Engineering Thermodynamics
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Week-11
Lecture-50
Thermodynamic Property Relations

Welcome to Thermodynamics property relation in Bhagcha. In the previous lecture, we learned about the Clapeyron equation in. And how they use the Clapeyron equation to extract enthalpy and vaporization. If you have PVT data. And especially we derived the Clapeyron Clausius equation. and solid vapor is useful. These equations are commonly used in industry. In this lecture, we will learn how to find the general relation for Cv, Cp delta U and dHO. This relation will be valid, there will be no approximation. We will simplify it through approximation, like adults and kids. and we will also learn how to derive the basic relations that we had assumed earlier. This lecture is quite mathematical, and you should watch it carefully. This is a very fundamental lecture and if you understand it, you can easily do the first principle calculation, you can start on a basic dependence like U depends on T or V or how to use these steps to get relations and how to use it when a problem statement comes you will know about this in this lecture Let's start with a basic definition. Whenever we talk about state, as you can see, sometimes we are told that property depends on only two parameters. But we have used this state postulate method. The state postulate means that if there is a simple system, which is compressible, the pressure changes as the volume changes. So, it is a simple compressible system. Any simple fluid like air or water, all these are simple compressible fluids. You can tell the state of the fluid, whether it is in solid or liquid state, with two independent intensive properties. The intensive properties that you have defined, like temperature, specific volume, temperature, pressure, the state of the fluid is satisfied. So, this is a simple compressible system. So, if we look at it from this logic, then you can find the properties of any system when you have two independent intensive properties. Like, the temperature and pressure are given, so you can find the other properties, u, h, etc. You can assume this. But the problem is that when we experiment, when we want to extract any property, it's not like we can extract any property. You can measure temperature, pressure, heat capacity, but when it comes to internal energy or entropy, then you can't easily measure it. You can't extract it. You can't directly measure its absolute value. That's why, to extract all these things, these are dependent, the rest are measurable properties. So, if you know PVT data, or you know Cv and Cp, then you can extract all these properties. That's why, the relation between them, internal energy, entropy, and enthalpy, We will know more about the relation between PBT data, CV, CP data in this lecture. We discussed this in the last lecture as well. We are moving forward with this.

Let's start. I have summarized your Maxwell relation here. These are the Maxwell relations. These are the four relations. These are the relations of generalized thermodynamic potential. You can see the exact differentials. and we have taken it using the same property. And we will use it wherever you want to change the entropy, wherever you feel that entropy partial derivative is available, and you can change it in PVT data, then we will use it. For example, if you have given this data, these expressions are given somewhere, then you can replace it with this, or if it is given, then you can replace it with this. The benefit of this is that we, PVT data, we can easily extract this data from experiments, so we can also extract its derivative. So you can get this information through experiments and through the equation of state. So, its use becomes more important. And so, we will replace it by using Maxwell Relation where your entropy will be used. We will replace it and then we will get the PVT data. Let's start. We have to find out these relations. First, let's see internal energy changes. Assume that U is the data we want. That is the data of TV. In this case, if you take its derivative. This is your partial relativity. And this is You know what this is? This is Cv So this is your relation, this is what you know. Now if you look at the first law, if you look at Thermodynamics first law, and this is your dU is equal to dS minus PdV this is called equation 2 here you can see that S in general this relation is directly to your first log signal. Since we are talking about internal energy changes, we will try to use all these things. As we said, if we have fixed Tb, two intensive variables, T and specific volume, this is the specific volume. Similarly, if we consider that S is Sb, then S will depend on T or Vb. So, if we write its differential form with partial derivative, then we get this relation. Okay. Now from here, 2, the expression 2 is the normal first law. This expression of any closed system will remain. Here we are not saying that temperature volume, you can say that change in the temperature volume closed system. It's just that this mass will not transfer. Its boundary is not interacting with the surrounding in the form of mass. But heat transfer can be done. So, this expression is standard expression.

Internal energy changes,

$$u = u(T, V)$$

$$dU = (\frac{\partial U}{\partial T})_V dT + (\frac{\partial U}{\partial V})_T dV$$

$$dU = TdS - PdV$$

$$S = S(T, V)$$

$$dS = (\frac{\partial S}{\partial T})_V + (\frac{\partial S}{\partial V})_T dV$$

$$dU = T\left[\left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T dV\right] - PdV$$

$$dU = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV - PdV$$

$$dU = T\left(\frac{\partial S}{\partial T}\right)_V dT + [T\left(\frac{\partial S}{\partial V}\right)_T - P]dV$$

$$C_v = T\left(\frac{\partial S}{\partial T}\right)_V dT \ or \frac{C_v}{T} = \left(\frac{\partial S}{\partial T}\right)_V$$

$$(\frac{\partial U}{\partial T})_T = T(\frac{\partial S}{\partial V})_T - P$$

$$(\frac{\partial U}{\partial T})_T = T(\frac{\partial P}{\partial T})_V - P$$

$$dU = C_v dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] dV$$

$$U = \int_{T_1}^{T_2} C_v dT + \int_{V_1}^{V_2} [T\left(\frac{\partial P}{\partial T}\right)_V - P] dV$$

We can use this. So, there is a dS relation in this also. there is a dS relation in this also. And this, which we said that this is a variable, TOV, we are using two variables. We are using it here also, in the U and finally here also. Now, we have taken out this relation from here. Now, this thing here, if this is 2, if you put this expression of dS in this equation 2, which is 3, So what is your dU? 3 in 2 ok so left hand side is du and this will be t this term which we are writing here minus pdv so I will expand this t ds by dt v dt plus t del s del v t dv minus pdv and this common. So, we can write it like this. du is equal to t ds by dt v dt plus. I have taken this common. Here you have your common. So, you can write it like this. So, this expression is 4. This equation is 4. Now, if you compare the first equation, which we saw, the first one, and the second equation, so, you will see that Cv is Cv is T del S by del T V and Cv by T is and second expression is del u by del vt this expression is this so del u by del vt is your T ds by dv T-b this is 5 now as I said if you want to get del u So, you will get this expression Cv. Okay, you will get this Cv. When you want to get the change in del u, then you integrate it, then you will get Cv. But to get the expression del u by del v, you have to get this expression. And in this expression, there is del s by del v. And you can change del s by del v from Maxwell relation. If you see this, del s by del v at constant T is this. This is the third expression that is visible. You can replace this with del P by del t constant V. So, this is del P by del t V minus constant V. So, this expression is given. So, this means that your du is okay. du is finally Cv plus del P by del t V minus P. So, this expression is yours. Finally, you put change in internal energy in two parts. One is your CV dt plus the second term is T del P by del T constant volume minus P.So suppose you want to take out change in U in two states, like T1 V1 to T2 V2, then how will you take it out? So change in U from state if T1 is given V1 and from here you are going to T2V2. So, if you think of it this way that it has to be integrated, then you can think of it in two ways. If this is your T1V1 diagram, then if it is at this point, suppose this is your T1V1 and here you have to go to T2V2. So, we can consider this as a constant volume, where you did T1 to T2, then you went from T1 to T2, and then from V1 to V2. When you can use this, it is called two paths. There are two paths involved in this. We call this path a thermodynamic path. You can also call it as, since these are T and V independent variables, so if we integrate this to dU, then it will be u2 minus u1 and this will be integrated cv dt t1 to t2 plus integrate this part t dp dt v minus p dv v1 to v2 So you can think of it as this part is T1 to T2 where we have kept V1 constant This is the first part And this part is V1 to V2 This is V1 to V2 but in this T will be equal to T2 and this is your second part So you can draw a relation like this This is the first part and this is the second part So from the alarm So what we did was, we defined the variable T and V as U as a function of U, T, V. Then we wrote the differential form on the partial derivative. Then we used the first law and the Maxwell relation. Then we got the final relation and showed how to change it. Similarly, you can bring other changes. You can also do it in TAP. Then you will get a different relation. Now we will talk about enthalpy changes. In this case, we will consider TAP. Because in enthalpy, we normally calculate changes related to TAP. When enthalpy comes, it usually comes according to pressure change. In the flow system, enthalpy is used more. But Methods and Mathematics can be applied if you want to use any other Intensive Variable. The second expression will come that is different. But you can use this systematic approach to get the expression. So, what we are showing are enthalpy changes in which finally, T and P will change, i.e. T1P1 to T2P2 will go. So now we are considering H as a function of TP.

$$h = h(T, P)$$

$$dh = (\frac{\partial h}{\partial T})_P dT + (\frac{\partial h}{\partial P})_T dP$$

$$dh = TdS + VdP$$

$$s = s(T, P)$$

$$ds = (\frac{\partial s}{\partial T})_P dT + (\frac{\partial s}{\partial P})_T dP$$

$$dh = T[(\frac{\partial s}{\partial T})_P dT + (\frac{\partial s}{\partial P})_T dP] + VdP$$

$$dh = T(\frac{\partial s}{\partial T})_P dT + [T(\frac{\partial s}{\partial P})_T + V]dP$$

$$C_p = T(\frac{\partial S}{\partial T})_P$$

$$(\frac{dh}{dP})_T = T(\frac{\partial s}{\partial P})_T + V$$

$$(\frac{dh}{dP})_T = -T(\frac{\partial V}{\partial T})_P + V$$

$$dh = C_P dT + [V - T\left(\frac{\partial V}{\partial T}\right)_P]dP$$

$$h_2 - h_1 = \int_{T_1}^{T_2} C_p dT + \int_{P_1}^{P_2} [U - T\left(\frac{\partial U}{\partial T}\right)_P]dP$$

Now note that dh is tds plus vdp. We had taken this expression out. It is the general expression of the thermodynamic equation. This Because it comes out from the first clause easily. By definition, UH is U plus PV. It will come out from there. Here we are saying that we will use this expression. And we also want to write this. Because S will come finally, we will have to define S as well. S is a function of TP. Two intensive variables are considered here. T and P. It means that we have to get delta H between changes in T and P. So here we defined again, s is equal to function of STP, so ds is your del s by del t at constant p dt plus del s by del p at constant t dp. Now this expression of ds, if you call it 9, then we will put it in this expression. So, dh is here T is the expression plus vdp So let's expand it Now let's take common. Tp is common. So, this is your T es by dt P dt plus T del s by del p constant T plus V dp. This is 10. Now what we do is 8 and compare. If we do 8 and 10. What comes out? If we compare 8 and 10, then CP will be T dS by dt at constant P and del H by del P at constant T This part will be taken out from this expression So this is your T del S by del P at constant T plus V Now, we can replace this term with del S by del P. As I said, wherever partial derivative is given, or entropy constant is given, you can use Maxwell relation easily. This is the Maxwell relation. And if you don't remember Maxwell relation, then as I said earlier, you can use it easily in such cases, that you basically, you can use the basic definition to get the answer. As you have used in the first step in the Quasi-Static equilibrium. So, simple du is equal to ds, tds minus pdv. Then you can say that the definition is A and g. A is your definition. u minus ds, from there your da comes out. minus sdt. then you have dh, you know dh is u plus pv, so dts plus vdp then you know g is equal to h minus ds, so dg is minus sdt plus vdp and then if you use exact expression, you will get it easily like in this case, it is del s by del p now del s by del p is So, del S by del P is the relation you get from G. del S by del P is the expression you get from the fourth expression. So, the fourth expression was this expression. This is del S by del P constant T is equal to minus del V by del T constant P. See, you get it from here. From here, you get it from this expression. del v by del t at constant p. So, we have written this expression here. So, this is your del H is equal to del V. So, in this case, how can you take dH? Cp dt plus this term. We are writing here, this is del H by del P constant T. So dt is there. So, this is the dt term. Here is V minus T. del V by del t at constant P. So, this expression is done. Now I call this 11. Now we have to take out change in enthalpy. We have to take out delta H. Where to go? T1 P1 to T2 P2. Okay? So, to take this out, we will integrate these 11. So H2-H1 is integrated. T1 T2 CpdT. Plus, as I said last time, there are changes in this, it depends on which path you take. But you can integrate it independently. There is no issue in this is your, So this is P. So here is your P1 to P2. So, these are two expressions. You can interrogate these expressions in any way. As I derived last time, suppose you have this Pt, you can also say that this is your state 1, so this is P1, this is T1, and this is state 2. This is T2, and this is P2. We can also say that We will go to constant pressure and then we will go to constant temperature and finally state 2. This can also happen. This is your first and this is your second. The second way is that we can go like this. This is your first dash, and this is your second dash. You can use this expression in both cases. In the first case, your temperature will be T1 to T2. So, this is the first expression. In the second case, the temperature will be T2. In the case of the dash case, the pressure will be P2. If the CP doesn't change much, then we will put the CP constant at P2. In the second case, T will be T1. You can think in any way, both the parts can be taken. So, this expression is valid. Entropy change has been done. We have derived it. Now we will talk about entropy change.

$$s = s(T, V)$$

$$ds = (\frac{\partial s}{\partial T})_V dT + (\frac{\partial s}{\partial V})_T dV$$

$$ds = \frac{C_V}{T} dT + (\frac{\partial s}{\partial V})_T dV$$

$$ds = \frac{C_V}{T} dT + (\frac{\partial P}{\partial T})_V dV$$

$$S_2 - S_1 = \int_{T_1}^{T_2} \frac{C_V}{T} dT + \int_{V_1}^{V_2} (\frac{\partial P}{\partial T})_V dV$$

$$S = S(T, P)$$

$$ds = (\frac{\partial s}{\partial T})_P dT + (\frac{\partial s}{\partial P})_T dP$$

$$S_2 - S_1 = \int dS = \int_{T_1}^{T_2} \frac{C_P}{T} dT - \int_{P_1}^{P_2} (\frac{\partial V}{\partial T})_P dP$$

So, here two relations will be obtained. Because if we assume that S is STV, depending on two intensive variables, in which if we take TV, then one relation will be obtained. If we take TP, then another relation will be obtained. And we are assuming this as a simple compressible system. So, these two relations will be obtained, delta S. So, if we take S is equal to STV, So, we have already done DS Comb Book. Let's take it like this. And we just found out that del S by del t is the constant volume of CV divided by t. So, this is your relation. Now this relation is partial derivative. del S by del V. If you see, if you don't know the Maxwell relation from here, and you know that this is your relation, then this is the relation that comes out. This is del S by del V at constant T is equal to del P by del T at constant V. So, this will come out as del P by del t at constant V so this is the Maxwellization why? note that del S by del V at constant T that will come as del P by del t at constant V ok as you might have learned earlier mx is like mx is del m by del x at constant y should be del n by del y at constant. and this is the relation you have given here so we were talking about this we will use this and we will get the expression of entropy that ds is cv by t dt plus del p by del t at constant v dv if we integrate this as s2 minus s1 so this comes as integral cv t So this is your 12, this is your 13, this is your relation. So now you have this relation which you can use. The second relation will come out when you consider this that if your S depends on TP. So in this case, in such a case, We will do the same as before ds is equal to del S by del t at constant p dt plus del S by del p at constant t dp Now this expression which you have seen we have seen this before in this you have said del S by del t at constant p is related to Cp so what is this? Cp by t at constant t plus del S by del p at constant Now this relation will also come from Maxwell relations See, del S by del P del S by del P at constant T will come minus del V by del T at constant P So this will come minus del V by del T at constant P So your dS is here Cp by T dt minus del V by del t at constant P dP P1 by P2 and this will be T1 by T2, this is integral this will be S2 minus S1 Now the question arises that which relation you should use so this will always depend on the data and on the thermodynamic condition so whenever you have a change in temperature and pressure, then this relation will be used if the temperature and volume are changed, then we will use another relation If you want to extract the data, you will understand the changes in the data and the relation to use it. So, this will be explained in some examples. Let's move forward. Now, it's time to talk about specific heat. The specific heat is what we normally write. So, we always said that this is a function of only temperature. It depends on the temperature only, not on the rest. But real gases, okay, real gases, are a function of specific volume or pressure. Or it must be at a higher temperature. So, the specific volume and pressure comes on it. When the pressure is very low, the volume becomes very high. Specific volume, so Cp and Cv are It reaches the ideal gas. It means that when the pressure is very low, Cv will approach Cv and Cp will approach Cp which is the specific volume of the ideal gas. We can do approximation in this. You can assume that by making this basis.

$$real\ gas = f(\frac{V}{P}, T)$$

At Low Pressure

Cv→Cv0

Cp→Cp0

$$dz = Mdx + Ndy$$

$$(\frac{\partial M}{\partial y})_x = (\frac{\partial N}{\partial x})_y; M = (\frac{\partial Z}{\partial x})_y, N = (\frac{\partial Z}{\partial y})_x$$

$$(\frac{\partial C_v}{\partial V})_T = T(\frac{\partial^2 P}{\partial T^2})_v$$

$$(\frac{\partial C_p}{\partial P})_T = -T(\frac{\partial^2 V}{\partial T^2})_P$$

$$C_P - C_{P0} = -T\int_0^P (\frac{\partial^2 V}{\partial T^2})_P \, dP$$

$$C_P - C_V = -T(\frac{\partial V}{\partial T})_P(\frac{\partial P}{\partial T})_V$$

$$C_P - C_V = -T((\frac{\partial V}{\partial T})_P)^2 \left(\frac{\partial P}{\partial T}\right)_V = \frac{VT\beta^2}{\alpha}$$

$$\beta = Volume\ Expansivity\ and\ \alpha = Isothremal\ Compressibility$$

$$\beta = \frac{1}{V}\ (\frac{\partial V}{\partial T})_P;\ \alpha = -\frac{1}{V}\ (\frac{\partial V}{\partial P})_T$$

Let's assume that the ideal gas is ideal. and finally increase the pressure and if we get any relation then we can integrate and get it out So if we want to get CV and CP then we can get it from CV0, CP0 and PVT data If we have all this data information then we can get it out So to get it out we need to and I will try to find out how to remove it. For this, we will have to find out a relation of the CWD derivative. We will have to find out the derivative. So, to find out the derivative, we will apply the exact next. The exactness, the basic mathematical condition of the property is exactness. From there we got the max relation. So, let's do one thing, first we start from the relation we got of CV and CP. The one we got here. So, we will apply the test of exactness. Please remind us about it. Suppose for example you have this z function of x and y, then you can write it in partial derivative and m is partial derivative. And if it is exact, then del m by del y constant x will be equal to del n by del x constant y. x at constant y and n is del z by del y at constant x. So, this condition is called as test of exactness. This is the test of exactness. If any property is exact, then this condition should match. So the expression of entropy that we had taken out, we had taken out two expressions first, this one, in which we said that ds is is your Cv by T dT plus del P by del T V this expression was taken from S is equal to S TV then the second expression was CP by T del T minus del V by del T at constant P dP was taken from S is equal to T P now if you put the exact next condition on it So what will come out? del Cv by del VT is equal to T del square P by del T square This is your constant value This is one relation And the second Cv is del Cv, Cp del Cp by del P at constant T is equal to minus T del square V by del t square constant p. So, these two relations have been derived. Now if you look at the second relation, if we integrate this, integrating So, this will be your CP minus CP0. And we will do this at constant temperature. So, we are changing the pressure. This is the left-hand side. What will happen to the right-hand side? Minus T, 0 to P, del square V by del T square P dP. and you can extract this right hand side if you have PVT data if you have PVT data, you will get second derivative another thing you can extract is CP minus CV relation I am writing this without derivations but you can extract it with exact derivations because you know the CP and CV relation you have been given the expression so by subtracting you can extract the cyclic relation minus 1 relation You can show that Cp minus Cv is T del V by del T P del P by del T V and you can show it further that minus T del V square by del T square constant P and D del P by del V T and there are two things in this expression you can see how the volume is changing as you change the temperature and the volume changes at constant temperature as you change the pressure. So, we define this term as beta. expansivity This is defined as 1 by V del V by del t by P Now see what is this If you change the temperature, how the volume changes at constant pressure and the second term is alpha which is isothermal Compressibility means that the temperature is fixed and the pressure is increased, then how the volume is changed. Now let's take the negative in this. Minus 1 by V, del V by del P at constant T. Now using these two relations, you can show this Cp minus Cv that this is your VT beta square. VT beta squared divided by alpha Cp minus Cv is VT beta squared by alpha and this relation is called as Mere Relation and there are two things in this first is that if your volume is not changing then beta is zero and in this Cp minus Cp will be equal to Cv second is that If you can show in the ITIL test that the right hand side Vt beta square is basically R. So, there are some conditions in this. This is a generic relation. You can easily show many things with this relation. Especially notice that alpha is always positive. That's why it has been used in minus. But beta can also be negative in some conditions. But because beta square is used, the right-hand side term is always greater than or equal to zero. So, these three things are important for us. so now we will try to do some examples. This is a very important relation, the May relation. I can write the May relation as well. Cp minus Cv is Vt beta squared by alpha. Alright, so now we move ahead. Let's try to understand some examples. Here we have to find the internal energy change which follows the Van der Waal equation of state. And Cv of the gas depends on this condition and follows this relation. C1 plus C2 multiplied by T. So, pay attention to this, we don't have to do anything. You have to use this relation. and we will integrate it and use this relation and we will use the Van der Waal equation and instead of Cv we will keep C1 plus C2t let's write this so u2 minus u1 is your integral Cv dt plus integral t del p by del t v minus p we had taken this relation I will from here the volume will change V1 to V2 and this is T1 to T2 Cv is C1 plus C2T and we have to take out del P by del T at constant V for this we have to take out equation of state and use the Van der Waal equation so what is van der waals equation RTV minus V minus A by V square and if you take this as partial derivative del P to del T constant volume so this is Rv minus V constant volume second term will be zero first term is simple dependent on temperature so this is what we get and now if you do del P by del T constant V minus P so this is your Rv-V minus P V plus A by V square this is left when it is cut off A by V square so you have to plug in this A by V square in this expression so this expression is U2 minus U1 integral C1 plus C2 t dt plus t1 to t2 plus v1 to v2 a by v2 dv so u2 minus u1 what is c1 t2 minus t1 plus c2 by 2 t2 square minus t1 square The second term is integral of 1 by V1 minus 1 by V2.

$$V_2 - V_1 = \int_{T_1}^{T_2} C_V dT + \int_{V_1}^{V_2} \left[T \left(\frac{\partial P}{\partial T}\right)_V - P \right] dV$$

$$P = \frac{RT}{V-b} - \frac{a}{V^2}; \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V-b}$$

$$T\left(\frac{\partial P}{\partial T}\right)_V - P = \frac{R}{V-b} - P = \frac{a}{V^2}$$

$$U_2 - U_1 = C_1(T_2 - T_1) + \frac{C_2}{2}\left({T_2}^2 - {T_1}^2\right) + a(\frac{1}{V_1} - \frac{1}{V^2})$$

So, this expression is your answer. This is your answer to the question, internal energy changes of a wonder wall gas. This expression comes from the result of the chemical. So, this is your first question. Now the second question is about your internal energy. But the question is to know that the function of the ideal gas or incompressible substance is only the function of temperature. We have to prove this. The dependence of internal energy on the ideal gas or incompressible substance is only on temperature. U is the function of T in the condition of the ideal gas and in the condition of the incompressible substance. We are writing the expression du is equal to cv dt. I am writing it as we have just read it. I am repeating it. After writing this expression, the first comes is the ideal gas. The equation of the ideal gas is listed. pv is equal to RT. This is the specific volume V here. We can write it like this. Because we have to take out del p by del V, we can take it out. p is equal to RT by V so it means that del p by del t at constant V is equal to R by V it means that this term T del p by del t V minus p we can write it T times this term is R by V minus p and this is This is RT by V which is nothing but P. So, P-P is 0. This means that the right-hand side is 0. This means that dU is CvdT. So now the question arises that CvdT has come out. But Cv can also depend on the volume. So here we will use the expression that we just showed, the test of exactness. To prove Cv, we will use this expression that Cv does not depend on volume. I have used this expression here. You have noticed that these two expressions were used. If we put the equation of state of the idle gas in the right-hand side term, then you will get the relation of Cv, which is partial derivative. So, if it does not depend on the volume, then the right-hand side should be 0. So, we are using this term. So in the case of idle gas, del P by del t is at constant volume R by V and if we take its second derivative then it will be zero which means del Cv by del V at constant T right hand side is zero so if this is zero then Cv is not a function of V so in this case U is as I have shown before Cv is a function of T So here we have shown that Cv is not a function of V and in this case Cv will be only a function of temperature because we are talking about note that we are changing TV in this function we have used this function so we are changing TV so in this case two variables are changing T and V Cv is not depending on V so it will depend on T because It is constant and the other second term is also gone In this case, it is shown that du is only a function of Cv and dt. Here it is shown that u is a function of only t. Because both the terms are Cv and temperature. If we consider it as incompressible, then what will happen in the second case? If we consider it as incompressible, then the volume is constant. The volume is constant. In such a case, we have already done it before, in such a case, Cp will be Cv. You can also take it out from the expression that we just took out from the main relation. So, Cp will be Cv. In such a case, if you put the expression du in such a case, then note that if Cp is Cv, then we, this is the second term, T del Pb because volume is not changing, so second term will be zero. So Cv dt plus this term is dv, this is zero. And this is C dt plus zero. Now a question arises that you can check in this too that what depends on temperature or pressure. Depending on temperature, depending on pressure or depending on volume, what will we do in this? In this case, we can also take CP. But in general, we use this expression. In this case, we will show that this term is also zero. Because if V is constant, then this term is zero. This term is del CP by del V. del CP by del P t minus t del square V by del t square P. This is zero. So, this is also zero. This means that CP is not a function of pressure. So, Cp is only a function of temperature. And C is only a function of temperature. So, you can show this in this way. So, in general, we have noted two important things. First, we have taken out the relations. Second, we have used the exactness relations. It is difficult to remember all these things, but if you have given this, you can easily remove it. But remember its derivation, how it has always been removed. Okay. Now we will do the last example in this lecture. That is the specific heat difference of natural gas. That means we have to prove it. Prove Cp minus Cv is R. So, Cp minus Cv is VtBeta2 by alpha and beta is minus 1 by V del V by del t at constant pressure Beta is plus 1 by V del V by del t at constant pressure and alpha is minus 1 by V del V by del p at constant temperature So if PV is equal to RT, then what is beta? So, beta is 1 by V. Because this will result in del V by del T at constant pressure. This is R by P. Because V is RT by P, del V by del T at constant pressure. So, this is the result. Similarly, 1 by V R P this is your R P V is equal to R T and this is your R T so beta is 1 by T For alpha, you have to take out del V by del P. So, we can take out del V by del P from here. In this case, we will get del V by del P is... del V by del P at constant T is minus RT by P square. So, what is alpha? Minus 1 by V, RT by P square. So minus 1 by V, what is RT? It is P. Vv, p2, Vv is cut, pp is cut, so minus 1 by p is cut actually this is minus, so minus is actually cut so 1 by p is done so Cp minus Cv is Vt, 1 by t and alpha is 1 by p ok so Cp minus Cv is Vt, beta is 1 by t, but beta is square So t will be t square in fact So I will write it again So Vt 1 by t square by 1 by p This will come This is beta square Okay So tt is cut So this is pV by t This is your R Okay So one thing you can do like this Like I did Second thing is that you can get it in direct relation In fact Cv which is minus cv Minus t del V by del t square You can also plug in the equation of state of ideal gas by removing this relation. So, you get this R. In general, CP minus CV is equal to R, which is the relation of ideal gas. Always remember that. But the exact relation is in the differential form. If your equation of state is given, then you can remove it in the exact relation. And CP minus CV depends on R. It will be debated, and it depends on your PVT data. You can assume that it is an idle gas at low pressure and high temperature. You can prove it in this way. I hope you understood this lecture. What are the general relations and how they have been taken out. And keep in mind that when you can assume ideal gas at only a few conditions. What are the relations? We will continue this discussion in the next lecture. In the next lecture, we will study about the Jules-Thomson coefficient. And general relations of real gas, how they are, how they are extracted. We will discuss a little about that, but we will not focus on it much. It is important to understand only about the Jules-Thomson coefficient. Let's meet in the next lecture. Till then, I take leave. Bye-bye.