Computational Fluid Dynamics

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Fundamentals of Discretization: Finite Volume Method (Contd.)

Till now, we have look into two broad types of discretization methods namely the finite elements method, and the finite difference method. We will next move on to the finite volume method, but before that let us try to compare different aspects of finite element method, and the finite difference method that we have learnt through the examples that we have undergone. If you start with some of the important features of the finite difference method, one of the important features of the finite difference method is it simplicity. So, you can write very easily the difference expressions from the derivatives, starting from the differential form without going to further manipulations by simply using Taylor series expansions of various functions.

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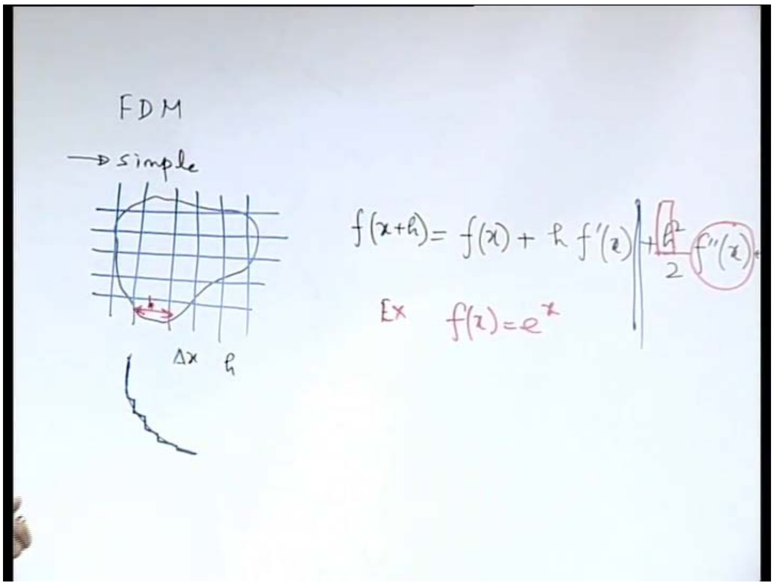

So, that is so finite difference method, it is simple enough, but there are issues associated, and the issues are as follows like, for example, if you are dealing with a very complex geometry, and you are trying to discretize this complex geometry using the finite difference method. Then you have to remember that finite difference method requires grid lines, which are stencils oriented in this mutually orthogonal manner. So, to represent a curved boundary, one may have to basically consider steps and it is therefore not very convenient to solve a problem with complex geometrics using the finite difference method; not that it is impossible, but one has to tediously create the geometry.

Now, this is about the geometry part, but there are certain other important issues, what are the important issues? The other important issues are as follows, first of all let us look into it from a pure mathematical view point, what does the finite difference method rely upon? It relies upon the fact that you expand a function in the Taylor series, and as the mesh size delta x or h whatever you call. As these tend to 0 in the limit, then higher and higher order terms in the Taylor series are more and more insignificant, so that you can truncate the Taylor series up to only a few number of terms and using those terms you can generate the difference equations; that is the basic concept that we use.

Now is it always true. So, remember that we write the Taylor series expansion as f (x plus h) is equal to f (x) plus h f dash x, we are considering function of a single variable as an example, in this way. So, if you truncate it up to this one; what is your expectation your expectation is that h is small, so that h square, h cube those will be smaller and smaller, but while stating that, we forget that there is a multiplier also, which is a higher order derivative of the function, what happens? if that multiplier itself is significant. So, let let us let us consider the problem in this way, if h is tending to 0 in a limiting sense that, if h is differentially small, then it does not matter, whether this is large, small, medium whatever because, if h is differentially small, then you can consider h square, h cube these to be subsequently almost all tending to 0, but the question is here, you are dealing with a finite mesh size right.

You are not dealing with a mesh size, where this h is limitingly small, it is small, but it is a finite small number. The length is not 0 or tending to 0, because you cannot deal with a number tending to 0 in computer arithmetic, you can deal with a small number, but not a number tending to 0, because then subsequent manipulations will be tedious or impossible. So, what you can do at most you can take a small number, which is not tending to 0, but very, very small. So, then if h is a small number, but not really tending to 0, if double dash x also has burden to play, in terms of dictating how small or how large the product will be. So, if you consider the function f double dash usually in finite difference analysis, when we doing a Taylor series expansion this part is often neglected.

We always highlight on this h or it is higher order powers as if, it is multiplying function has no role to play that is not the case. It is multiplying function has a big role to play, in a sense that this itself may be significant. Let us consider an example, let us say, that f (x) is equal to e to the power x. We choose this very simple function; I mean mathematically it is it is a very likable function, because it is derivatives are all the same. So, it is one of the simplest functions in that way to consider, physically we will see that there are certain examples, we will come across those examples later on in convection diffusion problems, where the variation itself may be of exponential nature, the variation of the field variable may itself be exponentially changing with x, but just for a from a pure mathematical perspective.

If f x is e to the power x, all it is higher order derivatives are e to the power x. So, if you cannot neglect the function itself, how can you neglect, it is higher order derivatives, so obviously you cannot neglect in such cases the higher order derivatives and then you rely on the smallness of h. So, if h is really very, very small, then you can get rid of the problem by truncating it up to and a few number of terms, if it is not so, then truncating the Taylor series up to a finite number of terms or up to a very few number of terms, can give rise to significant errors.

And that, is one of the drawbacks of a Taylor series based discretization method, where the Taylor series itself when truncated up to a few terms can give discrepancies, then you if you use that as a basis for discretization that is bound to give rise to some sort of discrepancies. So, that is so, it is simplicity in terms of using the Taylor series expansion sometimes may itself be a drawback, when the Taylor series expansion itself becomes becomes vulnerable to these these types of errors, if you truncate it, only up to a few number of terms. Now, these are not the only issues, we have to consider, certain other issues also for the method and to do that.

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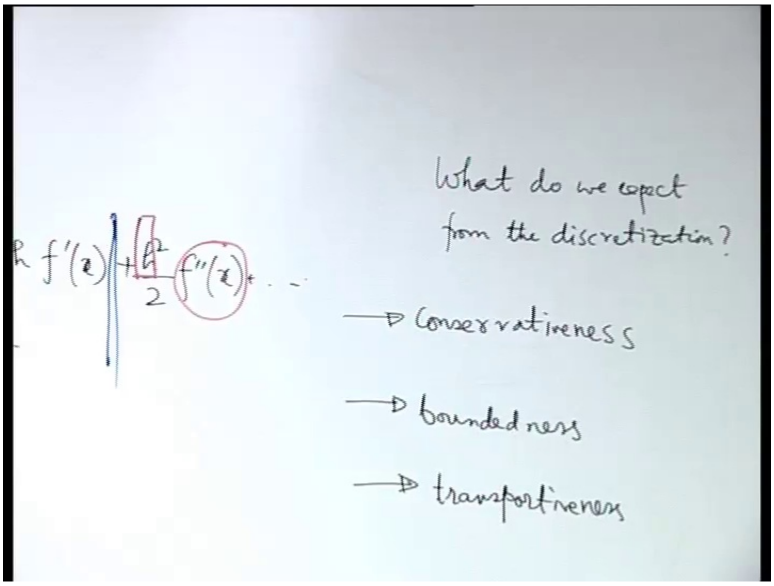

We will first consider that, what are the things that we expect out of c f d discretization method. So, what do we expect, it is important to assess the finite difference method in light of what we expect out of it. So, there are certain important characteristics that we expect, one is conservativeness, another is boundedness, another is transportiveness. Let us see that what are these and whether we can expect that the finite difference method will fulfill these requirements. Conservativeness from the name itself it is clear, that if the physical problem involves the conservation of a variable, the discretized equation should also reflect.

physically the same conservation that means, if you are having a collection of discrete equations all together summed up they should represent the conservation of the physical variable or what the entire control volume or over the entire domain, so to say. So that means, if it is a problem, where the physical requirement is the movement of conservation, then your discretized equations summed up together over the entire domain should ensure conservation of momentum over the entire domain that the equations must do.

Now, it is not guaranteed from the finite difference method, that it will do so, because it is just a mathematically expanded Taylor series, where we have never enforced any requirement of conservation therefore, it is not a necessity that the finite difference discretization will satisfy the requirement of conservativeness. And conservativeness is not inbuilt as a part of the discretization of the finite difference method, that we should remember.

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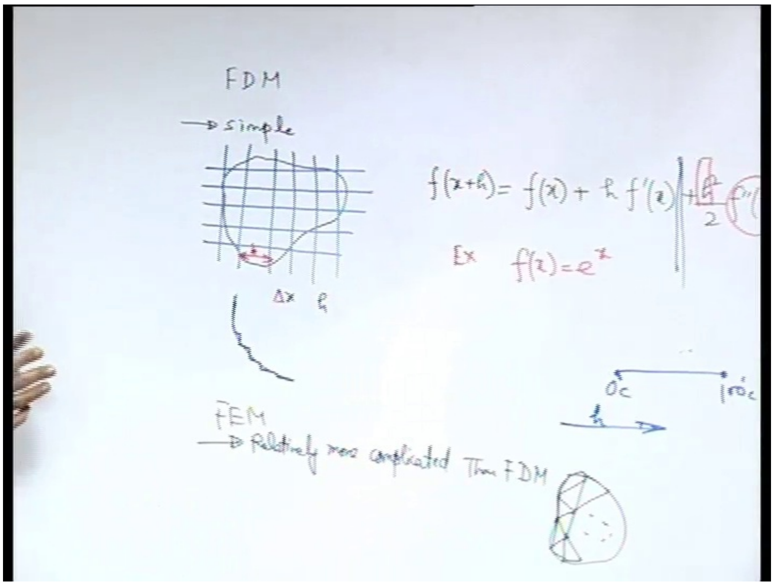

Then the other requirements one is boundedness, so what is boundedness, let us say that you have a rod, where the two ends are subjected to two different temperatures. Let us say that the left end is in touch with ice at 0 degree centigrade and the right end is in touch with steam at 100 degree centigrade. And let us say there is no heat source or heat sink and let us say that we are interested about the temperature distribution in this rod. So, when we say that we are interested in the temperature distribution in this rod, whatever we get in between, it should be bounded by it is boundary values, you cannot expect something less than 0 or greater than 100 in between.

So, it is it is a bounded nature of the solution, which comes from the physical description of the problem, so this is the physical description of the problem, that discretization also should ensure this boundedness, that is once you have discretized, your solutions should automatically come out, which would ensure this boundedness that also we have never ensured through that discretization policy of the finite difference method.

Then the third one is transportiveness, so transportiveness means, if there is a predominant direction of flow, then properties should be having a predominant transport direction based on the flow direction. For example, if it is a high reynolds number flow and if you if you have some property say enthalpy being transported. So, enthalpy will be predominantly transported along the direction of the flow and not predominantly along the opposite direction. Of course, it will have some tendency to get transported along the opposite direction because of diffusion, because diffusion is equally acting in all directions, but because it is the predominantly fluid flow that is governing the behavior for high reynolds number it is the inertia effect that is more dominant.

So, it will try to have effect of the transport predominantly in the direction of the flow, where the flow reynolds number is large. So, that property is called as transportiveness, of course we are not yet in a position to check the requirement of transportiveness, we will come back to that, when we we will be considering fluid flow as a part of the physical problem. The examples that we have sighted so far are based on pure diffusion type of problems like heat conduction problems. So, when we will be considering fluid flow problems as the examples, we will come and revisit this requirement of transportiveness. So, we can see that the finite difference method has certain pitfalls, of course it is simplicity is one of one of it is biggest biggest advantages, but otherwise it it has it is own limits or limitations.

Now, if you consider the finite elements method, of course I mean, if you compare the finite difference method with the finite elements method, finite elements method is relatively more complicated than the finite difference method, because you required to go through more rigorous formulation to derive the discretization equations. It is not as straight forward as writing the derivative terms directly in terms of the Taylor series expansion. So, it is relatively more complicated, it is not that it is complicated, but at least relatively more complicated than finite, but it it it is strongly based in a mathematical sense, because it it evolves from a general principle of error minimization. So, it it has it is strong mathematical basis, but physically it it is it is not intuitive that what it is talking about.

For example, if you consider a finite elements method, where you are considering a v formulation and then m formulation based on minimization of of functional. So, when you have the m formulation, let us say that the m formulation is there. So, it it must be having a physical meaning, so to say, because if you have a minimization of a functional then that minimization of the functional can be minimization of potential energy. For example, in a structural mechanics problem, if you have a structure, then the structure at equilibrium tends to minimize it is potential energy. So, for a structural mechanics problem, the functional that you minimize is actually, physically the potential energy of the system expressed in terms of differential expressions and integral expressions.

So, it is having a particular physical meaning, but for fluid flow and heat transfer, such a principle does not have any big relevance for fluid flow, heat transfer, mass transfer these types of problems. The more important governing guiding principle is the principle of conservation, like conservation of mass, momentum, energies, species all these things. So in that sense, the finite difference method the the sorry, the finite elements method is mathematically very rigorous, but does not carry the sense of conservativeness, explicitly through it is interpretation for fluid flow and heat transfer or mass transfer problems, but it has it is own advantages in a sense that, it can easily handle complex geometries.

So, it does not require a stencil of these structure type and you can have any arbitrary shaped element, and it is possible to design a mesh, that considers very complex geometries, that is very much possible. So, finite volume finite elements method is ideal in certain ways, but in in some sense it does not directly carry the sense of conservativeness. So, with these advantages and limitations of the finite difference and finite element methods in mind, we will move on to the next method, which is the finite volume method We will first state simply that, what does the finite volume method do, what is it is basic philosophy, what is it is basic principle and then try to assess it in comparison to the methods, that we have learnt before this.

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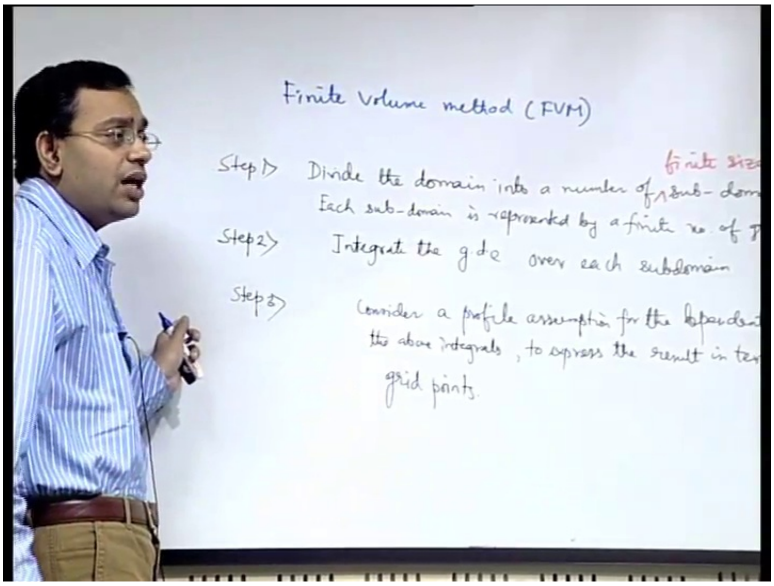

So, the finite volume method, what are the steps associated with the finite volume method. The step 1 is in any discretization divide the domain into a number of sub domains, in finite difference method remember we actually do not divide the domain into a number of sub domains, but essentially we get the corresponding effect by considering some grid points. So, the sub domains are essentially represented by grid points in the finite element method, in a finite difference method you do not have physical sub domains, but only a discrete set of grid points. In a finite volume method, it has a similarity with the finite element method in the way, that you can divide the domain logically into a number of sub domains. These sub domains are called as control volumes, remember in finite element method these are called as elements; in finite volume method these are called as control volumes.

Step 2 is integrate the governing differential equation over each sub domain, so divide the domain into a number of sub domains. We will amplify it with a very important consideration finite sized sub domains. So, this is just to demarked this one, with some standard derivations in analytical description of fluid mechanics or heat transfer, where we considered control volumes of infinitesimally small size, like having lengths delta x, delta y, delta g all tending to 0. Here, the dimensions of the control volume are not infinitesimally small, they are small, but finite. So, finite size sub domains, they integrate the governing differential equation over each sub domain. So, once you do that, we will show that, what you what you eventually get, you eventually get integral expressions, because you have integrated a differential expression. And in the integral expression you can evaluate the integral, provided you have an approximation of the variation of the function of as the as the dependent variable.

So, step three consider a profile assumption for the dependent variable, this is the interpolation function, so it has several logical similarities with the finite elements method, but it has more flexibilities than the finite element method that we will see. So, consider a profile assumption for the dependent variable, for evaluating the above integrals to express the result in terms of algebraic quantities at the grid points. So, remember here you have the domain divided into a number of sub domains, and each sub domain is represented by some grid points finite number of grid points, so these are like nodes in finite element method. So, what we have essentially done let us consider an example, to illustrate what we have essentially considered, but eventually out of this process, what we have achieved, we have come up with algebraic equations starting from a differential equation.

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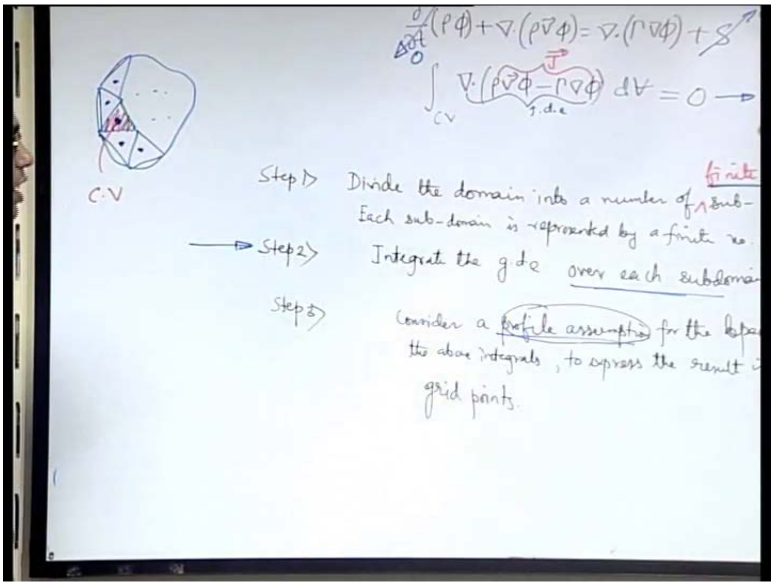

So, let us say let us let us consider an example, steady state convection diffusion with source equal to 0. So, remember the general transport equation we considered a steady state equation, so this term is equal to 0, and there is no source. If we want to solve this problem over a domain, say domain of this type, we divide the domain into a number of sub domains, in this manner. So you can see here, that your sub domains may be of whatever arbitrary shape and it is therefore, easy to tackle complex geometries.

In that way it can borrow the advantage of the finite elements method, because it considers logical elements which are called as control volumes, you can incorporate the effects of complicated geometries of the domain in this method which you cannot do, so easily in the finite difference method not that you cannot do, but you cannot do that elegantly. So, now let us consider, let us identify any any one of these elements, let us say this element, so then what is the next step integrate the governing differential equation over each sub domain. So, this sub domain remember this an element which is called as control volume. So, the terminology that we will use as no more an element, but a control volume and we will see that why logically this is called as a control volume.

So, then what we do we integrate this over the control volume. So, you integrate so remember that, this is your governing differential equation then it is possible to express this volume integral in terms of surface integral by using the divergence theorem. So, if you recall then what is this one physically physically it is the flux J net, where the first term is the advection flux and the second term is the diffusion flux, so we can write that integral of J dot eta d s, where s is the surface of the control volume at the control surface that is equal to 0, where this eta is the local direction normal at the location of the control surface that you are considering, the flux to be transported.

So, that is the step two that is you have integrated it over the domain and then you have simplified the integral by reducing the dimensionality by reducing it from the volume integral to surface integral, but to evaluate the integral, what you need to know, you need to know that how phi varies between the grid points. So, here you have to remember that when you have considered a considered a control volume, the control volume should be represented by certain points. So, let us say that it the centroid of the control volume, which is the corresponding grid point typically, in finite volume the representative grid points are located at the geometric centers of the corresponding control volumes.

Again this is not a necessity, it is a just a matter of convention for which people use and it is it is a convention, because it is it is mathematically much easier to handle with the centroid of a volume element, where you know where you know exactly where it is located, it is well defined and standardized. So, you can consider the grid points of different control volumes in this way. So, now you have to express these the phi appearing here, as a function of the values of phi at this discrete grid grid points. So, you would require a profile assumption, profile assumption means interpolation function, so phi as a function of position.

So, that interpolation function if you choose, then you can substitute and integrate it and once you integrate it, you will get the result as a function of phi at the discrete grid points, for each control volume it will come up with the corresponding grid point and it is neighboring grid points. We will come through illustrative examples to illustrate that, how it actually takes shape, but this is first the policy or the strategy of the method that we are going to highlight. Now, before going into more concrete example, where we show that how really we do it, there are certain important observations that we can note.

The key step in the finite volume method is really the step 2, where you integrate the governing differential equation over it is sub domain. So now, a question may arise, if you see that this was that overall balance of the quantity through advection and diffusion from which we derived the differential equation, if you recall that derivation of the general transport equation. Now, as if we are going back to the same route, so what we are gaining out of this, we had an integral expression for balance from that we derived a differential equation. Now, we are again integrating back the differential equation over the control volume.

So, a dilemma may arise that why we are doing it, once we had an integral expression for balance. We derived a differential equation from that, suddenly we are integrating it back, what we are gaining out of it, we should have gained nothing out of it, but actually we are gaining something, what we are gaining, remember that when we are considering a differential equation, the differential equation is valid at a point. So differential equation, when integrated is valid over a domain, here it is integrated over what it is integrated over finite sized control volumes, whereas when you first consider it derivation, you considered a infinitesimally small control volume. If you recall, we considered a box with delta x, delta y, delta g along the three three dimensions, where each of these were tending to 0 for the derivation of the general transport equation.

Now, we are integrating this same equation back, but not over such a differentially small control volume, but a control volume of finite size, so it is not exactly the same process the or the same point where we are coming back. We started with the control volume of differentially small size to derive the differential equation, but we are integrating the differential equation back on a control volume, which is no more that same differentially small control volume, it is a control volume of finite size, and that is why, the name finite volume method. So, it is based on finite sized control volumes that is why the name. So, this is the key step, now can you find any similarity of this step with the finite elements method, if you consider the finite elements method, there is the similarity of that with this key step, what is that?

We could write this particular step as follows, we could also write this particular step as this multiplied by a function w, where w is equal to 1, this w is the weight function. So, these can be perceived as a special type of Galerkin's method, where the weighting function is equal to 1. So you can see, that it can be thought it it can be perceived in in different ways, so the finite volume methods, since it can be perceived as a special case of Galerkin's method with the weighting function equal to 1, it borrows many of the important characteristics of the Galerkin's method in itself and that is one of the hallmarks of the method.

The other important feature is that, where does it is main advantage lie, see finite volume method is a method that we will we will go through in in much more details than the other two methods in this particular course. There is a reason behind this, that this is computational fluid dynamics as a course, so computational fluid flow heat transfer, these types of problems are the main focal themes of this particular course. And for these types of problems, now a days, people use mostly finite volume method, if you look into any commercial software that solves fluid flow heat transfer types of problems, all these software’s are written using the finite volume method as at as their basis. So, there must be something in finite volume method, which is good for fluid flow heat transfer, mass transfer these types of problems.

And what is that thing, that thing is that this method carries the physical meaning of conservation with itself, how does it do that. Remember that we are what we are doing, we are multiplying the governing differential equation or rather we are integrating the governing differential equation over the elemental volume. So, the governing differential equation, remember we have just stated that the differential equation is an equation which is valid at a point, so that means, it states the principle of conservation mathematically at a point, because we have derived them differential equation from the consideration of conservation of mass, momentum, energy whatever.

So, the differential equation when integrated over the domain will imply that we are now satisfying the same conservation principle over the entire domain in an integral sense. So, a over each sub domain, we are satisfying the same principle of conservation, which is conveyed by the differential equation. And when that is assembled over the entire domain; that means over the entire domain, we are physically satisfying the requirement of conservation, so the requirement of conservation; which is the basic physical route of deriving the corresponding governing equation, that particular route is followed, while deriving the mathematical form of the finite volume discretization, and that is why, it is so good and rather so well accepted by the fluid mechanics heat transfer community, because it satisfies the requirement of conservation by it is own simply implementation.

So, we can see that, the finite the other there is there are there are there are there are several other detailing or detail issues that we will come across like, one of the important issues is that, if you consider this profile assumption this profile assumption is not sort of requirement that comes through the interpolation functions in the finite elements method; that means, that in the finite element method, you you have a interpolation function and you use the interpolation functions subsequently for getting several other data from the calculations.

In a finite volume method, you use the profile assumption just to evaluate the integrals, once the integrals are evaluated the history of the profile assumptions is no more important, so; that means, it gives your freedom of choosing different profile assumptions for different terms or different interpolation functions of for different terms, because you do not require those interpolation functions any more for your subsequent analysis.

Whereas in finite elements method, you do not have that flexibility, because you; obviously, require the same interpolation function for subsequent post analysis of the system after you solve the system of algebraic equations. Now, with this background we will we will give a more concrete, but simple example of the implementation of the finite volume meth method through the one dimensional steady state heat conduction problem, the same prototype problem that, we have taken to illustrate the finite element and the finite difference method.

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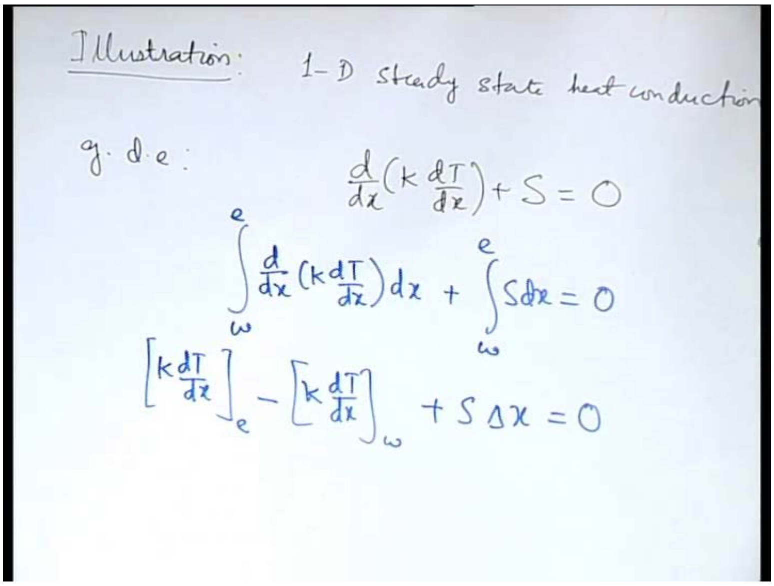

So illustration, one dimensional steady state heat conduction with a heat source, so what is the governing differential equation d dx of k dT dx plus S equal to 0. We have chosen this equation, because we have chosen the same equation to illustrate the other methods, so let us see that, how does it compare with the other methods through this same equation. Now, next what we will do, this is a one dimensional problem, so first we we should have a stencil of points is the finite volume stencil. So, in this stencil of points first what we will be doing, so this is a domain. So, we will divide the domain into a number of sub domains control volumes.

So, let us say, we divide the domain into these four control volumes just as an example. So, each control volume should be associated with a grid point, so the centroid of each control volume, we consider for the respective grid points plus you require the boundaries of the control volumes, because you also have to incorporate boundary conditions. So see, in addition to the centroids of the control volumes, you should consider grid points at the boundaries, because you have to have points through which you implement the boundary condition, then you can give this point some number say 1, 2, 3, 4, 5, 6 like that.

Let us isolate any any control volume out of it, and try to describe it is detailed features, so this is the standard nomenclature that one uses. So, if you have a control volume, it is centroid is the main grid point of the control volume, it is denoted by P. These are standard nomenclature universal nomenclature used for all finite volume descriptions; in any book, any standard text book of finite volume you and any paper or research article on finite volume you will see the same nomenclature. So, it is important that you start following this nomenclature, so the main grid point of the control volume is given by P

Now, you have the next control volume, so you have the immediately next neighbor, which we call as E and immediately that neighbor to the left we call as W, this is E for east and W for west. So, east west similarly, if it was a two dimensional it (( )) been N and S north and south and for three dimensional t and b top and bottom. So, we will come across two dimensional and three dimensional examples subsequently, but right now, we are confined to only one dimensional example to illustrate the method.

So, for 1 to 1 correspondence this is the point P, this is the point E, this is the point W corresponding to the particular control volume, then the control volume has a particular length this one. So, this length we give a name as delta x, if it is a two dimensional case in the y direction, it will be delta y like that, again these are standard nomenclatures. Then the distance between this P and E, We call it delta x small e, and distance between P and W delta x small w. And you can easily recognize that these locations, what are these locations these locations are nothing but, the phases of the control volume. So, this is the east phase, this is given by small e; this is the west phase, this is given by small w.

So, phases of the control volume are designated by lower case alphabets and the main grid points are designated by upper case alphabets; that is the standard nomenclature. So, this is the nomenclature for a one dimensional control volume. So, considering that control volume now, what is our first step, so we have divided the domain into a number of control volumes. Next step will be to integrate the governing differential equation over each control volume, so we have d dx of k dT dx, what is what are the limits of integration from where to where; from small w to small e. So, the integration becomes k dT dx at e minus k dT dx at w.

Let us consider that S as constant, as we have considered for the previous cases as well. So, then S will come out of the integral, so plus S into x small e minus x small w, that is delta x. Then the next step of the discretization is that, we have integrated, we have got the corresponding expressions, but these are still differential expressions to get corresponding algebraic expressions, we must have a profile variation for T as a function of x, so that, we can calculate dT dx. So, the problem boils down to some assumption on how T varies with x.

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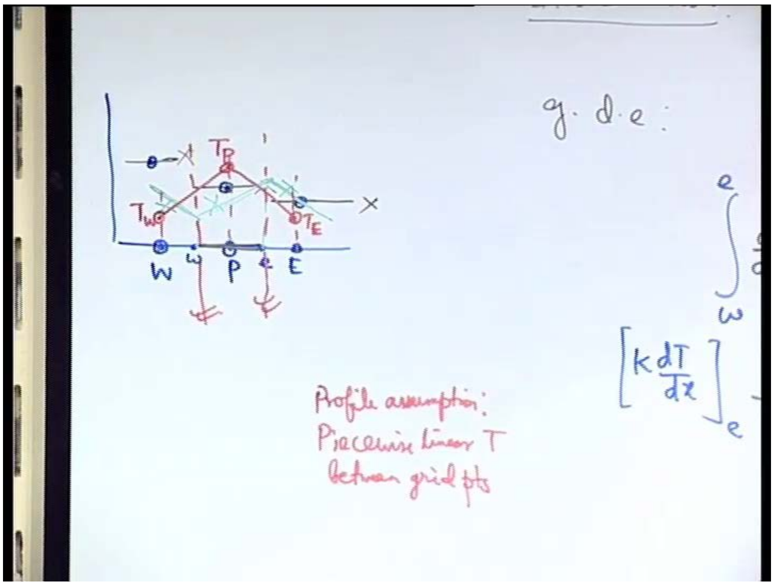

So, let us say that you have some point P, E, W. So, we have to make a choice, what is the simplest choice, that we could make see, you could make higher and higher order approximations of T as a function of x, say you can consider T as a very high order polynomial function of x for evaluating dT dx. We will abstain from that, but we will start from the opposite consideration that let us consider what is the simplest rather than, what is the most involved. So, if you have no other constraint, it could have been piecewise constant profiles of T, it could have been like. So, you have a control volume, which is your control volume, this is your control volume, so you could have a value of T in this control volume; a value of T in this control volume; a value of T in this control volume in this way.

Remember that these values could have been useful, only in terms of the values at the respective grid points. Otherwise it is artificial, only at the respective grid points, you consider these values to be corresponding to the corresponding temperatures. At all other points you need to interpolate basically, so it is not that, the same function that you are using for post processing, but here you cannot use these, because because of not the reason of such apparent discontinuities, but because of the reason that you require dT dx for your calculations. So, if T is a constant you cannot get dT dx out of it will be 0. So, this is not a valid profile assumption, in this case.

Then what could be the next possible choice piecewise linear not a constant constant is also linear, but non constant linear. So, let us let us let us see let us make a sketch of that one. Let us say this is the profile assumption that we choose in a arbitrary scales, so I mean do not be bothered so much about what are the exact values this is just a sketch. So now you can calculate dT dx, because you have T as a linear function of x, question is will it work, if yes why? if no why? Let us see that, where do you need to calculate dT dx; you need to calculate dT dx at small e and small w, so at small e and small w dT dx is discontinuous.

So, you can get a left hand derivative; if you consider this line, you can get a right hand derivative; if you consider this line and these two derivatives are different, because these two are different lines, so; that means, k dT dx is not continuous, physically what it means heat flux is not continuous, at a phase heat flux must be continuous, whatever heat flux enters the phase the same heat flux must leave the phase, but this discretization does not automatically ensure that, so that is also not a acceptable profile assumption, but you could make a linear profile acceptable in this way, instead of piecewise linear profile between the control volume phases, if you consider piecewise linear profile between the grid points.

So, this is piecewise linear profile between the grid points this is T w, this is T p, this is T E, which you need to solve. Now, will it work, this will work because here the discontinuity is here, but you do not have to calculate dT dx here, your dT dx calculation is at these locations at the phases of the control volumes, where the function is having continuous dT dx. So, this is the valid profile assumption, so with this as a profile assumption. So, profile assumption piecewise linear T between grid points.

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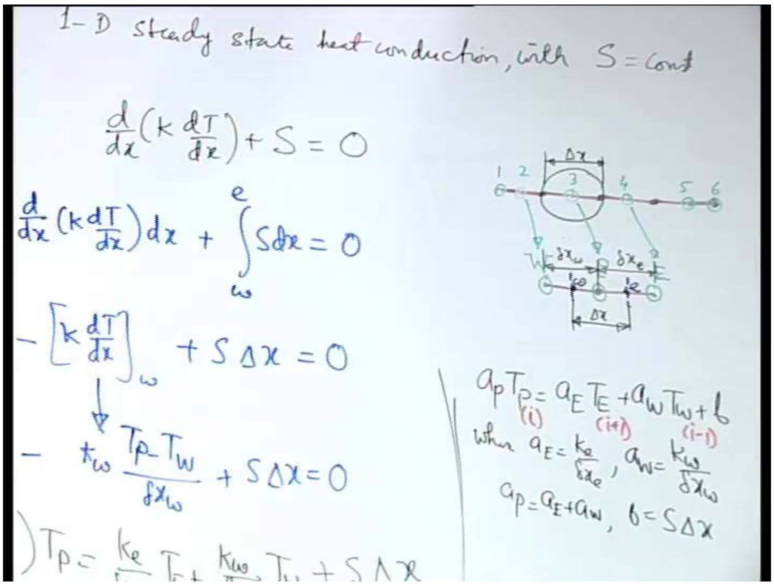

So, then what will be k dT dx at small e, because T is a linear function of x, it is T 2 minus T 1 by x 2 minus x 1 dT dx. So, it will be k at small e into T E minus T p by x e minus x p, what is x e minus x p, that is delta x small e, you see the nomenclature. Minus this term k w, what will be this one dT dx at small w, T p minus T E sorry T p minus T w by delta x w plus S into delta x equal to 0. So, you can assemble this equation and write (k e by delta x e plus k w by delta x w) T p is equal to k e by delta x e T E plus k w by delta x w T w plus S into delta x.

We can symbolically write it in this way a p into T p is equal to a E into T E plus a w into T w plus b, where a E is k e by delta x e, a w is k w by delta x w, a p is a E plus a w and b is equal to S into delta x. This is the finite volume discretized equation, you can write it in terms of indices by considering p to be i, e to be i plus 1 and w to be i minus 1. So, you can see that one particular grid point value of the variable at one particular grid point is expressed as a function of the value at it is neighboring grid points through algebraic equation. So, we have come up with a discretized algebraic equation from the governing differential equation, physically you can interpret different terms, what is k by delta x, it is the conductance at that particular location.

So, it is a conductance at the left phase, this is the conductance at the right phase like that this is the total generation within the control volume. So, it is a balance of fluxes and generation within the control volume. Now, one important question might come that well would have would it have been better, if you have considered a higher order interpolation function, say second order interpolation function.

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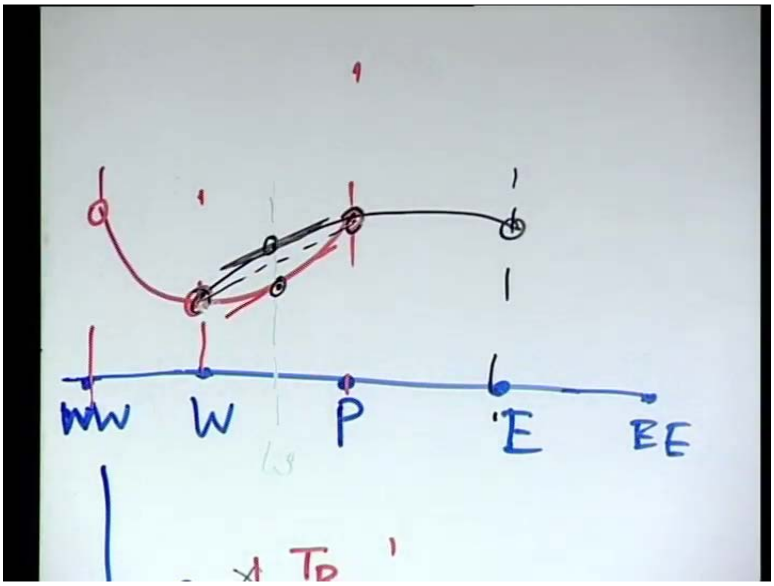

So, let us consider that, let us say that you have this point P; you have the point E, W and may be consider some additional points, because if you say want a quadratic interpolation function, how many points you require to generate a quadratic interpolation function. Three points just like you require two points to generate a linear interpolation function, so the interpolation could have look like this three values of temperature at these three points then similarly, for the points W, P and E taken at a time, it could have been this, so this is piecewise quadratic interpolation.

The question is will it worth, again let us try to see that what is dT dx at the control volume phase, so what is the phase, the phase is this one small w. So, how can you calculate dT dx here, it is the slope of the red line and similarly, it is the slope of the black line. It depends on which line you are considering are these two equal not necessarily, they are equal only when this small w is at the exactly at the meet point of capital W and capital P. There is a reason behind this, if you recall there is a theorem called as roll's theorem in calculus, what it says, is that if you have a cord like this.

Then, if you have a function which is continuously differentiable in between then the function at has at least one point in between, where the tangent is parallel to the chord that is the slope of the tangent is same as the slope of the chord. So, that point happens to be here, so here you can see that this is the chord. So, the function is parallel to the chord in the two cases only when the point where it happens to be parallel is the midpoint of the two. Otherwise, if the phase is not located at the midpoint of the two, but it is shifted these two different lines will have different slopes, these two different curves will have the same slope only when the point at which your considering the slope is the midpoint of the interval.

So, it gives us a very important lesson, what is that? A higher order interpolation function will not necessarily give you a more accurate result, this is this is something which is non intuitive. Intuitionally, we have always learned that a higher order interpolation function will give you a better solution. It is not necessary it is it will be true provided it accommodates all the basic physical requirements that is continuity of the flux across the phase.

So, we can see that if it is if this point is exactly at the midpoint of the interval then it will do, but if it is exactly not at the midpoint of the interval, then the fluxes calculated from the black profile and the red profile will be different and then; obviously, you will not get a continuous flux at the control volume phase. And that will give you a erroneous discretization, no matter it is higher order in terms of accuracy, but physically it is erroneous. So, we should not consider such discretization and sometimes a simple profile assumption like a linear profile assumption might turn out to be adequate. So we stop here, in this lecture and we will take it up from here in the next lecture, thank you.