Engineering Thermodynamics

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Week-07
Lecture-33
Compressor Work

Let's talk about the work of steady flow devices. Especially, let's talk about the compressor work. And the question is how to minimize the compressor work. In the compressor work, you have seen the integral of Vdp. This is the expression. And we believe that the control volume of kinetic energy and potential energy system does not change. Now, if you consider this particular work that is required, we have considered it as reversible. So naturally, we have taken this condition. Apart from this, if you want, you can minimize it more. By reducing the specific volume. And that will be reduced. If we talk about the compressor, then the temperature will be reduced. So, we normally try to reduce the temperature in the compressor. And what we do in this is that Let's see in which particular case, in which particular case, your compressor work will be the least. So first, let's take the isentropic process in which adiabatic reversible is there. We have assumed that in general, the steady flow is in adiabatic condition. We want to operate it. And we are assuming reversible, adiabatic reversible means the constant of the entropy, so the isentropic process will take place. In such a case, for the gas, the ideal gas system that we have considered, in that, the PV to the power k will be constant. What is k? C by Cv.

$$w_{rev,in} = \int_1^2 vdp$$

Isentropic process($pv^k = constant$)

$$w_{comp,in} = \frac{kR(T_2 - T_1)}{k-1} = \frac{kRT_1}{k-1}[\left((\frac{P_2}{P_1})^{\frac{k-1}{k}} - 1\right)]$$

Polytropic process($pv^n = constant$)

$$w_{comp,in} = \frac{nR(T_2 - T_1)}{n-1} = \frac{nRT_1}{n-1}[\left((\frac{P_2}{P_1})^{\frac{n-1}{n}} - 1\right)]$$

Isothermal process($pv = constant$)

$$w_{comp,in} = RT \ln (\frac{P_2}{P_1})$$

Now if you take this and plug it in, then you can get the compressor's work to be this, which you can see here. Isn't it? So, if you plot this in the PV diagram, then the isentropic process comes out this way. pressure 1 to pressure 2. Now the second is polytropic, which is a normally compressive process. Normally, N is less than K and more than 1. If you take it like this, then this expression comes out. And this line is the polytropic process. So, this curve is seen in the polytropic process. And if we do n equal to 1, then this PV is equal to constant which is your isothermal. Because PV is equal to nRT and T is constant, so PV is your constant. This is isothermal. In this case, this comes out. And when you plot this, then this will come out. This is your isothermal. Now in the previous case, there is no cooling in isotropic. No cooling. And there will be some cooling in this. There will be some cooling. In this case, you will have to keep the temperature very low. You will have to keep the heat very low. So, the most cooling is here. And if you see this, the area under the curve will be the lowest. So, as you see, the most work is required in the adiabatic compression, which is the PV equal to K isentropic process. And isothermal the compression is the least required because it has more cooling. So, the practical way of reducing the work of the compressor is basically to use the cooling. It will be more valuable and practical. Because no matter how much you do, this is an ideal scenario. 100% you can't remove the irreversibility. So, that's why the compressor usually has a cooling effect that is used to reduce the temperature so that the work input can be reduced. So, this is a practical way of managing the system. So, now I understand that cooling the gas when compressing is more beneficial. which reduces the work input of the compressor. That's why multi-stage compression with intercooling is commonly used. And, using an example, there are two stages in the steady flow compression process. In which the compression is between P1 and P2. So, in the first stage, your compression was between Px, and after that, the temperature at constant pressure Px has cooled down, because the temperature has increased, and after that, it has cooled down and reached T1. Again, the compression was between Px and P2. Alright? So, these are the two stages you have completed. So, this is the first stage, and this is the second stage. Now, you can take out the work. Total work will be W1 plus W2. Now after that you can identify what should be the intermediate pressure Px so that your total work, if you are doing multi-stretch compression through intercooling, how much should it be? So, if we take out the expression for this, W compressor in is equal to W compressor in 1, which is the first one and the second one is this one. And this expression, because this is You can extract it in the same way as you have done with VdP. So, this expression that you have used, the one with polytropic, if you use the same expression here, so in polytropic, in this expression, we have only done Px instead of P2, and in this case, when there is 2 in place of P1, then Px is there. So, these are your work expression is 1 and 2, polytropic process. But the intercooling is different. You can differentiate this with respect to Px. And then you can find out on which Px this work is your minimum. If you differentiate the two and take out the blue compressor for minimum, then this expression comes out. and it says that the ratio of the two stages, Px by P1, should be equal to P2 by Px. So, the pressure ratio across the stages should be the same.

$$w_{comp,in} = w_{comp\ 1,in} + w_{comp\ 2,in}$$

$$= \frac{nRT_1}{n-1}[\left((\frac{P_x}{P_1})^{\frac{n-1}{n}} - 1\right) + \frac{nRT_1}{n-1}[\left((\frac{P_2}{P_x})^{\frac{n-1}{n}} - 1\right)$$

Where, $P_x = (P_1P_2)^{1/2}$

Let's move ahead. Specifically, we will talk about the isentropic efficiency of steady flow devices. When we discussed the heat engine, it is a cyclic device. For this, we used a Carnot cycle, which we made a model system. According to that, we were trying to make any engine that reaches the same efficiency. as much as your ideal cycle. Similarly, these flow devices are specially made for the adiabatic condition. So normally it operates accordingly. So, what is the ideal model for flow devices? What will be the model process? So, since we have made it for the adiabatic condition, there is an ideal scenario for reversible. So, the adiabatic reversible condition It's not just an isentropic process. So, what I mean to say is that if we take an isentropic system, it becomes a model system. Now, if you make any device, flow devices, then the isentropic efficiency, how much you can reach it, that will be your actual efficiency. So, an isentropic efficiency, an isentropic process, an idle system, and your actual device. how much it can reach you, that will be your efficiency. So, there is no irreversibility in the isotropic process, and they understand it as an idol. So now if we have to find out how close you are to your process, then we will write a definition for that. So, the definition is basically for the parameter which will define to us that it is a measure How much is the hydraulic efficiency and how close is it to your ideal system. The closer you are to it, the better your device will perform. So, this is one way. So, our wish is that the closer you are to the ideal system, the better. Now the definition of this varies. It varies because the purpose of every device is different. So, we will discuss it.

Now let's talk about Turbine. If we consider Turbine, this is the ideal scenario, which is adiabatic reversible. This is the ideal model. And what will be the actual thing? There will be some irreversibility in it. Now the question is how do we find out how efficient or close is this actual device of ideal? So, this is your plot of enthalpy vs entropy. Because the pressure is less, your steam will expand, for example, and the pressure will be a little less. In fact, it should be enough so that the other changes of enthalpy will be converted into your work. So P2, if you take this, P1, if you take the input condition, then this is 1 here, this is corresponding enthalpy, this is your ideal. So, the isentropic process will drop vertically and exit at low pressure. So, the difference between the actual work and the actual work will be this. This is what you see in Ws. And if it is actual, it will deviate because the isentropic process will not happen. There will be some irreversibility. So, from here, the dotted line that you see is the actual process. And it will land on 2A. Because the inlet and outlet conditions are fixed. So, it will land on P2. Now if you see its enthalpy change, it is this. So, this means that this work is Wa. And pay attention that Wa is less than Ws. So, your work is less. Why? Because it is irreversible. If it is ideal, then it becomes maximum work, which is Ws. So, if you take both the ratios, then you will get that the actual work is Ws and isentropic. The turbine work is Ws, so Wa by Ws is eta, this is the isentropic efficiency of turbine. And eta, naturally you will understand that it will be less than 1.

$$\eta_T = \frac{actual\ turbine\ work}{isentropic\ turbine\ work} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Chage in K.e. and P.e. = 0

So, this way you can get the isentropic efficiency of turbine. That is, the actual divided by the ideal. And you can also get this according to enthalpy change, H1-H2A and divided by h1-h2s which comes out at isentropic condition which is enthalpy at exit condition. And we have also considered that there is no difference between kinetic energy and potential energy.

Now let's understand this by using an example of how to solve this problem. There are tables and the definition of efficiency is given. So here there is steam which is entering the turbine in this condition and which is exiting in this condition. And the total output of 2 MW is work. Now, what is to be taken out is the isentropic efficiency of the turbine and the mass flow rate of the steam flowing in the turbine. So, this is the question, we will solve it now. And I have already put some tables here. So that you can understand easily. One is table A6. So, we have done this. And the other table is A5. Which is not taken from the test book. Let's solve this question. To remove the turbine, you need h1 minus h2a is the actual turbine's eta and h1 minus h2s is the ideal turbine's isotropic condition Now we will get h1 from here and we need to remove two things from here h2a which is the actual turbine's condition and h2s when we assume s2 is equal to s1. So now we have to find out the state conditions for removing the h1 which is already given P1 and T1 and this is 3 MPa and 400°C and I have removed the superheated table of 3 MPa so if you see 400 on 3 MPa then you get this table and in this This is your V, the first column, the second is internal energy and the third is your enthalpy. So, H1 is 3231.7 kJ per kg. Now you can also remove S1 from this because S1 is your last column. So, 6.9235 kJ per kg per Kelvin. So, this is your state 1. Now we will consider state 2, which is actual. The actual is 50 kPa and 100 degrees Celsius. So, I saw table A6. I saw this column in which P is equal to 0.05 MPa, which means 50 kPa. This is 100 degrees Celsius, which is shown, is superheated. You will take out its enthalpy on this. Because you are interested in, h2a So this data comes out So h2a came out 2682.4 kJ per kg, so this came out easily Now the question arises that if we want to extract state 2S So how to extract it? So, we can understand that 2A is here. For example, 3 MPa was here and 2a is here. This is the actual process. It landed in 50 kPa superheated isobar. But 2S It is in a two-phase system or it can be here or anywhere. But since we have to take it out, let's find out how it comes out. So, what you have to do to do this is that the pressure is the same. The pressure of 2S is 50 kPa. Temperature will not be known in isentropic. So, we only know the condition that S2 comes out S1. Because we have put this condition. because of the isentropic conditions so we have to understand this very carefully that this state 2S is isentropic and S2 is equal to S1 and we have put this here now this condition means you have to see that S2 S1 which is 6.9235 where does it land? at 50 kPa so Pay attention to 50 kPa, because it has more entropy than Superheated. So definitely it is not superheated. If we look at 50 kPa table A5, it comes out here. Yes, that's it. And our value S1 6.9 and 235 unit kilo joules per kg per Kelvin So, this lies between these two. Notice that if you take this at 50 kPa, then this is less than this in superheated conditions. This S1 is also less than this. Which is in saturation conditions. Superheated is definitely less. So, this means that this is in the two-phase region. What we saw in 50 kPa, this one we saw in table A5, so here you get that this is between the two. S1 is equal to S2S S is equal to S2S This means that your S1T state 2 is equal to S2S You can extract the quality from this To extract the quality you have to do X2S Basically we want S2 but we have put S here because we are assuming isotropic so it is identifying and this will come out as S2S-SF divided by SFG So what is S2S? This is S1. What is Sf? This is your Sf. And what is Sfg? The difference is 6.5. This is Sfg. So, if we insert this value, it comes out as 6.9235-1.0912 divided by 6.5019 which is 0.897. So, this means that this is close to vapor but is vapor. 2 phase region and this is shown in the diagram. So, this is your 2S system. Now you have X2, which means you can also get S2. So, if you put the value from this line, you will get the value of Hf. This is your Hf. And this is your Hfg. If you insert this in this equation, you will get 2407.9 kJ per kg. So, you will get eta. Now when we put it back in the equation, the value will come out as 66.7%. Now the second question is how much is the mass flow rate of steam? To get this out, you will apply the energy balance, which is E in equal to E out, because it is steady. flow system and E in is my m dot h1 E out is your m dot h2 plus workout, right? Workout is the actual one we are talking about so this will come out let's rearrange it m dot h1-h2 is your Wout. Wout is already given to you. This is 2 MW. So, this is your 2 MW. So here h1-h2 is given to you. So, this is your W dot out 2 MPa. You can write it like this. 2 MW and 1000 kJps is 1 MW so you can rearrange it h1 and h2 you know, h2 will take 2A in it so this will come out M dot which is 2 x 1000 kJps divided by h1-h2a So when you put these values in it, then this will come out. 3.64 kg per second. So, we solved this problem in this way. We used tables, basic expressions and efficiency and energy balance. Let's move forward and discuss compressors and pumps. If we talk only about compressors, then we have to work on the work of compressors. The pressure is less, and the exit will be more. So, this is an example, again, this is enthalpy vs entropy. This is the initial point and when we take the isentropic condition, then this straight line will be vertical, which will align with P2. And the actual line will go further and towards the upper side, because due to irreversibility, you will have to work more to achieve that pressure. Now look at the area under the curve, this is your WA, it is doing more work, which is actual. And this is less than Ws under the isentropic condition. That's why eta is Ws divided by Wa. Because it is less than 1, so actually WS will be less, and WA will be more. And you can also use this as a compressor. So, you can write it in the gas form as S2S-H1 which is the actual isotropic and this one is the actual. So, you can write it in the form of delta H. And when used for pumps, you can use Ws in the form of simple V delta P. Because it is isotropic, you can assume it as constant, and you can also get VdP from the alarm. And when you want to get the actual, you have to use S2a minus h1. As we said, we usually cool the compressor to minimize its work. That's why this efficiency is not in the case of non-adiabatic. Because this efficiency is in the case of isentropic, which is adiabatic and we are talking about reversible. When you don't have adiabatic, then this expression will not be used. In such cases, as non-adiabatic, in which you have constant temperature, where heat is continuously used or if you are cooling then heat is extracted heat is extracted or cooling water is used for example in such a case your eta will be Wt divided by Wa and in which Wt is your work input for reversible isotherm. So, this is the total work that we are doing. The condition is that we are maintaining the temperature of reversible isothermal. But it is reversible. And of course, divided by W1 means actual condition. So, in such cases, you will have to use this. Let's move forward with this topic. We won't give an example, but we want to show you how problems can come from this side. As we did in the tub, if you are in the form of gas, then you will have to use gas tables. For example, this is your air compressor with this inlet condition and outlet pressure. And its steady flow rate is given already. If its isentropic efficiency is given to you 80% i.e. eta C is 0.8 then if it is given to your isentropic efficiency, then what is exit temperature? i.e. T2 and what is its total work input? Now to remove this you can remove eta c. Eta c will be h2s-h1 divided by h2a-h1. The actual temperature will definitely be high. 2S will be at a lower temperature because more temperature means more work. So, this is the initial condition 1, which was at 12 degrees Celsius, which is 285. So, if this condition is temperature versus entropy, then you see the graphical form like this. From 100 kPa to 800 kPa, this is a straight line, this is an isentropic condition. And this dash line is actual. The basic definition is that you have to use the table. This is the table. And the third one is that you have to use the energy balance.

So, if you use energy balance, the total amount of work you have done is simple m dot multiplied by enthalpy It has changed. In this, I have used A, and it has become actual. So, you will have to take out the actual enthalpy. And the relation of S2s and H is that you can get a lot from A17. You will get enthalpy from the temperature. You can also find the enthalpy conditions in the pressure relation later. You can solve this problem. The example is of a book. You can see how it can be solved directly. But there is not much difference in this problem. It is fundamentally of a turbine. So, you can do it at home with an alarm. Now let's move this discussion forward. This will be the last problem that we will take.

In this lecture, we will discuss about Nozzle. A nozzle is a very common study flow device. Basically, it has kinetic energy changes. The purpose of this is to change the kinetic energy of the fluid. That is why its isentropic efficiency is the kinetic energy of the actual nozzle that is coming out of the exit. and divided by the one which is coming out in the isentropic condition. In this expression given H vs S, you can notice that the pressure was initially at 1 and the pressure will be less on the exit. The same flow work is converted into kinetic energy. And this is your actual kinetic energy which is of nozzle in the exit. and its ratio is called isentropic kinetics. As I was saying, this is your initial inlet P1 and when it expands, when it exits the nozzles, it does it at low pressure. And that's why its kinetic energy changes, it increases. But actually, in the actual process, if you notice, it deviates too isentropic. So, its final enthalpy is actually in the initial condition. The nozzle does not work normally. You can balance this in the energy balance,

$$\eta_N = \frac{actual\ KE\ at\ nozzle\ exit}{isentropic\ KE\ at\ nozzle\ exit} = \frac{{v_{2a}}^2}{{v_{2s}}^2}$$

$$h_1 = h_{2a} + \frac{{v_{2a}}^2}{2}$$

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Assume that the inlet kinetic energy is almost negligible. In this case, you can use this expression and get eta of the nozzle which will be h1 minus h2a. which is of actual nozzle and this denominator is h1 minus h2s this will come out of isentropic so this tells that the difference of kinetic energy is simply delta H which is shown in this is actual and this is in isentropic condition so in general what will happen is for example by considering the example that if it is coming at 950 Kelvin, then if the actual nozzle is there, then its temperature will be more due to the isentropic condition, because of the friction. And because of this, its speed will also be a little less due to the isentropic condition.

Now let's understand this by using the example. We will do the example in this. So, you have air 200 kPa 950 Kelvin and we initially believe that the velocity is almost negligible from the final velocity which is the exit velocity. The efficiency of the air nozzle is 0.92 and the final pressure given in the exit condition is 110 kPa. We have to get the maximum possible exit velocity, i.e. we have to get V2, i.e. at the maximum, i.e. at the isentropic condition. And the exit temperature and the actual exit velocity, i.e. we have to get the exit temperature, we have to get T2. We have to find V2 which is the maximum. So, the maximum will be when it is in a isentropic condition. And we have to find V2a. We are assuming that it has a constant specific heat property. Now, to understand this... To understand this, we have to... derived for the isentropic process for the air, we will have to use that. Because if you are talking about maximum exit velocity, then this system, your nozzle, device, it has a steady flow but in this, the reversibility should be zero. That is, in reversible adiabatic condition. So, this adiabatic is saying, but if we assume it to be reversible, then this will be your maximum possible exit. In that condition, we are talking about the isentropic process. So, we get a relation of T2 by T1. Okay. This will be P2 by P1. And this is k minus1 by k. Now, k is your ratio Cp by Cv. Now we will get a constant specific heat in this. So, in this, we can also get it from the table. Now this temperature is 950 Kelvin is used. So, we have a trick in this. If we want to get Cp and Cv, we need to know the exact condition to get the k. So, how to get the average of T2 and T1? We will take the average of T1 and T2 for K. So, if we assume that the final temperature is 850. So, let's assume that t-average is 850. Let's assume this. So, the corresponding value of K can be taken out from the table. It comes out as 1.349. The specific Cp of the table comes out as 1.11. And since Cp and Cv are related, Cp is equal to Cv plus R, so you can take out K. Now from here you have T2 by T1 and P2 by P1 but this is for isentropic conditions so we will write T2s and P2s. Because it tells that you have to write for the isentropic process so this will come out from here because you know T1 and P2 is 110 whether it is for actual or isentropic both are same for P2, 110 kPa so this is your 110 kPa and temperature is 1950 K T2s is 814 K if we look at it again T average is 882 K we assumed 850 now, there is a difference of 30 degree but we will neglect it because we have to solve the problem. The first thing is that even if we repeat it, the changes are very small. Even if you try, the changes will be very small. So, we will neglect it for now and assume that this is the temperature. And the second thing is that you have to take out the exit maximum velocity. So, for that we take energy conservation. So, per unit mass. So, E in is equal to E out. So, in this we will get h1 plus V1 2 square is equal to h2 plus V2 square. Now, in this E out we will take isentropic condition. So, from here you will get, and this is also isentropic condition. From here you will get V2s which is your 2h1 minus h2s. Now, since this is a gas, we can also take this as your Cp average T1-T2s. Now, we have taken the Cp average, which we said is at 850 Kelvin. So, this CP average will come out to be 1.11, this unit is kilojoules per kg Kelvin. This is 950 And T2 is T2s Which is 814 So if we use this V2 will come out Which is V2 So Which is maximum 549 Meters per second And to take out actual To take out actual temperature You can use eta directly Because eta is given So eta is h1minus h2a Divided by h1 minus h2s and this is Cp average T1-T2a divided by Cp average T1-T2S ok this is 0.92 we will assume that there is no change in Cp so we can cancel it so this will come out 0.92 950-T2a and this is 950-814 which we have taken out T2s And from here T2a is 825 K which is more than 11 degrees So you can solve the problem like this You can also understand that the temperature will be more in the case of irreversible And you can also see in the graphical form that in the case of T2s, the temperature is less and T2a is more I hope you understood the first part of the isentropic process. And how we are making the isentropic device idle and getting the efficiency of any flow device. In the next lecture, we will understand about entropy balance. How to use it to generate entropy in a system. See you in the next lecture. Till then, bye.