shall describe a little later, this area not only this opening we have to consider the width also then we will find the area of this orifice and then this is the system pressure, this is the pressure here so we get this equation.

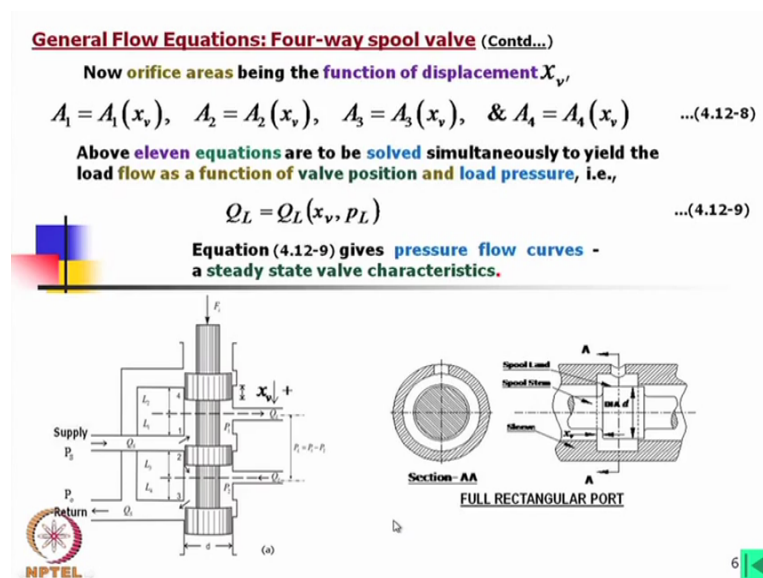
Similarly Q_2 will be $C_{d2} A_2 \sqrt{2 \rho (P_s - P_2)}$, in this case again the A_2 might be only the clearance area when it is in this form. So this basically this will be a leakage flow. And Q_3 is $C_{d3} A_3 \sqrt{2 \rho P_2}$ because P_0 we may consider is equal to 0, or else it is like that suppose in this system here we have a pressure let us consider 11 Mega Pascal is the pressure if you measure in a pressure transducer or a simply a pressure gauge.

Suppose you have noticed it is 11 Mega Pascal and in the return side as there are filter and other things you may find that this is Mega Pascal. So what you would do your P_s should be that 11 minus 1 is equal to 10 Mega Pascal whereas P_0 is equal to 1 minus 1 is equal to 0. We first while we are considering this pressure and this pressure we first consider that. So P_0 we

put into 0 or P s is the differential pressure between these two that is why equation is in this form.

Similarly Q 4 also can be written in this form and in above four equations P is the system pressure. The drain pressure or outlet pressure P 0 is consider to be negligibly small in comparison to the other pressures, although we have written in this form but basically we should consider this P s is the differential pressure and P 0 is 0, am I clear? Okay now here another thing I would like to mention that C d1, C d2, C d3, C d4 we have consider four different coefficient of discharge through A 1, A 2, A 3 but it can be shown that such orifices for a valve is such that these all four coefficient of discharge are more or less equal and in general a single value is consider for that, okay.

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No orifice areas being sorry now orifice areas being the function of displacement x_v what it is this area basically that opening into the displacement will give the area of this orifice. So what we write A 1 the area which we have consider in the equation is A 1 x_v actually it is function of x_v . Similarly A 2 is equal to A 2 x_v , A 3 is equal to A 3 x_v and A 4 is equal to A 4 x_v , okay.

Now I have shown here that what is x_v and what is the spool motion as well if you look into this this is a full rectangular port look at this spool here are the radial clearances, this diameter is the land diameter, this is the movement x_v in this direction we have consider, okay. Now what will be this area of the orifice, this x_v distance this x_v distance into the periphery that means πd so this must be function of x_v .

Now how many equations we have developed there seven and here actually we may consider in this with this equation number there are four so totally 11 equations are to be solved simultaneously to yield the load flow as a function of valve position and load pressure. Now Q_L as well is the function of $Q_L \times v$, into P_L because this load flow that will definitely vary when $x \times v$ varies as well the load pressure will also vary.

Now equation this equation gives pressure flow curve if we develop this equations from there we will find the pressure flow curve, a steady state characteristics and the steady state valve characteristics this means that here we should understand what the equations we have developed these are you have seen the root is there, etc. So due to that these are highly nonlinear and when you put into the dynamics it will be highly nonlinear but at the vicinity of operating point that means actually we would say that depending on the speed suppose we are moving an actuator with an with load and with a certain speed that means we need a certain amount of flow, it might be the same flow again can be moved at a lower speed.

Now depending on that we will first open the orifice to the extent to have the desire flow, okay after that what we need we need to control that velocity suppose we need a constant velocity in that case say it has open by 1 millimetre then what you have to in control what we have to find that this spool we have to control about this 1 millimetre very accurately so the flow is controlled to have the desire motion desire velocity, what happens if the actuator moves with accelerations then we have to decelerate this by just closing that one a little bit and then again we have to open it.

So that sought of control is different issue of course but the our analysis valve analysis will be at the vicinity of that flow and at that zone we can linearize all such equations and we can solve.

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General Flow Equations: Four-way spool valve (Contd...)

The theoretical analysis of Q_L considering all nonlinearities is very tedious.

However, in the vast majority of cases the valve orifices are matched and symmetric.

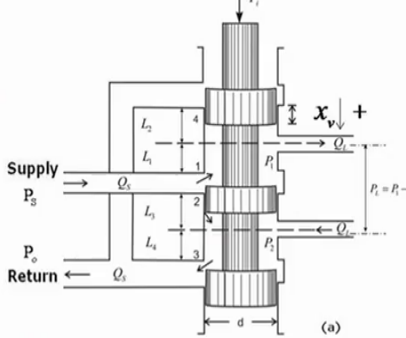
For matched orifices:

$$A_1 = A_3 \quad \dots(4.12-10)$$

$$A_2 = A_4 \quad \dots(4.12-11)$$

For symmetric orifices:

$$A_1(x_v) = A_2(-x_v) \quad \dots(4.12-12)$$

$$A_3(x_v) = A_4(-x_v) \quad \dots(4.12-13)$$


(a)

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The theoretical analysis of Q_L considering all nonlinearities is very tedious, which I have mentioned. However, in the vast majority of cases the valve orifices are matched and symmetric this is another issue we should discuss. Now what is called matched and symmetric that is two things are there one is called matched and one is called symmetric. By equations for matched orifice A_1 is equal to A_3 , say this is A_1 and this is A_3 that means when this spool will create this port area A_1 and the port area A_3 they must be equal. So we have to take care in manufacturing A_1 should equal to A_3 and A_2 must be equal to A_4 .

In this case what is A_2 ? A_2 is nothing but this clearance and it is a capillary passage although it is orifice is a capillary passage but this actually means if we move in the opposite directions then for the same x_v or minus x_v A_2 must be equal to A_4 . So that is the definition of matched orifice. Now what is symmetric orifice? $A_1 x_v$ and A_2 minus x_v this what I have explain in the opposite directions for whatever may be the value of x_v they will be equal, due you understand? This is you have to visualize this what is meant by matched and what is meant by symmetric, symmetric means is dependent on the spool motion, matched mean is a static condition any general conditions, okay.

So if these two are satisfied then again we can many this what 11 equations we can minimize them actually if you look into the unknowns and the equations these are sufficient but handling 11 equations at a time will be difficult. So we are trying to minimize those and that is why it is preferred that from the control point of view also we make the ports are matched and symmetric.

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General Flow Equations: Four-way spool valve (Contd...)

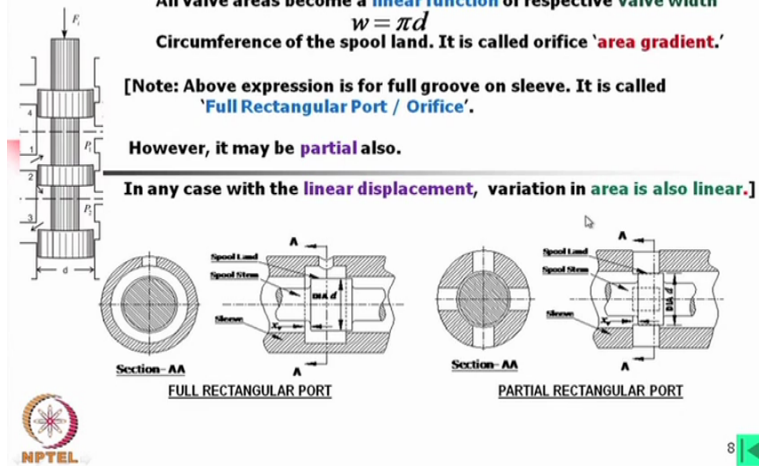
Then at null: $A_1(0) = A_2(0) = A_0$... (4.12-14)

All valve areas become a **linear function** of respective valve width
 $w = \pi d$
 Circumference of the spool land. It is called orifice '**area gradient**'.

[Note: Above expression is for full groove on sleeve. It is called '**Full Rectangular Port / Orifice**'.

However, it may be **partial** also.

In any case with the linear displacement, variation in area is also linear.]



The diagrams illustrate the internal structure of a four-way spool valve. The top part shows a longitudinal section of the spool with four ports labeled 1, 2, 3, and 4. Below this, two cross-sectional views labeled 'Section-AA' are shown. The left view is labeled 'FULL RECTANGULAR PORT' and shows a spool with a full groove. The right view is labeled 'PARTIAL RECTANGULAR PORT' and shows a spool with a partial groove. Both views include labels for 'Spool Land', 'Spool Groove', and 'Orifice'.

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Then at null at null position $A_1(0)$ is equal to $A_2(0)$ and may be $A_3(0)$ is equal to $A_4(0)$. All valves areas become a linear function of respective valve width, now this is slightly looks like a unknown term what is valve width. Here if you look into this w is called valve width which is nothing but πd this means that these all these orifice area can be calculated by $x v$ into the periphery of this valve land which means this port opening port is always a rectangle and that rectangle is having length is equal to πd and width is equal to $x v$, okay or $x v$ minus the overlap portion (Δ) (31:10).

Now this popularly called as area gradient, now in this case any orifice we call the area gradient. In this case in this rectangular port we call area gradient is equal constant is equal to πd . So to get an area opening definitely this will be linear and that will be function of $x v$. Above expression is for full groove on sleeve, this πd we have used it is called full rectangular port or full rectangular orifice.

Now here I have shown that usually see this part is sleeve, this is put inside the valve body, what we find that in valve body there will be connection to the inlet, outlet, etc, etc, pressure side, tank side and at that position there will be hole, it might be only one hole is provided on the sleeve whereas on the body there is again rectangular groove so that oil when is coming to that path that can go inside wherever may be the inlet or outlet, do you understand this?

Inside this we have a groove like this, okay say due to this we have got this view we have a groove and what we find there is small clearance and while this valve is moving we consider this $x v$ into that πd is the area of the orifice and we call it full rectangular port. However, it

may be partial also, how it is done and why it is needed? Now here I have shown that partial rectangular port.

What is the difference I think you can recognize from this drawing this is perfect engineering drawing, here we have made a groove inside the sleeve. Let us consider this width and this width is also same, okay in this groove we have made and this groove throughout the periphery of the inside sleeve instead of that what is made that in that sleeve the holes are made say this is one through hole, this is another through cross hole you can say and these are again made rectangular.

Now making a rectangular hole is difficult in a sleeve but this is done you will find that those who are manufacturing these valves these holes are made, how it is made any idea? First of all you can make a through drill hole and through which you can broach you know this broaching tools there will be multiple tools will be there say from a circular hole to rectangular hole in the broach you will find any cell you will find that may be with a first broach is circular with a four corners diagonally opposite it is cutting some material, next will be slightly more cutting of the material and finally you will find a square broach will pass through that and that will make the hole.

For one or two prototypes you can nowadays use the EDM technology to make that rectangular hole. Now the question is that why we would go for such small rectangular hole in comparison to the groove the reason is that this spool diameter imagine this spool diameter, this spool diameter usually for very large flow 30, 40 meters per minute this diameter will be around 10 or 12 millimetre.

Now 12 millimetre into π will be around 36 millimetre and suppose you have given 0.5, $x \cdot v$ is equal to 0.5 so 36 into 0.5 about 18 millimetre square is it or not 18 millimetre that 18 millimetre square area is very large for controlling the flow. So to reduce that we need to go for such small rectangular groove so that flow is coming in through small opening in that case clearly w will not be πd rather you can have suppose this angle is say 20 degree, so this will be 20×180 into d by 2, okay.

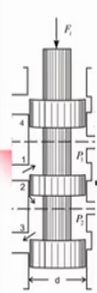
In that way we will get that each area this each width that into four say we have used four that might be there might be six holes like that, 4 holes, 6 holes it might be like that and from there we can find out w is equal to that much. However, we have consider a one area in that

case for differential groove we will consider the number of holes into this width is the total width and coefficient of discharge is more or less same in all cases, okay.

So am I clear this part that these are widely used in this control valves instead of this full port. In any case with the linear displacement variation in area is also linear we need this linearity that is why we should not go for any other configurations. If you look in little details these corners are slightly rounded so when it is really opening at the beginning slight variation in area also I mean it is not linear but that can be neglected because at the beginning the pressure difference is high so it can be may be one can optimize that if there is a circular arch at the corner what should be the circular arch for better performance that can be done but in reality there will be a circular arch but that can be neglected for the practical calculation, okay.

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General Flow Equations: Four-way spool valve (Contd....)





In case of servo valve special care is taken in manufacturing to make valve ports matched and symmetric. Also, matched part assembly practice is adopted.

In case of matched and symmetric ports, also:

$$Q_1 = Q_3 \quad \dots(4.12-15)$$

$$Q_2 = Q_4 \quad \dots(4.12-16).$$

Now in case of servo valve special care is taken in manufacturing to make valve ports matched and symmetric. I have explained what is symmetric, what is matched. Also, matched part assembly practice is adopted, what is matched part assembly? I have visited the moves control they are very I mean they are pioneer in manufacturing servo valves. Now what I observed who are matching, who are assembling the sleeves and spool. In a tray they have taken at least 10 spool and 10 sleeves and then in each sleeve they are putting their spool inside and they are measuring they are manufacturer in a batch.

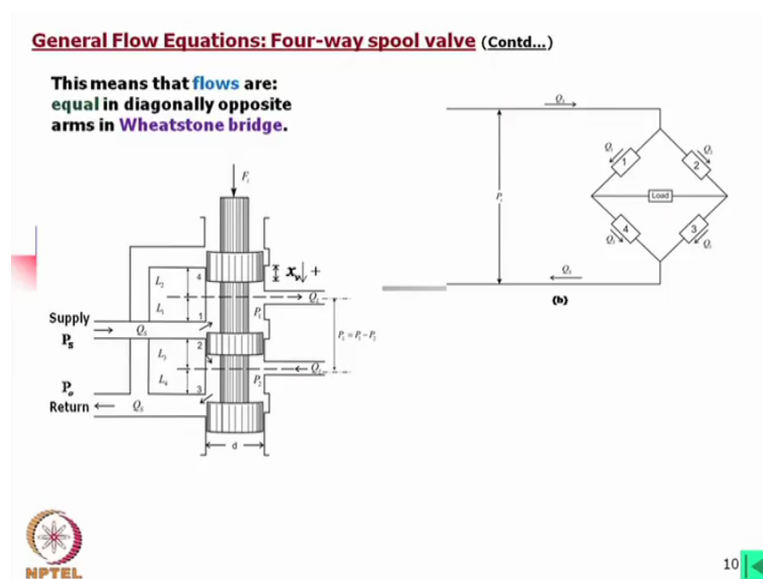
But for a sleeve they are putting 10 sleeves inside and measuring the distances and after that I found that they are making a pair then it might be out of the 10 sets they accept only 6 or 7,

others they reject not reject mean not scraped these are again kept for another batch and then again they are matched, then I asked that why it is done? Actually in case of servo valve this is so sensitive to this sleeve and spool dimensions that if there is slight error the whole valve has to reject.

In fact the servo valve is like that because servo valve means it is completely inside feedback control is there, no external control over there. So this has to be very accurate, in case of proportional valves it is not that you can externally control but in case of servo valve it is not possible. So matched part assembly is most important there just this is for your information but in reality we need ideally we need matched and symmetric ports.

Now in case of matched and symmetric ports also Q_1 is equal to Q_3 and Q_2 is equal to Q_4 , we can write this Q_1 is equal to Q_3 and Q_2 is equal to Q_4 .

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Therefore we can say this means the flows are equal in diagonally opposite arms in Wheatstone bridge. This is equal and this will be equal, okay.

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General Flow Equations: Four-way spool valve (Contd...)

Substituting equations 4, 6 and 10, i.e.,

$$Q_1 = C_{d1} A_1 \sqrt{\frac{2}{\rho} (P_s - P_1)} \quad \dots(4.12-4)$$

$$Q_3 = C_{d3} A_3 \sqrt{\frac{2}{\rho} P_2} \quad \dots(4.12-6)$$

$$A_1 = A_3 \quad \dots(4.12-10)$$


into 15

$$Q_1 = Q_3 \quad \dots(4.12-15)$$

We get:

$$P_s = P_1 + P_2 \quad \dots(4.12-17)$$

Equation 16 may be similarly treated to give the same result.



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Now we now I consider equation 4, 6 and 10. So we have consider this three equations and then we have put into Q 1 is equal to Q 3 then what we get finally we get P s the system pressure is nothing but P 1 plus P 2, if you equate this you will find that P 1 is P 2, obviously we have consider that coefficient of discharge they are equal so they are cancelling A 1, A 3 will also cancel if we write like this, this equal to this then squaring this 2 by Rho, 2 by Rho all such thing will go and we will ultimately get P s is equal to P 1 plus 2 and similarly equation 16 also from the other equation Q 2 is equal to Q 4 we will arrive into the same equation.

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General Flow Equations: Four-way spool valve (Contd...)

From eqns. 3 and 17, i.e. :

$$P_L = P_1 - P_2 \quad \dots(4.12-3)$$

$$P_s = P_1 + P_2 \quad \dots(4.12-17)$$

We get,


$$P_1 = \frac{P_s + P_L}{2} \quad \dots(4.12-18)$$

And,

$$P_2 = \frac{P_s + P_L}{2} \quad \dots(4.12-19)$$

At no load: $P_1 = P_2 = \frac{P_s}{2}$ as $P_L = 0$.

Now, as the load is applied pressure increases in one line and decreases in other line by same amount.



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So from equation 3 and 17 these two equations we get two very important equations that P_1 will be always equal to P_s plus P_L by 2. Now here again I would like to mention we have taken Four-way three lands spool valve, these are matched and symmetric and in that case what we find P_1 is equal to P_s plus P_L by 2 that is why I told that if system pressure is 10 Mega Pascal other one will be half of that to get this value, you can equate like this.

And P_2 is equal to P_s by P_L by 2 they will be I think these equations we have made a mistake it is P_s minus P_L by 2, this will be minus. So at no load P_1 will be is equal to P_2 is equal to P_s by 2 because P_L is equal to 0. Now as the load is applied pressure increases in one line and pressure decreases in other line, okay that is obvious this is easy to understand that this has to be.



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General Flow Equations: Four-way spool valve (Contd...)

Considering the flow (see Fig.) the following flow relations are obvious.

$$Q_s = Q_1 + Q_2 \quad \dots(4.12-20)$$

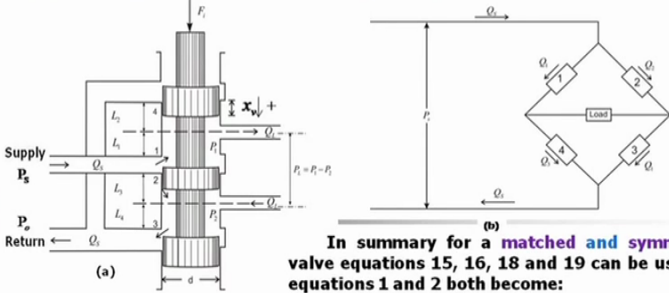
$$Q_s = Q_3 + Q_4 \quad \dots(4.12-21)$$

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Considering the flow the following flow relations are obvious that Q_s is equal to Q_1 plus Q_2 , Q_s is equal to Q_1 plus Q_2 and Q_s is equal to Q_3 plus Q_4 .

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General flow equations: Four-way spool valve (Contd...)





In summary for a **matched and symmetrical** valve equations 15, 16, 18 and 19 can be used and equations 1 and 2 both become:

$$Q_L = C_d A_1 \sqrt{\frac{1}{\rho} (P_s - P_L)} - C_d A_2 \sqrt{\frac{1}{\rho} (P_s + P_L)} \quad \dots(4.12-22)$$

Similarly equations 20 and 21 can be derived as.

$$Q_s = C_d A_1 \sqrt{\frac{1}{\rho} (P_s - P_L)} + C_d A_2 \sqrt{\frac{1}{\rho} (P_s + P_L)} \quad \dots(4.12-23)$$

In summary for a matched and symmetrical valve equations 15, 16, 18 and 19 can be used and equations 1 and 2 both become Q_L is equal to $C_d A_1 \sqrt{\frac{1}{\rho} (P_s - P_L)} - C_d A_2 \sqrt{\frac{1}{\rho} (P_s + P_L)}$. Now you can look into this we have used only one C_d because we have assumed they will be same. Similarly equations 20 and 21 can be derived as $C_d A_1 \sqrt{\frac{1}{\rho} (P_s - P_L)} + C_d A_2 \sqrt{\frac{1}{\rho} (P_s + P_L)}$, you see this is Q_L is defined by this, this minus this and Q_s is defined by this plus this, okay.

Now we will find some so we have only developed that what might be the relation of the flow pressure in a valve which is which can be used for linear control or not just on of control the same spool we will find that is a four way sorry four port three way valve same spool can be used but in that case we did not accurately maintain such the port configurations, in that case just we make either full flow or it is 0 flow, we do not control the mid-way. But these analysis what we have present that is for the controlling at the mid-way to I mean controlling the orifice area we are controlling the flow rate as well as pressure.

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Linearized Analysis of Valves
Valve Coefficients

First of all, **pressure flow curve to be linearized at the vicinity of an operating zone.**
 This will help in predicting the dynamics at the same operating zone.

Consider equation 4.12-9, i.e.:

$$Q_L = Q_L(x_v, P_L)$$


Expanding it in **Taylor series** about a **particular operating point** – say Q_{L1} .

$$Q_L = Q_{L1} + \left. \frac{\partial Q_L}{\partial x_v} \right|_1 \Delta x_v + \left. \frac{\partial Q_L}{\partial P_L} \right|_1 \Delta P_L + \dots$$

Considering at the vicinity of the operating point which is real in many operating cases-

$$Q_L - Q_{L1} = \Delta Q_L = \left. \frac{\partial Q_L}{\partial x_v} \right|_1 \Delta x_v + \left. \frac{\partial Q_L}{\partial P_L} \right|_1 \Delta P_L \quad \dots(4.12-24)$$

The equation (4.12-24) is **very useful in valve analysis.**

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Now here to get the results I mean how much force is required, what will be the transient, etc we should use some more useful coefficients, what are these? First of all pressure flow curve to be linearized at the vicinity of an operating zones, we have shown that pressure flow curves which is equal to Q_L is equal to Q_L is a function of x_v and P_L that equations. So we now expand that equations and we will find that how it can be solved.

Now this will help in predicting the dynamics at the same operating zone, I have explained that we cannot analysis this valve for all operating conditions, we have to consider a particular operating point and at the vicinity of that we have solved the equations. Now in real control what is done depending on the position of the valve these parameters are calculated and that is say for example we are using a proportional control valve with respect to that either there will be chart from experimental data or theoretical data from there at a particular position what are the parameters that we will take into account and then there will be the computer calculations it might be very fast because of the linearity in the equations and then that will be given to the feedback.

In case of servo valve, these are automatically done in the electronic devices put inside or may be some other mechanical devices also. Now considering equation 4.12-9 that this is the as I told that this is pressure flow equation and expanding we shall now expand it in Taylor series at an operating point where let us consider the initial desired flow is Q_{L1} , then Q_L is equal to Q_{L1} plus $\left. \frac{\partial Q_L}{\partial x_v} \right|_1 \Delta x_v$ plus $\left. \frac{\partial Q_L}{\partial P_L} \right|_1 \Delta P_L$ plus then there will be $\frac{\partial^2 Q_L}{\partial x_v^2} \Delta x_v^2$ etc, etc Taylor series as you know but in reality the second order terms will be negligibly small so that part we can neglect.

So therefore for that particular operating zone where we need a flow of Q_L and after that this will be Q_L plus something minus something for that operating zone, okay. What you can write for that operating zone? We can write Q_L minus Q_{L1} is equal to ΔQ_L that is the difference is equal to this part and we have neglected other part. So the equation 4.12-24 is very useful in valve analysis, in fact we should remember this equation.

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Linearized Analysis of Valves- Valve Coefficients (Contd....)

Equation (4.12-24) is the **valve characteristic equations** where we define **two derivative terms** as follows.


The **flow gain** is defined by-

$$K_q \equiv \frac{\partial Q_L}{\partial x_v} \quad \dots(4.12-25)$$

The **flow-pressure coefficient** is defined by-

$$K_c \equiv -\frac{\partial Q_L}{\partial P_L} \quad \dots(4.12-26)$$

We shall examine later that $\frac{\partial Q_L}{\partial P_L}$ is always **negative** for all valves and therefore, in **reality** K_c becomes **positive**.



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Now equation 24 is the valve characteristics equation where we define two derivative terms as follows these are very important, we all the fluid for control engineer they will say that what is flow gain, flow gain is equal to K_q is equal to $\Delta Q_L / \Delta x_v$, what we are looking into that equations, equation 24. And another term the direct unit term is that flow-pressure coefficient that is K_c is equivalent to minus $\Delta Q_L / \Delta P_L$, okay. So two components two coefficients flow gain and flow-pressure coefficient you should remember this.

Here interestingly this is having minus sign, why minus sign is there? We shall examine later that $\Delta Q_L / \Delta P_L$ is always negative what does it mean we have put this minus sign so that K_c is always positive because we use this positive term in further equations but why this is negative why this term will be always negative because if pressure increases flow decreases. So these are reverse function that is why always a minus term will come negative term will be here. So this negative with this real negative value if you put it there K_c will be always positive.

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Linearized Analysis of Valves- Valve Coefficients (Contd....)

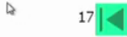

Another useful quantity is the –"pressure sensitivity" which is defined as.

$$K_p \equiv \frac{\partial P_L}{\partial x_v} \quad \dots(4.12-27)$$

Above three equations can be related as follows:

$$\frac{\partial P_L}{\partial x_v} = \frac{\text{Flow gain}}{\text{Flow Pressure Coefficient}} = - \left[\frac{\partial Q_L / \partial x_v}{\partial Q_L / \partial P_L} \right]$$

Finally, $K_p \equiv \frac{K_q}{K_c} \quad \dots(4.12-28)$



Now another term this is also we use in electro hydraulic valve analysis where accurate control is required which is called pressure sensitivity. But this pressure sensitivity is actually derived term from that two derivatives, okay. So how this is you can see clearly if you have look into the other equations, this can be derived as $\partial P_L / \partial x_v$ is equal to flow gain by flow pressure coefficient is equal to this and finally we would say K_p is equal to K_q by K_c , here as these valves that this value will be positive. So we are considering only the positive signs do not confuse with this here no negative sign is there because already after you are calculating these you will find this value.

Now these three values are normally provided by the manufacturer. So if you use a valve you will find these are three coefficients are mentioned there or else you can also calculate if you get the data.

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Linearized Analysis of Valves- Valve Coefficients (Contd....)

With the preceding defined characteristics of the valve the pressure-flow curves becomes in linearized form as:

$$\Delta Q_L = K_q \Delta x_v - K_c \Delta P_L \quad \dots(4.12-29)$$



This is applicable to all valves whether it is a spool, flapper or otherwise.

These three coefficients are called as valve coefficients and are extremely important in determining stability, frequency response, and other dynamic characteristics of a control valve.

It is to be noted that flow gain directly affects the open loop gain and thereby valve stability.

The flow-pressure coefficient directly affects the damping ratio.

The pressure sensitivity accounts for the ability to breakaway large friction load with little error.



With the preceding characteristics of the valve the pressure flow curve becomes in linearized form as ΔQ_L is equal to $K_q \Delta x_v$ plus $K_c \Delta P_L$ that K_q is the flow gain and K_c is pressure strength sensitivity. In this term we can write down the equation that means if these two values are known for a valve if the valve characteristics are fully defined in a operating zone, that means for each and every operating zone you should have this values available in your hand.

Then you know what will be the displacement and this pressure you can measure or pressure you can calculate to have the load flow. Now once this is calculated then our actual interest will be this one if we can give this much displacement then automatically this will be at the state. So this is the control feature of this spool valves. Now you see this actual equations are nonlinear and complicated but we have arrived into the simple form and if these two characteristics, these two coefficients are known then we can do it but in normally when we go for selecting the valve what we should look into.

So this is applicable to any valve analysis, in case of flapper valve of course their orifice are different there that gradient area gradient will be different but still this analogy will be applicable to that. Flapper valve will come sometimes later but spool valve we are now we are analysing. These three coefficients are called as valve coefficients and are extremely important in determining stability, frequency response and other dynamic characteristics of a control valve, okay.

It is to be noted that flow gain directly affects the open loop gain and thereby stability that means flow gain is directly controlling the stability of the valve. Whereas flow pressure coefficient directly affects the damping ratio. The pressure sensitivity accounts for the ability to breakaway large frictional load with little errors. So we shall look into this all these values while we are selecting a valve.

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Linearized Analysis of Valves- Valve Coefficients (Contd....)

The values of the valve coefficients depend on valve operating point.



The most important operating point is the origin of the pressure flow curves (i.e., at Q_L, P_L and $x_v = 0$),

because system operation usually occurs near this region.

At that region the valve flow gain is largest, giving a high system gain and the flow-pressure coefficient is smallest, giving a low damping ratio.

Hence, from stability point of view this operating point is critical. Importantly, a system, which is stable at this region, remains stable and quite at all other operating points.

The valve coefficients evaluated at the operating point are called the null valve coefficients.

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The values of the valve coefficients depend on the valve operating point, that I have mentioned this will vary different point. The most important operating point is the origin of the pressure flow curves, why it is most important? Usually what we need that a load has to be made in one direction and next movement it has to move in the other direction also so definitely operating point near the null point is most important, okay.

So we analyse for that point and we should call that this I have already explained because the system operation usually occurs near this region. And at that region the valve flow gain is largest, giving a high system gain and the flow pressure coefficient is smallest, giving a low damping ratio, we need high damping ratio actually but at that zone it will be there but. Hence from stability point of view this operating point is critical. Importantly, a system which is stable at that region remains stable and quite at all other operating points.

So if we analyse a valve for that region if you find that performances good we can consider at the other points for larger flow in the same direction, right directions or left directions it will be stable. The valve coefficients evaluated at the operating point are called the null valve coefficients, you see this null position actually may be the neutral and zero positions but

when we are considering a particular operating zone then for ideal position for that amount of flow say Q_L , say 30 litre per minutes we will say that x_v position is the null point and about that point the coefficients are called the null valve coefficients, okay.

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Summary

**General Flow Equations (Four-way spool valve),
Linearized Analysis and Valve Coefficients:**

For Matched and Symmetrical Ports

The Load Flow:

$$Q_L = C_d A_1 \sqrt{\frac{1}{\rho} (P_s - P_L)} - C_d A_2 \sqrt{\frac{1}{\rho} (P_s + P_L)}$$

The System Flow:

$$Q_s = C_d A_1 \sqrt{\frac{1}{\rho} (P_s - P_L)} + C_d A_2 \sqrt{\frac{1}{\rho} (P_s + P_L)}$$


Three Valve Coefficients are:

Flow gain: $K_q \equiv \frac{\partial Q_L}{\partial x_v}$ **Flow-Pressure Coefficient:** $K_c \equiv -\frac{\partial Q_L}{\partial P_L}$

Pressure Sensitivity: $K_p \equiv \frac{\partial P_L}{\partial x_v} = \frac{K_q}{K_c}$


Expression for Pressure-Flow Curves becomes in linearized form as:

$$\Delta Q_L = K_q \Delta x_v - K_c \Delta P_L$$

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- D. McCloy & H. R. Martin, 'Control of Fluid Power', Longman Group Ltd., UK, 1973, ISBN 0 582 47003 x.

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So general flow equations and linearized analysis valve coefficients in summary you can say that matched and symmetrical port, this is the load flow and this is the system flow if possible try to remember these equations and the valve coefficients are this is the flow gain, this is the flow pressure coefficient and this is the pressure sensitivity and expression for pressure flow curves become in linear form as ΔQ_L is equal to $K_q \Delta x_v - K_c \Delta P_L$, okay this is the flow gain and this is the pressure coefficient, okay. And with I end today's lecture and I have followed mainly the hydraulic control system the book, hydraulic control system by

Merritt and however, some knowledge of the orifice and others have more detailed in Martin and McCloy. So I suggest to read these book, thank you.