Computation Fluid Dynamics

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Lecture No. # 37

Discretization of Navier Stokes Equations (Contd.)

In our previous lecture, we were discussing about the simple algorithm for solution of fluid flow equations. We derived various important equations in the algorithm and we can summarize the algorithm again which we did in the previous lecture as well just for recapitulation.

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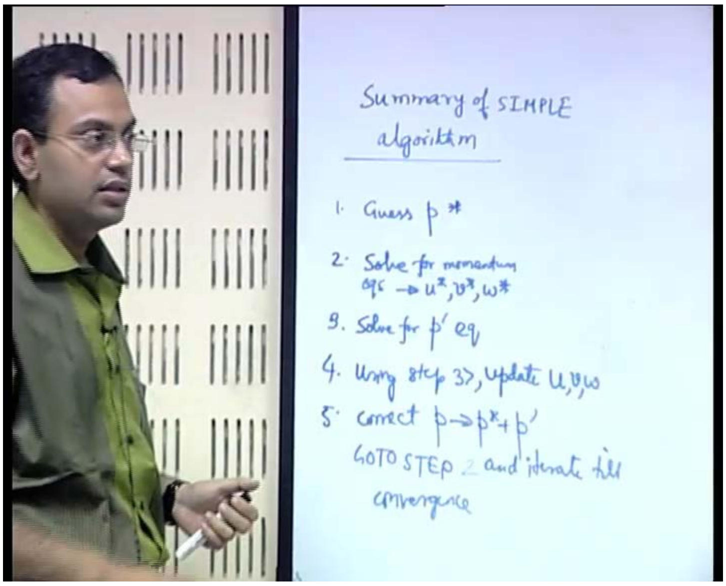

So, in the simple algorithm, what you do? You guess the value of the pressure. From that you solve for the momentum equation. Then, you solve for a pressure correction equation which you derive from the continuity equation and then, using the pressure correction equation, you update the velocity and the pressure and go to the step one. Go to the step, basically go to step two it should be because your pressure is already guessed, so go to step two and iterate till convergence.

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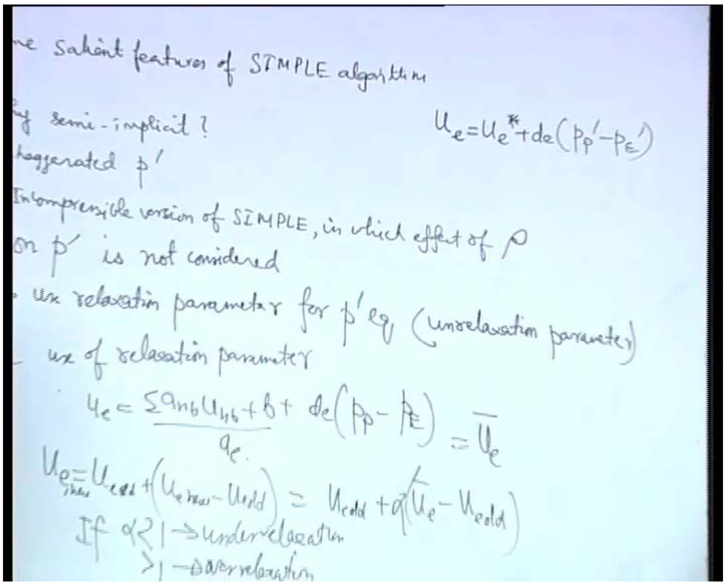

Now, let us discuss some salient features of the simple algorithm. Some features we have already discussed that why it is semi-implicit. So, remember that the key formula in the simple algorithm is the velocity correction and the pressure correction formula. So, for example, the velocity correction is u v equal to u v star plus d e into p p dash minus p e dash.

So, the velocity correction is a function of the pressure correction at the neighbouring grid points, but not the explicit function of the velocity correction at the neighbouring grid points. So, in that way the pressure correction carries the sole burden of correcting the velocity. So, the velocity correction could in principle be a combination of velocity correction due to neighbouring grid points plus pressure correction due to neighbouring grid points. Since, the velocity correction due to neighbouring grid points is not considered, that is omitted, so pressure correction at the neighbouring grid points is taking the sole responsibility of correcting the velocity. In doing so, it is exaggerating the pressure correction. So, that is one important consideration.

Then, the third important characteristic is that point to note is that pressure correction at a point or velocity correction at a point could also be a function of density if it were a compressible flow. Now, here in the simple algorithm which we have described, this we have described basically for incompressible flows, so that we have not used any relationship between the pressure correction and the density. So, this is incompressible version of simple algorithm.

On pressure correction is not considered. So, this we have to keep in mind that the simple algorithm that we have described, that needs to be suitably extended for compressible flows. It is not otherwise fit for compressible flows because of the non-consideration of the density dependence on pressure correction. Now, when we have an exaggerated pressure correction, what it does is something as follows. So, when it has an exaggerated pressure correction, the new value of pressure which is predicted on the basis of pressure correction is always exaggerated from the value that you would have expected. Whatever value of pressure that would have expected, the correction is more severe to take it away from that.

So, how would you bring the pressure back to a non-exaggerated corrected value? So, you can do that in certain ways. One is use relaxation parameter for pressure correction equation. So, what is a relaxation parameter? For example, if you wanted to use a relaxation parameter for momentum equation, how you would have done it. So, just as an example use of relaxation parameter, so a e u e is equal to sigma a n b u n b plus b plus a e into p p minus p e. So, u e is equal to sigma a n b u n b plus b by a e, this into d e. So, let us call this as u e bar. So, we could write u e is equal to u e old plus u e new minus u e old. So, this is something that we can write irrespective of any constraint, that is u e. This u e new is equal to u e old plus new minus old, right.

So, now in place of u e new which should be u e bar, we can write this u e old plus u e bar minus u e old, just different name. So, this is the same equation. Just we in place of u e new, we have put u e bar to explicitly mention the expression. Next, what we do is, we multiply this by a relaxation factor alpha. This type of policy we have adopted in the relaxation techniques for system of solving systems of algebraic equations. So, the same thing that we are doing here just in the context of solution of these discretized equations is that, when we have this relaxation parameter alpha by this, you control the change. So, u new is equal to u old plus the change. So, if this alpha is less than 1 that means you are slowing down the change. So, in this way, you can if alpha is less than 1, it is called as under relaxation parameter. If alpha is greater than 1, it is over relaxation.

So, in case of pressure correction equation, what type of relaxation you want to use, under relaxation or over relaxation? Under relaxation. So, when you say use relaxation parameter for pressure correction equation, we may amplify a bit more and say that it is under relaxation parameter, ok.

Now, there are few other facts that remain about the simple algorithm which are what we discussing and one of the key issues is the boundary conditions for pressure correction equation. See pressure correction equation has evolved as one of the artefacts of this algorithm. So, we need to discuss more specifically on the pressure correction equation, its various criticalities. So, one of the criticalities is how to apply the boundary conditions. So, boundary conditions for pressure correction equation.

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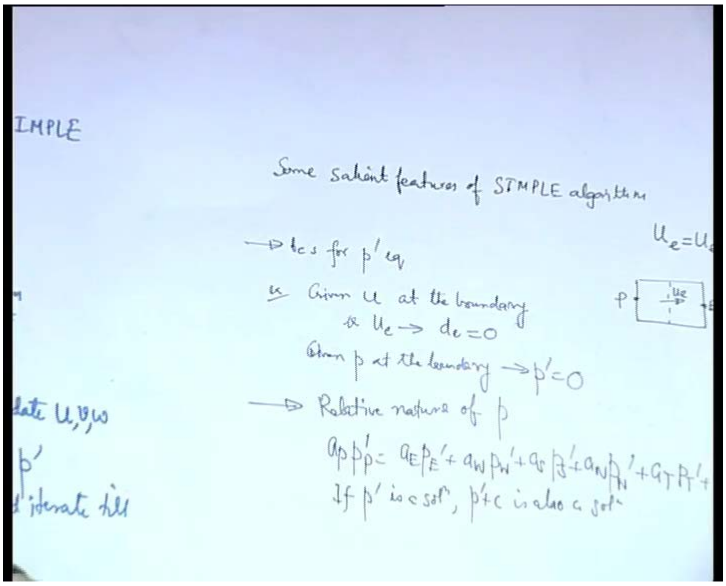

Now, there are several types of boundary conditions possible. We can discuss two important boundary conditions. One is given value of pressure at the boundary and there is given value of velocity, normal component of velocity at the boundary. So, if you have given velocity at the boundary, then how do you reflect that in in your implementation of the algorithm? How do you reflect that? See when you have a given velocity at the boundary or where ever, then add that particular location. You do not require any velocity correction, right. Since, you do not require any velocity correction, but you you may require pressure correction. So, this term may be there, but you want u v equal to u v star.

As an example, if you have specified velocity at e and that means, that you must have d e equal to 0. Example, u v, then d e equal to 0 which is equivalent to a e equal to 0 or it is better to say, d e equal to 0 because it directly covers the case without going through the original coefficients. If you have given pressure at the boundary, then obviously pressure correction equal to 0.

The other very important characteristic of the pressure correction equation is the relative nature of pressure. So, if you recall the pressure correction equation is a p p p prime is equal to some a e p e prime plus a w p w prime a s p s prime plus a n p n prime plus a t p t prime plus a b p b prime plus b, where the convergence criteria is that b is the mass source term from continuity equation. So, b equal to 0 is the convergence criteria that when b equal to 0, that means you have come up with a continuity satisfying velocity field. So, when b equal to 0, 0 on convergence, then we can see that if p dash is a solution of the equation, then p dash plus c is also a solution and that follows from a p is equal to sigma a n b.

So, you can get non-ambiguous solution of pressure correction. Of course, you can get a unique solution if you set pressure correction at a particular point equal to a reference value and you can calculate the pressure corrections are other points based on that particular reference value, but otherwise, there is no (( )) about the value of the pressure correction. The reason is that in an incompressible flow when you are not considering a density dependence of pressure correction, it is the difference in pressure that is important for driving the flow and that difference in pressure is reflected in terms of the difference in pressure correction and the difference in pressure correction does not matter. Its value does not matter irrespective of the value of c. It will come out to be the same because for example, if you have p P prime minus p E prime, it is same as p P prime plus c minus p E prime plus c.

So, that plus c will not matter. It is difference in pressure or difference in pressure correction. They are equivalent. So, that is what is mattering. In fact, p dash equal to 0 is a trivial solution of this equation and that is quite physically intuity because 0 pressure correction means, as if you have come up or you have arrived at converge pressure where there is no further correction of pressure necessary. When there is 0 pressure correction, the source term, the mass source term in this equation becomes 0, ok.

So, it is very important to keep in mind that this pressure pressure correction equation has these very important specific characteristic. The other thing is that it does not have any one way type of coordinate behaviour. So, it is essentially an electric type of equation, where the effect of pressure variation at a point is failed at all neighbouring points, but if you consider flow which has a strong unidirectional property, like a case where you are having boundary layer type of approximation that, you do not have any pressure gradient in the wide direction within the boundary layer.

So, in that case, you have a one way type of behaviour and also, when you have a high velocity compressible flow in a particular direction, so in that case also you are adding directionality to the equation in terms of making it a one way system. That depends on the mach number of flow. So, it all depends on what is the mach number of flow for compressible flows. For incompressible flow, until and unless you have a special situation like the boundary layer where you have gradient of pressure at one particular direction equal to 0, you do not have such situation. In the normal case, you have the pressure correction at a point is affected by pressure correction at all other neighbouring points, ok.

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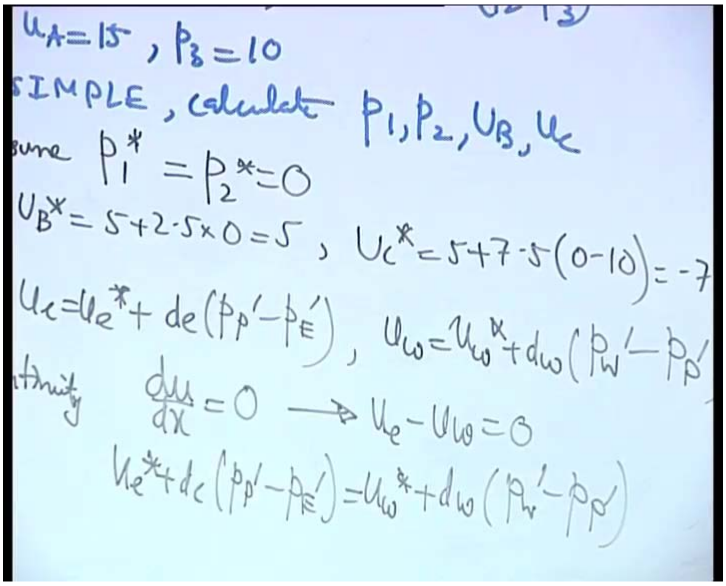

Now, with this background, let us try to work out 1 or 2 examples on the simple algorithm. It is a one-dimensional problem and you have given the momentum equations for the grid points B and C. U B is equal to 5 plus 2.5 into P 1 minus P 2, U C is equal to

5 plus 7.5 into P 2 minus P 3. The boundary conditions are U A equal to 15, P 3 equal to 10. Using simple calculation procedure, calculate P 1, P 2, U B and U C. So, let us mark the given values in the figure just for convenience. U A equal to 15 with some arbitrary units, P 3 equal to 10 and the grid points are equidistant. So, what would be the first step? Guess P star, right. So, your first step is to guess what the value of the pressure at each point is. So, how will you guess the value of P star? You can make some intelligent choice and even if you do not make any intelligent choice, any arbitrary choice will also do. So, let us just make any arbitrary choice just to see that how it functions.

So, let us say that P 1 star equal to P 2 star equal to 0. Really, it is a wrong, very wrong choice if you can see. If you have a flow here like this with a pressure gradient driven, then you you do not expect that these two pressures will be equal, but just let us see that how the algorithm behaves with this. So, when you have this one, then next step will be to solve for momentum equations to get u star, v star and w star. So, the step 2, if you solve for the momentum equation, then what is U B star? 5 plus 2.5 into 0 that is 5. U C star is equal to 5 plus 7.5 into 0 minus 10, so that is minus 70.

Next will be to solve for the pressure correction equation. So, in this simplified case, let us derive the pressure correction equation from the continuity equation. So, you have, if you consider grid point e, where you have the velocity u e, so you have u e is equal to u e star plus d e into p p dash minus p e dash and u w is equal to u w star plus d w into p w dash minus p p dash. So, continuity equation is, you assume one-dimensional and rho equal to constant. So, continuity equation d u d x equal to 0. If you integrate it with respect to x, what you get? U v minus u w equal to 0. So, you have u v star plus d e into p P prime minus p E prime is equal to u w star plus d w into p W prime minus p P prime.

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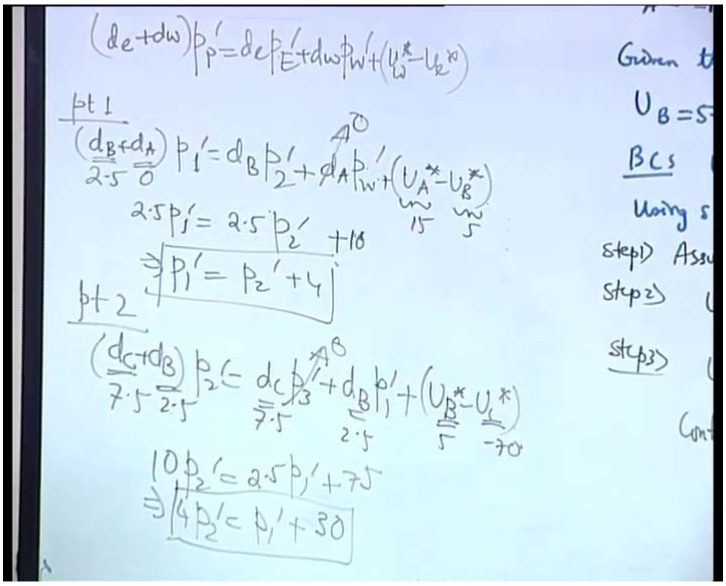

So, you have d e plus d w p P prime is equal to d e p E prime, sorry d e p capital E prime plus d w p capital W prime plus u w star minus u e star. So, in place of d e and d w, what are the values you will be putting? It depends on what are the grid points for which you are solving. So, what are the points for which you will solve? One is for 1, another is for 2. So, you have one control volume for pressure correction as this, another control volume for pressure correction as this. So, if one is the point p, then what is e and small e and small w? Capital A is small w and capital B is small e. So, you can write for point 1, d B plus d A into p p prime is equal to d B p B prime plus d A p A prime plus U A star minus U B star.

What is d B? What is the value of d B? What is the momentum equation given? U B equal to it is of the form U B equal to U B star plus d B into p 1 minus p 2. So, d B is 2.5. What is d A? D A is 0 because U A is given. So, 2.5 p p prime is equal to 2.5. In place of p, we will put 1, p 1 prime, sorry this instead of B and A, these will be the numbers. So, this will be d B. Instead of p e prime, what will be that? 2, p 2 prime? Remember, this d refers to the face of the control volume and this number refers to the corresponding main grid point. So, it is not the same. Just mistakenly I have written it to be the same, it is not the same.

So, what will be the w prime? W prime will be, so it does not matter because you have d A equal to 0. There is no such point and clearly reflected by d A equal to 0. So, 2.5 p 1 prime equal to 2.5 p 2 prime. What is u A star minus u B star? So, u A star is 15, u B star is 5 plus 10. So, that means you have p 1 prime is equal to p 2 prime plus 4. This is one equation.

Then, consider point 2. Let us write it straight away. So, for point 2, it will be d C and d B. So, what is d C? So, let us write d C plus d B into p 2 prime is equal to d C p 3 prime plus d B p 1 prime plus U B star minus U C star. So, let us substitute the values. d C is 7.5, d B is 2.5. So, 7.5. This is 2.5, but even if it is 7.5, it does not matter because p 3 is given. So, p 3 prime is 0. Then, U B star minus U C star. U B star is 5 and U C star is minus 70. So, you have 10 p 2 prime is equal to 2.5 p 1 prime plus 75. That means 4 p 2 prime is equal to p 1 prime plus 30. Now, you can calculate. Let us do it at the top. Now, you can calculate p 1 prime and p 2 prime.

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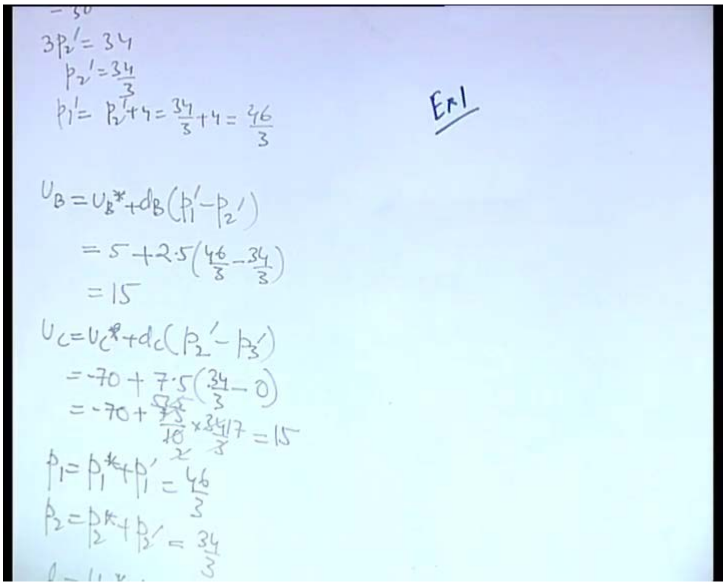

So, you can write 4 p 2 prime. You can just solve these equations. In place of p 1 prime, you can write 4 p 2 prime plus 30 equal to p 2 prime plus 4 minus 30. So, 3 p 2 prime is equal to 4. That means, p 2 prime is 34 by 3 and p 1 prime is p 2 prime plus 4. So, 34 plus 4 by 3, so 46 by 3. So, once you obtain the pressure corrections, next step will be.

What will be the next step? So, you have to update the velocity based on the pressure correction. Now, you have U B equal to U B star plus d B into p 1 prime minus p 2 prime. What is U B star? 5 plus d B 2.5 into 46 by 3 minus 34 by 3. So, this is equal to 12 by 3, that is 4 into 2.5 is 10 plus 5 is 15 and you need to calculate U C also. U C equal to U C star plus d C into p 2 prime minus p 3 prime. U C star is minus 70 plus d C 7.5 into p 2 prime minus p 3 prime. So, let us just simplify it a bit. 75 by 10 into 34 by 3. 5, 5, 17, 15.

So, once you calculate this, next you should check for convergence. That is if the mass source term is 0, then you need not calculate anything else and you can simply write what are the pressures as the old pressures plus the pressure correction and stop your calculation there. So, anyway, first let us write the pressures, p 1 is equal to p 1 star plus p 1 dash. So, that is equal to 46 by 3, p 2 equal to p 2 star plus p 2 dash. That is equal to 34 by 3. What is B? B in the momentum equation was u w star minus u e star. So, for control volume 1, there are two control volumes.

For control volume 1, B is equal to U A star minus U B star which is equal to 0. For control volume 2, B equal to U B star minus U C star. This is equal to 0. That means it has converged. So, it is a very interesting thing to note that irrespective of your initial choice, your initial guess in the pressure value, the simple algorithm in a one-dimensional problem has converged just in one step of the calculations. One of the important insights towards that is that the continuing continuity satisfying velocity field in that case ensures the same velocity at all points, which is readily arrived at. So, there is no mass source term that remains.

The other important thing to observe is that, it is p 1 prime minus p 2 prime. That is important. So, p 1 prime minus p 2 prime equal to 4. That was the sole important thing for dictating what U B is. So, even if p 1 prime was p 1 prime plus c and p 2 prime was p 2 prime plus c that would have not changed or created any difference. Similarly, even if p 2 prime was p 2 prime plus c and p 3 prime was p 3 prime plus c that would not created any difference. From where that plus c could have come? It would have come from different initial guess of p 1 prime and p 1 star and p 2 star. So, if would have guessed something different, what you would have expected is that the final value due to difference in pressure. The final value of pressure may be the final value, so pressure corrections may be different, but the differences in pressure corrections will still remain the same. The differences in pressure correction will not change. The velocities will remain the same; just the values may change depending on how you start with your initial guess, ok.

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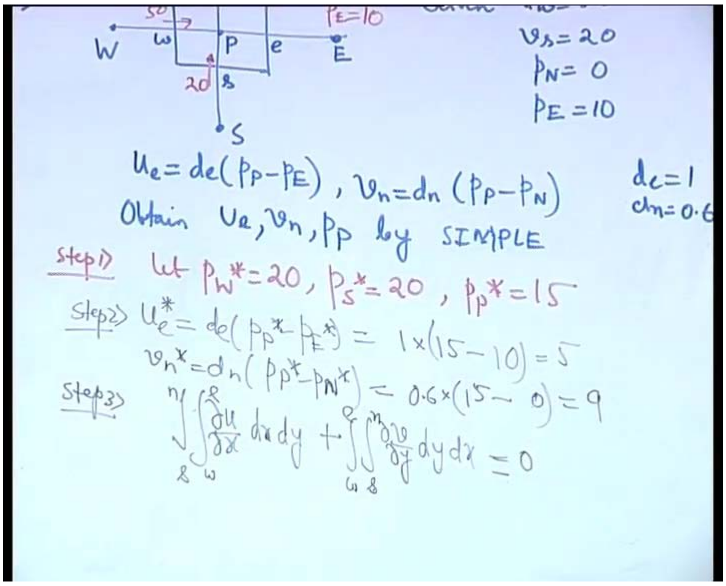

Let us work out another problem. This is a two-dimensional problem with uniform grid spacing along x and y, that is delta x equal to delta y. It is given that u w equal to 50, v s equal to 20, p N equal to 0, p E equal to 10 and the momentum equations are given as follows. U e is equal to d e into p p minus p E, v is equal to d n into p p minus p N, where d e equal to 1 and d n equal to 0.6. Obtain u e, v n, p p by simple algorithm.

So, let us just mark in the figure, what are the parameters given. U w equal to 50, p s equal to 20, p n equal to 0 and p e equal to 0, sorry 10. So, what will be your step 1? Guess pressure value at the points where pressures are not given, that is capital P, capital W and capital S. So, let us just for example, let p w star equal to 20, p s star equal to 20 and p p star equal to 15 based on some arbitrary guess values. Based on these guess values, you can next calculate what is u v star, sorry u e star? What will be u e star? This is step 2. It will be d e into p p star minus p e star.

So, 1 into 15 minus. What is p e star? P e star is same as p e, which is given equal to 10, this is equal to 5. Then, v n star is equal to d n into p p star minus p n star. So, 0.6 into p p star is 15 minus p w star is 20, sorry p n star is 0. So, that is equal to 9. This is N star c. So, you have calculated the guess values corresponding to the momentum equation for velocity. What will be the next step? Next step is to derive the pressure correction equation. So, it is a two-dimensional problem incompressible flow.

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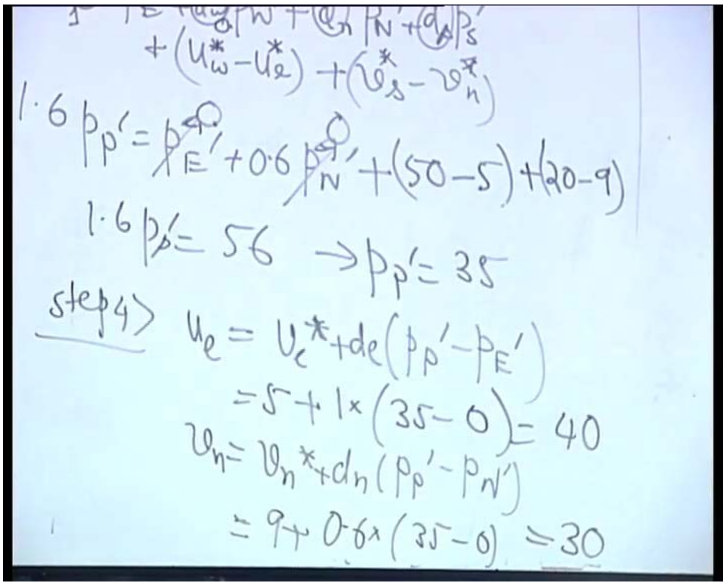

So, we have integrated the continuity equation. So, it will be u v minus u w into delta y plus v n minus v s into delta x equal to 0. Delta y and delta x are the same. In place of u e, you can write u e star plus d e into p p star minus p E star. In place of u w, u w star plus d w p w star, sorry p w dash. These are dash. Corrections, p w dash minus p p dash. Similarly, in place of v n, v n star plus d n into p P dash minus p N dash. In place of v s, v s star plus d s into p S dash minus p P dash equal to 0. So, if you assemble it, it will be d e plus d w plus d s plus d n, p P dash is equal to d e p E dash plus d w p W dash plus d n p N dash plus d s p S dash plus u w minus u e plus v s minus v n is the mass source term.

So, let us substitute these values now. What is d e? D e is equal to 1. What is d w? U w is given, so d w is 0. What is d s? V s is given, so d s is 0. What is d n? D n is 0. 6. So, this is 1, d w is 0, d n is 0.6 and d s is 0. These are stars, right, u w star minus u e star plus v s star minus v n star. So, 1.6 p P prime is equal to p E prime plus 0.6 p N prime plus u w star. What is u w star? U w star is given. What? That is u w that is 15 minus. What is u v star? U v star is 5 plus v s star 20 minus v n star. V n star is 9 and since, at E and N, p values are given, so p E prime equal to 0 and p N prime equal to 0 because p E and p N are given. So, there corrections are 0. So, you have 1.6 p P prime equal to 50 minus 5, that is 40, 50 plus 20 is 70 minus 14, that is 56. So, if you solve for p P prime, p P prime will be equal to 35. So, once you have calculated p P prime, what will be the next step?

So, step 4. U v is equal to u v star plus d e into p P prime minus p E prime. So, 5 plus 1 into p P prime minus p E prime. 35 minus p E prime is 0. So, this equal to 40. Then, v n equal to v n star plus d n into p P prime minus p N prime. So, what is v n star? 9 plus d n 0.6 into 35 minus 0. So, what will be this? 30. So, the unknown velocity is we have obtained as the new u E equal to 40 and new v N equal to 30. So, what is the new pressure? So, p P.

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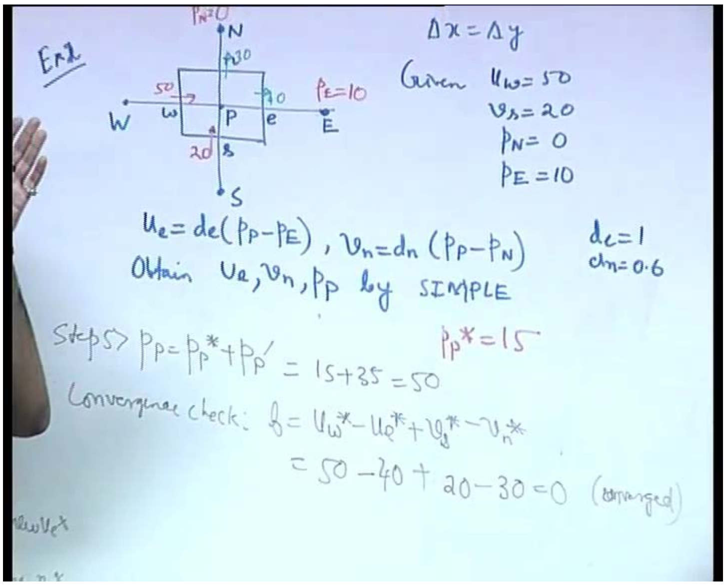

Step 5, p p equal to p p star plus p P prime. So, 15 plus 35, that is 50. Next convergence check what was b here? U w star minus u v star plus v s star minus v n star. U w star is 50 that is given. Minus u v star is and this is new u v star, this is new v n star. So, u v star is 40, v s star is 20 minus v n star is 30. So, this is equal to 0. That means it has converged. So, you can see that the total rate of inflow is same as total rate of outflow about this control volume. So, that means the calculations in this iteration whatever have been obtained, these are the converged values for u v, v n and p p. So, we will stop here today and in the next class, we will consider a slightly improved version of this algorithm known as revised simple or simpler algorithm. That we will see in the next class.

Thank you