Computational Fluid Dynamics
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Module No. # 01
Lecture No. # 04
Energy Equation and General Structure of Conservation Equations

We were discussing about the conservation equations and next we will move on to the energy conservation equation. So, till now we have discussed about the continuity equation, the momentum equation. And we have seen that, if you recall that, these equations are sort of coupled in nature, because you have unknowns and equations matching only when these equations are compatible in number, as the number of unknowns.

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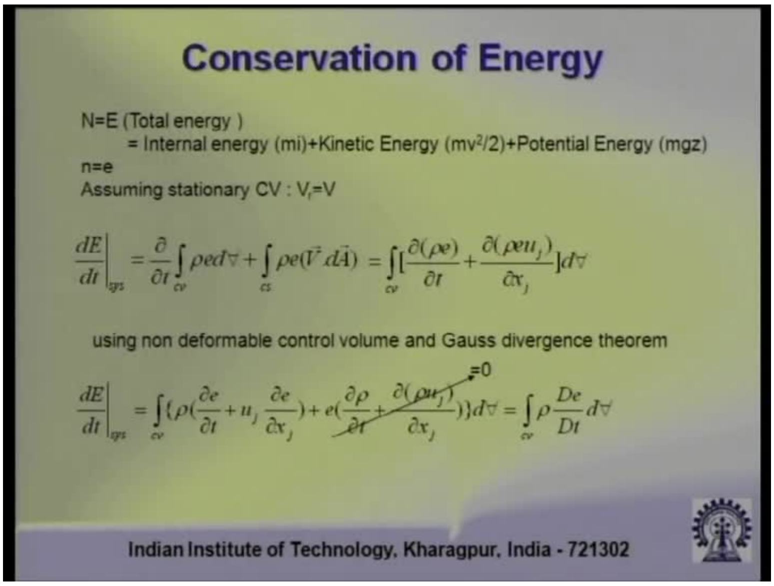

Next, we will consider the energy conservation equations. So, let us try to refer to these slides, where we sort of see a quick derivation of the energy equation. To do that, first of all we consider that we have the total energy of the system, which we need to conserve. So, we call capital E as the total energy, which is the sum of the internal energy, kinetic energy and potential energy. So, in the Reynolds transport theorem, we substitute our variable capital N, the extensive property as the total energy of the system. And we can substitute that total energy in the left hand side. In the right hand side, what we do? We use the specific total energy; that is, total energy per unit mass, which we call as small e.

So, in the right hand side we use this small e. And this is the transient term, and this is the outflow minus inflow of energy term. What you can see that we can make further simplifications to this term, first term, what we do is we put the time derivative inside the integral by considering it to be a non-deformable control volume, that is the first thing that we do. Then, the second thing that what we do is, we write this area integral in terms of a volume integral by using the divergence theorem. So, using these two, we come up with this simplified form. In this simplified form, we have both the terms appearing in the integral inside the integral term.

Next, what we do? Next, we try to simplify this term. So, the left hand side is as it is, the right hand side we are expressing it in terms of a non-conservative form. So, we have seen it through the example of the momentum conservation equation that how we express the conservative form in terms of a non-conservative form, here also we do the same thing. And here what comes out is the total derivative of the energy, capital DD t of e, by using the continuity equation. So, with this understanding now, it is important for us to express the left hand side.

See, why we have done all this? The right hand side gives the rate of change with respect to time plus the rate of change with respect to position, as you see it across a control volume. The left hand side talks about the same thing for a system. So, when you say that what is the corresponding expression for a system, we can use the first law of thermodynamics for a system.

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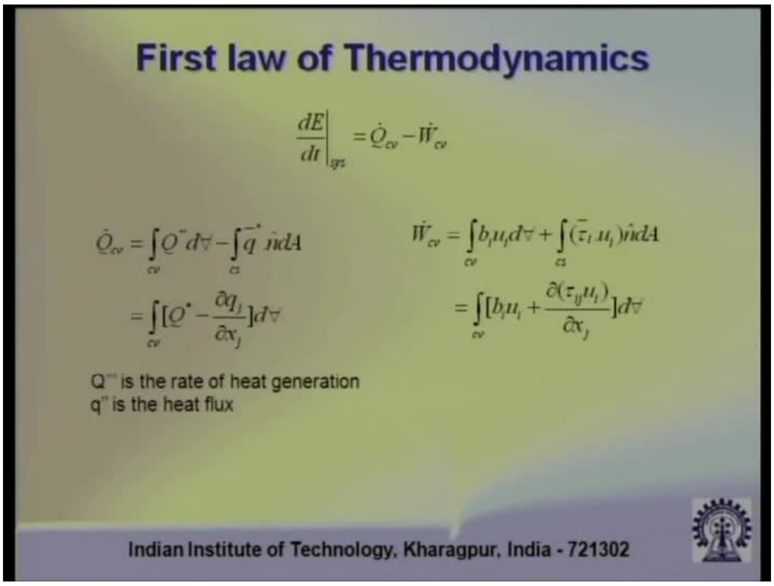

So, to do that we will go into the next slide, where we use the first law of thermodynamics. So, in the first law of thermodynamics we are writing the total rate, the rate of change of the total energy of the system as the rate of heat transfer to the system minus the rate of work done by the system. So, here we have to remember that work done by the system we are considering as positive and heat transfer to the system is considered to be positive. So, if it is in the opposite sign, it will come out to be algebraically a negative quantity.

Now, let us consider the heat transfer and the work done term separately. So, in the left column of this view-graph you have the expression for the heat transfer. So, when you have the expression for the heat transfer, there are just like you have two types of forces, body force and surface force, for heat transfer you have two types of heating, one is the volumetric heating which is a heating over the entire volume of the body, another is the surface heating.

So, the first term it considers the volumetric heating. So, capital Q tripe prime is the heat generation per unit volume. Again, we consider generation to be positive, so it is a heat sync, it will be negative. So, we can express it as a volume integral. The next term is a surface heating term which we express in terms of the heating flux. So, small q double prime is the heat flux vector. So, that is the rate of heat transport per unit surface area normal to the area per unit time. So, we can express the total heat transfer because of that by using this area integral. We have used a negative sign because a heat flux out of the control surface is actually a negative heat transfer, because heat transfer to the system is what is positive by our sign convention. So, that is why the negative term there.

Now, you can express this area integral in terms of a volume integral by using the divergence theorem. So, you can express the total heat transfer term in terms of a volume integral, that is the simplification with regard to the heat transfer term. Next, simplification with respect to the work done. Again, work done is because of work done due to body force plus work done due to surface force. So, what is the work done due to body force? You can write this as the body force per whatever, you either you express it per unit mass or per unit volume, does not matter, accordingly it will be either rho b i or b i whatever it is, and it is basically it is the body force dot product with the velocity.

So, you have to remember that it is the rate of what. So, it is just like the power, dot product of force and velocity. So, that is why b i u i. So, it is a dot product. Then you have the next term which is the surface force. Surface force, we have seen that you can express the surface force as the tau i vector dot eta d a and that dot with a velocity vector will give the work done due to the surface force. So, that is why that times u i. Again you can express the area integral in terms of a volume integral. So, you are getting this particular term for the work done for the control volume.

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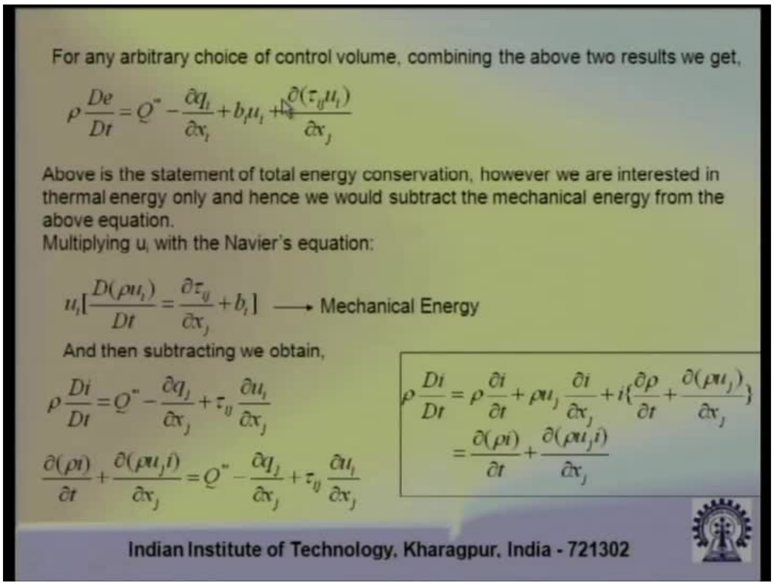

So, next what we can do is, for any arbitrary choice of the control volume we can combine these results; that is, we can write the volumetric term, we can write the surface term for both the work done and the heat transfer and then express the left hand side and the right hand side in this following manner. It is a statement of total energy conservation that we have to remember. So, a statement of total energy conservation means it considers the internal energy plus kinetic energy plus potential energy.

So, the left hand side is rho into the total derivative of the specific energy, the right hand side you have the volumetric heating term, the surface heating term, the work done due to body force and work done due to surface force. So, it is just it is nothing, but a differential form of expressing the first law of thermodynamics for a control volume, nothing more than that. Now, for the energy equation, for deriving the energy equation, we are ultimately interested for the thermal energy, not the total form of energy. So, what we have to do is, we have to subtract the mechanical energy from the total energy to get the thermal energy.

So, how do you get the mechanical energy? So, if you look into this equation, this is nothing but the Navier’s equation, the term between the square bracket is nothing but the Navier’s equation. This Navier’s equation, if you sort of take a dot product of velocity, so that will give nothing, but the mechanical energy. Because it is just like, if you consider u d u d t, this is d d t of u square by 2 where from the kinetic energy comes, similarly the potential energy will also come out from the conservative forces which appear in the energy equation.

So, what we do is, we make this manipulation for the mechanical energy. So, we manipulate on the Navier’s equation to get the mechanical energy and we have the total energy expression already there, we subtract the mechanical energy equation from the total energy equation, so what is remaining is a governing equation for thermal energy. So, that is the expression which appears here.

So, again when we are having this expression, remember this we have now eliminated the kinetic energy and potential energy. So, it was internal energy plus kinetic energy plus potential energy, small i is the specific internal energy; that is, internal energy per unit mass. So, we are left with that and that we can again switch from a non-conservative form to a conservative form by using the continuity equation.

In the box which is given in the right, what is shown is how you can transform the non-conservative form with the total derivative of internal energy into a conservative form where you also use the continuity equation. So, when you do that you are left with this equation, where you are now having internal energy as a variable, and right hand side you have the heat flux and the stress tensor with the rate of deformation tensor.

To proceed further ahead we have to relate the stress tensor with the rate of deformation tensor. So, again this is a general form of the energy equation, which is valid for even non-Newtonian fluids. But when you consider specific type of fluid, say the Newtonian fluid, then what you basically have to express is how do you write tau i j in terms of the rate of deformation. So, that distinguishes one fluid with the other. Let us move on to the next slide to understand that.

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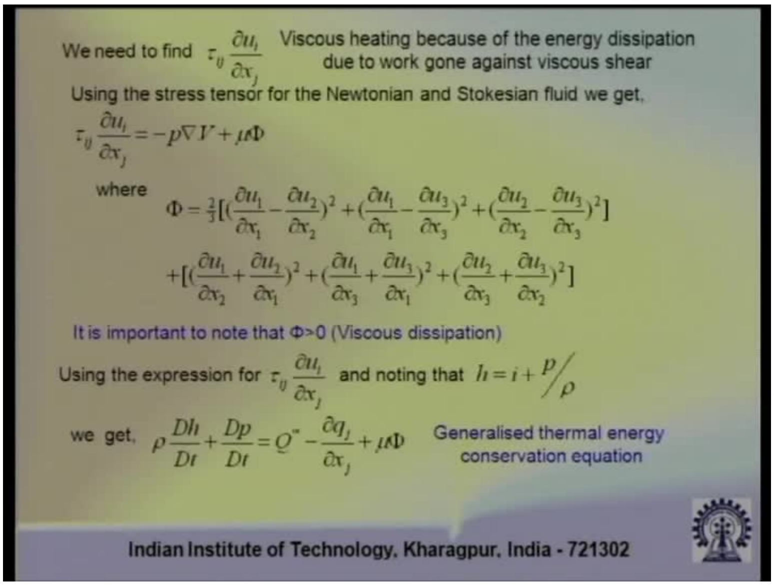

So, we need to find out tau i j del u i del x j. What is physically this? Physically this is the viscous heating because of energy dissipation due to work done against viscous shear. So, if you have a viscous fluid, there is a shear between the various fluid elements. Just like if you rub your palms, you will see that these get heated. Similarly, fluid elements, when you have a shear between various elements, because of viscous effects there is a local heat dissipation and that increases the internal energy of the system, so that it gets heated. So, this is viscous heating because of energy dissipation due to work done against viscous shear.

So, you can write, you can simplify that expression, how do you simplify that expression. For Newtonian and Stokesian fluid, we have already derived what is tau i j. Now, you can substitute that tau i j in the expression tau i j into del u i del x j. Once you do that and make an algebraic simplification, all those effects are combined now in the terms which are shown in the next equation.

So, in the next equation what is shown. You have these tau i j del u i del x j, which contains a work pressure work term plus a viscous term. So, minus p into divergence of the velocity vector is a sort of pressure work because of the volumetric dilation. And the second term is the so called viscous dissipation term, which is viscous heating because of energy dissipation due to work done against viscous shear.

You can see that this quantity phi is always a positive quantity, because it is a form of squares of certain numbers, so it shows that because of viscous effect there is always irreversibility in the system which gives always rise to heat generation, it does not give rise to cooling, it will always give rise to heating. So, it is important to note that this viscosity dissipation term is always positive.

We are proceeding step by step. So, we have expressed the heating terms in terms of the velocity and their gradients. So, once we have done that, next is we again concentrate on the left hand side of the equation where the expression was written in terms of internal energy. Our final objective is to write the energy equation in terms of temperature, because that is the most intuitively understandable measurable quantity. So, we have first written in terms of internal energy, next we will write it in terms of enthalpy, and then we will write it in terms of temperature to show the different forms of the equation.

So, let us see how to do that. So, when we convert the equation from internal energy form into enthalpy form, we require to utilize the well-known relationships in thermodynamics. So, what we do is, we write h equal to, where h is the enthalpy, specific enthalpy is equal to the specific internal energy plus p by rho, that is pressure into specific volume, that is pressure by density.

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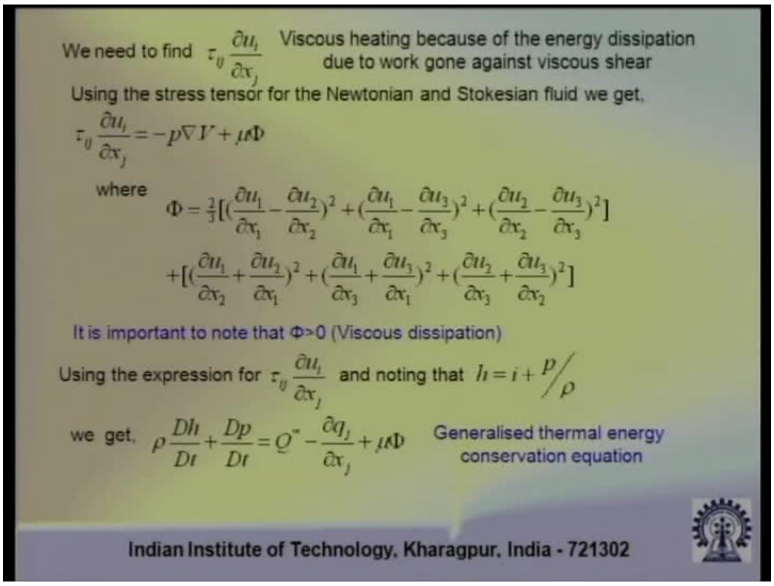

So, we are now writing this expression in terms of the enthalpy by substituting the enthalpy in terms of internal energy pressure and density; that is, the generalized thermal energy conservation equation. So, if somebody is interested to write the equation, generalized equation in terms of enthalpy, then this is the general form. But if we are interested for further simplifications, we have to relate the enthalpy with pressure and temperature. So, we have to express the enthalpy gradient in terms of pressure gradient and temperature gradient. So, what we do, we just write a thermodynamic relationship for the enthalpy.

So, if you recall from your basic understanding of thermodynamics, that for a simple compressible pure substance any intensic property can be written as a function of two other independent intensic thermodynamic properties. So, we can write the specific enthalpy as a function of temperature and pressure.

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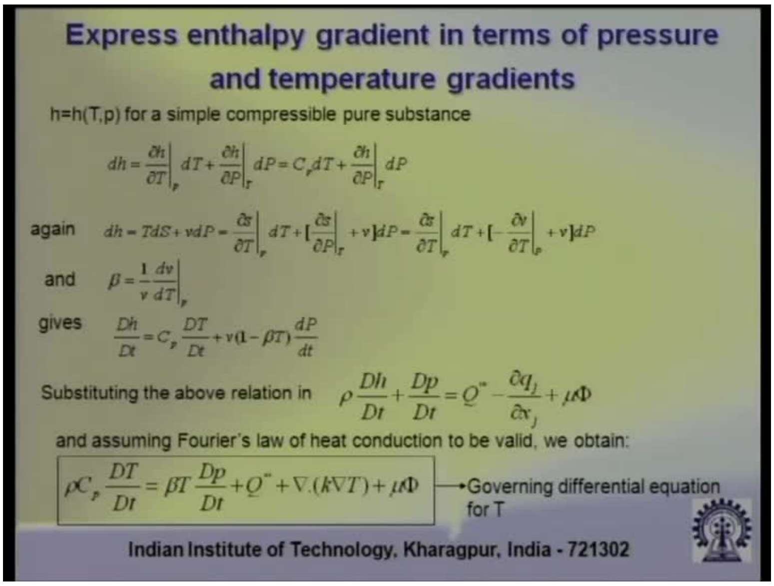

So, we can write, using the rule of partial derivative that dh is del h del T into dT plus del h del P into dP. So, by definition del h del T at constant pressure is equal to c p. So, this is c p dT plus del h del P at constant temperature dP. Now, we are using the thermodynamic relationship, T dS equal to dh minus vdP. So, it is a property relationship. So, it is valid no matter what kind of process you are considering.

So, if we use this T dS equal to dh minus vdP, so you can write dh equal to T dS plus vdP. And we are expressing the entropy S again as a function of temperature and pressure. So, what we are doing is, we are writing this del s del T at constant pressure into dT plus del s del P at constant pressure into dP, and then we add the vdP term. Now, for further simplifications what we do is, we write this entropy, the temperature dependence of entropy term in terms of other measurable quantities through one of the Maxwell's relationship. So, what we write is, we write this gradient in terms of, if you have this as s and t, what are the two other variables which will come in the Maxwell's equation, p and v. So, t and s if it is there, so this particular term, so it will be partial derivative of v with respect to the temperature, that will be the corresponding term of course, with a minus sign, that is one of the four Maxwell's relationships.

And you can use the definition of volumetric expansion coefficient beta, this is change in volume, per unit volume for each degree change in temperature. So, if you combine all these, then it is possible to write the total derivative of specific enthalpy in terms of the total derivative of temperature and the total derivative of pressure, that is what is expressed by these terms. So, it is essentially an outcome of exercise of using thermodynamic relationships, it is nothing more difficult than that.

Now, once you do that, you can combine that relationship with the energy equation right hand side which you have already derived and come up with the general governing differential equation for temperature, where we have expressed the heat flux using Fourier's law of heat conduction. So, the heat flux is proportional to the negative of the temperature gradient is the Fourier's law. And it is again a linear constitutive relationship, where the heat flux is linearly proportional to the negative of the temperature gradient. So, just like you have the Newton's law of viscosity for momentum transport, it is a similar thing for heat transfer, where here the proportionality constant is the thermal conductivity k.

So, if the thermal conductivity k is appearing here, then you can see that you have a form of the conservation equation, the energy conservation equation, where this is a term which is the total derivative of temperature in the left hand side, you have rho into C p which has evolved from these transformation of enthalpy to temperature. So, you have the density and the specific heat. The right hand side, you have one term due to the pressure work. So, this is beta T into total derivative of pressure, this is somewhat related to volumetric dilation work.

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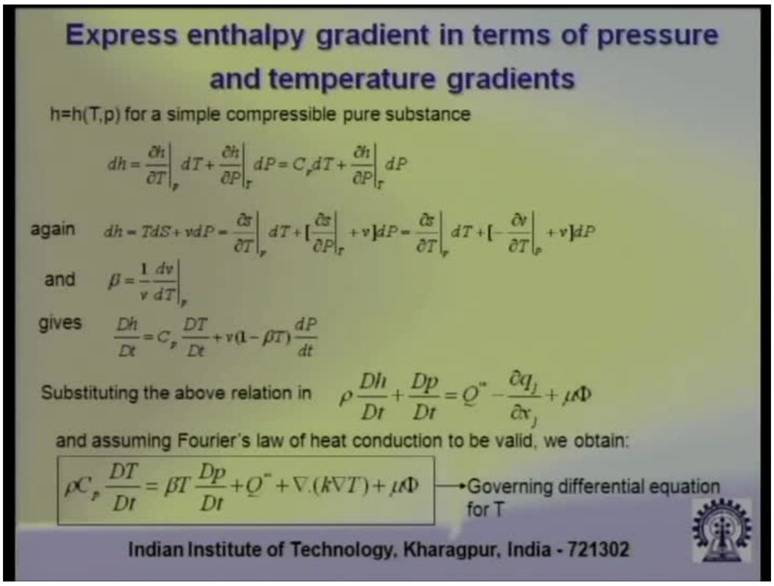

Then, you have the heat generation rate per unit volume, then this is the heat conduction term and this is the viscous dissipation term. In many of the problems in heat transfer, because of low velocity gradients, the viscous dissipation term is neglected and if compressibility effects are not important, then volumetric dilation term is also neglected. So, in the right hand side, you are commonly left with the heat conduction term and the heat generation term per unit volume.

So, we have seen the energy equation as specific example of a conservation equation. So, how many conservation equations we have seen? We have seen three different conservation equations, the continuity equation, the momentum equation and energy equation. All these equations we have derived by using a common synergy. So, what we have done, we have started with the Reynolds transport theorem, we have expressed all the terms in terms of either volume integrals or area integrals, wherever area integrals appeared; we converted those into volume integrals by using the divergence theorem, and then combined all those terms in the total integral expression and obtained a differential expression from the integral expression by considering the choice of the control volume to be arbitrary, that is the general principle that we have followed.

Now, once we have been successful in mathematically deriving the general forms of these equations, one thing we must keep in mind, that since it is possible to derive these equations in a general form, there must be a synergy in these equations somehow. Where lies the synergy? The synergy lies in terms of the fundamental principle, what is the fundamental principle? It is a conservation of something. It may be a conservation of mass, it can be a conservation of momentum, it can be a conservation of energy or whatever, but it is basically conservation of a physical variable.

So, once you are considering the conservation of a physical variable, then it has certain important mechanisms and these mechanisms do not differ from one case to another. For example, if you consider the transport of heat from one point to the other, it has two types of mechanisms, one is conduction, another is advection. So, one is dependent on the gradient of the variable, another is dependent on the bulk fluid motion.

Similarly, if you have momentum transport also, you have momentum transport due to advection, you also have momentum transport due to diffusion of momentum, because of gradients of velocity, that is nothing but the viscous effect. So, there is a synergy between all these, and to understand what is the synergy let us try to derive a general form of the conservation equation.

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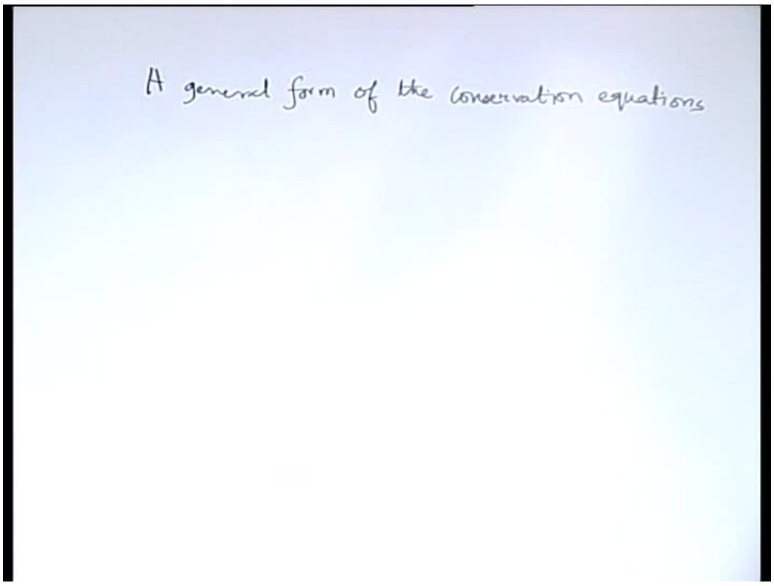

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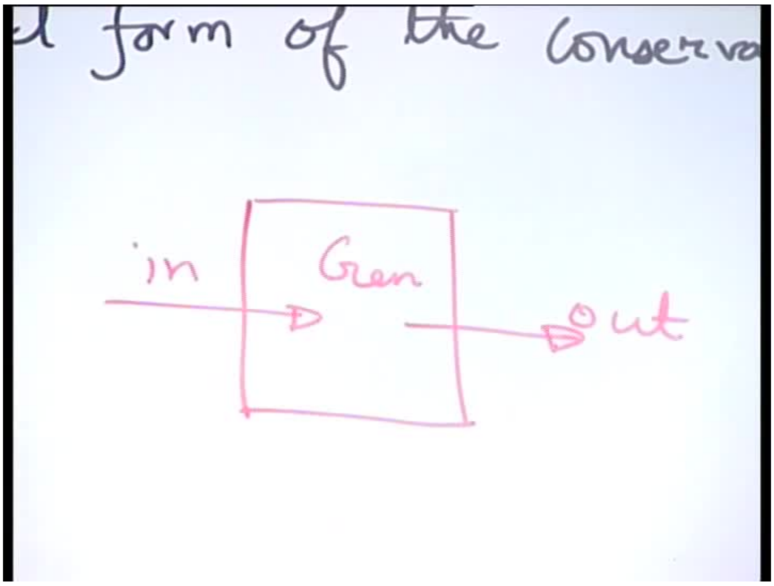

So, a general form of the conservation equations. To derive that, we will start with our basic principle of conservation, that if you consider a control volume you have something in, something out, something generated and rate of in minus out plus rate of generation is equal to rate of change with respect to time inside the control volume.

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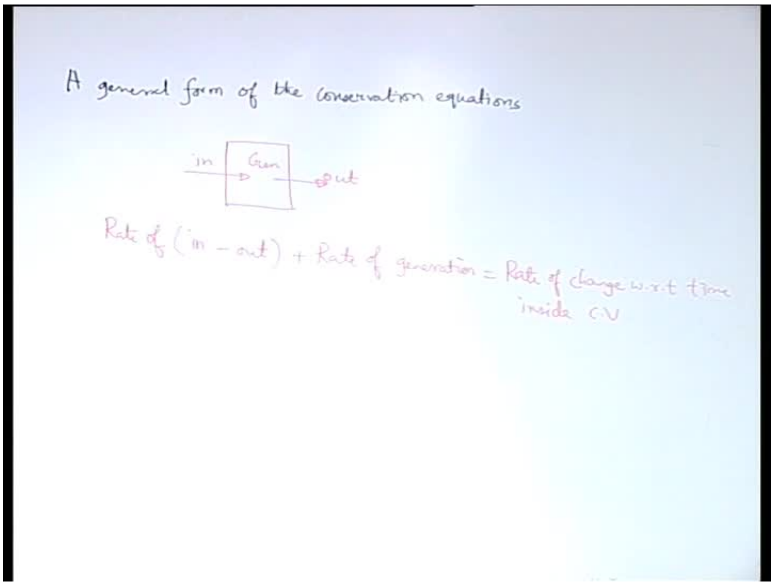

This is the basic statement of conservation that we discussed in our introductory lecture. We gave an example through a bank account balance and it does not matter what kind of balance we are talking about, it is the same philosophy that works. Now, what we will do is? We will try to express this qualitative statement in terms of a mathematical quantification.

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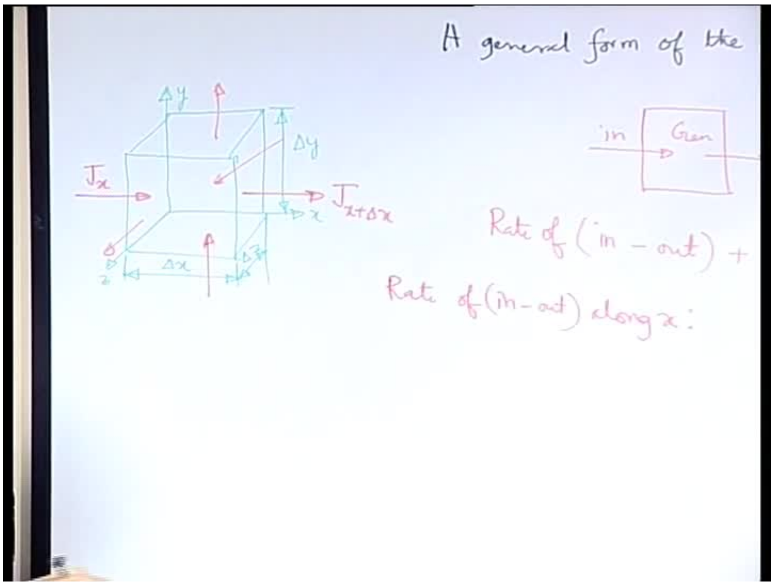

So, we will consider a control volume, let us make a sketch of a control volume, for convenience let us consider a control volume of a rectangular shape. So, let us say that you have a small rectangular control volume with the dimensions as delta x, delta y and delta z. And let us write the quantities, rate of in minus out, rate of generation and rate of change with respect to time inside the control volume by referring to this. To do that, we first introduce a flux that enters and leaves the control volume. So, let us say that J x is the flux that enters the control volume through the left face, flux of what? See, we are trying to be a bit abstract now, we have seen special examples of mass momentum energy conservation, now we are trying to be clever, we are not committing whether it is mass conservation, momentum conservation, energy conservation, what conservation that we are not committing, what we are saying is that it is transport of some variable. So, what is that variable? Let us say that phi is the quantity per unit mass that is transported and J represents flux of that quantity.

So, J x subscript x indicates that it is a flux along the x direction. So, J x is the flux that enters the left face and the flux that leaves the right face is J at x plus delta x. If the left face is x, x equal to x, the right face is x equal to x plus delta x. Similar fluxes do occur definitely for y and z directions, but just for simplicity we are writing only or we are detailing only what happens along one direction and we will extend that concept to the other directions. So, let us write what is rate of in minus out along x, let us write that.

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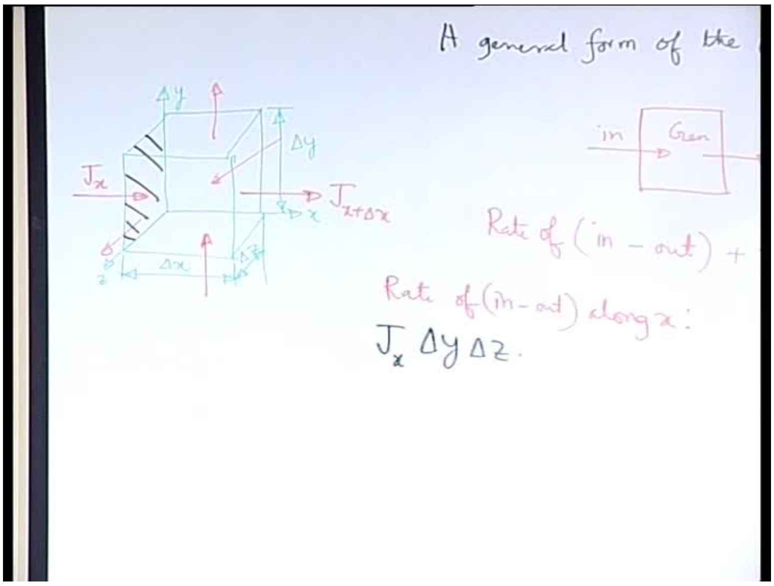

So, what is rate of in along x? Remember J x is the rate per unit area. So, J x into delta y into delta z, which is the area of the shaded face. The opposite face has the same area, but what changes is J x becomes J at x plus delta x.

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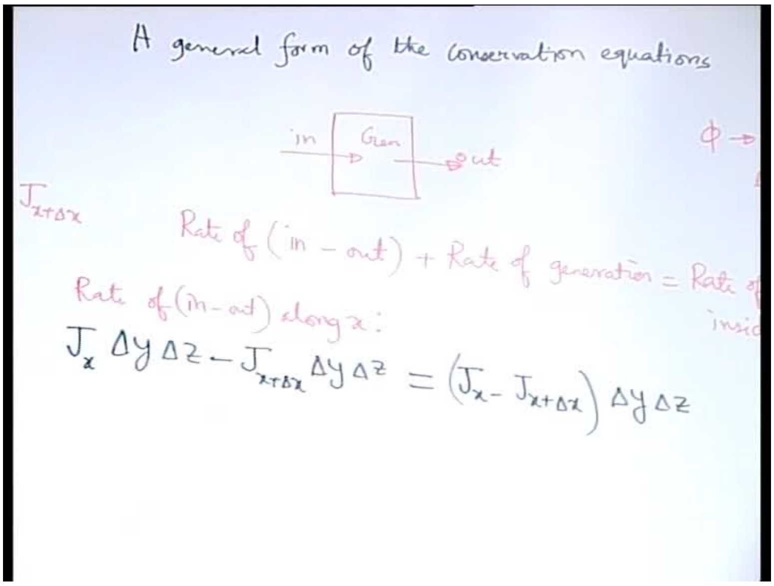

So, minus J x plus delta x delta y delta z. So, what is this? This is J x minus J x plus delta x…

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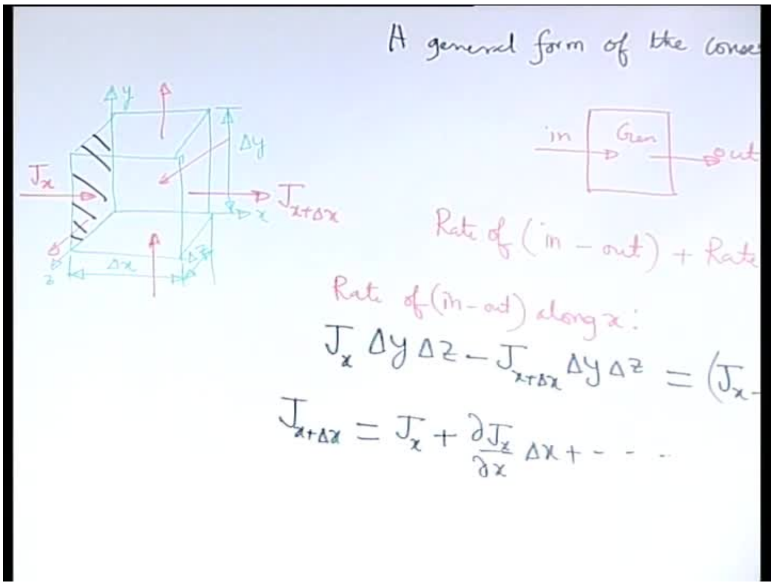

Now, you can express J x plus delta x in terms of J x by using the Taylor series expansion. So, you can write J x plus delta x as J x… We are not writing the higher order terms because we will eventually take the limit as delta tends to 0, when those terms will be vanishingly small.

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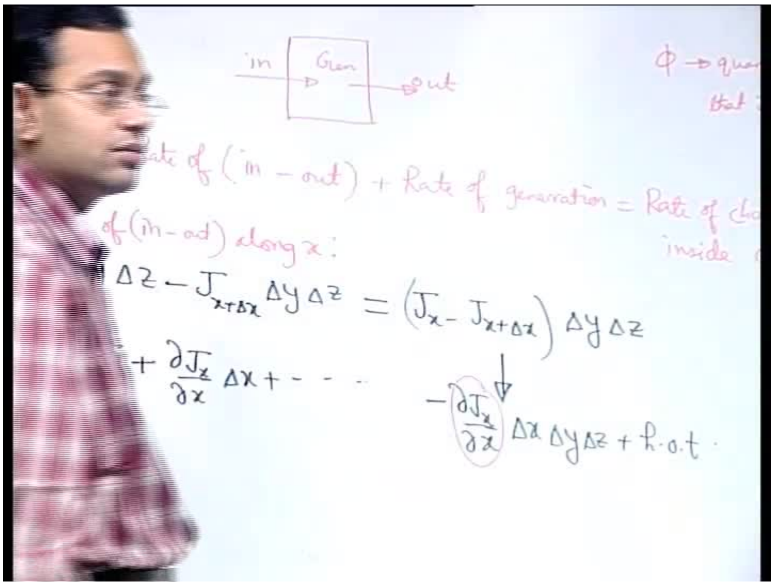

So, you can write this particular term as… plus higher order terms. So, those will involve delta x square, delta x cube like that. So, this is rate of in minus out along x, you can see that the x specific term is this one. So, you can have similar term for y and z, only x will be replaced with y and z.

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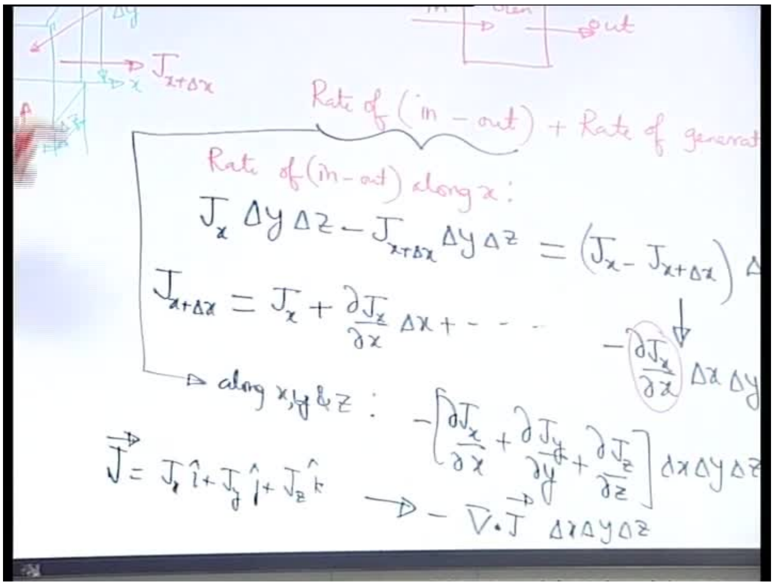

So, we can write the total rate of in minus out along x, y and z. So, if you consider the flux as a vector with components as J x, J y and J z, then this is divergence of the flux. So, this is nothing, but minus of… So, we have been able to mathematically express to some extent the rate of in minus out. Next is rate of generation. So, for rate of generation, usually one expresses it in terms of the rate of generation per unit volume.

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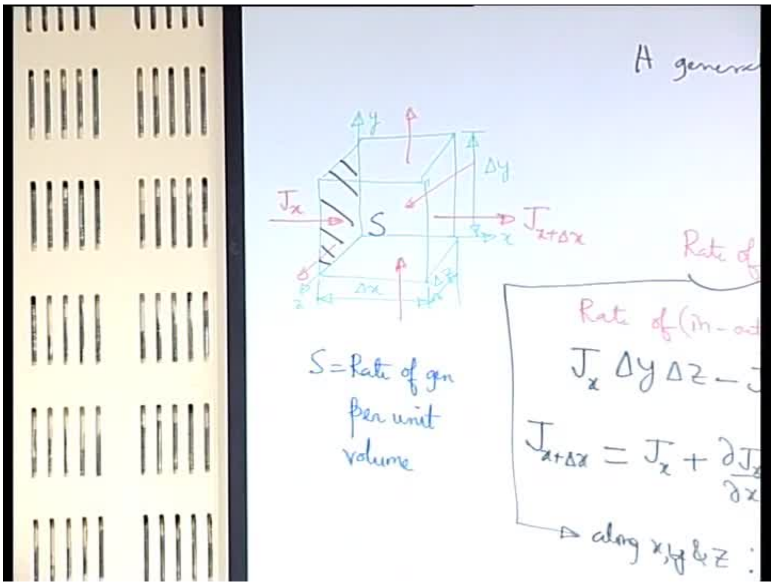

So, let us say that S is rate of generation per unit volume. So, if S is the rate of generation per unit volume, then what is the total rate of generation? S into delta x into delta y into delta z, very simple and straightforward.

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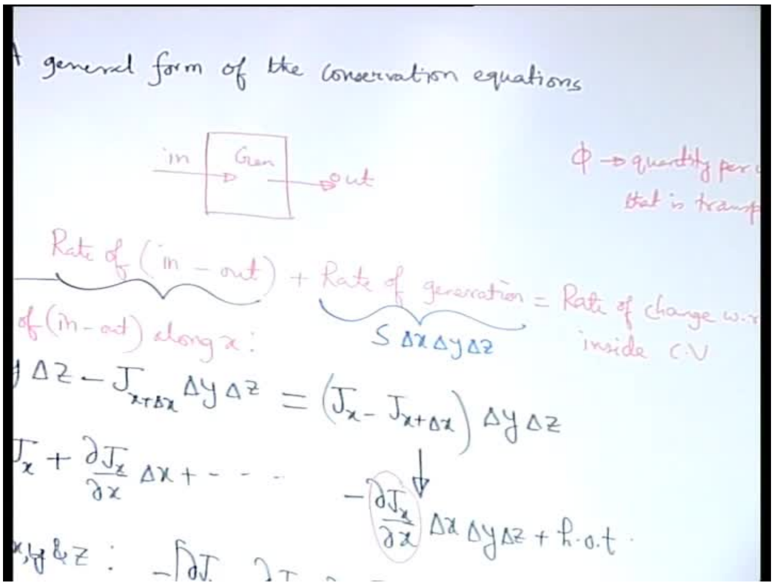

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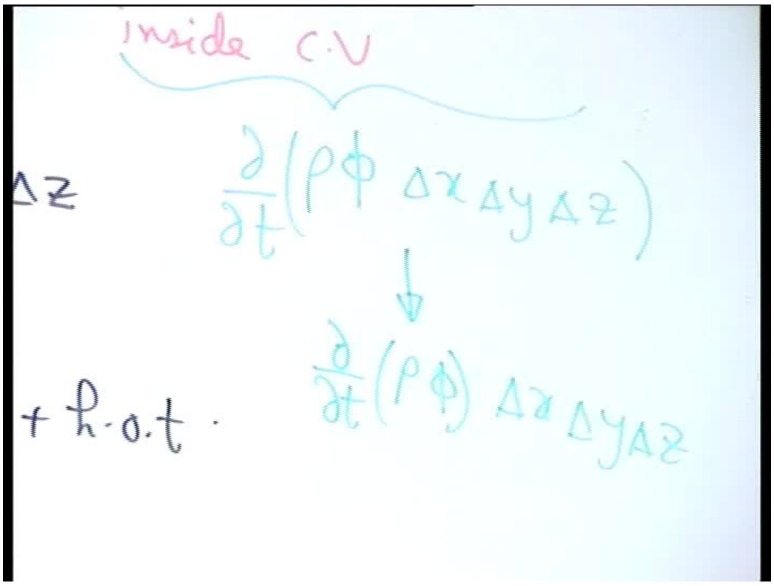

Then finally, the last term in the right hand side, that is rate of change with respect to time inside the control volume, this term. First of all, what is the rate of change? It is the rate of change of the total quantity. What is the total quantity inside the control volume? Phi is the quantity per unit mass. What is the mass within the control volume? Rho into delta x into delta y into delta z. So, rho delta x delta y delta z, this is the total quantity inside the control volume, partial derivative of this with respect to time is the rate of change with respect to time.

Again just look into the analogy with Reynolds transport theorem, here we are using the partial derivative, it is actually the term within the control volume, you can clearly see it is inside the control volume. So, we are fixing the position that is the control volume and finding out the rate of change with respect to the time. So, this is… Because delta x delta y delta z, these are sort of fixed quantities, you can take those outside the derivative. So, if you equate the left hand side and the right hand side, what you are left with? You are left with certain terms and delta x into delta y into delta z these are cancelled from both sides.

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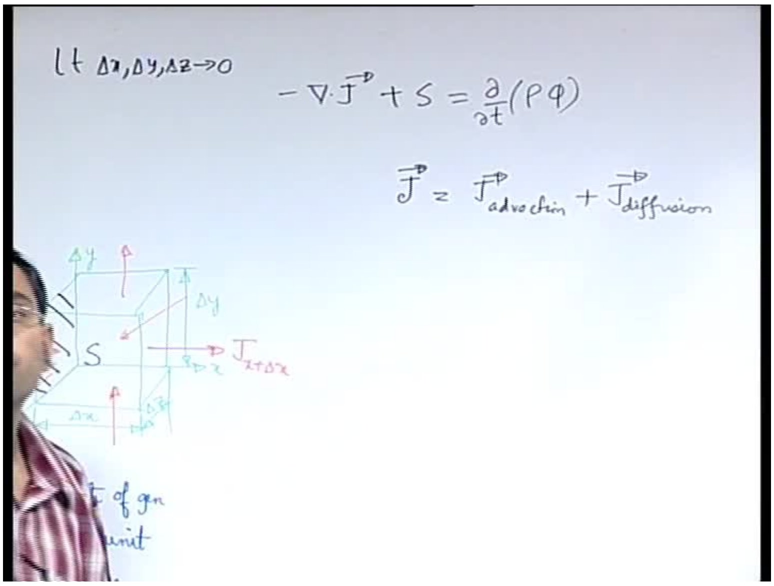

Because we are taking the limit as delta x delta y delta z are tending to 0. Therefore, these are not actually equal to 0, these are vanishingly small. So, you can cancel these from both the sides and you are left with this particular equation. Minus divergence of J plus s… The next big question that we have to answer is that what is this J? Till now we have abstracted this J from any physical consideration, we have just considered that it is a flux, in one way it is good because if we have a flux that is being transported over the surface, the control surface in any physical problem, we can express the physical problem in this mathematical form. But the mechanism of the transport of the flux is important and let us see that how we can sort of incorporate that in this particular equation. So, when we say J, to be more physical in the problems that we are considering the transport phenomena problems, J will be sum of J advection and J diffusion. Let us spend a couple of minutes to understand that, what we mean by advection and what we mean by diffusion.

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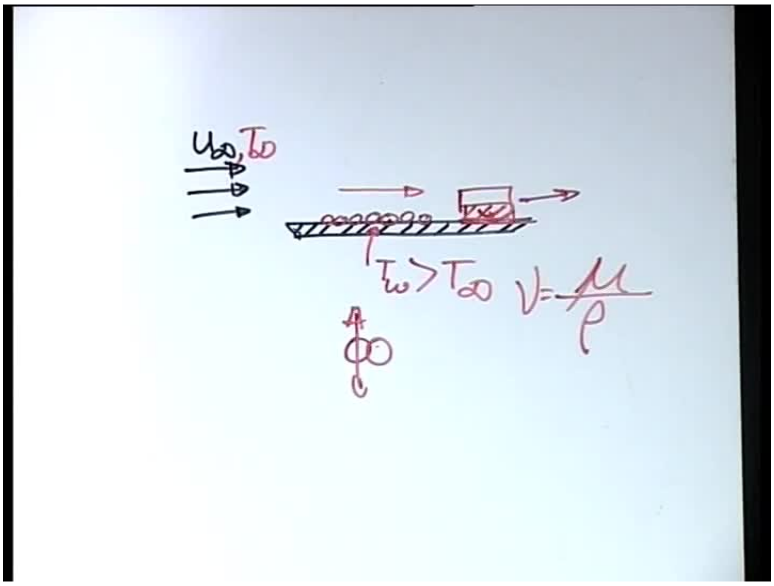

So, let us say that you have the classical example of boundary layer development on a flat plate. So, you have a fluid coming from first stream and with a uniform velocity, say u infinity. Let us say that this plate is a heated plate and this is a cold fluid, so this has a temperature to infinity, the plate has a temperature t, say by some mechanism you are heating the wall of this plate, say by a strip heater you are heating it where the wall temperature is always maintained greater than the first stream temperature. What is the objective? The objective is to heat the fluid which is coming on the plate.

So, as you see that as the fluid flows over the plate, then what happens? There is a transport of heat along the actual direction, what is the predominant mechanism. Typically if the velocity of this is quite high, then there is a bodily movement of the fluid from one point to another point, and whatever heat is going to the fluid, that is being transported from one point to the another point because of this bodily movement.

But the first question that we have to answer is that, how does the heat reach the fluid? Of course, if you consider the fluid which is there adhering to the solid boundary, this is stationary by mostly boundary condition, until and unless there are special circumstances when the boundary condition is violated. In the general case, it is a mostly boundary condition.

So, under that mostly boundary condition, you will have stationary molecules, on an average the fluid layer is fixed. So, when you have the fluid layer fixed, it cannot be true that by bulk motion this fluid layer is transporting energy. So, how can energy be transported from this layer to the outer layer? That is by mechanism of conduction. So, you have in the first layer there is a conduction which sort of gives this thermal energy which is available at the plate to the bulk fluid, to the outer fluid, and then this outer fluid transports it along different directions through a combination of conduction and the bodily movement of the fluid. So, the bodily movement of the bulk fluid that carries energy from one point to another point, it is very much analogous to the example of what a conveyor belt does, say in an airport.

So, you put a suitcase on a conveyor belt. So, what it basically does? It transports the suitcase from one point to another point. The conveyor belt is like a fluid flow and the suitcase is like a packet of energy, so to say. And the conveyor belt is transporting that from one point to the another point by its bodily motion. So, this type of transport where the transport of the quantity, the quantity can be mass, momentum, energy, whatever is taking place by virtue of bodily motion of a fluid element, then you call that as advection.

On the top of that, you also have a transport where it does not depend on the bulk motion. For example, heat conduction, heat conduction does not depend on the bulk motion of molecules. So, one of the mechanisms can be that the molecules are in random thermal motion and in that way they are vibrating with respect to their mean position and by that they are possibly interacting with the surrounding molecule and exchanging their energy, that is how heat is being transferred. Although there is no bodily movement of the molecules from the respective mean position, only they are instantaneously vibrating. So, that type of transport we call as diffusion.

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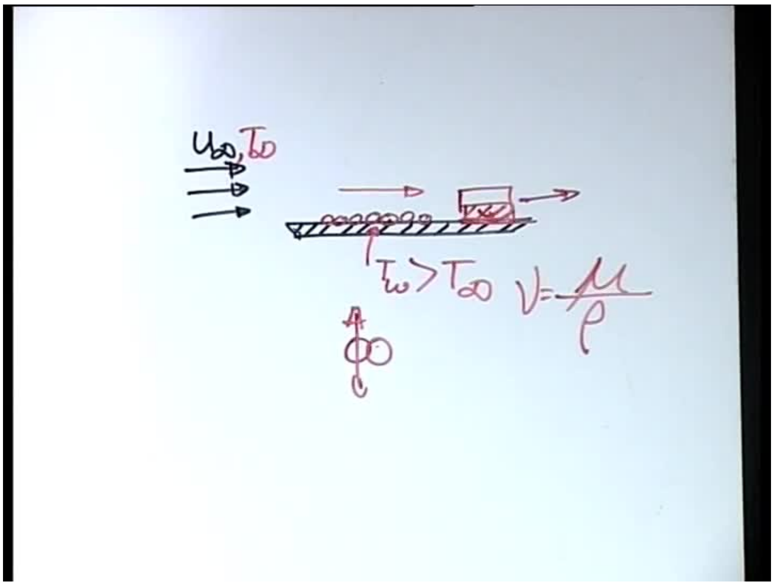

So, conduction of heat in a way is also called as heat diffusion. So, if you have a diffusion, you can have a diffusion of heat, as an example you can also have a diffusion of momentum. How can you have diffusion of momentum? So, you have because of the effect of the plate, you have a disturbance in the flow, disturbance in the fluid velocity. Now, if you go to the next fluid layer, which is there. This fluid layer has two opposing effects on it, one is the upper fluid layer tries to make it push forward, the other is the wall tends to slow it down. So, this layer feels the effect of the wall directly. So, its bottom is sort of anchored to the plate, but the top one tends to move towards the right a little bit because it is somewhat away from the wall.

What happens to the next fluid layer? It is not directly in contact with the wall. If it is not directly in contact with the wall, then it would have been transported with a free stream velocity, but it does not do that, because it also feels the effect of the wall. How does it feel the effect of the wall? Not directly in contact with the wall, but implicitly through this intermediate fluid element. So, this intermediate fluid element is like a carrier which carries the message that there is a wall from the wall towards the outer fluid and the fluid must possess a property by which this message of momentum disturbance is propagated inside the fluid, that property is the viscosity of the fluid.

So, you have a momentum disturbance that is being transported from the wall to the bulk fluid and the viscosity is the property that does that. But is it the viscosity that is solely important? It is not so because the fluid also has a tendency to retain its momentum, that is if it is moving at a fast rate it tends to move at a fast rate, if it is slow it tends to move at a slow rate and the momentum disturbance sort of changes this. So, viscosity is the disturbance in momentum. There is something which is an indicator of the sustenance of its momentum and that is the density of the fluid. Because density of the fluid is an indicator of the mass of the fluid element, which is a measure of the inertia of the fluid. So, it is the viscosity relative to the density, what is important, not the viscosity alone, which we call as kinematic viscosity, this is also interpreted as a momentum diffusivity.

So, what we have interpreted from here is that, there is a disturbance and there is a diffusive mechanism that makes the disturbance to propagate inside the system, so that other points in the system also feel the effect of this disturbance. Similarly, thermal conductivity of the fluid or solid whatever, it allows the material to feel the effect of heating or cooling at any other point, that effect propagates inside through a mechanism of conduction through thermal, by sort of using the property thermal conductivity of the fluid.

Now, there is a synergy between all these. No matter whether you are talking of viscosity or you are talking of thermal conductivity, these are material properties, but these properties are not the sole determining factors for the flux. The corresponding fluxes are related to the negative of the gradient of the corresponding variables, just like the heat flux is proportional to the gradient of the temperature, the momentum flux is proportional to the gradient of the velocity.

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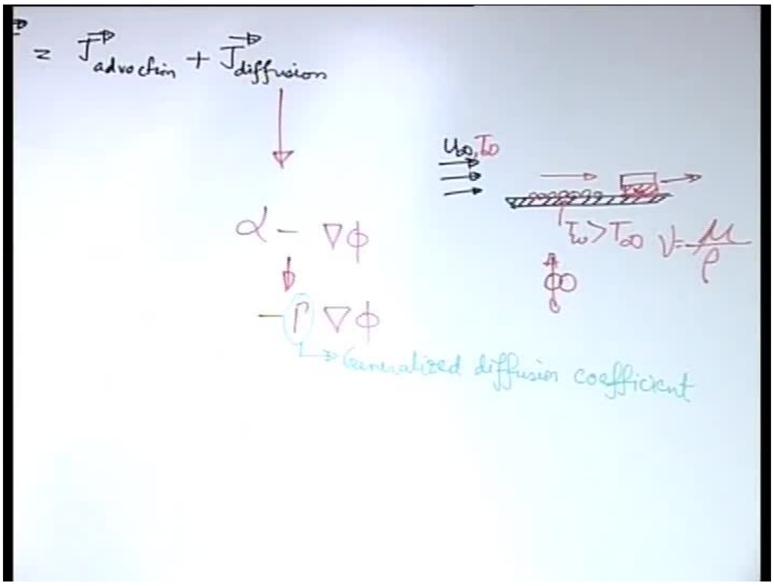

So, in this way what we can see is that, if you generalize that as a diffusion flux, the diffusion flux is proportional to negative of the gradient of the variable that we are considering to be transported. Remember phi is the scalar quantity that we were considering, just like temperature or one component of the velocity, it is any scalar quantity per unit mass that is being transported. So, the diffusion flux is proportional to the negative of the gradient of the particular variable which is being transported. This is the constitutive law, just like Fourier's law of heat conduction or Newton's law of viscosity, this is a general way of expressing all those laws in a common framework. So, we are considering diffusion as a mechanism, where the constitutive relationship is following this type of behavior. If it is a different type of behavior, we can express it mathematically, but because this is the most common behavior, we are giving this as the generic example. So, we can write the diffusion flux as minus a constant of proportionality times the gradient in the variable phi, where this gamma is called as the generalized diffusion coefficient. So, for momentum transfer it is something, for heat transfer it is something, it is just different for different transports. Let us come to the advection flux.

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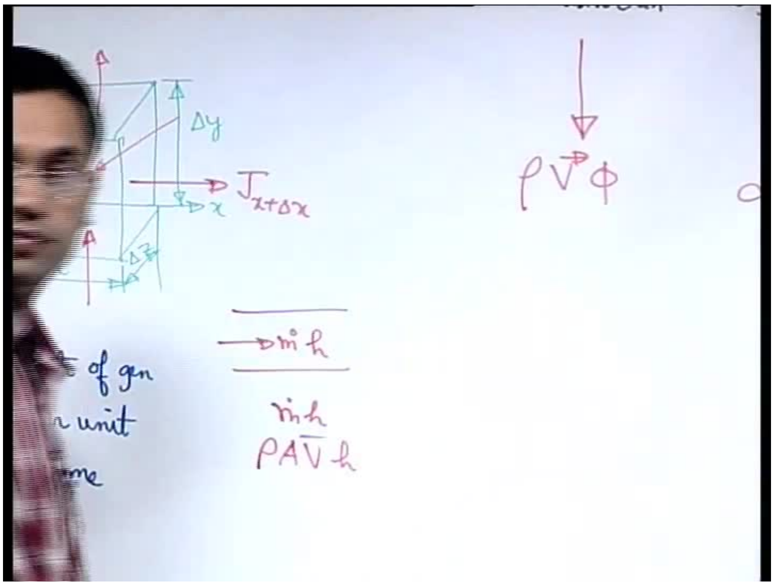

To understand that let us say that you have a, just let us take an example, let us say that you have a pipe, and let us say that m dot is the mass flow rate across each section of the pipe, and let us say that h is the specific enthalpy. Then what is the rate of transport of thermal energy because of the mass flow? m dot into h. m dot is rho A into average velocity into h. If you consider it at a point, then instead of average velocity you replace by local velocity. So, what is the flux corresponding to this? Remember this is advection, this is advection of heat. So, what is the flux corresponding to this? Divided by the area. So, rho into velocity into the variable per unit mass. So, it is rho into velocity into phi because this is flux, this is per unit area. So, you can follow from this example that why it should be rho V phi.

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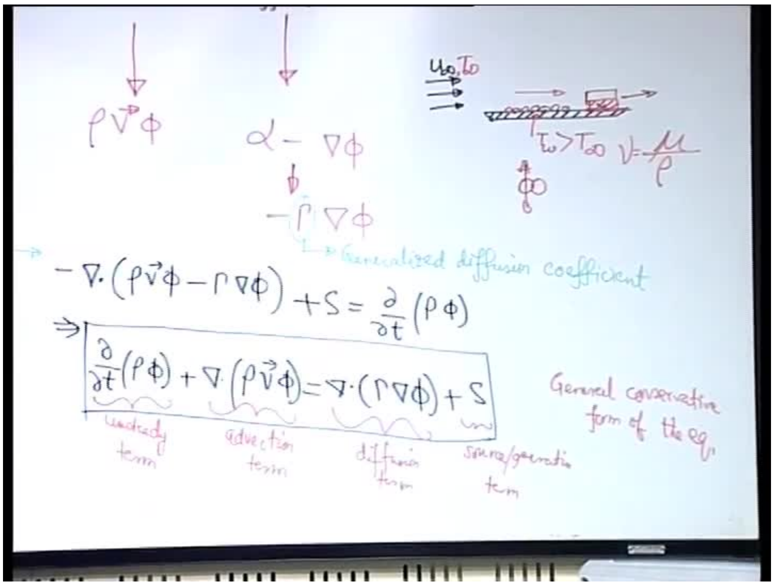

So, we can substitute that in the equation for the flux. So, minus… This equation is like the heart and soul of the CFD course that we are going to undergo. See, this is an equation which does not tell you whether it is mass conservation, momentum conservation, energy conservation whatever, it just tells that it is a general conservative equation where it conserves something. Irrespective of what is that something, it comes up with different terms with different physical interpretation. This is the unsteady term, this is what, this is the advection term, because this has originated from advective flux, this is diffusion term and this is source or generation term, and this is the general conservative form of the equation.

You can see it is a conservative form because rho is inside the derivatives, it is not outside the derivatives. So, now, because it is a general form it will assume special forms depending on what conservation we are talking about, is it mass conservation, momentum conservation, energy conservation. Let us consider some example where we show that indeed our special conservation equation, like the mass, momentum, energy all fall in this general framework. Let us try to show that. So, what we can see here, that in this equation there are certain things which are which are generic. We have not committed what is phi, we have not committed what is gamma and we have not committed what is S. So, depending on what we choose for phi, gamma and S, we can represent different equations.

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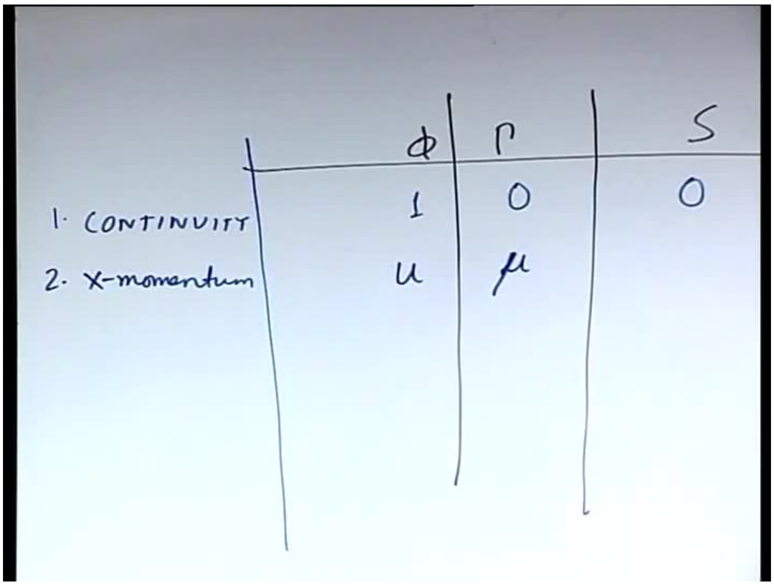

So, let us try to make a table, where we represent different cases based on what is phi, what is gamma and what is S. So, the first example that we will consider is a continuity equation. What is phi in the continuity equation? What we want to conserve? Mass, phi is a quantity per unit mass. So, it is 1. What is gamma in the continuity equation? We do not have any diffusion. 0. And what is the source? 0. Let us say x-momentum equation. What is phi? u. What is gamma? Mu. What is S?

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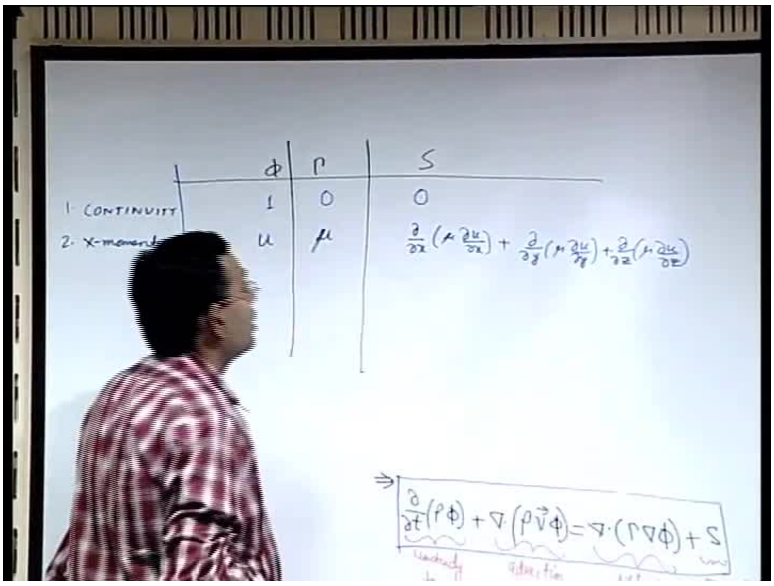

So, you have one term… which is the collection of basically these three terms. What is that? Del del x J of mu del u i del x J, it has these three terms.

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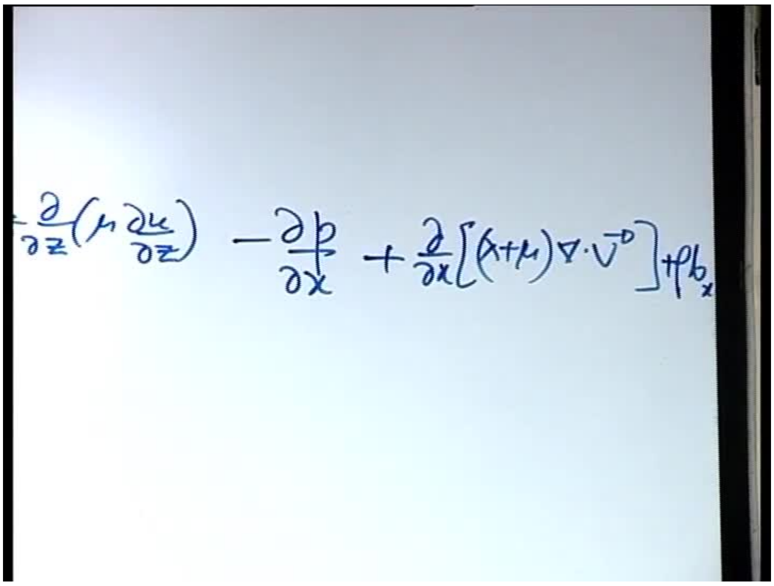

Then you have pressure gradient term. And if it is a compressible flow, you have an additional term. So, you have a viscous term, pressure gradient term and volumetric term, all these are body forces. Similarly, you can write y-momentum and z-momentum, I am not repeating those, just you have to replace u with v and w. Yes if there is some extra body force, then you have this one. So, you may also have extra body force beyond this.

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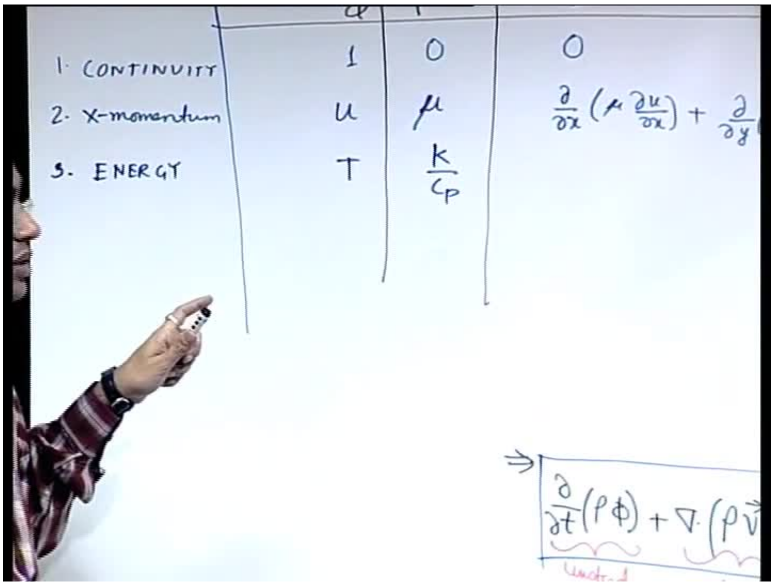

Let us consider energy equation. How do we write the energy equation? It depends on what is the variable in terms of which we are writing, are we writing in terms of enthalpy, temperature, internal energy. Let us say we are interested to write in terms of temperature. So, if we write this here t, then what will be this.

See, in the left hand side you have rho Cp t, and here you have k. Rho remains inside because it is a conservative form. So, it becomes k y Cp. This is not same as thermal diffusivity or alpha, alpha is k by rho C p. But it carries a similar physical meaning, only thing is there we do not have rho, here because rho is clubbed with t, it is there as a conservation form because we want to cast all the equations in the same form, but it carries a similar meaning as in the thermal diffusivity. Just like the kinematic viscosity, it carries the meaning of what? It carries the meaning of the propagation of momentum disturbance relative to the sustenance. Here it is a propagation of the heat conduction relative to the thermal storage capacity. Because more C p means it can store more thermal energy, on the other hand more k means it can dissipate more thermal energy. So, it is the relative ability of dissipation with respect to storage. For momentum transport, it is important in terms of the viscous effect. For heat transfer, it is in terms of the conductive nature.

So, here also it could have been mu by rho, but because it is a conservative form, the rho is clubbed within the equation, so that is why it has not appeared. But physically here the important parameter is this by rho, here also this by rho. Because rho is retained within the derivative that is why they do not mathematically appear, but physically they are very important. And what will be this one?

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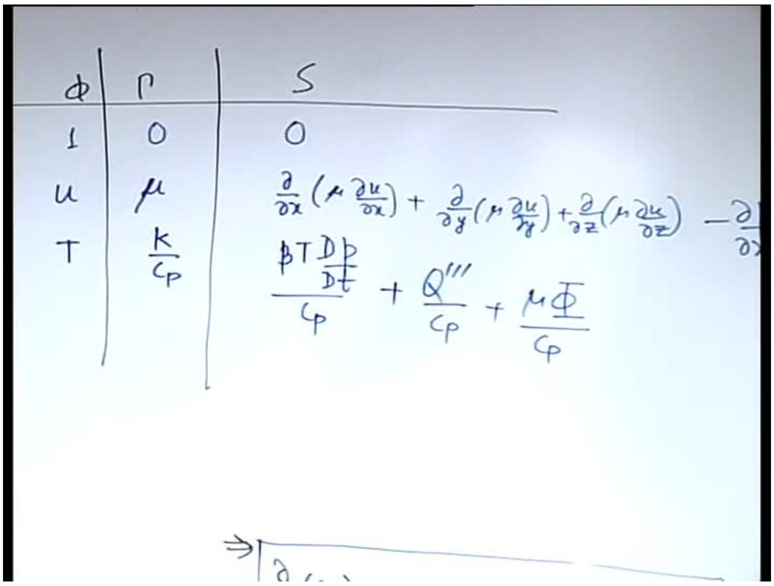

So, you have a beta TDpDt divided by C p plus volumetric heat generation by C p plus viscosity viscous dissipation by C p. Only one term sorry because this term is already there in this gamma. So, very sorry this particular term that we have already kept, these three terms are already considered here. So, these are not the source terms right, so only these terms.

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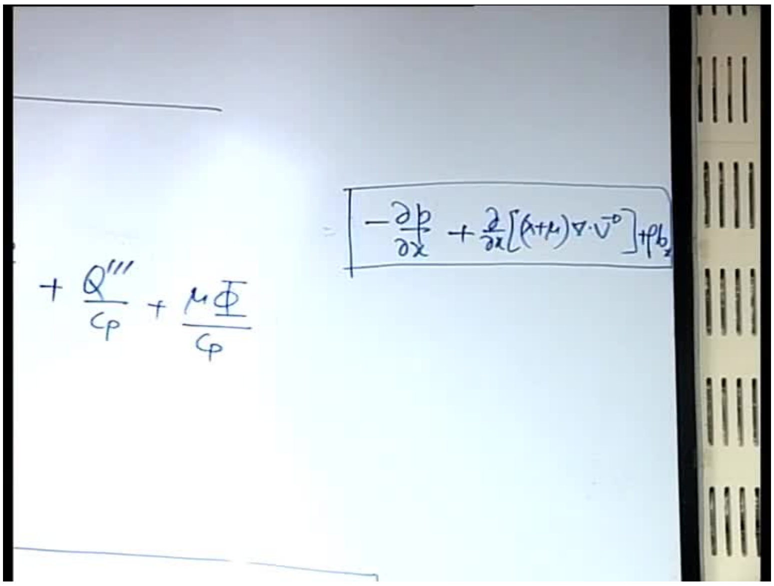

So, of course, those terms are there in the right hand side, but those terms are considered already in this diffusion coefficient. So, what we can summarize from here? We can summarize that we can cast all these equations in a general conservative form, where depending on the choice of the variable, we could have phi as different variables, we could have gamma as different and we could have S as different. Of course, if there are other parameters like for example, you could have lengthy expression for the body force because of some other physical considerations, but it is cast in a generic form.

Here also you can have like complicated mechanisms by which you have volumetric heating. For example, by but dual heating, but that also you can cast in this form. Similarly, transport of a single species in a mixture, say you have a mixture of a and b, in which you have a being transported in b. For that also we call it as a species conservation. So, for that also you can write this similar equation. So, we can see that if we can understand how to solve this equation in this generic form, then what is possible for us is to apply that generic solution methodology for solving continuity, momentum, energy whatever equation. It is not so important because mathematics will not directly understand that physically whether it is mass, momentum, energy whatever conservation so long as it is fitted in this particular framework.

So, our objective for the subsequent part of this course will be to see that how we can solve partial differential equations of this generic form. Because if we know how to do that, that means we know how to solve equations which are pertinent for computational fluid mechanics, heat transfer, mass transfer, and not only that even in other electro-magnetics and other special applications, where you still can write the governing equation in this form.

It is a skill that very complicated physical situations, how still you can write the general equation on this form. If you can do that, then you can use a general platform or a software which can solve partial differential equations of this form. And you can use that for solving your systems of equations.

So, in the next class we will start with the classification of the partial differential equation. See that this is not all, this is the generic form, but again what are these physical interpretations, accordingly the partial differential can be classified in different ways and accordingly the solution technique may vary. That we will do in the next class. Thank you.