Computational Fluid Dynamics
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Lecture No. # 09
Weighted Residual Approach and Introduction to Discretization

In our previous lecture, we were discussing about the weighted residual method, and its basic understanding. So, let us take up from there.

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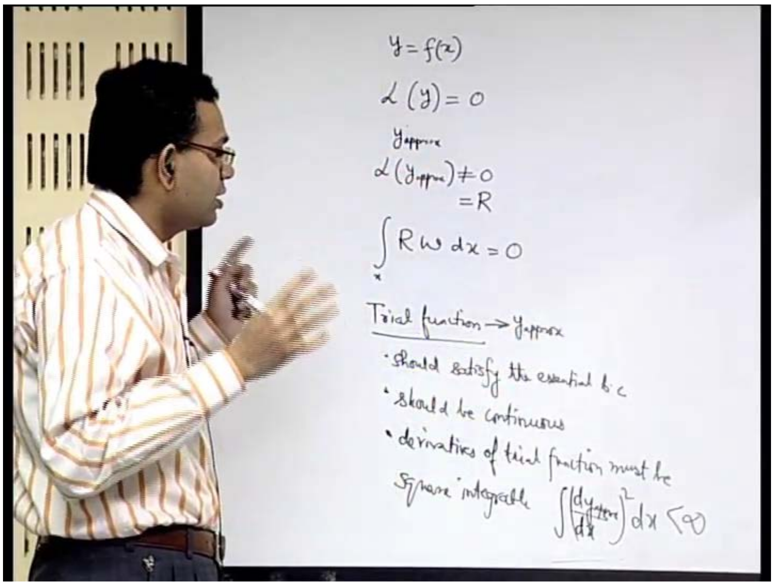

Let us say that you have governing differential equation with a linear operator on y, L y equal to 0. So, if you now substitute y approximate, then L of y approximate will not be equal to 0. And we call this equal to some residual which is non-zero. Our objective is to have a method which tries to minimize this residual. So, what is done? In the weighted residual method is this residual is multiplied by a weighting function, and it is integrated over the entire domain. So, here if it is a one-dimensional problem the domain is x; where y is a function of x, integrated over the domain, and this is said to zero. So, the error is minimized in an weighted integral or weighted residual sense.

Now, the big question remains that what should we take as the approximating function. So, the approximating function is a trial, it is a guess. So, you are trying with a function, and trying to make sure that the the function is trying to make sure that the function is appropriate in a way that it minimizes the error in an weighted integral sense. Now, let us see how we how we can achieve that. We will look into an example, and try to follow that example to see that how that is achievable, but before that we learn certain terminologies, and concepts which we borrow on to the examples.

So, first example first terminology is the trial function. The trial function is nothing but the y approximate, and in this trial function the y approximate can be chosen in many ways. We will see later on that polynomial is one of the convenient methods or one of the convenient choices of the function. But you it need not always be a polynomial function, it can be any function provided it satisfies certain characteristics. What are those conditions; so, the trial function should satisfy the essential boundary condition should be continuous. Again we should keep in mind, that it should satisfy the essential boundary condition is one of the very key requirements. Essential boundary condition is basically, what is the value of the variable for a second order problem as an example.

So, for higher order problems - it can also be the derivative. Whatever it is we have seen earlier through the variational formulation, that what do we mean by essential boundary condition. And that essential boundary condition should be such that, it should be or rather the choice of the trial function should be such that it satisfies the essential boundary condition. It should be continuous, and when once we say it should be continuous, it should be continuous over the domain of the definition of the function. And derivatives of trial function must be square integral.

So, derivative square integral means for example, if you have y dy approximate dx whole square integral of that one must be less than infinity. So, it should be a bounded integral - square integral means, integral of square of the derivative, basically you are doing. So, this type of requirement is a very very important, and stringent requirement. It shows an un it shows a roundedness of the function. So, so that its integral is not unbounded. Why this type of thing is necessary, because if you have say an operator a y, y bilinear operator, then often the bilinear operator can take the form of this square. And if it is integrated, its integral should be less than infinity. So that, it that such functions are bounded, that is very very important.

Now, you can also have higher order derivatives. We have to see that what is the highest order derivative if necessary. If you have a variational formulation for a second order differential problem, you may at most require a first order derivative continuity, because you have already reduce the order of continuity by integration by parts. So that, your derivative requirement - highest order derivative requirement for satisfaction of the continuity is the first order derivative for a second order problem.

Similarly, for a fourth order fourth order problem, it would be a second order highest highest order derivative continuity requirement. So, if it is a second order problem, then we only requires up to the dy dx type of term for your continuity requirement. You do not require, any higher order derivative for corresponding to your variational formulation. However, you will require that, if you do not go through the variational formulation.

So, you you may also have the weighted residual method implemented without going through the variational formulation, because variational formulation gives us a clue that you have to multiply this with a weighting function. Now, weighting function is having a same meaning as that of a variation or a similar meaning as that of a variation. But it does not mean that, you have to go through the roots of variational formulation.

We will see that you can have any arbitrary choice of the waiting function of of course, consistent with certain constrains. But the whole understanding is that for a second order problem also, if you do not go through the variation formulation, you will require a second order derivative continuity. If you do not reduce the continuity requirement through integration by parts. So, the variational formulation is sometimes with formulation, because you reduce the or you weaken the requirement of the order of the continuity of the approximating function. But if you do not required, if you if you do not employ that as a strategy, and simply do the integration without doing integration by parts; then obviously, you required the continuity up to the highest order derivative.

So, it is not the first order derivative for a second order problem, it is then a second order derivative for a second order problem. This we must keep in mind. So, depends on through which route we are we are going, are we going through integration by parts route or we have straightaway integrating without integrating by parts.

Now, this type of function. So, here the first order derivative is square integrable, we call it an H 1 function - in functional space. So, there could be several types of functions, like H 1 function, H 2 function like that. So, H 1 function in a functional space is such that that its first derivative is square integrable. If it is H 2 function then its second derivative has to be square integrable like that. So, this superscript stands for the order of the derivative which is square integrable. Now, this is the trial function requirement. Now, what about the requirements of the weighting function.

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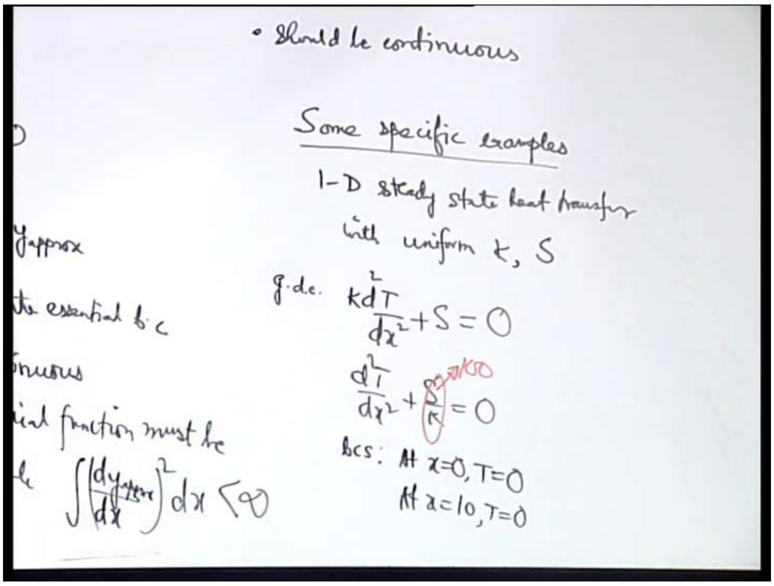

That is W. It should satisfy homogenous part of the essential boundary condition. What is homogenous part of the essential boundary condition? So, let us say that you are having a boundary value problem with y at 1 of the boundary is equal to 5. So, the weighting function should satisfy its corresponding homogenous part. So, the weighting function should satisfy the requirement that, it is equal to what? It is equal to 0. Homogenous means, this right hand side is zero. So, why it is so; see if y is specified, then variation in y is 0, and the weighting function has the similar meaning as that of variation in y.

And therefore, the weighting function is 0 at the points where the functions function itself is defined. So, that is why wherever you have the essential boundary condition, that means the that function is defined. The corresponding variation which is reflected through the weighting function is zero. So, that should be satisfied by the weighting function, that requirement should be satisfied by the weighting function. Weighting function also should be continuous. You see, when we are designing the trial function and the weighting function, we are bothering ourselves so much, about the essential boundary condition - not the natural boundary condition. Because the natural boundary condition is automatically incorporated through the formalisms of the variational method. So, you do not have to forcefully incorporate the natural boundary condition that is why it is called as natural boundary condition. So, you do not have to forcefully incorporate it.

Now, this is a brief background. Now with this background let us try to understand, that how differently you can choose the trial function, and the weighting function based on some particular methods. So, some specific examples.

Let us say, we are interested to solve a heat transfer problem - one-dimensional steady state heat transfer with uniform thermal conductivity, and heat source yes. So, if that is the case, we have seen earlier that the governing equation becomes this equal to 0, this is the governing differential equation, so that d 2 t dx 2 plus S by K equal to 0. To be able to solve this problem numerically, let us assume some value of S by K in consistent units. Let us call let let us say this is 100; what should be the unit of this, say this is what is the unit of d 2 t dx 2.

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Kelvin per...

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Meter square. So, this is like 100 Kelvin per meter square say. We are not putting the unit here, but it is with consistent units. And let us say that the boundary conditions are such that at x equal to 0, you have T equal to 0, and at x equal to 10, T equal to 0.

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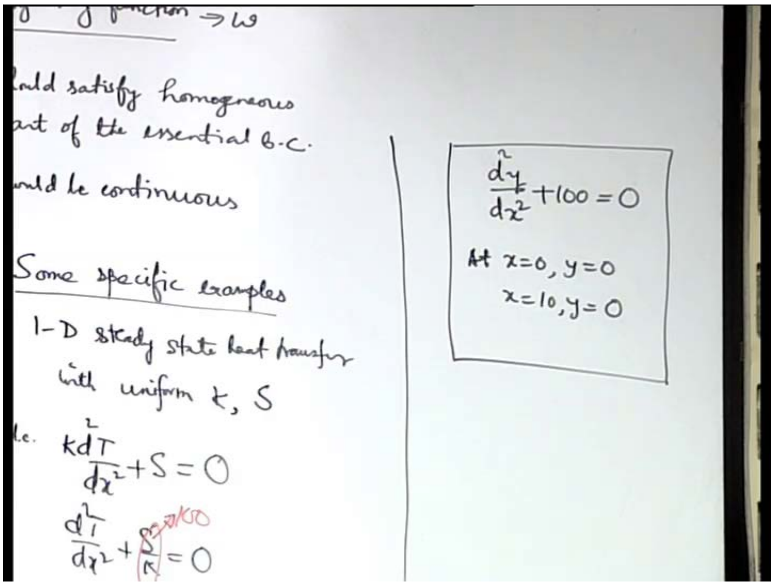

So, our problem becomes, a problem of solving a differential equation a very simple differential equation of this form - d square y dx by dx is square plus 100 equal to 0. And at x equal to 0, y equal to 0 at x equal to 10, y equal to zero. So, this is the problem that we intend to solve.

So, when we intend to solve it, there are various ways in which you can approximately, remember we intend to solve it approximately. The objective of this demonstration is not to show that how to solve it exactly, because that is very simple in straight forward. So, let us let us not bother about that, let us bother about the approximate solution. So, for approximate solution, we need what if you have to go through the method of weighted residual, you need a trial function and you need a weighting function. So, there are different methods, accordingly you can have different trial, and weighting function. So, let us take one example: The first example will be the lease square method.

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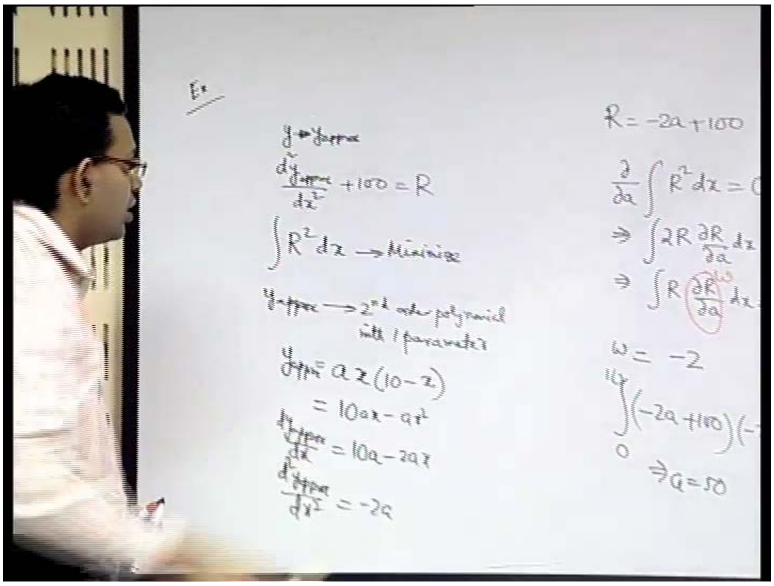

So, in the least square method, what we will do? We will try to minimize the error in a least square sense. So, what we will be doing - First of all in place of y, we have some y approximate, that is there in all methods. So, it there is a trial functions. Now, you substitute the trial function in the governing equation. So, you have this plus 100, this is equal to the residual R. So, what you are interested in - you are interested to minimize you are interested to minimize the value of this square of R. So, how do you do it? You find out what is R square? Integrates it over the domain, and try to minimize that. So, when you say that, you are interested to minimize R square - you mean integrate to minimize the sum of R square.

So, what you are trying to do, at each and every x there is a corresponding error, R is the error. You are finding square of the error at each and every x, and summing it up. If it was discrete points, it would have been a sigma or summation, but but because it is a continuous function, you have an integration. So, you are having. So, this is... So, physically what is the meaning of this; physically you are finding the sum of the squares of residuals at each and every point. Why square, because if you just find out the residual itself, and find the sum of it, then it can be misleading. Because some of the residuals may be positive, some of the residuals may be negative, the total sum may be 0 giving a false illusion that as if the error is 0 or error is very small, but actually that may be, because of nullification of some positive error, with some negative error.

Alternatively, one could actually do with the mod of the residual, but because the algebra with the mod is tedious, one does not go through the mod, but one basically finds the square which anyway is always positive. And therefore is indicative of the magnitude of the error, and then try to minimize that. So, these now need to be minimized. Now, when it is to be minimized, there must be some unknown parameter through which it needs to be minimized. So now, let us come to the choice of the approximating function. It can be many, now one of the choices could be that you choose a polynomial. So, if you choose a polynomial, remember it is not a must. It is just an example, say we consider it to be a second order polynomial with one parameter.

So, what can be its form? So, what are the requirements that it should satisfy. It should be continuous; obviously, we will when we consider polynomial by nature it is continuous. So, we do not have to bother about the continuity of it, but most importantly it should satisfy the essential boundary conditions. That means, it should be a polynomial of such type, that at x equal to 0 - it should be 0, and at x equal to 10 - it should be 0. And it is a second order polynomial by our choice. Remember you could also choose higher order or lower order polynomials, and general expectation is that higher order polynomials will give you less error, because you have considered more number of terms. And with one parameter means, there will be one unknown parameter which you are interested to find out.

So, general form of the polynomial can be x into 10 minus x, because this is a second order one, you have constructed it by deliberately choosing it product of two first order function. So, that it is a second order function. Now, it satisfy the essential boundary condition and you consider a parameter a; which is a single parameter, which is a multiplying parameter. You want to find out this a, such that this is minimized. So that, this is minimized. So, this is your y approximate. So, once you have this y approximate - next is what your R then. So, let us just open it up. So, 10 a x minus a x square dy approximate dx. The second order derivative minus 2 a. So, what will be the value of R? So, R becomes minus 2 a plus 100.

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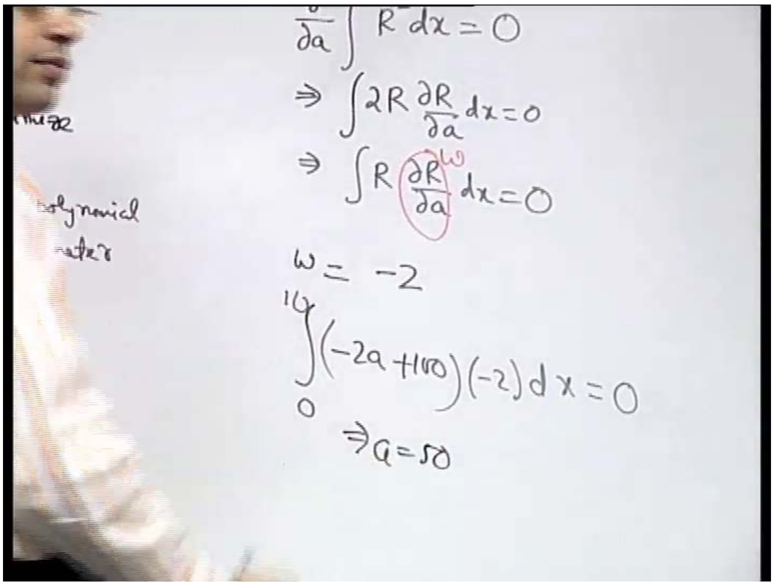

Now you have to minimize this one R square dx integral of that; that means, you have only one parameter here a. So, if this needs to be minimized, then it is derivate with respect to a is 0. Because a is the only unknown parameter. So, that means you can write, you can take this derivate inside the integral, because the limits are functions of x which are not dependent on a. So, you can take this inside the integral, and you can write this as 2 R.

So you can see, that although it is a least square method by name it is not a weighted residual method by name, but by implication it is a special case of the weighted residual method, where the weighting function is nothing but del R del a. So, let us find out the weighting function. Weighting function is...

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Minus 2. So, integral of minus 2 a plus 100 to minus 2 dx, the limit of x equal to 0 to 10 that is equal to 0; that is the outcome of our formulation least square method which we have seen may be perceived as a special case of the weighted residual method. So, from here what is the value of a. So, minus 2 a plus 100 will be equal to; that is a coefficient, because the integrant itself is not 0. So that means a is equal to 50. So, we get one method as the least square method.

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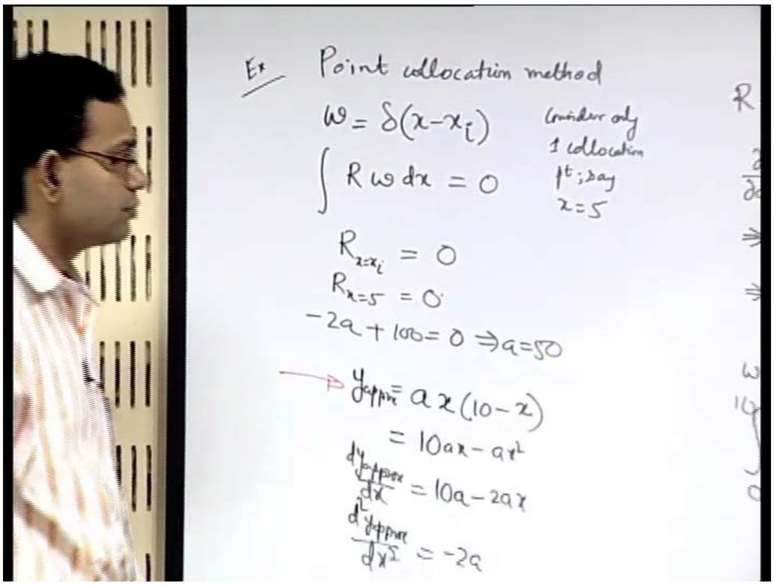

Next, let us take the second example where we consider method called as point collocation method. So, point collocation method what we do? We try to have the weighting function as a special function. So, we do not go through this route. We have the weighting function as the direct delta function, at some chosen points which are collocation points. What is the whole idea? The whole idea is that, you try to satisfy the value of the function, at some chosen points given by x equal to x i. So, this is the delta function. So, when you have this one, then what is the result of the integral? So, you have integral R W dx equal to zero. So, what is R? R is... So, it it depends on again that what is your trial function. Let us say we keep the trial function same. This is the different method, but it does not mean that we have to forcefully choose a different trial function, we can keep the same trial function.

We have understood one thing, how this how this method is different choice of the weighting function is different. So, different methods are basically formulated based on different philosophies by which the weighting functions are formulated. Of course, you can have different trial functions, but that is not a necessity, but as you go from one method to the other, these are all special cases of the weighted residual method. You have to consider the concept of the weighted weighting function in somewhat different way.

So, when you have this one. So, basically what it becomes R at x equal to x i is equal to 0. So, if there are many such points, collocation points then it would be a summation of that. So, but here we consider only one collocation point. So, point collocation collocation method, in this example we consider only one collocation point. Again this is an example say x equal to 5. So, collocation point is a point, where you make the function satisfy its requirement; that means, at that point the error is exactly equal to 0. We are this is the ideal thing. In in in general we are trying to minimize the error, minimize the sum of the residual in in some weighted sense. But here, in the collocation method you are only focusing your attention on some specific points, and trying to make sure that the corresponding errors are zero.

So, at x equal to 5, you consider you are considering only one collocation point. Here if you consider multiple collocation points, it will be sum of the residuals at those collocation points. But here you consider only one collocation point. So, R at x equal to 5 has to be equal to 0. So that means, you have minus 2 a plus 100, here R is such a function which is not a function of x. So, does not matter what is the choice of your collocation point, but that is because of this very special simple example. Otherwise R could itself depend on x. So, minus 2 a plus 100 equal to zero. That means, a equal to 50. So, this has given the same solution. The third example example 3, we consider as Galerkin's method.

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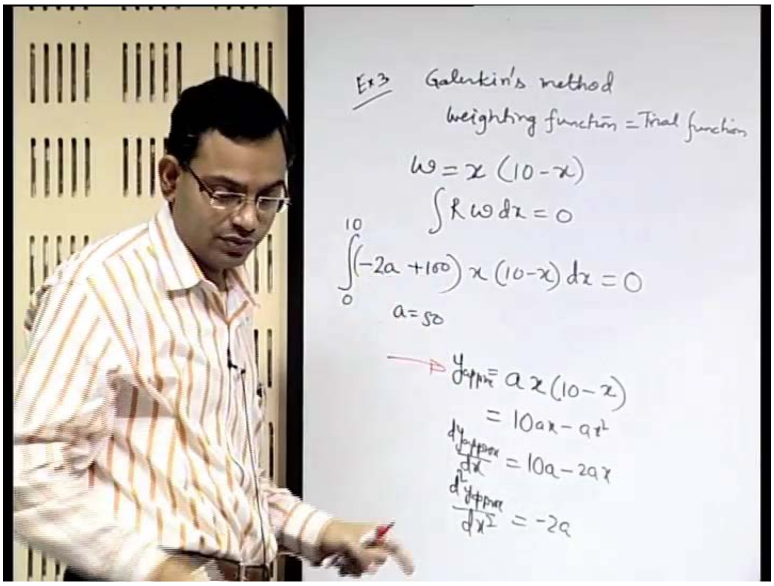

In the Galerkin's method - so, we are seen in the previous methods - previous two methods, we have chosen the weighting function in some special way. Galerkin's method also does it in a very special way, it considers the weighting function same as the trial function. So, it considers same form of the two functions. So that, your sort sort of degree of freedom is somewhat reduced in terms of choosing the weighting function. Once you choose the trial function, you have automatically chosen the weighting function. You cannot choose it anything beyond the form of the trial function. So, if you want to see that how that can be implemented, let us consider the same y approximate.

So, the weighting function, remember the weighting function we consider without the parameter. So, we take weighting function we consider as x into 10 minus x, we do not consider a as a part of the weighting function. Because it is the weighting function equal to trial function in the sense of the form. Its if if you just keep the form that is sufficient, because a is just a constant multiplier.

So, if you multiply it with multiply the function with a x into 10 minus x, it is as good as multiplying it with x into 10 minus x, because integral of R w dx equal to zero. So, if you put a or if you do not put a, it does not matter. Because eventually a is not equal to 0. So, just for simplicity we just, but if you are interested to to write it in the full form, you can write it a. But we are not writing a a here, because eventually it is the form of the weighting function that matters.

So, let us now do this exercise. So, integral of if you choose the same approximating function minus 2 a plus 100 into x into 10 minus x dx equal to 0. So, what should be the value of a? a a will be 50, because the remaining part of the integration if you take this out of the integral, remaining is just a number. Integral of x into 10 minus x dx from 0 to 10 will be just a number. So that means, effectively minus 2 a plus 100 will be equal to 0, that is a equal to 50.

Now, we have seen 3 examples. And in all these 3 examples we have come through the roots of the weighted residual technique, which has some resemblance with the V form or the variational formulation. Now, remember that we had different forms of the governing equation, we had the original form as the differential form or the D form, we could reduce it to a V form, and subsequently we could also reduce it to a M M form or the minimization form. So, let us see, that whether we can tackle the same problem by utilizing the concept of the M form. Again using some approximating function, but not using this form, but using the M form. So, let us see that how we go through the roots of the M form.

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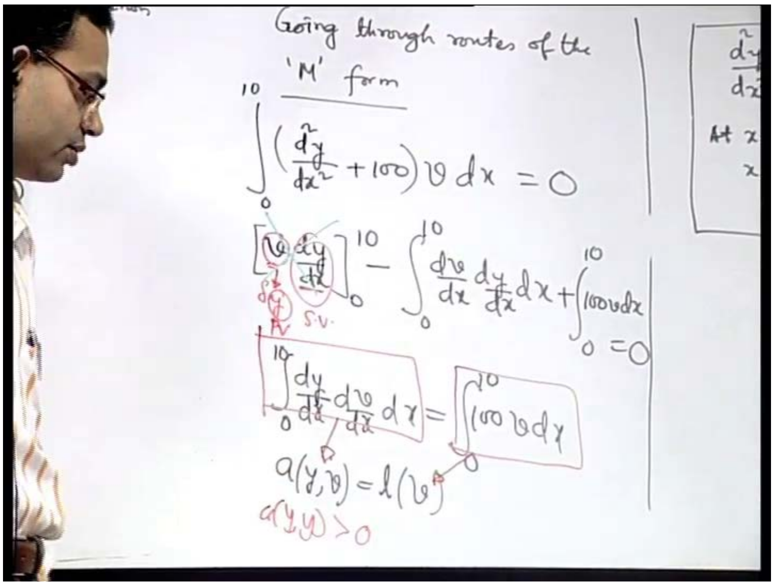

So, before doing that one important thing that we should keep in mind, that no matter whether it is the least square method or the point collocation method or the Galerkin's method, that these are not the only methods. These are just some examples that we have we have consider, you could have different other methods where you you may choose different forms of the trial functions. Like there is a very commonly used method called as spectral method. In spectral method, what want does is one uses the truncated form of an infinite series as a trial solution.

So, you could have some special polynomial functions or or special functions which are of forms of the infinite series. And in the special functions of the form of the infinite series you truncate it up to a finite number of terms, and that you call as a spectral method so to say. So, we are not going into all those methods, but these are just to illustrate that how you can make a weighted residual formulation given the philosophy of a method.

Now, the M formulation also is similar, we could have several types of methods derived from the M formulation, but we will consider only one type of example. But before that let us, derive the M form of the equation. So, we have done it similar similar case earlier.

So, let us do it quickly. So, to derive the M form we first derive the V form. So, we integrate it with respect to x by multiplying it with V, and set it to zero. So, then what we do, we integrate it by parts. So, V into dy dx minus integral of dv dx into dy dx equal to 0. So, you have come up with a boundary term V into dy dx. Now, let us try to answer some questions. What is the variable on which you are giving the essential boundary condition. It is y, because V is variation of y - variation in y. So, that is called as the primary variable. Primary variable is the variable on which you are giving the variation in the boundary term. So, you have two types of variables. One is primary variable, another is secondary variable. Primary variable is this one, I mean if this is delta y, then this y is the primary variable, not V.

And secondary variable is dy dx. Specifying the primary variable at the boundary is essential boundary condition, specifying the secondary variable at the boundary is natural boundary condition. Here both the boundary conditions are essential boundary conditions. So that means, you have V equal to 0 at both x equal to 0, and x equal to 10. So, these term is no more important becomes zero. So, once these term becomes 0, you are left with integral of dy dx, dv dx dx between 0 to 10 is equal to integral of 100 V dx. This is of the form a y V is equal to l V. Where a is a bilinear operator, and l is a linear operator. So, this is a, and this is l.

What are the requirements that these are satisfying, these are continuous functions not only that a y V equal to a V y. So, a is symmetric, and a is bilinear, because in place of y if you substitute alpha 1 y plus alpha 2 V, and in place of V you substitute beta 1 y plus beta 2 V. Then you can see that you can expand it in the form of functions which are individually linear in each slot. We have seen that what do you mean by a bilinear operator, and here it satisfies that.

Similarly, this l is a linear operator. So, in place of V you can substitute say alpha y plus beta V, and you can see that it is alpha into l of y plus beta into l of V. So, this that is a linear operator, but is it sufficient to derive the M form on the basis of symmetry, and bilinearity and linearity or you require something else. You require additionally that it must be a positive definite operator. That means, a y y must be greater than 0. And that is indeed the case, because if you a y y means you replace V with y. So, it becomes dy dx whole square dx integral of dy dx whole square dx is greater than 0. Because dy dx whole square is positive, and the domain of the integral integration is also 0 to 10.

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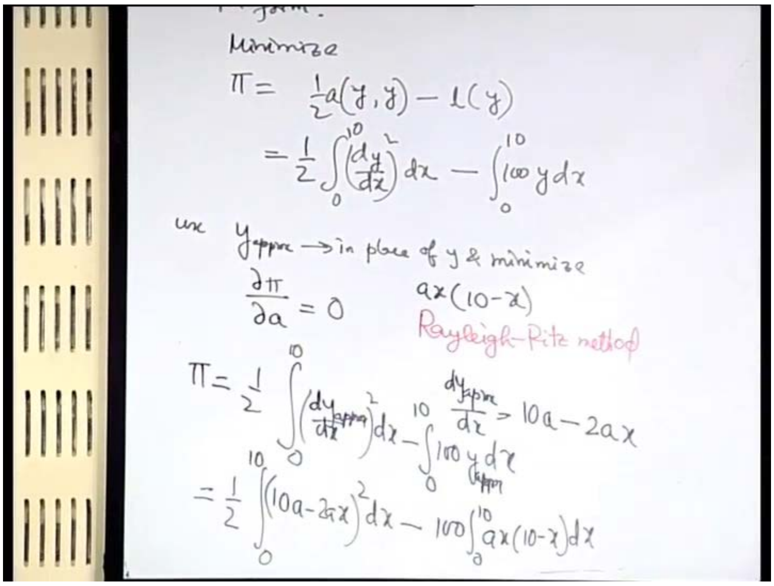

So, you have a y y greater than zero. That means, it is positive definite operator. So that means, its M form exists. So, M form is what is the M form? Minimize phi equal to half a y y minus l y. We have derived it earlier, in one of our previous lectures. It is very easy to derive it straight away from here by substituting V equal to delta y, and it will come out to be delta of this phi equal to zero. That means, minimize phi. So that means, minimize half integral of dy dx whole square dx minus 100 y dx. Now, if you approximate, so use an approximate function, in place of y and minimize phi, then that is one way in which you can achieve this minimization through an approximating function. So, here also you are having a trial function, but what is the difference with the with the others methods like the Galerkin's method, you do not have any weighting function here.

You have just a trial function, and you do not require any weighting function. But you had to pay a price or rather you had to have a requirement satisfied, because of which you can do it. That it is a positive definite operator - if it were not a positive definite operator, you could not have been able to come to this form. So, here you have come to a form where it is not only a minimization problem, where you require only a trial function not a weight function, and also there continuity of the trial function is one order less than that of the ordinary differential equation equation originally given.

(( )) half a y.

Half a y y yes, this is half a y y.

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So, now you try to minimize this one, and set its derivative with respect to a equal to zero. So, you can see that you can do that. So, we are considering y approximation y approximate same as a into x into 10 minus x, the same form. So, your objective is to minimize this based on the choice of this approximating function. Your approximating function, it could be up to continuous up to first order derivative for this case. Here of course, it is continuous up to second order derivative, that was very much essential for up for the weighted residual methods that we have used. Why, because there we could not utilize or rather there you did not utilize the integration by parts, and the related reduction in the order of the continuity requirement.

Here you have already utilized integration by parts. So, the continuity requirement is one order less. Similar thing, you can do for a fourth order equation, then you can come up with up to second order derivative. So, you can reduce, you can use functions approximating functions with reduce requirement of continuity. And no weighting function is necessary. So, that is one of the advantages of this method - and this method is known as Rayleigh-Ritz method.

So, let us do this exercise integral of. So, what in place of dy dx, we will be approximating approximating it with dy approximate dx.

So, then phi will be half of dy approximating dx whole square minus 100 y dx minus, here also y is y approximate. In place of y approximate a x into 10 minus x dx. So, let us complete this one.

So, the remaining part is simple algebra, if you help me then we can do it a bit quickly.

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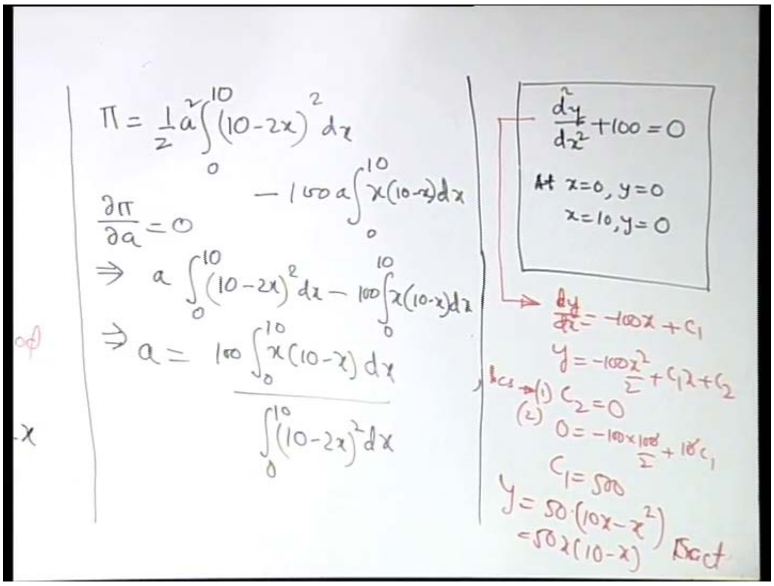

There will be one a square, then 10 minus 2 x whole square dx 0 to 10 minus 100 a. So, you have del phi del a equal to 0. That means which implies a is equal to. So, you can complete this exercise, this is a very simple exercise. And at the end, if you do the integrations there is no surprise, here also you should get a equal to 50. Question is why we have got a equal to 50 for all cases. So, it is very easy to come to that conclusion for that you just, complete the analytical solution of this equation. So, you integrate it once dy dx is equal to minus 100 x plus c 1, and y is equal to minus 100 x square by 2 plus c 1 x plus c 2.

So, apply the boundary conditions at x equal to 0, y equal to 0. That means, c 2 equal to 0. Second boundary condition at x equal to 10, y equal to 0. So, you can cancel one 10 here. So, c 1 is 500. So, y becomes minus 50 sorry 50, if you take 50 as common, then 10 x minus x square. So, 50 x into 10 minus x.

You can see that this is a special case where the approximate solutions, and the exact solution are the same. This is the exact solution. The reason is that at the exact solution is itself a second order polynomial form. And you have chosen the trial function also to be a second order polynomial form. So, we have used four methods actually. And in all the four examples, we have use the same trial function which is a second order polynomial function. Our objective was that what is the unknown coefficient in that trial function, so that the error is minimized in some way. In three of the cases, we have gone through the root of the weighted residual method, in another case we have gone through the root of the Rayleigh-Ritz method. But whatever was the method the objective was to minimize the error, based on a particular choice of the trial function and that choice of the trial function happens to be the exact form.

Therefore, what you can see here is that this coefficient 50 is coming out to be same, no matter whether you are using the actual actual exact solution or the approximate solution. In reality that will not happen for a problem, because you do not know what is your exact solution. So, your exact solution form will always be different in reality than your approximating function form. Only in a accidental case or where you know the solution, and you are just going for a check in such cases it may be same, but in general not.

So, what we have learnt out of these. We have learnt that how to use the M form or the V form to get a corresponding approximate solution of the differential equation. But one important thing is that these approximate solutions are global in nature, that means if you have the domain like this, you have an approximating function which you intend to make to to be made valid over this entire domain. So, you have this y approximate as a x into 10 minus x, that is a global form of the function that you approximate for as a solution over the entire domain. However, what you could do to improve upon your solution, you could use a higher order polynomial. Here it is not necessary, but there are some problems or in fact, for most of the problems your accuracy of the solution will go up, if you use a higher order polynomial.

But higher order polynomial will mean more calculations, and more tedious calculations. So, how you can reduce those calculations in a way, in a more effective way, you consider the domain to be locally divided into number number of sub domains. And you consider lower order polynomials for approximating solutions over each of these sub domains. So, what you are doing is, you are now not considering a global solutions, but a local solution. Because you are considering a local solution, the your entire global solution it may be whatever very complicated, but when you consider a small part of the domain, then this may be fitted by lower order functions. Whereas, if you wanted to do it in one shot over the entire domain, you would have required a much higher order polynomial function.

So, you are reducing the requirement of the order of your polynomial function, but the cost that you have to pay now is that, now you are sort of dividing your domain into a discrete number of sub domains. And for each sub domain you are going for a lower order polynomial to approximate your solution. That is the advantage, but the limitation now is that, you are no more having a continuous variation, but you are having only discrete points, and your solution is fitted within this discrete points. So, this is the basic concept of discretization. So, let us sum it up, that what is discretization?

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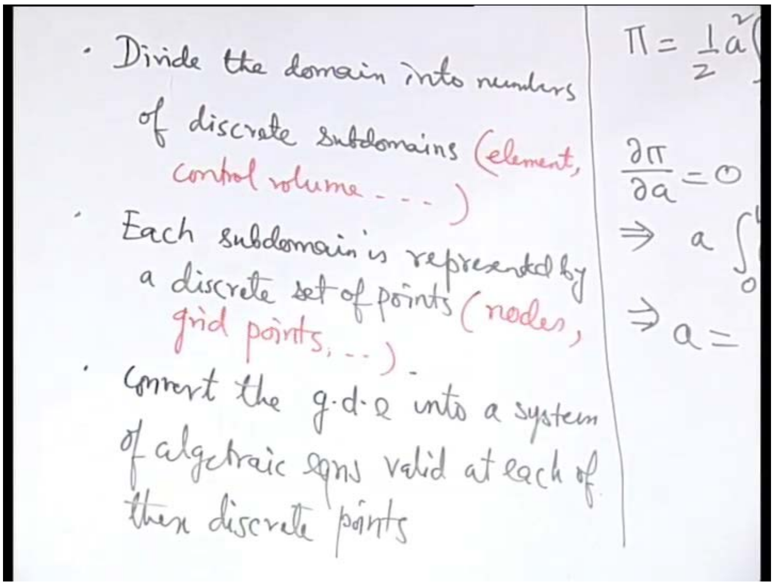

Divide the domain into numbers of discrete sub domains. The names of the sub domains may be element, control volume depending on the particular method, that you are choosing. So, once you divide the domain into a number of sub domains, then each sub domain is represented by a discrete set of points. So, the continuous variation of the function is lost, these discrete sets of points these are called as grid points or nodes, depending on again the method that you have chosen. And then your objective is to convert the governing differential equation into a system of algebraic equations valid, at each of these discrete points. That is the task of the method itself.

So, we can see that you can have different methods in principle. You can have for example, we will see finite difference methods, finite element method, finite volume method that we will see in our subsequent lectures. But first and foremost it is important to appreciate the general philosophy of these methods. Say what you are doing? You are losing the continuous nature of the solution, and considering the solution to be obtained for a discrete set of sub domains or points. And for each of these points, you are writing, you are tending to write a system of algebraic equations. Your objective is to convert the ordinary differential equation or partial differential equation, whatever into a system of algebraic equations. And there you apply the particular method that you are intending it, it could be finite element, finite difference or finite volume. How we can do that that is where the heart, and soul of the computational fluid dynamics lies. And that we will take up in our subsequent lectures, thank you.