Computational Fluid Dynamics

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Lecture No. # 36

Discretization of Navier Strokes Equations (Contd.)

In the previous lecture, we introduced some of the inter cases related to the solution of the Navier stroke equations. And we came across the situation where, if we were interested to represent the pressure gradient term in terms of an integrated discretized term then that created some problem.

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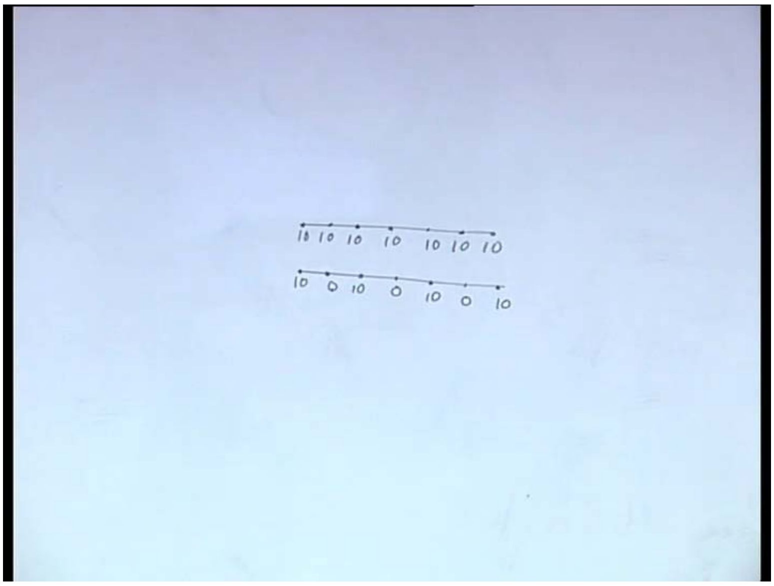

Let us just quickly relook into the problem. So, if you had a pressure field like this say at all points pressure is having a value same value say 10 and another case, where you have alternating values of pressure say 10 0 10 0 10 0 like that say a wavy type of pressure profile. These two we have shown that these two are interpreted in the same way.

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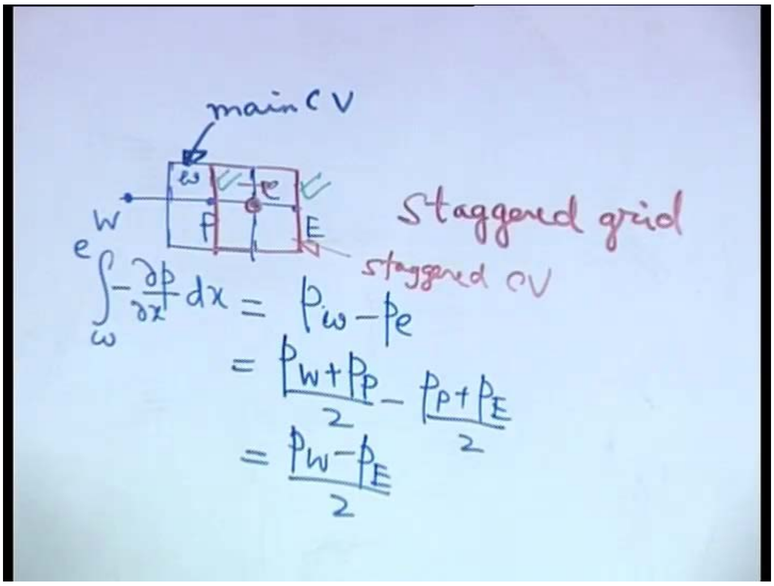

Why are these two are interpreted in the same way, because if you consider a grid point p with it is neighbouring grid points W and E then integration of the corresponding pressure gradient term from small w to small e will be P small w minus P small e for uniformly distributed grids. If we interpolate pressure at the faces in terms of the pressure at the control at the grid points corresponding to the main control volumes then this is that is effect of p E gates eliminated effect of P p gates eliminated.

So, when effect of P p gates eliminated if this happens to with the point p then the flow here is flow through this control volume flow across this control volume is driven by the differences of these two pressures, which is sort of erroneous because it does not demarked the situations represented in the upper figure and in the lower figure in one case there is no pressure gradient in another case there is a pressure gradient in the flow. But, this discritization does not capture that pressure gradient. So, it is a fault we have also shown by example that the discritization of a of the continuity equation also shows a similar discrepancy.

Now, how to get rid of this discrepancy to understand that we have to understand that what is the primary origin of this? The primary origin of this discrepancy are as follows, you have a first order derivative term you are having to discritize this first order derivative term by integrating it over the control volume and there you are having values of the variable here. The variable is pressure in the continuity equation it may be the velocity whatever may be the variable that particular variable is appearing at the control volume phase. At which you do not have the values defined you have the values defined at the grid points capital P, capital E, capital W like that. Because, at this point small e and small w these are not explicitly defined these need to be defined through interpolation and that interpolation has created the problem.

So, as a remedy one could suggest that well let us consider a situation where you do not have to interpolate these variables. That means, the flow is driven by this pressure difference and the pressure difference itself is the pressure difference between two grid points where pressure is defined. So, let us consider that this is a grid point where pressure is defined this is a grid point where pressure is defined.

So, instead of p small w minus p small e this type of an expression if you have p capital P minus p capital E this type of an expression then velocity at the points velocity at the location of the small e is governed by physically the pressure difference between capital P and capital E. So, it also is intuitively physical that velocity at a particular phase is governed by the pressure difference between the neighbouring grid points and in that case you do not have to interpolate the values of the variables of pressure because, you are considering pressure at the respective grid points where they are originally defined.

But then when you consider pressures at these locations you are no more considering velocities at those locations, but you are considering velocities at a shifted location. So, you are now considering a different grid point choice for velocities and for pressure or any other scalars. So, effectively what you are doing you had a original control volume which was your main control volume like this. Now, for velocity what you are doing you are shifting this control volume.

So, what becomes the velocity control volume this becomes the control volume for velocity where the velocity is defined now, not at the main grid points originally of course, at the main grid points. You can find out the velocity by interpolation but, fundamentally those are defined at the interfaces of the main grid points, which are the grid points corresponding to the velocity evaluation. So, we have now some new grid points and these are demarked in a way that you have some grid points for velocity calculation and some grid points for pressure or other scalar calculation.

The grid point for the velocity calculation happens to be oriented along the interface line of the main control volume. So, in a way if, you consider this as the grid point for velocity then the grid point for velocity is shifted from the grid from the main grid points this is called as staggered grid. So, in this particular figure this blue coloured control volume is called as main control volume and the control volume shown in the red colour that is called as staggered control volume. So, the staggered control volume is staggered with respect to the main control volume for calculation of velocity.

So, why do you require a staggered grid you can employ a staggered grid with the objective that once you have a velocity at small e that is directly driven by the pressure difference between p at capital p and the p at capital e. So, you do not have to interpolate the pressure gradient term because, you directly have the information of pressure at p and pressure at e. How that we will see but, once you have the information at them main grid points that is good enough to consider the for the momentum equation for the small e point.

So, this is. So, you can see that the staggered control volume is shifted with respect to the main control volume a staggered with respect to the main control volume for x momentum. It is shifted along x for y momentum it is shifted along y.

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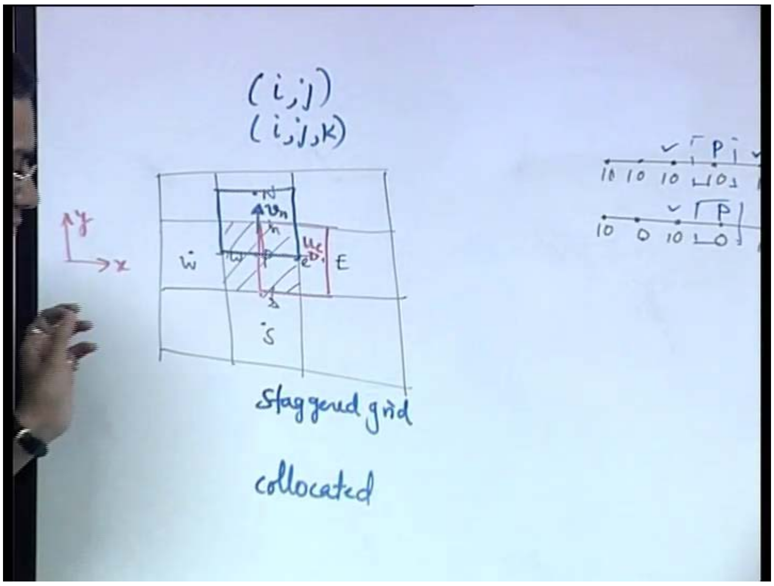

So, let us just consider a general profile to show that how do you have how can you consider the layout of a staggered control volume. Say these are the main control volumes, this is p this is e just for example, w n s small e small w small n small s. So, what is the main control volume the main control volume is this one for the point p. What is the staggered control volume the staggered control volume is this one for u for x component of velocity carefully observe that, when you have a staggered control volume for x momentum equation. It is staggered only along x, but it is not staggered along y along y it is boundaries are coincident with the original control volume or main control volume. Now if you want to have a staggered control volume for the y direction.

So, this is the staggered control volume for evaluation of v n velocity at small n. So, you can see that for a y momentum equation the control volume is now staggered along y, but not staggered along x. So, staggered in staggered grid control volume is not simultaneously staggered along x and y depending on which momentum equation it is whether it is x or y or even the other direction. Whatever is the direction in which you are solving the velocity component in that direction only you are having the stagger or shift in a control volume.

So, this is like the standard architecture of a staggered control volume. Now, the advantage we have seen that what the advantage is associated with the staggered control volume. That you need not interpolate the pressure at the control volume phases anymore because you can directly use the values of the pressure at the main grid point. So, with the staggered grid point arrangement you calculate only velocity velocities at the staggered grid points. But, for other values of the scalars like pressure or may be temperature or any other variable for that you refer to the main grid point only.

So, you have to use two types of control volumes that is one of the limitations that is you have to have a book marking or accounting of two different types of data sets one data set for the layout of the main control volume, another data set for the layout of the staggered control volume. So, extra book keeping is necessary which and it is significantly extra because the main and the staggered control volumes are having their grid points and their dimensions all those things are different.

So, you have to account for them separately and use them for different purposes staggered control volume for solution of the momentum equations and main control volume for solution of the other equations. Now, originally when these methods of solution of fluid flow equations were developed the main control volumes were creating problems. And for the velocity calculation and that is why the staggered control volumes came into the picture and all those were invoked by considering such a structured nature of a grid layout that means, you have the grid phases or the grid lines oriented either along x or along y or if in a 3 dimensional problem along g.

So, in such a situation you can represent anyone of the grid points or control volume phase centroids as in terms of index two indices for two dimensional problems i and j three indices for a three dimensional problem i, j and k, where I may be for orientation along x j may be along y and k may be along g.

So, if you have such a structured layout of the grid system then it is known as a structured mesh for solving problems involving complicated geometry. It is not always convenient to use a structured mesh because, that may not be able to represent the domain boundaries properly and many real life industrial problems have complicated geometries for example, many complicated moulds in a casting problem.

So, if you have a mould filling or in a in a complicated shape mould. So, for that the boundary of the domain may be of complicated shape. So, it is therefore, not a must that you have to use such a structured system in fact, in these days for most of the problems where complicated geometries are involved. We use unstructured grid systems later on in we will briefly look into that how to descritize the equations for unstructured meshes till now we have used the descritization for structured meshes.

Now, if you have unstructured meshes we will go into that more details later on but, unstructured meshes do not have a preferential orientation of gridlines along x y or g Therefore, staggering in a particular direction has no meaning for unstructured meshes not only that. We have seen here that even in a structured mesh the problem with the interpolation is because of using a very simple linear interpolation for pressure. So, may be this problem could be avoided if we had used a more complicated interpolation scheme for pressure.

So, the whole idea is that not to develop a prejudice that the staggered grid is a must the staggered grid is one of the elegant ways of getting rid of the problem with this interpolation. But, alternatively one could avoid staggered gridding with an understanding that staggering requires extra bit of accounting and for unstructured mesh staggering does not have any particular relevance.

So, staggering could be avoided at that cost one could have a special pressure interpolation scheme. So, that this type of problem does not arrive at interface so, In fact there are many such schemes where you do not use staggered grids. But, you use the same grid point for calculation of velocity and pressure but, you just use special interpolation techniques for interpolating pressure at the phases of the main control volumes.

So, in such cases the layout is known as collocated grid. So, this is staggered grid. Staggered grid means the grids for the velocity calculation and calculation of other scalar variables they are staggered with respect to each other whereas, collocated grids means co located; that means, the grids for velocity and pressure are located at the same location.

So, you have the possibility of having collocated grid increasingly more when there is no particular physical meaning of staggering such as in unstructured meshes and the many difference E F D software is have provisions of both using staggered grid as well as collocated grid interestingly. the basic philosophy of solution of fluid flow does not change if you go from collocated (( )) from a staggered grid parodying to a collocated grid parodying only some finer issues like some interpolation of schemes these types of things change.

So, it is it may be convenient to describe the fluid flow algorithm through fluid flow solution algorithm by either using staggered grid or collocated grid. For understanding for simplicity in understanding we will use the staggered grid for demonstrating the various fluid flow solution algorithms. But, keep in mind that it is not absolutely a must to use staggered grid and one can also use a collocated grid and still implement analogal algorithms for solving fluid flow problems.

So, why staggered grid it is clear staggered grid is not a must that is also clear and using if we use staggered grid what are the cost that we have to pay and what the advantages are that we get at that in expense of that. These are certain things we have discussed till now, we have bothered. So, much about representation of the pressure gradient term but, we have not yet got a clue of where from to get the value of pressure this is a very critical bothering on us because, you do not have a separate governing equation explicitly for pressure although pressure happens to be a dependant variable that you need to solve. So, where from you get the pressure.

So, in an iterative environment whenever we do not know something we make a guess. So, let us say that in the momentum equations at least if, we do not know the pressure. We guess the value of pressure if we guess the value of pressure then in the momentum equation the only unknowns remain to be the velocities and those velocities may be solved.

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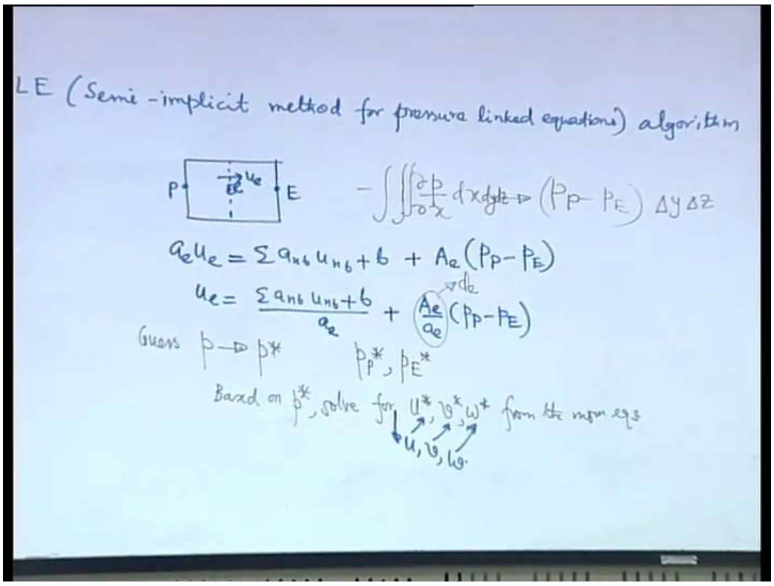

So, let us try to form a strategy the strategy that we will be outlining is broadly known as simple algorithm or semi implicit method for pressure link equation. So, we will go through steps to develop the algorithm first let us try to understand the philosophy of this algorithm. So, if you consider the momentum equation remember the momentum equation is just like a convection diffusion equation with the pressure gradient as a source term.

Let us say that we are interested to solve the momentum equation with respect to a staggered control volume. So, this is e this is p this is capital E. So, if you write the corresponding descritized equation descritized equation without the pressure gradient source term would have been a e u e. Now, the grid point value are interested to solve for the velocity is x momentum equation the corresponding velocity that you are interested to solve is u.

So, a e u e is equal to sigma a n b u n b plus b this would have been the case without any pressure gradient term this b represents the source term different from pressure gradient term. So, this can this can represent any other body force now where what are these u n b s u n b s are the corresponding staggered grid point locations in the neighbours. So, these are also coincident with the phases of some other main control volumes in the neighbouring locations.

Now, this plus now you will have to have the effect of pressure gradient. So, you have minus pressure gradient term along x that is the body force term per unit volume in the x momentum equation. So, what you have to do you have to integrate this with respect to the control volume and once you integrate with respect to the control volume. What you will get you will get a term in this form.

So, if you integrate it with respect to first say x. So, what you will get you will get p P minus p E, now that again you integrate with respect to y. So, in a way this A e represents what it represents the delta y times if it is a three dimensional problem now we will try to solve a three dimensional problem. So, over this course we have solved many one dimensional problems may be a few two dimensional problems. Now it is a good time to introduce the concept through a three dimensional problem which is the most general fluid flow problem that we can think of.

So, if you have a three dimensional problem you are integrating it with respect to y and so this will become this one. So, delta y into delta is nothing, but the area of the phase across which the fluid flow is taking place. So, generically we call this as A e. So, we can write u e is equal to sigma a n b u n b plus b by a e plus this one. So, this we give a name d e with subscript small e these are sort of universal nomenclatures given in all books.

So, I am trying to follow the same nomenclature as that in given in any of t he text books that you can follow it from the books also. Now you have the momentum equation which you are ready to solve provided you know what pressure is. So, what you do is if you do not know pressure you guess a value of pressure of course, you do not know what is what is the value of pressure in a general problem until and unless you have a specific case where it is a pressure driven fully developed flow those types of cases are not general one general case you do not know what is the value of pressure. So, you guess the value of pressure.

So, when you guess the value if pressure you call this as p star so, that means, you know what is p P star p E star etcetera. Based on the values of p P star and p E star, if you substitute it here you can solve the momentum equation because now you have only velocity, but that will not be the correct velocity because the pressure itself is there which is substituted is not correct.

So, it will give some approximate velocity. So, based on p star solve for u e solve for u star v star w star from the momentum equation. So, this basically solve for u v w that u we call as u star, v we call as v star and w we call as w star. So, if you are interested to write the corresponding y momentum equation just by looking into the analogy.

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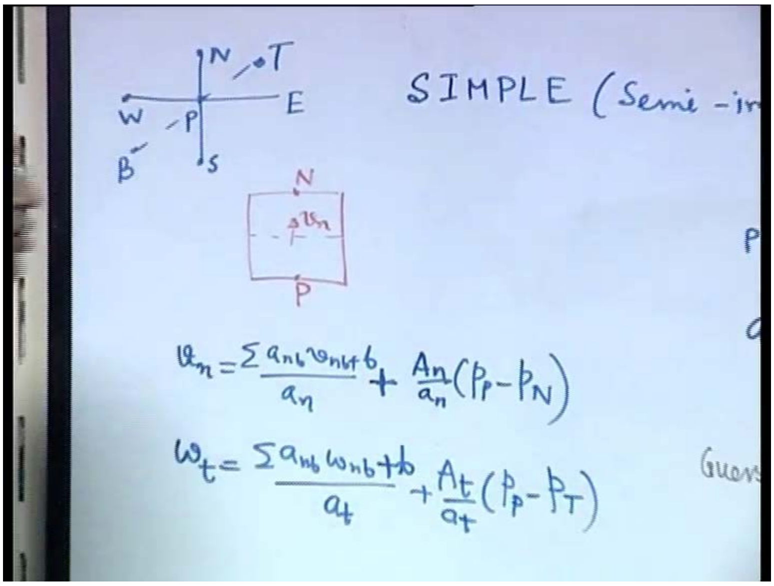

You can write what will be this one u n v n is equal to sigma a n b v n b plus b by a n plus it will be A n by v n sorry A n by small a n into p P minus p N the corresponding control volume will look like this and this is v n this is p this is N similarly, it is a three dimensional problem.

So, the layout is like this east west north south and then in a third direction top and bottom. So, the other momentum equation what will be this w t is equal to sigma a n b w n b where u v w are the components of velocity along x y and plus b by a t plus A t by a small t p P minus p T.

So, you can see that the equations are structured in the same way. So, if we consider anyone that will give us enough idea of how to deal with the others that is why the one dimensional problems are. So, important if you look into it very carefully you will see that in a structured system it is not true for an unstructured system, but, in a struck structured system.

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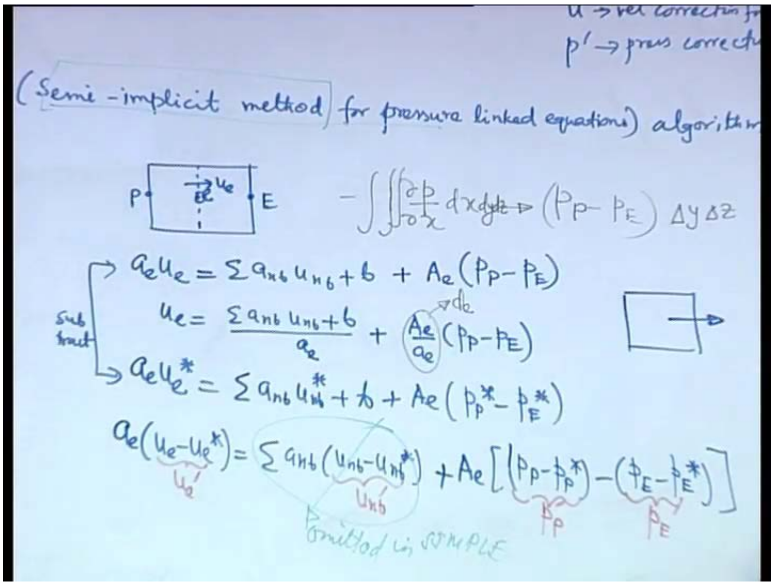

If you have a control volume phase no matter whether the control volume is two dimensional, three dimensional whatever across the phase locally the flux is one dimensional that is how the structured control volumes are oriented. And that is why we can easily extend one dimensional formulation to two dimensions and three dimensions the same (( )) hold for unstructured mesh for which separate considerations are necessary.

Now, let us come back to this solution of u v and w. So, we get u star v star and w star now these u star v star and w star are not correct velocities. So, they need to be corrected. So, if you consider a e u e star that will satisfy this particular equation with p equal to p star right. So, a e u e star is equal to sigma a n b u n b star plus b plus A e into p P star minus p E star where the star is not the correct solution and this is supposed to be the correct solution if p is correct.

So, if you subtract these two what you get you get a e into u e minus u e star is equal to sigma a n b u n b minus u n b star plus A e into p P minus p P star minus p E minus p E star this we call as u minus u star we call this as u dash which is in words called as velocity correction. So, this is the correction of velocity to bring the accurate in accurate velocity to the accurate velocity. So, u e star is the inaccurate one and u e is the correct one u e prime is the correction necessary to bring the inaccurate one to the correct one this is similarly, u n b prime this is p P prime this is p E prime remember the pressures are also guessed. So, the pressures were not the correct values.

So, you have the difference of these the inaccurate and the actual pressure is termed as the pressure correction. So, u dash is the velocity correction and p dash is the pressure correction. Velocity correction for u and p dash is pressure correction. Now, what you can see from here let us try to understand the physics of this equation. So, this equation says that the velocity correction at a point is a combine effect of the correction of velocity at the neighbouring points plus correction of velocity correction of the pressure at the neighbouring points.

Now, in the simple algorithm what is done this particular term is omitted. Now you can readily ask that, if you omit a term in a equation the equation becomes inaccurate right but, then even after omitting this. How do you expect the solution to converge see the rationale behind this is like this the velocity correction at a point is burdened by two things, one is the velocity correction at the neighbouring points another is the pressure correction at the neighbouring points.

So, if you do not consider the velocity correction at the neighbouring points and still you can achieve the same correction that is possible provided the pressure correction takes the sole burden of even making up for omission of this one. So, so it is like some x equal to y plus g now y is no more there. So, itself is becoming plus y to make up for this effect. So, let us say that there are two friends in a particular group and there is an assignment which is given where the group students has to solve. Now, as usual one of the friends will do the whole work the other friend will sleep.

And this is this is what is very common but, at the end they show a group assignment right and may be the person who has not done anything he is smart enough to present it in such a way that people will believe that he or she has only done it. But, in general there will be some lazy persons who have not done it.

So, this is that lazy extreme lazy now, this is a poor guy who takes all the burden of this correction who takes all the burden of working for the assignment now in doing. So, what happens is that this guy becomes fatted. So, this guy tends to over express himself to solve the problem, try he is trying to go beyond his own capacity to solve the problem he is going beyond the time limit or whatever available in the day to solve the problem because the other one is not giving a helping hand. So, in this way what happens is that here also this has to be achieved this term is not contributing anything this term is contributing in this way the velocity correction is achieved, but, the pressure correction is exaggerated pressure correction is over expressed.

So, the pressure correction that comes out of it is not a very accurate one it is many times over expression of whatever it would have been if this term was already considered. But this term is not considered to keep it to keep the framework in a particular form in which we have solved all the descritized equation and that we will see later on. So, that is an objective of dropping this term if you do not drop this term the equation becomes implicit.

Because, the velocity correction cannot be explicitly expressed in terms of the pressure correction then the velocity correction is explicitly expressed in terms of the neighbouring velocity corrections and the pressure correction. But, still the velocity correction at a point has some implicit dependence on the velocity correction.

At the neighbouring points, how the velocity correction is dependent on the pressure correction at the neighbouring points and once pressure correction is there at the neighbouring points that will incur a velocity correction. So, that is why this is called semi implicit remember it has. So, semi implicit method for pressure linked equations.

So, we have first seen that what is origin of this semi implicit method remember this has nothing to do with the time implicit explicit or crank Nicolson method that we have considered. So, this implicit or explicit or semi implicit nature has nothing to do with time descritization. So, so just clarify this concept. The part is pressure linked equation. So, how which equations are pressures linked that we will now come to that stage.

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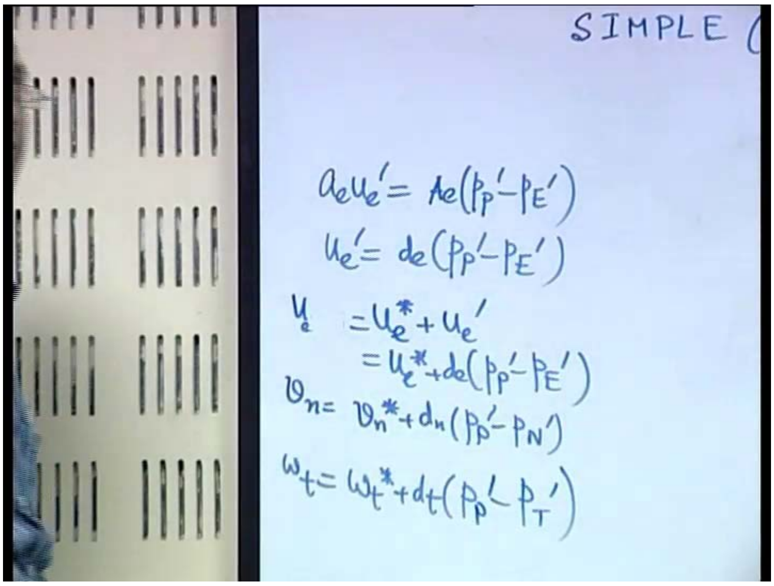

So, before that once this term is omitted how you can write this equation you can write a e u e prime is equal to A e into p P prime minus p E prime. So, u e prime is equal to d e into p P prime minus p E prime. So, you have u new is equal to u e mu is equal to u e star plus u e prime that is u e star plus d e into p P prime minus p E prime. So, in this way you can. So, let us omit this subscript new because in iteration anything which is calculated recently is the new one.

So, let is not explicitly write it as new similarly, you have v n is equal to v n star plus d n into p P prime minus p N prime w t is equal to w t star plus d t into p P prime minus p T prime like this. So, these are called as velocity correction formula. Still in this formula you do not know what the pressure corrections are if, you know what the pressure corrections you can implement this formula are.

So, that golden question remains that where from you get that pressure correction now the understanding is that we can get the pressure correction from this consideration that no matter whatever is the velocity field that we get out of this correction we expect that that should satisfy the continuity equation. So, during the iterations the velocity fields should evolve in a manner with an attempt or effort to satisfy the continuity equation. So, we should try to get a pressure field which is which again confers to a continuity satisfying velocity field. So, the pressure field should be corrected in a way such that the velocity field satisfies the continuity equation. So, the continuity equation acts like a governing equation for correction of pressure. So, that is how the continuity equation is utilised and it gives us the answer to the question that we do not have an explicit governing equation for pressure. But, where from we update pressure is the continuity equation which we take equivalently as an equation that governs us the variation of pressure during iterations.

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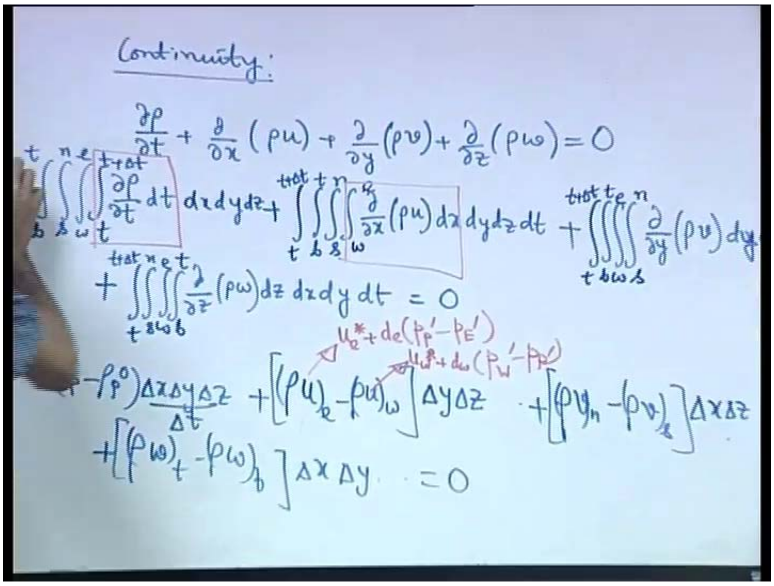

So, let us now consider the continuity equation.

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This is the general continuity equation. So, let us now integrate the continuity equation the first question that you give an answer is this once we integrate this equation. We integrate this equation over a control volume that is the finite volume method. We integrate it with respect to which control volume main control volume or staggered control volume main or staggered it is not staggered why because, the objective of using this equation to solve for the pressure correction not for the velocity.

So, you are not going to solve for the velocity from this equation, but you are going to solve for pressure correction which is a correction in pressure which is a scalar. So, other than velocities whatever you solve for that the reference control volume is the main control volume. So, you integrate the continuity equation with respect to the main control volume not the staggered control volume. So, this is the main control volume you integrate this with respect to the main control volume. So, let us complete that integration. So, now there are four integrations for x y z and time. (( )) Class the last term. So, let us put the limit of integration from time t to t plus delta t x small w to small e y small a to small n and g bottom to top small b to small t. So, do not confuse the time t with the top t incidentally they are the same symbols, but we are running out of symbols. So, let us of course, it is clear that these are not the same. (( )) so, let us now integrate these 1s first this term what is the profile assumption we will take piecewise constant.

So, rho p minus rho p not those multiplied by delta x delta y delta z plus then let us consider this as the first integration so, rho u e minus rho u w so, rho u e minus rho u w into delta y delta z delta t plus rho v n minus rho v s delta x delta z delta t plus rho w t minus rho w b delta x delta y delta t equal to 0. We can divide all the terms by delta t as we commonly do for time dependant problems and then what will be the next step next step will be to substitute the velocity correction formulas in this expression. So, in place of u v we will substitute u v star plus d e into p P dash minus p E dash what will be the corresponding u w.

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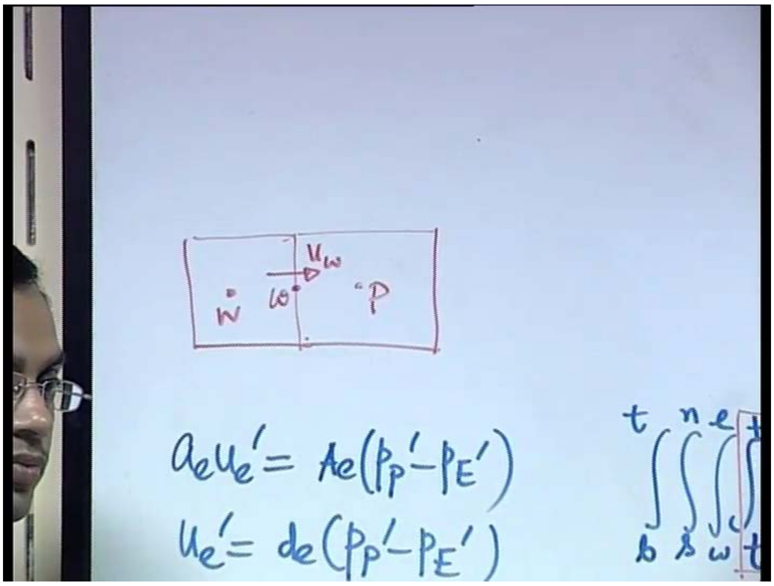

So, this is w this is capital w this is u w. So, it will be u w star plus d w into P w prime minus p P prime. So, in the flow direction from left to the right the flow is occurring. So, it is a pressure between w and P other terms will be similar let us write for one term and we can extend it easily for the other terms.

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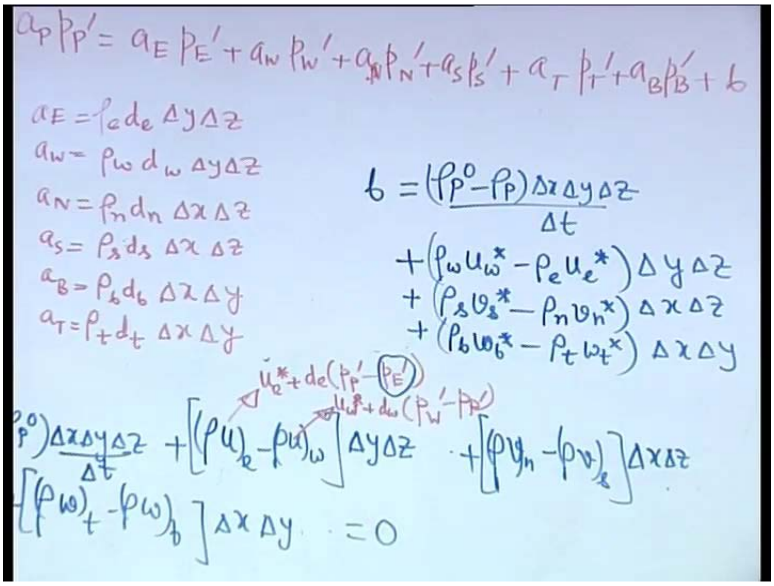

So, if you do that you will get an equation in the form a P p P dash is equal to a E p E dash plus a W p W dash plus a N p N dash plus a S p S dash plus a T p T dash plus a B p B dash plus b. So, let us write what is a E from here what will be a E which term will contribute to a E only the coefficient for this one. So, rho e d e delta y delta z what is a w rho w d w delta y delta z. Now if you know these two you can write all four other coefficients. So, a N rho n d n multiplied by what see as are x deductions. So, they are multiplied by delta y into delta z this is y deduction. So, delta x delta z, a B is equal to rho b d b into delta x delta y, a T is rho t d t delta x delta y then what is b.

So, you can see that this is taken in the other side to make it positive right it was minus rho e d e like this. So, it was taken to the other side. So, this term will also have to be taken to the other side to make it as a part of v. So, rho p not minus rho p delta x delta y delta z by delta t then the contributions of u stars will come, plus rho w u w star minus rho e u e star into delta y delta z plus rho s v s star minus rho n v n star delta x delta z plus rho b w b star minus rho t w t star delta x delta y. So, this equation is called as the pressure correction equation which is actually derived from the continuity equation.

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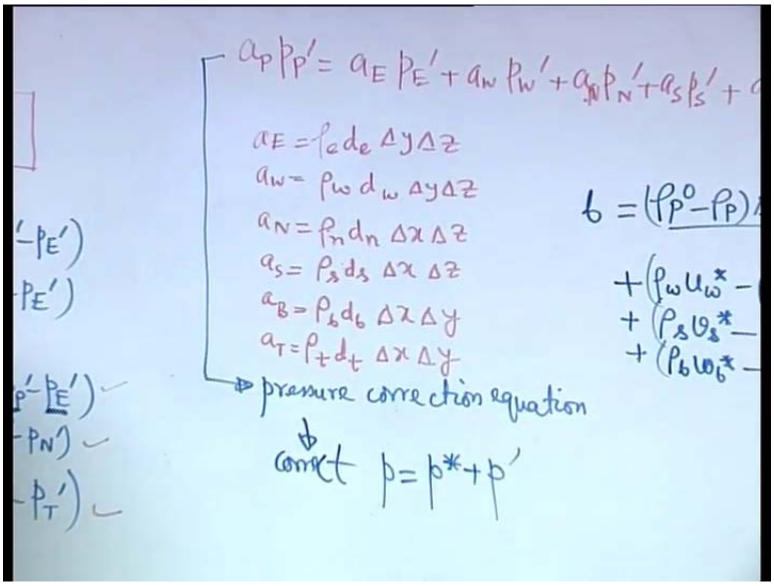

So, in the pressure correction equation you are having a source term what is this source term you can clearly see that rho u w star rho u e star is like that into delta y into delta z is the net flow rate in minus out along x similarly, this is net flow rate in minus out along y this is net flow rate in minus out along z.

So, this the last three terms in b is sum totals of the flow in minus flow out and this is the rate of change of mass inside the control volume. So, the b expression is nothing, but, a measure of the satisfaction of the conservation of mass. So, if the mass conservation within the control volume is satisfied then what will happen, if mass conservation in the control volume is satisfied b will be equal to 0 and once b is equal to 0 all the pressure corrections will be trivially equal to 0.

So, pressure correction equation is also an implicit indicator of satisfaction of mass conservation and that is indeed the case because we have to derive it through the continuity equation. So, once you have this pressure correction equation next you can use this pressure correction equation to correct pressure p star plus p prime.

So, what you can see here is that now you can iterate on this you started with an initial guess for pressure now you have an updated value of pressure with this updated value of pressure you again solve for momentum equation. And go through this route till you find out a convert solution that is still the pressure correction is equal to 0.

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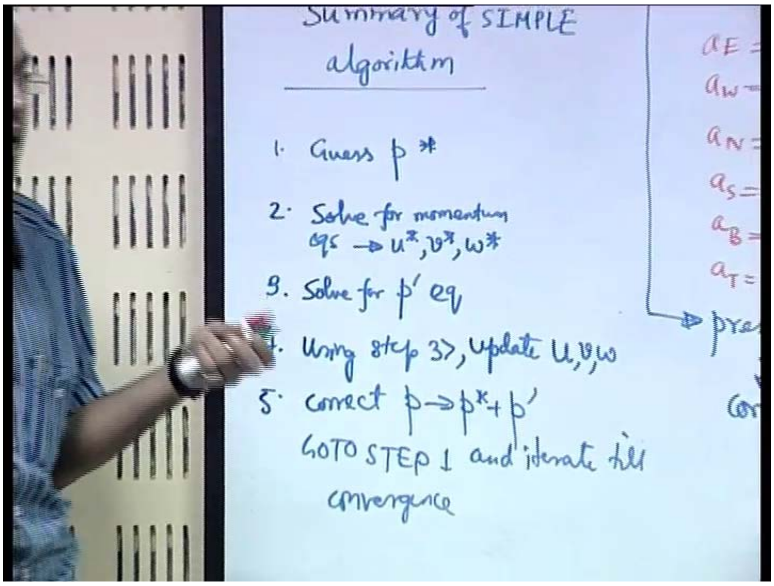

So, let us summarize the simple algorithm summary of simple algorithm step one is guess pressure p star then once you guesses pressure then you solve for momentum equation. So, what you get is u star v star w star then, you need to correct the velocities. So, what you need for that you need for that the pressure correction equation. So, solve for pressure correction equation then using step three you solve for you update velocity u v w, remember u equal to u e is equal to u v star plus d e into p P dash minus p E dash.

So, once you know p dash you can update u then correct p as p star plus p dash then go to step one and iterate till convergence. So, we have learned this algorithm this is very commonly used algorithm to solve fluid flow problems in the next class. We will try to make a bit of a more assessment of this algorithm try to go for an improved version of this and workout some examples on these algorithms. That we will do in the next class. Thank you.